第四节 热力学和统计力学

Questions 1-2

The Maxwell distribution of molecular speeds in a gas is given by

$$n(v) = Av^2 e^{-mv^2/2kT}$$

where A is constant.

1. The most probable speed is

(A) $\sqrt{\frac{2kT}{m}}$

(B) $\sqrt{\frac{3kT}{m}}$

(C) $\sqrt{\frac{8kT}{m}}$

(D) $\frac{3}{2}kT$

(E) $\sqrt{2\pi mkT}$

解：所谓最可几速度，指出现几率最大的速度，即n(v)最大。由

$$\frac{dn(v)}{dv}\Big|_{v_P} = 0$$

，

得

$$2Av_p e^{-mv_p^2/2kT} - \frac{mv}{kT}Av_p{}^3 e^{-mv_p^2/2kT} = 0$$

。

解得

$$v_p = \sqrt{\frac{2kT}{m}}$$

。

选(A)。

2. The root-mean-square speed is

(A) $\sqrt{\frac{2kT}{m}}$

(B) $\sqrt{\frac{3kT}{m}}$

(C) $\sqrt{\frac{8kT}{m}}$

(D) $\frac{3}{2}kT$

(E) $\sqrt{2\pi mkT}$

解：由能均分定理

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$

，

均方根速度为

$$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{m}}$$

。

选(B)。

3. For an ideal gas, the specific heat at constant pressure C_p is greater than the specific heat at constant volume C_v because the

(A) gas does work on its environment when its pressure remains constant while its temperature is increased

(B) heat input per degree increase in temperature is the same in processes for which either the pressure or the volume is kept constant

(C) pressure of the gas remains constant when its temperature remains constant

(D) increase in the gas's internal energy is greater when the pressure remains constant than when the volume remains constant

(E) heat needed is greater when the volume remains constant than when the pressure remains constant

解：理想气体在定压过程中如果温度升高，体积会膨胀，对外做功。为了做这部分功气体要吸额外的热，因此定压比热大于定容比热。选（A）。

4. The number of ways of distributing N distinguishable molecules in K states, with N_1 in state 1, N_2 in state 2, etc., is

(A) $N - \ln N - N - \sum_{i=1}^{k} N_i \ln N_i - N_i$

(B) $(N_1!N_2!...N_k!)^{-1}$

(C) $N_1!(N_2!)^2(N_3!)^3...(N_k!)^k$

(D) $N!/(N_1!N_2!...N_k!)$

(E) $(N_1!N_2!...N_k!)/N!$

解：可能状态数为

$$C_N^{N_1} C_{N-N_1}^{N_2} \cdots C_{N-N_1-\cdots-N_{k-1}}^{N_K}$$

，

计算方法为

$$\frac{N!}{N_1!(N-N_1)!} \cdot \frac{(N-N_1)!}{N_2!(N-N_1-N_2)!} \cdots\cdots \frac{(N-N_1-\cdots N_{k-1})!}{N_k!(N-N_1-\cdots N_k)!}$$

。

答案显然为 $N!/(N_1!N_2!...N_k!)$ 。选(D)。

5. An engine absorbs heat at a temperature of 727 °C and exhausts heat at a temperature of 527 °C, if the engine operates at maximum possible efficiency, for 2000 joules of heat input the amount of work the engine performs is most nearly
(A) 400 J
(B) 1450 J
(C) 1600 J
(D) 2000 J
(E) 2760 J
解：可以证明，Carnot热机具有最高的效率，且不由工作介质决定。工作在温度为T_1 和 T_2（$T_1 > T_2$）的热源间的Carnot热机效率为

$$\eta = 1 - \frac{T_2}{T_1}$$

。
所以最大做功为

$$W = Q_{input}\eta = (1 - \frac{T_2}{T_1})Q_{input} = (1 - \frac{800}{1000}) \times 2000 = 400\text{J}$$

。
选(A)。

6. In a certain group of N particles, the number of particles with speeds in the range dv is given by dN = kv dv, where k is a constant. The speeds are in the range between zero and a maximum value v_{max}. The average speed of the particle is
(A) $\frac{1}{4}v_{mzx}$
(B) $\frac{1}{3}v_{mzx}$
(C) $\frac{1}{2}v_{mzx}$
(D) $\frac{1}{4}v_{mzx}$
(E) $\frac{1}{4}v_{mzx}$
解：关键是别忘了归一化，或者说除以总粒子数。

$$\overline{v} = \frac{\int_0^{v_{\max}} v \cdot kvdv}{\int_0^{v_{\max}} kvdv} = \frac{2}{3}v_{\max}$$

选（D)。

7. In terms of the Boltzmann constant k, the classical constant-volume specific heat per molecule of helium gas is
(A) $\frac{k}{2}$
(B) k
(C) $\frac{3k}{2}$
(D) 2k
(E) 3k
解：由能均分定理，温度为T的经典系统，每个自由度的能量为 $\frac{1}{2}kT$。氦气为单元子分子，只有3个自由度，所以每个分子能量为 $\frac{3}{2}kT$，比热为 $\frac{3k}{2}$。选(C)。

