

## 第三节 光学和波动

1. In a hologram a photographic plate contains a wave pattern that produces a three-dimensional picture when illuminated with monochromatic coherent light from a laser. When only half of the photographic plate is illuminated, which of the following is true of the resulting picture?

- (A) Only half of the picture is seen.
- (B) The picture is still seen, but is less distinct than before.
- (C) The picture is still seen, but is smaller than before.
- (D) The color of the picture is changed.
- (E) The picture is inverted.

解：全息成像的一大特点是由全息片的任一部分均可成像，因为全息片上纪录的内容为干涉条纹，而条纹以所载信息的频率在全息片上重复排布。但因为用全息片的局部成像会丢失高频信息，所以图像将会变模糊。选 (B)。

2. Waves on a string propagate with a speed  $v$  and are represented by giving the displacement  $y$  as a function of  $x$  and  $t$ . Which of the following is NOT a possible wave or pulse? ( $c$ ,  $b$ , and  $k$  are constants.)

- (A)  $y = ac^{-b(x-vt)^2}$
- (B)  $y = ac^{-b(x-vt)^2} \sin k(x+vt)$
- (C)  $y = a \cos k(x-vt)$
- (D)  $y = ae^{-bx^2} e^{-b(vt)^2}$
- (E)  $y = a / \cosh b(x-vt)$

解：由偏微分方程理论，函数  $f(x \pm vt)$  满足波动方程

$$\frac{\partial^2 f}{\partial t^2} = v^2 \frac{\partial^2 f}{\partial x^2}。$$

所以答案选 (D)。

3. A converging lens of focal length 4 centimeters is

used as a magnifier. If an object is placed 3 centimeters from the lens, what is the magnification?

- (A) 3
- (B) 4
- (C) 6
- (D) 12
- (E) 24

解：由凸透镜成像公式

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}，$$

得像距为

$$v = \frac{uf}{u-f} = \frac{3 \times 4}{3-4} = -12 \text{ cm}。$$

放大倍数为

$$\left| \frac{v}{u} \right| = 4。$$

选 (B)。

4. A source of 1-kilohertz sound is moving straight toward you at a speed 0.9 times the speed of sound. The frequency you receive is

- (A) 0.1 kHz
- (B) 0.5 kHz
- (C) 1.1 kHz
- (D) 1.9 kHz
- (E) 10 kHz

解：Doppler 效应

$$v' = \frac{c}{c-v} v = \frac{1}{1-0.9} \times 1 = 10 \text{ kHz}。$$

选 (E)。

5. Two coherent sources of visible monochromatic light form an interference pattern on a screen. If the relative phase of the sources is varied from 0 to  $2\pi$  at a frequency of 500 hertz, which of the following best describes the effect, if any, on the interference pattern?

- (A) It is unaffected because the frequency of the phase change is very small compared to the frequency of visible light.
- (B) It is unaffected because the frequency of the

phase change is an integral multiple of  $\pi$ .

- (C) It is destroyed except when the phase difference is 0 or  $\pi$ .
- (D) It is destroyed for all phase differences because the monochromaticity of the sources is destroyed.
- (E) It is not destroyed but simply shifts positions at a rate too rapid to be detected by the eye.

解：干涉光源的相对相位改变会使干涉条纹移动，只要想一下零级条纹就知道了。500Hz 的改变频率人眼无法分辨，因为人眼的视觉暂留时间约为 25s。选 (E)。

#### Questions 6-7

Light of wavelength 5200 Angstroms is incident normally on a transmission diffraction grating with 2000 lines per centimeter.

6. The first-order diffraction maximum is at an angle, with respect to the incident beam, that is most nearly

- (A)  $3^\circ$
- (B)  $6^\circ$
- (C)  $9^\circ$
- (D)  $12^\circ$
- (E)  $15^\circ$

解：由光栅公式

$$\sin \theta = k \frac{\lambda}{d}$$

对于一级衍射， $k = 1$ ，所以

$$\sin \theta = \frac{\lambda}{d} = \frac{5200 \times 10^{-10}}{0.01/2000} = 0.104。$$

角度很小，近似

$$\theta \approx \sin \theta = 0.104 = 5.96^\circ。$$

选 (B)。

transmission diffraction grating：透射光栅。

7. If the width of one slit is reduced to  $\frac{w}{2}$ , what

happens to the interference pattern of the light from the two slits?

- (A) It remains the same except that it has lower intensity.
- (B) It remains the same except that it is replaced.

(C) It still has intensity  $I_0$  at  $\theta = 0$ .

(D) It no longer has minima with zero intensity.

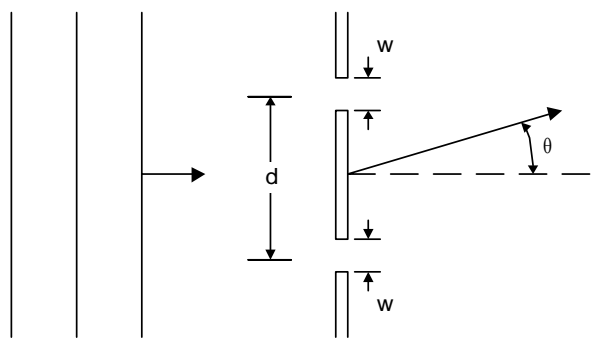
(E) It has zero intensity at  $\theta = 0$ .