Question 8-9 refer to the following processes involving systems labeled by numbers 1 through 8.
A bar of iron (1) at 300K is brought into thermal contact with a body (2) at 400K, the two being thermally isolated from all other systems.
An ideal gas (3) is compressed reversibly while in contact with a reservoir (4), the two being thermally isolated from all other systems.
A body of water (5) freezes reversibly.
A container of water (6) is stirred and its temperature increases by 1K.
A chemical reaction takes place in an isolated system (7).
A Carnot engine (8) operates in a cycle.
8. For which of the following systems does the entropy decrease?
(A) 1
(B) 4
(C) 5
(D) 6
(E) 7
解：熵是态函数，两个被可逆过程联系起来的态之间的熵变为

$$\Delta S = S_2 - S_1 = \int_1^2 \frac{dQ}{T}$$

。
先对各个系统逐一分析：
（1）吸收热量，熵增加；(2）放出热量，熵减少。二者组成系统与外界无热交换，也无相互做功，熵不变。
（3) 被可逆压缩，条件不足以判断温度的变化情况，无法确定熵值变化情况，从而（4）也无法判断。但是二者组成系统虽然与外界无热交换，但外界对气体做功，所以系统熵增加。
（5）向外放热，体积膨胀对外做功，熵值降低。

（6）由于外界做功，使其温度上升，熵值增加。
（7）会发生熵变，等于生成物的熵值减去反应物的熵值。
（8）Carnot热机一个循环后回到原来状态，熵不变。
选（C）。

9. For which of the following systems does the entropy increase?
(A) 2
(B) 3
(C) 8
(D) 1 and 2 combined
(E) 3 and 4 combined
解：选（E）。原因见上题分析。

10. Upon freezing at 271.3°C and one atmosphere pressure, liquid bismuth expands and releases heat. It can correctly be inferred from the facts that
(A) bismuth is a metal.
(B) solid bismuth will float on water.
(C) the entropy of the solid is greater than that of the liquid.
(D) the Gibbs free energy of the solid is greater than that of the liquid at the freezing point.
(E) raising the pressure at 271.3°C will cause solid bismuth to melt.
解：铋确实是一种金属，但不可能从题目所给条件推知，所以不能选（A）。同理（B）也推不出来。由于铋凝固时放热，体积膨胀对外做功，所以熵值减小，自由能降低，（C）、（D）不对。选（E）。可以和水在冰点的性质类比。

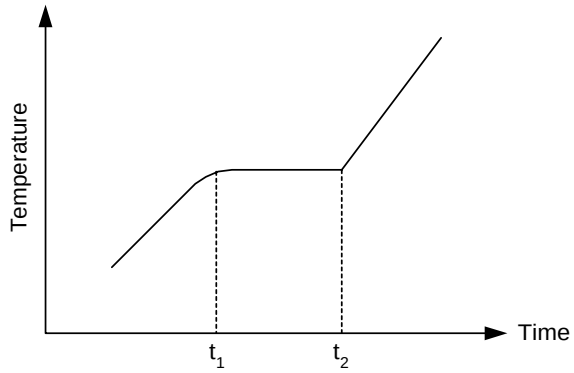
11. How many ways can two indistinguishable particles obeying Bose-Einstein statistics be arranged among three states?
(A) 3
(B) 6
(C) 9
(D) 12
(E) 15
解：由于二者不可分辨，而且可同处在一个态上，所以可能情况为（态1，态1）、（态2，态2）、（态3，态3）、（态1，态2）、（态2，态3）、（态3，态1）。答案选（B）。

12. The root-mean-square speed of oxygen and nitrogen molecules in air at room temperature is closest to
(A) 100 m/s
(B) 500 m/s
(C) 3000 m/s
(D) 15,000 m/s
(E) 50,000 m/s
解：因为氧气和氮气都是双原子分子气体，每个分子自由度为5，所以室温下能量为 $\frac{5}{2}kT$，

$$\frac{5}{2}kT=\frac{1}{2}m\overline{v^2},$$

$$v_{rms}=\sqrt{\overline{v^2}}=\sqrt{\frac{5kT}{m}}=\sqrt{\frac{5RT}{M}}=\sqrt{\frac{5\times 8.31\times 300}{32\times 10^{-3}}}\approx 624\text{m/s}$$

。
选（B）。



13. Electric power is used to heat and melt 3 kilograms of a certain material. The graph of temperature vs. time for the process is shown above. A current of 10 amperes at a potential difference of 100 volts is used and the time between t_1 and t_2 is approximately 17 minutes. The heat of fusion of the material is most nearly
(A) 80 J/kg
(B) 970×10^2 J/kg
(C) 144×10^3 J/kg
(D) 340×10^3 J/kg
(E) 539×10^3 J/kg
解：相变潜热为

$$T=\frac{Q}{M}=\frac{UIt}{M}=\frac{100\times10\times17\times60}{3}=340\times10^3\text{J/kg}$$

。
选（D）。

14. For a system in which the number of particles is fixed, the reciprocal of the Kelvin temperature T is given by which of the following derivatives? (Let P=pressure, V=volume, S=entropy, and U=internal energy.)