解：将干涉缝看作是三条宽为  $w/2$  的狭缝的组合，缝间干涉因子为（复数表示）

$$\tilde{N}(\theta) = \sum_i e^{ikL_i} = 1 + e^{ikd} + e^{ik(d-w/4)}，$$

干涉因子的模总是大于零的，因此不会出现完全相消的暗斑。选 (D)。

#### Questions 8-9



A plane wave of monochromatic light with wavelength  $\lambda$  in vacuum is incident on two slits of equal width  $w$ , as shown schematically above. A pattern is observed on a screen a large distance away. The intensity of the light at  $\theta = 0$  is initially  $I_0$ .

8. If one slit is blocked the intensity is

(A)  $\frac{I_0}{4}$

(B)  $\frac{I_0}{2}$

(C)  $\frac{I_0}{\sqrt{2}}$

(D)  $I_0$

(E)  $2 I_0$

解：在  $\theta = 0$  处，两波干涉相加。设每个波振幅为  $A$ ，则

$$I_0 = (A + A)^2 = 4A^2。$$

仅剩一个波时， $I = A^2$ 。选 (A)。

9. When the apparatus is immersed in a medium with

index of refraction  $n$ , the interference pattern is unchanged if light is used whose wavelength in vacuum is

- (A)  $\frac{\lambda}{n}$   
 (B)  $\frac{\lambda}{\sqrt{n}}$   
 (C)  $\lambda$   
 (D)  $\lambda\sqrt{n}$   
 (E)  $\lambda n$

解：设介质折射率为  $n$ ，则其中单色光的波长为其真空中波长的  $1/n$ ，频率不变，从而波速为  $c/n$ 。本题中若光在真空中波长为  $\lambda n$ ，则在介质中为  $\lambda$ ，与原来情况相仿。选 (E)。

10. The following functions may represent the wave disturbance  $f(x,t)$  in a one-dimensional elastic medium in terms of position  $x$ , time  $t$ , and positive constants  $A$ ,  $a$ , and  $b$ . Which function represents a traveling wave moving in the negative  $x$ -direction?

- (A)  $f(x,t) = A \sin(ax + bt)$   
 (B)  $f(x,t) = A \sin(ax - bt)$   
 (C)  $f(x,t) = A \cos ax \cos bt$   
 (D)  $f(x,t) = A \sin ax \sin bt$   
 (E)  $f(x,t) = A \sin ax \cos bt$

解：(C)、(D)、(E)均表示驻波。它们最明显的特点是各点的振幅不同。(A)、(B)表示行波，符合形式  $f(x \pm vt)$ 。其中 (B) 沿正方向传播，(A) 沿负方向传播。选 (A)。

11. Unpolarized light passes through two polarizers whose optical axes are vertical. The intensity of the emerging light is  $I_0$ . If a third polarizer is placed between the polarizers so that its axis is at an angle  $\theta$  with the vertical, the intensity of the emerging light is

(A) zero for all  $\theta$   
 (B)  $I_0$

- (C)  $I_0 \cos \theta$   
 (D)  $I_0 \cos^2 \theta$   
 (E)  $I_0 \cos^4 \theta$

解：注意前两个偏振片平行排列，开始时第二个偏振片其实不起作用，所以入射光经过第一个偏振片后强度为  $I_0$ 。加入第三个偏振片后，经过第一个偏振片后光矢量要进行两次投影，出射强度为

$$I = A^2 = (A_0 \cos \theta \cos \theta)^2 \\ = A_0^2 \cos^4 \theta = I_0 \cos^4 \theta$$

选 (E)。

12. Although the sky is blue in the daytime, sunsets are red because

- (A) the Sun emits more red light in a forward direction, but more blue light when at a larger angle with the forward direction  
 (B) the Sun emits more red light than blue light in the evening  
 (C) the index of refraction of red light in air is greater than that of blue light  
 (D) there are many ions in the upper atmosphere  
 (E) red light is less strongly scattered by air molecules than blue light

解：天空为什么是蓝色最早由 Rayleigh 解释。Rayleigh 散射定律说，当散射体的尺度比光的波长小时，散射光强与波长的四次方成正比。因为大气满足以上条件，所以白光中的短波成分（蓝紫色）遭到的散射比短波成分（红黄色）强烈得多，散射光因为短波富集而呈蓝色。日落时光所经大气距离加大，白光中短波成分被更多的散射掉，直射的日光中剩余更多的是长波成分，因此偏红。选 (E)。

13. An observer looks through a slit of width  $5 \times 10^{-4}$  meter at two lanterns a distance of 1 kilometer from the slit. The lanterns emit light of wavelength  $5 \times 10^{-7}$  meter. The minimum separation of the lanterns at which the observer can resolve the lantern lights is most nearly

- (A) 0.01 m  
 (B) 0.1 m  
 (C) 1 m  
 (D) 10 m

(E) 100 m

解：根据 Rayleigh 判据

$$\theta_{\min} = 1.22 \frac{\lambda}{D},$$

$$l_{\min} = S \theta_{\min} = 1.22 S \frac{\lambda}{D},$$

$$= 1.22 \times 10^3 \frac{5 \times 10^{-7}}{5 \times 10^{-4}} = 1.22 \text{ m}$$

最短距离应大于上述值。作为近似，选 (C)。

14. An antireflection coating with index of refraction 1.25 is put on glass with index of refraction 1.56, as shown above. If the coating is designed for light incident normally on the glass with wavelength in air of 400 Angstroms, which of the following thicknesses of the coating will result in minimum reflection?