(A) $\left(\frac{\partial P}{\partial V}\right)_S$

(B) $\left(\frac{\partial P}{\partial S}\right)_V$

(C) $\left(\frac{\partial S}{\partial P}\right)_U$

(D) $\left(\frac{\partial V}{\partial P}\right)_U$

(E) $\left(\frac{\partial S}{\partial U}\right)_V$

解：热力学基本方程

$$dU = TdS - pdV$$。

把U看作S、V的函数U = U(S,V)，其全微分为

$$dU = \left(\frac{\partial U}{\partial S}\right)_V dS + \left(\frac{\partial U}{\partial V}\right)_S dV$$。

比较以上两式得

$$\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_V$$。

选（E）。

15. A large isolated system of N weakly interacting particles is in thermal equilibrium. Each particle has only 3 possible nondegenerate states of energies 0, ε, and 3ε. When the system is at an absolute temperature T>>ε/k, where k is Boltzmann's constant, the average energy of each particle is

(A) 0

(B) ε

(C) $\frac{4}{3}\varepsilon$

(D) 2ε

(E) 3ε

解：由Maxwell分布，

$$\overline{E} = \frac{0 + \varepsilon e^{-\frac{\varepsilon}{kT}} + 3\varepsilon e^{-\frac{3\varepsilon}{kT}}}{1 + e^{-\frac{\varepsilon}{kT}} + e^{-\frac{3\varepsilon}{kT}}}$$。

当 ε << kT 时，$e^{-\frac{\varepsilon}{kT}} \to 1$，有

$$\overline{E} = \frac{4}{3}\varepsilon$$。

选（C）。

16. In a gas of N diatomic molecules, two possible models for a classical description of a diatomic molecule are:

Model I　　Model II

Rigid Dumbbell　　Springy Dumbbell

Which of the following statements about this gas is true?

(A) Model I has a specific heat $c_v = \frac{3}{2}Nk$.

(B) Model II has a smaller specific heat than Model I.

(C) Model I is always correct.

(D) Model II is always correct.

(E) The choice between Models I and II depends on the temperature.

解：在低温下，两个原子间距离这个振动自由度被冻结，此时可取模型I。当温度较高时，这个自由度被激发，须采用模型II。答案选（E）。答案(A)不对，模型I中两个原子共有5个自由度，由能均分定理，

$$c_v = \frac{5}{2}Nk$$。

17. Consider a system of N noninteracting particles confined in a volume V at a temperature such that the particles obey classical Boltzmann statistics. If the temperature is lowered to the point at which quantum effects become important, the pressure of the gas may differ depending on whether the particles are fermions or bosons. Let P_F be the pressure if they are fermions, P_B be the pressure if they are bosons, and P_C be the pressure the particles would exert if quantum effects are ignored. Which of the following is true?

(A) $P_F = P_B = P_C$

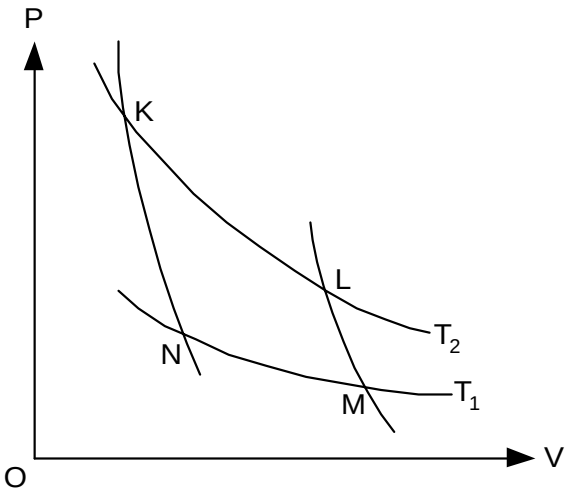
(B) $P_F > P_C > P_B$

(C) $P_F > P_B > P_C$

(D) $P_F < P_B < P_C$

(E) $P_F < P_C < P_B$

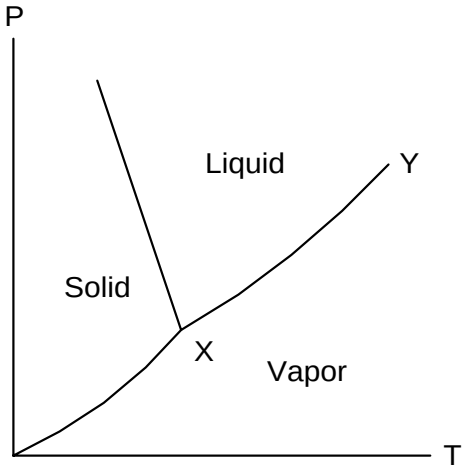
解：Fermion有相互排斥的等效作用，压强变大；而Boson有相互吸引的等效作用，压强变小。选（B）。可以联想一下白矮星中子星中惊人的简并压，分别由电子和中子造成，均为Fermion。而Bose-Einstein凝聚则发生在Boson之间。