- (A) 400 Å
- (B) 800 Å
- (C) 1600 Å
- (D) 3200 Å
- (E) 6400 Å

解：由于增透膜的折射率大于空气的，小于玻璃的，所以光在增透膜的两个表面上发生反射时都有半波损，作用抵消。所以厚度  $d$  满足

$$2n_c d = (k + \frac{1}{2})\lambda, \quad k = 0, 1, 2, 3, \dots$$

对于本题，

$$d = \frac{k + \frac{1}{2}}{2 \times 1.25} \times 400 = (2k + 1) \times 80 \text{ Å}。$$

选 (A)，对应于  $k = 2$ 。

15. In an ordinary hologram, coherent monochromatic light produces a 3-dimensional picture because wave information is recorded for which of the following?

I. Amplitude

II. Phase

III. Wave-front angular frequency

- (A) I only
- (B) I and II only
- (C) I and III only

(D) II and III only

(E) I, II and III

解：所谓全息照相，指记录下全部信息，包括波前的相位和振幅（实际为振幅强度的相对分布）。选 (B)。

16. The dispersion law for a certain type of wave motion is  $\omega = (c^2 k^2 + m^2)^{\frac{1}{2}}$ , where  $\omega$  is the angular frequency,  $k$  is the magnitude of the propagation vector, and  $c$  and  $m$  are constants. The group velocity of these waves approaches

- (A) infinity as  $k \rightarrow 0$  and zero as  $k \rightarrow \infty$
- (B) infinity as  $k \rightarrow 0$  and  $c$  as  $k \rightarrow \infty$
- (C)  $c$  as  $k \rightarrow 0$  and zero as  $k \rightarrow \infty$
- (D) zero as  $k \rightarrow 0$  and infinity as  $k \rightarrow \infty$
- (E) zero as  $k \rightarrow 0$  and  $c$  as  $k \rightarrow \infty$

解：由群速度公式

$$v_g = \frac{d\omega}{dk},$$

本题中

$$v_g = \frac{d\omega}{dk} = \frac{c^2 k}{\sqrt{c^2 k^2 + m^2}},$$

$k \rightarrow \infty$  时， $v_g \rightarrow c$ 。选 (E)。请同时牢记相速的公

式  $v_p = \frac{\omega}{k}$ 。

17. It is necessary to coat a glass lens with a nonreflecting layer. If the wavelength of the light in the coating is  $\lambda$ , the best choice is a layer of material having an index of refraction between those of glass and air and a thickness of

- (A)  $\frac{\lambda}{4}$
- (B)  $\frac{\lambda}{2}$
- (C)  $\frac{\lambda}{\sqrt{2}}$
- (D)  $\lambda$
- (E)  $1.5\lambda$

解：为发生最大透射，从薄膜前后两表面反射的

光波应干涉相消，厚度  $d$  需满足

$$2d = \left(n + \frac{1}{2}\right)\lambda ,$$

$$d = \left(\frac{n}{2} + \frac{1}{4}\right)\lambda .$$

选 (A)，对应  $n = 0$ 。此题未考虑半波损，这只当  $1 < n < n_{\text{glass}}$  时成立。

18. Unpolarized light is incident on two ideal polarizers in series. The polarizers are oriented so that no light emerges through the second polarizer. A third polarizer is now inserted between the first two and its orientation direction is continuously rotated through  $180^\circ$ . The maximum fraction of the incident power transmitted through all three polarizers is

(A) zero

(B)  $\frac{1}{8}$

(C)  $\frac{1}{2}$

(D)  $\frac{1}{\sqrt{2}}$

(E) 1

解：光经过第一个偏振片后为线偏振光，强度为原来的  $1/2$ （不是  $1/4$ ，这地方容易出错），

$$I_{\text{out}} = \frac{I_0}{2} \sin^2 \theta \cos^2 \theta ,$$

显然  $\theta = 45^\circ$  时， $I_{\text{out}}$  最大，为  $\frac{I_0}{8}$ 。选 (B)。

19. The screen of a pinhole camera is at a distance  $D$  from the pinhole, which has a diameter  $d$ . The light has an effective wavelength  $\lambda$ . ( $\lambda \ll D$ ) For which of the following values of  $d$  will the image be sharpest?

(A)  $\sqrt{\lambda D}$

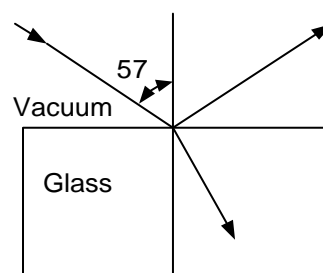
(B)  $\lambda$

(C)  $\frac{\lambda}{10}$

(D)  $\frac{\lambda^2}{D}$

(E)  $\frac{D^2}{\lambda}$

解：考试现场很可能不知道这个公式，不用慌，从量级上进行一下分析即可。 $\lambda \sim 10^{-7}\text{m}$  量级， $D \sim 10^{-2}\text{m}$  量级，而小孔的直径介于二者之间。选项中 (B)(C)(D) 答案太小， $10^{-7}\text{m}$  量级的小孔怎么能用来照相。选项 (E) 答案太大。选 (A)。



20. A ray of unpolarized monochromatic light traveling in vacuum is incident on a flat plate of glass at an angle equal to Brewster's angle ( $57^\circ$  for this case), as shown above. Which of the following statements is NOT correct?

(A) The tangent of the angle of incidence is equal to the index of refraction of the glass plate.

(B) The angle between the reflected ray and the refracted ray is  $90^\circ$ .

(C) The refracted ray is partially plane polarized.

(D) The reflected ray is 100% linearly polarized.

(E) The plane of vibration of the electric vector in the reflected ray is parallel to the plane of incidence of the light ray.