18. In the cycle shown above, KL and NM represent isotherms, while KN and LM represent reversible adiabats. A system is carried through the Carnot cycle KLMN, taking in heat Q_2 form the hot reservoir T_2 and releasing heat Q_1 to the cold reservoir T_1. All of the following statements are true EXCEPT:

(A) $Q_1/T_1=Q_2/T_2$.
(B) The entropy of the hot reservoir decreases.
(C) The entropy of the system increases.
(D) The work W done is equal to the net heat absorbed, Q_2-Q_1.
(E) The efficiency of the cycle is independent of the working substance.

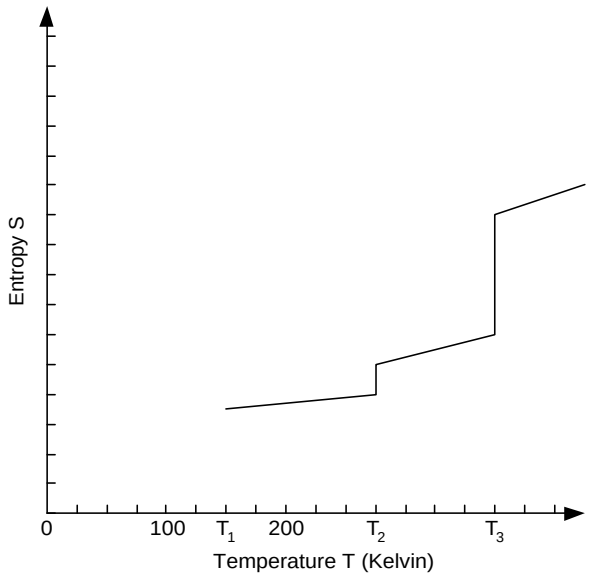
解：熵是态函数，所以经历一个循环后回到原来状态，熵值不变。选（C）。注意题目中所指系统指循环系统，不包括热库等。大家可以用本题复习一下Carnot循环，其他几个选项均是重要结论。



19. For the substance represented by the diagram above of pressure P versus temperature T, which of the following statement is true?

(A) The substance expands when it freezes.
(B) In the vicinity of point X, the vapor is more dense than liquid.
(C) Point X represents the critical point for the substance.
(D) The segment of the graph from X to Y represents the fusion curve for the substance.
(E) At any temperature above that of the triple point, the distinction between liquid and gas disappears.

解：我们在X点左侧的范围内做等温直线。从液相沿等温线变为固相时，压强变小，说明发生了膨胀。选（A）。三相点附近气体密度可以大于液体的，（B）不对。X是三相点，而不是什么相变点，二者概念不同，（C）不对。X到Y一段曲线是汽化曲线，（D）不对。在Y点以上，而不是在X点以上，气相液相的区别消失，（E）不对。



20. The graph above shows an entropy s versus temperature T curve for isobaric for a certain substance. The S and T scales are linear. Which of the following statement is NOT correct?

(A) The vertical portion of the curve at temperature T_2 represents a first-order phase transition.
(B) The vertical portion of the curve at temperature T_3 represents a second-order phase transition.
(C) The latent heat for the phase change at T_2 is less than the latent heat for the phase change at T_3.
(D) The specific heat at constant pressure in the region between T_1 and T_2 is less than that in the region between T_2 and T_3.
(E) The entropy approaches zero if the temperature is continuously lowered so that T approaches zero.

解：一级相变指相变点两相的化学势连续，但化学

势的一阶导存在突变，且存在相变潜热。T_2处的直线表示相变前后熵值发生了变化，所以为一级相变。而在二级相变中，相变前后熵值不变，没有相变潜热和比容的突变。T_3处的直线与T_2处相同，也为一级相变，只是熵值改变更大，潜热比T_2处的相变大，如（C）所述。所以选项（B）不对。由比热容公式

$$c_p = T\left(\frac{\partial s}{\partial T}\right)_p ,$$

而T_1到T_2一段斜率较小，比容比较小。选项（C）正确。（E）是热力学第三定律的要求，正确。选（B）。

21. A mole of ideal gas initially at temperature T_0 and volume V_0 undergoes a reversible isothermal expansion to volume V_1. If the ratio of specific heats is $c_p/c_v = \gamma$ and if R is the gas constant, the work done is

(A) zero

(B) $RT_0(V_1/V_0)^{\gamma}$

(C) $RT_0(V_1/V_0 - 1)$

(D) $C_v T_0[1-(V_0/V_1)^{\gamma-1}$

(E) $RT_0 \ln V_1/V_0$

解：等温膨胀，做功为

$$W = \int_{V_0}^{V_1} PdV = \int_{V_0}^{V_1} \frac{RT_0}{V} dV = RT_0 \ln V_1/V_2 ,$$

其中用到了气体方程

$$PV = nRT ,$$

$$P = \frac{RT}{V} 。$$

选（E）。

Questions 22-23

The Maxwell distribution of molecular speeds in a gas is given by

$$n(v) = Av^2 e^{-mv^2/2kT} ,$$

where A is a constant.

22. The most probable speed is

(A) $\sqrt{\frac{2kT}{m}}$

(B) $\sqrt{\frac{3kT}{m}}$

(C) $\sqrt{\frac{8kT}{\pi m}}$

(D) $\frac{3}{2}kT$

(E) $\sqrt{2\pi mkT}$

解：Maxwell速度分布律下常用的结论如平均速率、最可几速率、方均根速率应熟记。

最可几速率满足条件

$$\frac{dn}{dv} = 0 ,$$

解之即得。

$$2Ave^{-mv^2/2kT} - \frac{mv}{kT} av^2 e^{-mv^2/2kT} = 0 。$$

选（A）。

23. The root-mean-square speed is

(A) $\sqrt{\frac{2kT}{m}}$

(B) $\sqrt{\frac{3kT}{m}}$

(C) $\sqrt{\frac{8kT}{\pi m}}$

(D) $\frac{3}{2}kT$

(E) $\sqrt{2\pi mkT}$

解：接上题。方均根速率的计算式为

$$v_{rms} = \sqrt{\frac{\int_0^{\infty} v^2 n(v) dv}{\int_0^{\infty} n(v) dv}} = \sqrt{\frac{3kT}{m}}$$

其中积分利用分部积分法，分子分母相约。选（B）。

24. The observed specific heat of the electrons in a metal is much smaller than classical (i.e., nonquantum) statistical mechanics would indicate. The reason for this directly related to

(A) special relativity

(B) The Pauli exclusion principle

(C) the indeterminacy principle

(D) Hund's rule

(E) the principle of least action

解：电子是Fermion，满足Pauli不相容原理，服从Fermi统计。相应的比热容计算的结果表明，在一定温度T下，只有Fermi面附近大致为k_BT能量范围内的电子受到热激发，激发能 ≈ k_BT。因此量子统计的比热容结果比经典值小很多。选（B）。

25. The results derived from the Bose-Einstein statistics approximate those derived from the Fermi-Dirac statistics under which of the following conditions?
(A) The number of quantum states thermally accessible to each particle is much greater than the number of particles.
(B) The number of quantum states thermally accessible to each particle is approximately equal to the number of particles.
(C) The number of quantum states thermally accessible to each particle is much less than the number of particles.
(D) The particles have spin zero.
(E) The quantum states are degenerate.
解：Fermi-Dirac统计和Bose-Einstein统计为

$$a_l = \frac{\omega_l}{e^{\alpha+\beta\varepsilon_l} \pm 1}$$

由此可以看出，当参数α满足条件

$$e^{\alpha} >> 1$$

时，分母中的±1就可以忽略，这时两种分布都过渡到Boltzmann分布，显然此时 $\alpha_l << \omega_l$，对所有的l。此条件也称为经典极限条件。选（A）。

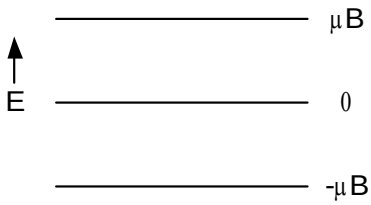
26. Copper has a compressibility $\kappa = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T = 8\times10^{-7}\,atm^{-1}$ and coefficient of thermal expansion $\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P = 6\times10^{-5}\,K^{-1}$ at 300K. A sample of copper initially at 1 atmosphere is heated 1K at constant volume. What is the final pressure?
(A) -74 atm
(B) 0.987 atm
(C) 1.013 atm
(D) 43 atm
(E) 76 atm
解：利用热力学等式

$$\left(\frac{\partial p}{\partial T}\right)_V\left(\frac{\partial V}{\partial p}\right)_T\left(\frac{\partial T}{\partial V}\right)_p = -1$$

从而有

$$\left(\frac{\partial p}{\partial T}\right)_V = -\frac{1}{\left(\frac{\partial V}{\partial p}\right)_T\left(\frac{\partial T}{\partial V}\right)_p} = \frac{\beta}{\kappa} = 75\text{atm/K}$$