解：由 Brewster 角的定义

$$i_B = \tan^{-1} \frac{n_2}{n_1} = \tan^{-1} \frac{n_g}{n_v} = \tan^{-1} n_g ,$$

(A) 正确。设  $\theta$  为折射角，由

$$\sin i_B = n_g \sin \theta ,$$

$$\sin \theta = \frac{n_g}{\sin i_B} = \frac{n_g}{n_g / \sqrt{1+n_g^2}},$$

$$= \frac{1}{\sqrt{1+n_g^2}} = \cos i_B$$

所以

$$\theta = 90^\circ - i_B。$$

(B) 正确。

由 Fresnel 公式，不管入射光的偏振态如何，反射光总是线偏振。故 Brewster 角又叫全偏振角，(D) 正确。Brewster 角还有一个性质，当入射角  $i_B$  时，反射光中的 p (与入射面平行的方向) 分量为 0。所以反射光的电矢量只有 s (与入射平面垂直) 分量。选项 (E) 认为电矢量在入射平面内，错误。选 (E)。

21. The angular separation of the two components of a double star is 8 microadians and the light from the double star has a wavelength of 5500 angstroms. According to the Rayleigh criterion, the smallest diameter of a telescope mirror that will resolve the double star is most nearly

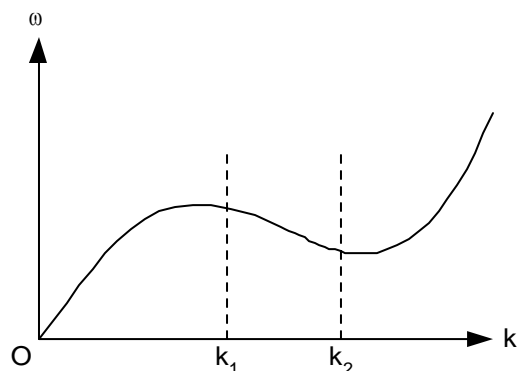
- (A) 5mm
- (B) 2cm
- (C) 10cm
- (D) 50cm
- (E) 2m

解：由 Rayleigh 判据

$$\theta = 1.22 \frac{\lambda}{D},$$

$$D_{\min} = 1.22 \frac{\lambda}{\theta} = 1.22 \frac{5500 \times 10^{-10}}{8 \times 10^{-6}} = 0.084 \text{ m}。$$

选 (C)。



22. The dispersion curve shown above relates the angular frequency  $\omega$  to the wave number  $k$ . For waves with numbers lying in the range  $k_1 < k < k_2$ , which of the following is true of the phase velocity and the group velocity?

- (A) They are in opposite directions.
- (B) They are in the same direction and the phase velocity is larger.
- (C) They are in the same direction and the group velocity is larger
- (D) The phase velocity is infinite and the group velocity is finite.
- (E) They are the same in direction and magnitude.

解：相速度  $V_p$  和群速度  $V_g$  的公式为

$$V_p = \frac{\omega}{k}, \quad V_g = \frac{d\omega}{dk}。$$

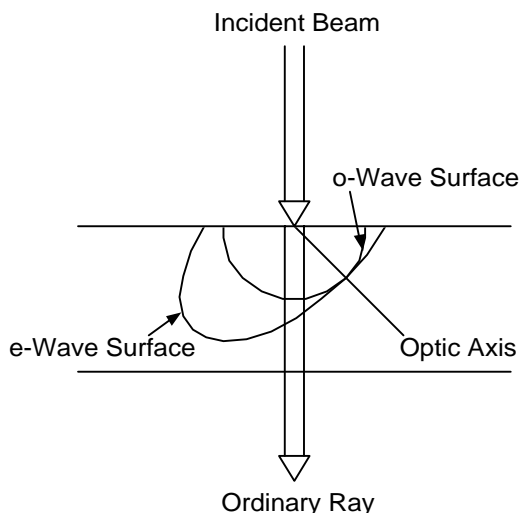
当  $k_1 < k < k_2$  时， $\omega(k)$  曲线的切线斜率为负，即

$$V_g = \frac{d\omega}{dk} < 0。$$

因为  $\omega(k)$  在第一象限， $\omega$ 、 $k$  均为正值，所以

$$V_p = \frac{\omega}{k} > 0，$$

二者方向相反。选 (A)。

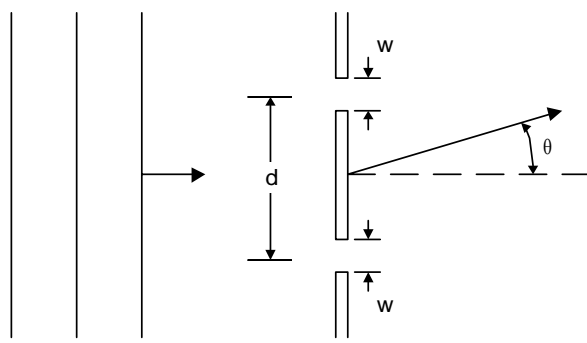


23. A beam of light is incident from above on a slab of doubly refracting crystal such as calcite whose optic axis is in the plane of the paper as shown above. The beam is split into an ordinary ray (shown) and an extraordinary ray which is displaced. Also shown are the Huygens wave surface for the ordinary and extraordinary waves. Which of the following statements is true?

- (A) The slab is a quarter-wave plate.
- (B) The ordinary index of refraction  $n_o$  is less than the extraordinary index of refraction  $n_e$ .
- (C) The extraordinary ray is deflected to the left.
- (D) The extraordinary wave is unpolarized.
- (E) The ordinary ray is polarized with the electric vector in the plane of the paper.

解：e 波的 Huygens 波面为一椭圆，且光轴方向为短轴。所以  $v_e > v_o$ ，从而 e 波的折射率小于 o 波，所以选项 (B) 不对。另外 e 波和 o 波均为线偏振波，二者振动方向垂直。O 波电矢量的振动方向与主平面（晶体中某条光线与晶体主轴组成的平面，在本题中为纸平面）垂直，e 光电矢量的振动方向在主平面内。选项 (D) (E) 不对。由 Huygens 作图法，从入射光束与界面交点向 e 波的 Huygens 波面作切线，切点在光轴左侧，所以 e 波向左偏折。选 (C)。

#### Questions 24-25



A plane wave of monochromatic light with wavelength  $\lambda$  in vacuum is incident on two slits of equal width  $w$ , as shown schematically above. A pattern is observed on a screen a large distance away. The intensity of the light at  $\theta = 0$  is initially  $I_0$ .

24. If one slit is blocked, the intensity at  $\theta = 0$  is

- (A)  $\frac{I_0}{4}$
- (B)  $\frac{I_0}{2}$
- (C)  $\frac{I_0}{\sqrt{2}}$
- (D)  $I_0$
- (E)  $2I_0$

解：由对称性分析可得两条狭缝在衍射屏中心产生的分振幅是同相位叠加的，设一条狭缝贡献得分振幅大小为  $A_0$ ，则有

$$I_0 = (2A_0)^2。$$

遮住一条狭缝后，剩余光强为

$$I' = A_0^2 = \frac{I_0}{4}。$$

选 (A)。

25. When the apparatus is immersed in a medium with index of refraction  $n$ , the interference pattern is unchanged if light is used whose wavelength in vacuum is

- (A)  $\frac{\lambda}{n}$

(B)  $\frac{\lambda}{\sqrt{n}}$

(C)  $\lambda$

(D)  $\lambda\sqrt{n}$

(E)  $\lambda n$

解：在真空中光程等于相应的几何路程长度  $l$ ，相应的干涉条件为

$$l = i\lambda'$$

其中  $i$  为干涉级数。当整个仪器浸入媒介物后，光程为相应的几何路程长度的  $n$  倍，相应的干涉条件为

$$nl = i\lambda。$$

对比以上两式可以发现，当

$$\lambda' = \frac{\lambda}{n}$$

时，对相同的干涉级次有相同的几何路程  $l$ ，即对应相同的干涉图样分布。选 (A)。

26. In a hologram a photographic plate contains a wave pattern that produced a three-dimensional picture when illuminated with monochromatic coherent light from a laser. When only half of the photographic plate is illustrated, which of the following is true of the resulting picture?

- (A) Only half of the picture is seen.
- (B) The picture is still seen, but is less distinct than before.
- (C) The picture is still seen, but is smaller than before.
- (D) The color of the picture is changed.
- (E) The picture is inverted.

解：全息照相原理简介：全息照相以干涉衍射等波动光学规律为基础，过程分记录、再现两步。全息图与普通照相底片不同，它记录的是物体各点的全部光信息，包括振幅和位相，而且图中每一局部都包含了物体各点的光信息。相应它要求光源有很高的时间和空间相干性。

由以上简介可以得知，当全息图只剩下一半时，仍然可以完整反映物像，只是相应的细节分辨率有一定的损失。

选 (B)。

27. Waves on a string propagate with a speed  $v$  and are represented by giving the displacement  $y$  as a function  $x$  and  $t$ . Which of the following is NOT a possible wave? ( $a$ ,  $b$ , and  $k$  are constants.)

(A)  $y = ae^{-b(x-vt)^2}$

(B)  $y = ae^{-(x+vt)^2} \sin k(x+vt)$

(C)  $y = a \cos k(x-vt)$

(D)  $y = ae^{-bx^2} e^{-b(vt)^2}$

(E)  $y = a / \cosh b(x-vt)$

解：波动性的基本要求是存在一个以波速  $v$  传播的等相面。以上五个  $y$  方向的位移方程中，显然 (C) 描述的是以  $y$  轴为对称轴向两边衰减振荡的振动，没有波动性。选 (C)。

28. An electromagnetic disturbance propagates along the  $y$ -axis through a medium of index of refraction  $n$ . At time  $t$ , the equation of the electric field in the medium is represented in MKS units by the following equation.

$$E = E_0 \sin(5 \times 10^6 \pi y - 5 \times 10^{14} \pi t + \pi/6)$$

The index of refraction  $n$  of the medium is most nearly

- (A) 6
- (B) 5
- (C) 4
- (D) 3
- (E) 2

解：此媒介中的光速

$$v = \frac{\omega}{k} = \frac{5 \times 10^{14}}{5 \times 10^6} = 10^8 (m/s)，$$

因此媒介的折射率

$$n = \frac{c}{v} = 3。$$

选 (D)。

29. A small object is moved along the optical axis of a spherical concave reflecting surface of radius  $R$ .



Which of the following best describes the nature of the image for object distance of  $3R/4$  and  $R/4$ , respectively, from the mirror?

$$\frac{3R}{4} \quad \frac{3R}{4}$$

- (A) Real and erect                      Real and inverted  
(B) Real and inverted                  Virtual and erect  
(C) Real and inverted                  Virtual and inverted  
(D) Real and erect                      Virtual and erect  
(E) Virtual and erect                   Virtual and inverted