。
以下计算即得。选（E）。



27. A set of energy levels for an atom in a magnetic field of magnitude B is shown above. For a collection of such atoms, what fraction is on the lowest state at temperature T?
(A) 1
(B) $e^{-\mu B/kT}$
(C) $e^{\mu B/kT}$
(D) $e^{\mu B/kT}/(e^{\mu B/kT}+e^{-\mu B/kT})$
(E) $e^{\mu B/kT}/(1+e^{\mu B/kT}+e^{-\mu B/kT})$
解：根据Boltzmann统计，原子处于能量为E的能级上的几率

$$p_E \propto e^{-E/kT},$$

因此在三个能级上的几率比为

$$p_-:p_0:p_+ = e^{\mu B/kT}:1:e^{-\mu B/kT},$$

且总几率和为1。选（E）。

28. How many ways can two indistinguishable particles obeying Bose-Einstein statistics arranged among three states?
(A) 3
(B) 6
(C) 9
(D) 12
(E) 15
解：两个粒子占据同一个量子态的可能性总数共3种，占据不同量子态的可能性总数（考虑到粒子的不可区分性），为

$$\frac{1}{2}C_3^2 = 3$$

种。故一共有6种占据方式。选（B）。

29. A photon gas such as results from the “big bang” undergoes an adiabatic expansion during which VT^3 is constant. An ideal gas would obey the same law of adiabatic expansion if its ratio of specific heats $c_p/c_v = \gamma$ were

(A) 1/3
(B) 3/4
(C) 4/3
(D) 3/2
(E) 3

解：理想气体的绝热过程方程为

$$pV^{\gamma} = const,$$

利用状态方程改写形式可得

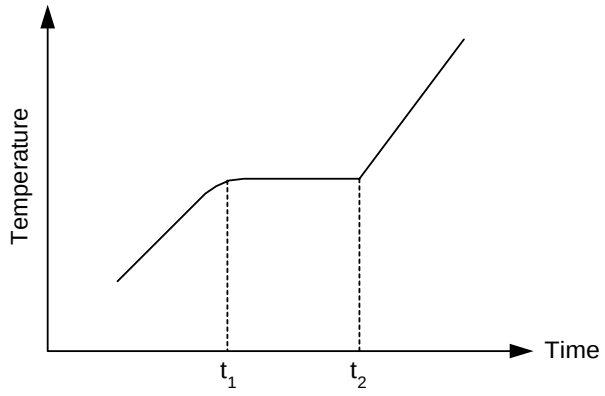
$$VT^{\frac{1}{\gamma-1}} = const,$$

因此有

$$\frac{1}{\gamma-1} = 3,$$

$$\gamma = \frac{4}{3}。$$

选（C）。



30. Electric power is used to heat and melt 3 kilograms of a certain material. The graph of temperature vs. time for the process is shown above. A current of 10 amperes at a potential difference of 100 volts is used and the time between t_1 and t_2 is approximately 17 minutes. The heat of fusion of the material is most nearly

(A) 80 J/kg
(B) 970×10^2 J/kg
(C) 144×10^3 J/kg
(D) 340×10^3 J/kg
(E) 539×10^3 J/kg

解：t_1到t_2之间被加热物体保持温度不变，进行相变，所吸收的热量全部用来熔解。

$$L = \frac{Q}{m} = \frac{UIt}{m} = \frac{100\times10\times17\times60}{3} = 340\times10^3 \text{ J/kg}$$

选（D）。

31. A mixture of one mole of helium (atomic weight = 4) and one mole of argon (atomic weight = 40) is in thermal equilibrium in a container at room temperature. The ratio of the rms speed of a helium atom to the rms speed of an argon atom, v_{He}/v_{Ar}, is most nearly equal to

(A) 1
(B) 0.3
(C) 1
(D) 3
(E) 10

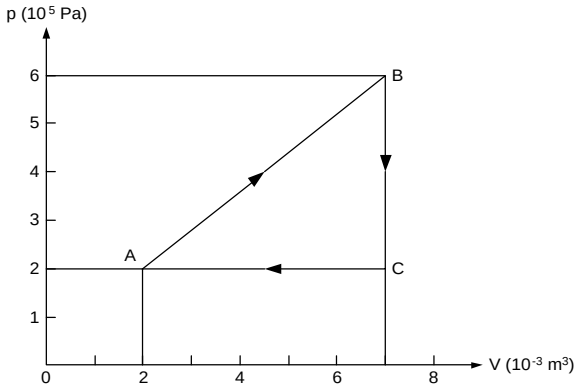
解：根据Maxwell速度分布率计算的气体分子方均根速率

$$v_{rms} = \sqrt{\frac{3kT}{m}}。$$

故两种气体的方均根速率之比

$$\frac{v_{He}}{v_{Ar}} = \sqrt{\frac{m_{Ar}}{m_{He}}} = \sqrt{10} \approx 3。$$

选（D）。



32. A gas is taken through the cycle A→B→C→A, as shown above. What is the net work done by the gas?

(A) 2000 J
(B) 1000 J
(C) 0 J
(D) –1000 J
(E) –2000 J

解：在p-V图上循环曲线所包围的面积就是气体所作

的净功，顺时针为正。

$$A = S_{A-B-C-A} = \frac{1}{2}(V_C - V_A)(P_B - P_C) = 2000\text{J}$$

。

选（A）。

33. The number of ways of distributing N distinguishable molecules among k states, with N_1 in state 1, N_2 in state 2, etc., is