解：直接利用几何光学中的球面镜成像公式

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R},$$

其中  $s$ 、 $s'$  分别为物距和像距。 $s = 3R/4$  时， $s' > 0$ ，是倒立的实像； $s = R/4$  时， $s' < 0$ ，是正立的虚像。选 (B)。

30. White light is normally incident on a soap film that has air on both sides. The reflected light is orange. If one assumes that the index of refraction of the film is 1.33 and that 6000 Angstroms is a typical wavelength of orange light in air, a possible thickness of the film is most nearly

- (A) 1500 Å  
(B) 3000 Å  
(C) 3375 Å  
(D) 4225 Å  
(E) 4500 Å

解：考虑到前后反射表面存在半波损失，橙色反射光干涉加强的条件为

$$2nd = \left(i + \frac{1}{2}\right)\lambda$$

代入题中给出的数据，当  $i = 1$  时  $d = 3375$  Å。选 (C)。

31. A rocket ship is moving away from Earth at a high speed. A spectral line from a source on the rocket ship is shifted from a wavelength  $\lambda$  to a wavelength  $4\lambda$  for an observer on Earth. If  $c$  is the speed of light and the shift is assumed to be due to the relativistic Doppler effect, the velocity of the rocket ship relative to Earth is most nearly

- (A)  $\frac{1}{4}c$   
(B)  $\frac{2}{5}c$   
(C)  $\frac{1}{2}c$   
(D)  $\frac{3}{5}c$   
(E)  $\frac{15}{17}c$

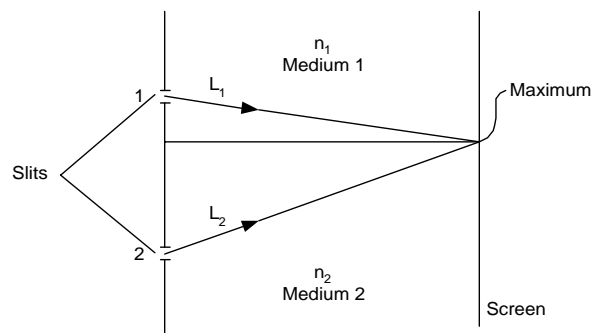
解：利用相对论性的横向 Doppler 效应公式

$$\omega = \frac{\omega_0 \sqrt{1 - \beta^2}}{1 - \beta} = 4\omega_0,$$

解之可得

$$\beta = \frac{15}{17}.$$

选 (E)。



32. Coherent monochromatic light passes through two narrow slits and forms an interference pattern on a distant screen. The space between the slits is filled, as shown above, with media having different indices of refraction  $n_1$  and  $n_2$ . Light from slit 1 travels a distance  $L_1$ , passing only through medium 1 to reach the maximum shown in the figure. Light from slit 2 travels a distance  $L_2$ , passing only through medium 2, to reach the same point. What is a possible relationship between  $L_1$  and  $L_2$ ?

- (A)  $L_1 = \frac{n_1}{n_2} L_2$   
(B)  $L_1 = \sqrt{\frac{n_1}{n_2}} L_2$

$$(C) L_1 = \frac{n_2}{n_1} L_2$$

$$(D) L_1 = \sqrt{\frac{n_2}{n_1}} L_2$$

$$(E) L_1 = (1 + \frac{n_1}{n_2}) L_2$$

解：对荧光屏上的中心极大光强位置，从两条狭缝传播过来的光的光程差为零。

$$\Delta l = n_1 L_1 = n_2 L_2 ,$$

$$L_1 = \frac{n_2}{n_1} L_2。$$

选 (C)。

33. Radio signals emanating from a distance quasar at a frequency of 30 gigahertz are received with a dish antenna 10 meters in diameter. With what precision can the direction of the quasar be determined?

(A)  $10^{-9}$  radian

(B)  $10^{-6}$  radian

(C)  $10^{-3}$  radian

(D)  $10^{-1}$  radian

(E) The precision cannot be determined from the information given.

解：盘状天线所能分辨的极限角

$$\delta\theta = \frac{\lambda}{D} = \frac{c}{\nu D} = \frac{3 \times 10^8}{30 \times 10^9 \times 10} = 10^{-3} \text{ rad}。$$

选 (C)。

34. The sky appears blue because

(A) red light from the Sun is absorbed by the Earth's atmosphere

(B) dust particles in the Earth's atmosphere preferentially scatter red light

(C) molecules in the Earth's atmosphere preferentially scatter red light

(D) molecules in the Earth's atmosphere preferentially scatter blue light

(E) oxygen molecules have absorption lines in the red region of the spectrum

解：根据 Rayleigh 散射定律，大气分子中散射强度跟波长的四次方成正比，因此波长较长的红光较容易被四处散射，从而穿过大气到达地面的可见光主要是短波段的，因此看起来呈蓝色。选 (C)。

35. When two thin lenses of focal lengths  $f_1$  and  $f_2$  are placed in contact, the focal length  $f$  of the combination is given by the relation

$$(A) f = f_1 + f_2$$

$$(B) f = f_1 - f_2$$

$$(C) f = \frac{f_1}{f_2}$$

$$(D) f = \frac{f_2}{f_1}$$

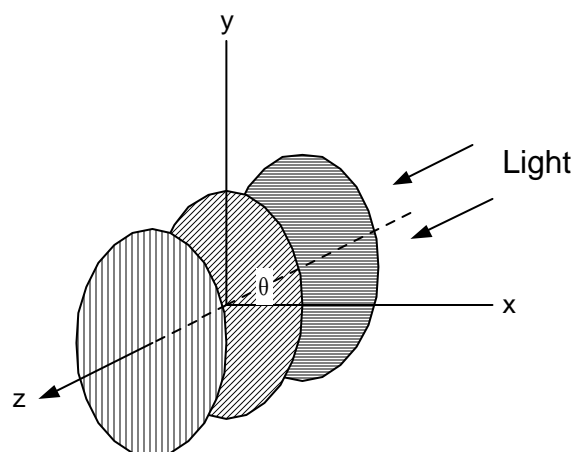
$$(E) f = \frac{f_1 f_2}{f_1 + f_2}$$

解：对密接的薄透镜组，其复合透镜的焦距可由 Gauss 公式求出

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} ,$$

$$f = \frac{f_1 f_2}{f_1 + f_2}。$$

选 (E)。



36. Unpolarized light of intensity  $I_0$  travelling in the +z direction passes successively through three perfect polarizers. The first of these passes light polarized along the x-axis, the middle one passes light polarized at an angle of  $\theta$  relative to the x-axis, and the last passes light polarized along the y-axis. The intensity of light emerging from the third polarizer is

- (A) 0  
 (B)  $\frac{1}{2}I_0$   
 (C)  $\frac{1}{2}I_0 \sin^2 \theta$   
 (D)  $I_0 \cos^2 \theta$   
 (E)  $\frac{1}{8}I_0 \sin^2 2\theta$

解：自然光经过第一个偏振片，光强变为  $\frac{I_0}{2}$ ，偏振方向沿 x 轴。经过第二个偏振片之后，光强为  $\frac{I_0 \cos^2 \theta}{2}$ ，偏振方向转了  $\theta$  角。经过第三个偏振片之后，偏振方向沿 Y 轴，光强为

$$I = \frac{I_0 \cos^2 \theta}{2} \cos^2 \left( \frac{\pi}{2} - \theta \right) = \frac{1}{8} I_0 \sin^2 2\theta。$$

选 (E)。

37. A transmission diffraction grating gives first-order diffraction for 5000-angstrom light at an angle  $\theta$  with respect to the original direction of propagation. If the grating diffracts all wavelengths, one would also find which of the following at angle  $\theta$ ?

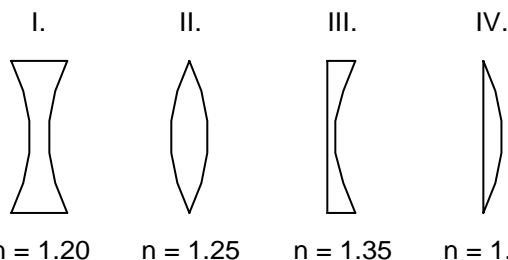
- (A) Second-order diffraction for 1250A  
 (B) Second-order diffraction for 10,000A  
 (C) Third-order diffraction for 2500A  
 (D) Fourth-order diffraction for 1250A  
 (E) Fourth-order diffraction for 20,000A

解：根据衍射公式，

$$d \sin \theta = n\lambda。$$

对固定的  $\theta$  角， $\lambda = 5000\text{A}$  时， $n = 1$ ；故当  $\lambda' = 1250\text{A}$

时， $n' = 4$  成立。选 (D)。



38. To render parallel a converging beam of light propagating in water with index of refraction  $n = 1.30$ . Which of the following lenses might be used in the water?

- (A) I only  
 (B) I or III  
 (C) I or IV  
 (D) II or III  
 (E) II or IV

解：图中 I、III 是凹透镜，II、IV 是凸透镜。利用 Gauss 逐步成像的定性计算可得，对于凹透镜，当玻璃折射率大于媒介折射率时，对光线有发散作用；当玻璃折射率小于媒介折射率时，对光线有汇聚作用。对于凸透镜的情况测刚好相反。选 (C)。

39. Let  $\hat{x}$ ,  $\hat{y}$ , and  $\hat{z}$  be three orthogonal unit vectors. The expression

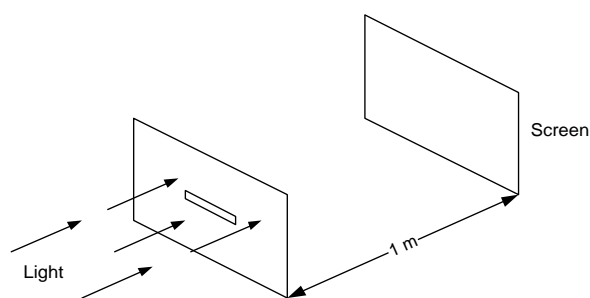
$$u(r, t) = 2(\hat{x} - \hat{y}) \exp\{(6\hat{z} \cdot \vec{r} + 3t)\}$$

represents a traveling wave that is

- (A) transversely polarized and propagating in the negative z direction  
 (B) transversely polarized and propagating in the positive z direction  
 (C) circularly polarized and propagating in the negative z direction  
 (D) circularly polarized and propagating in the positive z direction  
 (E) longitudinally polarized and propagating in the  $\hat{x} - \hat{y}$  direction

解：x 方向和 y 方向的振动反号，位相差为  $\pi$ ，因此是线偏振的。对比指数上的传播因子

$e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}$ ，知  $\mathbf{k}$  是沿 -z 方向的。选 (A)。