(A) $N \ln N - N\sum_{i=1}^{k}(N_i \ln N_i - N_i)$

(B) $(N_1!N_2!....N_k!)^{-1}$

(C) $N_1!(N_2!)^2(N_3!)^3....(N_k!)^k$

(D) $N/(N_1!N_2!....N_k!)$

(E) $(N_1!N_2!....N_k!)/N!$

解：N个可区分分子的总排列数为N!，其中每个量子态内的N_i!种排列对应的是统一中分布，总的分布可能性

$$n = N!\Big/\prod_i N_i!$$

。

选（D）。

34. For an isotropic two-dimension metal, the Fermi surface is a circle of radius k_F in momentum space. If the energy of an electron of mass m and wave vector k is $\varepsilon = \hbar^2 k^2/2m$, then the number of states per unit energy (density of states) at the Fermi energy ε_F is

(A) proportional to $\sqrt{\varepsilon_F}$

(B) proportional to ε_F

(C) proportional to $1/\varepsilon_F$

(D) proportional to $1/\sqrt{\varepsilon_F}$

(E) independent of ε_F

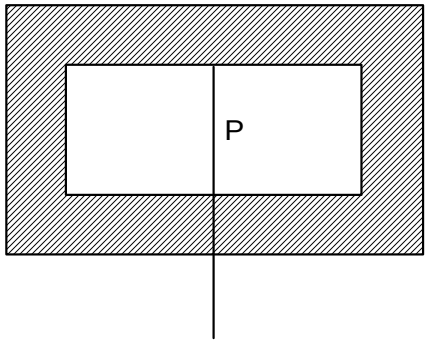
解：每个量子态在k空间中占据的体积数为$8\pi^3/V$，能态密度

$$N(E) = \frac{V}{8\pi^3}4\pi k^2\frac{dk}{dE}$$

$$\propto \varepsilon\frac{d\sqrt{\varepsilon}}{d\varepsilon} = \sqrt{\varepsilon}$$

。

选（A）。



35. Gas at standard temperature and pressure is initially in the left-hand compartment of the device shown above. The fight-hand compartment is initially evacuated. The volumes of the two compartments are the same, and the entire device is thermally insulated from its surroundings. If the partition P separating the two compartments is rapidly pulled out sideways, which of the following will be true?

(A) For an ideal gas, the temperature will be lower after the expansion.

(B) For a real gas, the temperature will be the same after the expansion.

(C) The final temperature is determined solely by the specific heat ratio c_P/c_V.

(D) The final pressure must be known before the final temperature can be computed for an ideal gas.

(E) The internal energy will remain constant for any gas, real or ideal.

解：自由扩散运动过程中，气体与外界没有热交换，也没有对外做功。因此，根据热力学第一定律

$$\Delta U = W + Q,$$

气体的内能不变。选（E）。

36. Heat dQ flows from a body at temperature T_1 to a body at temperature T_2. The total change in entropy of the two body is equal to

(A) $dQ(1/T_1 + 1/T_2)$

(B) $dQ/(T_1 + T_2)$

(C) $dQ/(T_1 - T_2)$

(D) $dQ(1/T_1 - 1/T_2)$

(E) dQ/T_2

解：根据熵的定义式计算

$$dS = \frac{-dQ}{T_1} + \frac{dQ}{T_2} = dQ\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$。

选（D）。

37. A gas consists of N_1 atoms in a state of energy E_1 and N_1 atoms in a state of energy E_2. If the temperature T of the gas is 300 K (kT=1/40 eV), the energy difference $E_1 - E_2$ is 4×10^{-3}eV. The ratio $(N_2–N_1)$ / (N_1+N_2) is most nearly
(A) 1/10,000
(B) 1/100
(C) 1/12
(D) 1/3
(E) 1/2
解：根据Boltzmann分布，

$$N_E \propto e^{-E/kT}$$，

因此

$$\frac{N_2 - N_1}{N_2 + N_1} = \frac{e^{-E_2/kT} - e^{-E_1/kT}}{e^{-E_2/kT} + e^{-E_1/kT}} = \frac{e^{\Delta E/kT} - 1}{e^{\Delta E/kT} + 1}$$

$$= \frac{e^{0.16} - 1}{e^{0.16} + 1} \approx \frac{0.16}{2.16} \sim \frac{1}{100}$$。