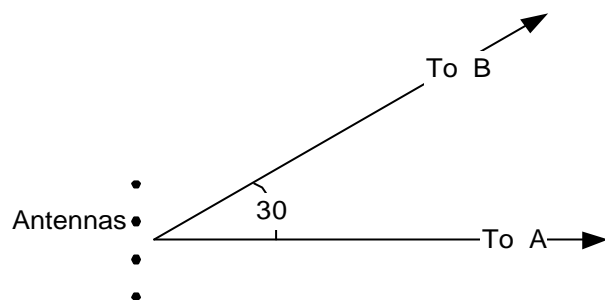
40. A slit of vertical height  $1 \times 10^{-4}$  meter and horizontal width  $4 \times 10^{-4}$  meter is a distance of 1 meter from screen, as shown above. Light of wavelength 5000 Å is incident on the slit. At the screen, the pattern consists of a

- (A) vertical line of dots
- (B) horizontal line of dots
- (C) grid of dots with vertical spacing 4 times larger than the horizontal spacing
- (D) grid of dots with vertical spacing 4 times smaller than the horizontal spacing
- (E) grid of dots with equal horizontal and vertical spacing

解：狭缝衍射的角宽度

$$\Delta\theta \cong \frac{\lambda}{d},$$

因此峰的宽度  $d$  越小，对应的衍射斑的分布越稀。对于矩形孔的衍射，可看作两个垂直的狭缝衍射图样互相调制的结果。沿宽度方向的扩展比沿长度方向的扩展要稀。选 (C)。



View from above

41. The four equally spaced vertical radio antennas shown above transmit equally and in phase at  $10^6$  hertz. If signals of equal intensity are received at A and B, each 50 kilometers away, a possible spacing

between two adjacent antennas is

- (A) 100 m
- (B) 150 m
- (C) 300 m
- (D) 350 m
- (E) 600 m

解：当

$$d \sin \theta = n\lambda$$

时，从每个天线发射的信号达到 B 时对于到达 A 附加的相因子都是  $2\pi$  的整数倍，因此叠加的干涉形式没有改变。n = 1 时，得

$$d = \frac{c}{f \sin \theta} = 600 \text{ m}。$$

选 (E)。

42. The speed of sound in an ideal gas is related to the temperature  $T$  of the gas. This speed is proportional to

- (A)  $T^{\frac{1}{4}}$
- (B)  $T^{\frac{1}{2}}$
- (C)  $T$
- (D)  $T^{\frac{4}{3}}$
- (E)  $T^2$

解：波动方程的普遍形式

$$v = \sqrt{\frac{dP}{d\rho}}。$$

把声振动的传播看成是绝热过程，物态方程是

$$\frac{P}{P_0} = \left( \frac{\rho}{\rho_0} \right)^\gamma，$$

其中  $\gamma$  是定压比热和定容比热之比。则

$$\left( \frac{dP}{d\rho} \right) = \gamma \frac{P}{\rho}。$$

设  $M$  是气体的摩尔质量，则由 Clapeyron 方程

$$PV = \frac{m}{M} RT$$

得

$$\frac{P}{\rho} = \frac{P}{m/V} = \frac{RT}{M}。$$

所以

$$v = \sqrt{\frac{dP}{d\rho}} = \sqrt{\gamma \frac{P}{\rho}} = \sqrt{\gamma \frac{RT}{M}}。$$

选 (B)。

43. Two harmonic transverse waves of the same frequency with displacements at right angles to each other can be represented by the equations

$$y = y_0 \sin(\pi t - kx),$$

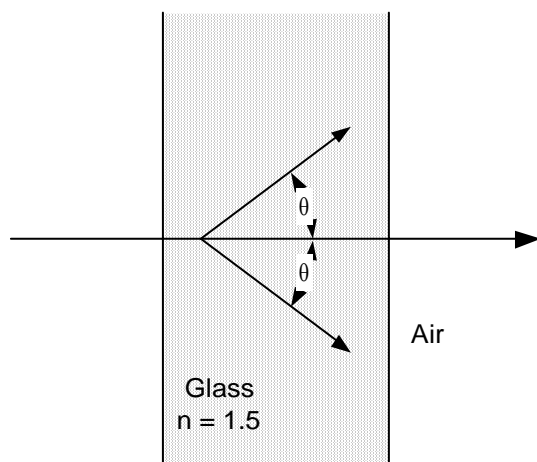
$$z = z_0 \sin(\pi t - kx + \phi),$$

where  $y_0$  and  $z_0$  are nonzero constants.

The equations represent a plane-polarized wave if  $\phi$  equals

- (A)  $\sqrt{2}$
- (B)  $3\pi/2$
- (C)  $\pi/2$
- (D)  $\pi/4$
- (E) 0

解：同一平面内两个频率相同方向垂直的简谐振动的合成情况有以下几种：当二者相位差为  $\pi$  的整数倍时，为线偏振；此外则为椭圆偏振。特例是当二者振幅相同，相位差为  $\pi$  半奇数倍时，为圆偏振。选 (E)。



44. A Fast charged particle passes perpendicularly

through a thin glass sheet of index of refraction 1.5. The cone of Cerenkov light, emitted in the glass at angle  $\theta$ , is incident on the glass-air interface at the critical angle for total reflection. The speed of the particle is

- (A)  $\frac{2}{3}c$
- (B)  $\frac{4}{5}c$
- (C)  $\sqrt{\frac{5}{9}}c$
- (D)  $\sqrt{\frac{2}{3}}c$
- (E)  $\sqrt{\frac{4}{5}}c$

解：由全反射角定义

$$n \sin \theta = 1, \\ \sin \theta = \frac{1}{n} = \frac{2}{3}。$$

另一方面

$$\cos \theta = \frac{c}{nv_s} = \sqrt{\frac{5}{9}},$$

$$v_s = \frac{2}{3} \sqrt{\frac{9}{5}}c = \sqrt{\frac{4}{5}}c。$$

选 (E)。