选（B）。

38. The specific heat of a metal at low temperatures, T < 10 K, varies as $c = \gamma T^m + AT^n$, where the first term arises from the free electrons and the second from the photons. The values of the exponents are
(A) m = 1, n = 1
(B) m = 1, n = 2
(C) m = 1, n = 3
(D) m = 2, n = 2
(E) m = 2, n = 3
解：在低温极限下（$T<<T_D$，$T<<T_F$），根据Debye T^3定律，晶格比热（来源于声子）

$$C_V \propto T^3$$；

根据Fermi气体模型，电子比热容

$$C_V \propto T$$。

选（C）。

40. Assuming that all the planes have the same reflection coefficient for sunlight and the same emission coefficient, which of the following relationships would be expected between the planets' average temperatures T in Kelvin and their distance R from the Sun?
(A) $T \propto R^{-2}$
(B) $T \propto R^{-1}$
(C) $T \propto R^{-1/2}$
(D) $T \propto R^{1/2}$
(E) $T \propto R^{2}$
解：假设行星的吸收系数为γ，根据热平衡的条件有

$$\gamma \frac{W}{4\pi R^2}\pi r^2 = \gamma a T^4 4\pi r^2$$，

$$T \propto \left(R^{-2}\right)^{1/4} = R^{-1/2}$$。

选（C）。

41. A mass m of water at temperature T_1 is mixed with an equal mass of water at temperature T_2 adiabatically. If C_p is the specific heat of water, the total entropy change of the universe equals
(A) $mc_p \ln[(T_1+T_2)/\sqrt{T_1T_2}]$
(B) $2mc_p \ln[(T_1+T_2)/\sqrt{T_1T_2}]$
(C) $2mc_p$
(D) $mc_p(T_1-T_2)^2/(2T_1T_2)$
(E) $mc_p(T_1-T_2)/\sqrt{T_1T_2}$

解：由于两种水的质量一样，比热容也一样，最终达到热平衡时升温与降温相等，末温为

$$T = \frac{T_1 + T_2}{2}$$，

总的熵变化为

$$\Delta S = mc_p\left(\int_{T_1}^{T}\frac{dT}{T} + \int_{T_2}^{T}\frac{dT}{T}\right) = mc_p \ln\frac{T^2}{T_1T_2} = 2mc_p \ln\frac{T}{\sqrt{T_1T_2}}$$

。

选（B）。

Questions 42-43

An ideal diatomic gas is initially at temperature T and volume V. The gas is taken through three reversible processes in the following cycle: adiabatic expansion to the volume 2V, constant volume process to the temperature T, isothermal compression to the original

volume V.

42. For the complete cycle described above, which of the following is true?

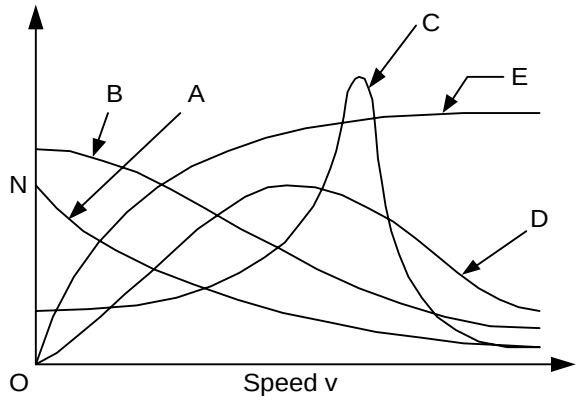
(A) Net thermal energy is transferred from the gas to the surroundings.
(B) The net work done by the gas on the surroundings is positive.
(C) The net work done by the gas on the surroundings is zero.
(D) The internal energy of the gas increases.
(E) The internal energy of the gas decreases.

解：对于本题所涉及的循环，第一个过程绝热膨胀，对外做功，吸热为零，内能下降，温度降低；第二个过程等容升温，吸收热量，做功为零；第三个过程等温压缩，外界对气体做功，气体内能不变，向外放热。在P—V 图上，代表一个过程的一条线下方的面积等于气体对外或外界对气体做功的绝对值，方向为体积变大则对外做功，反之外界对气体做功。本题中绝热线位于等温线下方，等温压缩中功的绝对值大，所以整个循环外界对气体做功。选项（B）、（C）不对。由于内能为状态量，而系统循环后气体回到原来状态，所以内能不变。（D）、（E）不对。如前所述，外界对气体做功，而气体内能不变，所以要对外放热。选（A）。

43. Which of the following statements about entropy changes in this cycle is true?

(A) The entropy of the gas remains constant during each of the three processes.
(B) The entropy of the surroundings remains constant during each of the three processes.
(C) The combined entropy of the gas and surroundings remains constant during each of the three processes.
(D) For the complete cycle, the combined entropy of the gas and surroundings increases.
(E) For the complete cycle, the entropy of the gas increases.

解：题目说了可逆过程，总熵变始终为零。选（B）。



44. Which of the curves in the graph above best represents the distribution of speeds of the molecules in an ideal gas at thermal equilibrium?

(A) A
(B) B
(C) C
(D) D
(E) E

解：由Maxwell分布率

$$n(v) = Av^2 e^{-\frac{mv^2}{2kT}}$$

。

v = 0 时，$n(v) = 0$，只有（D）、（E）符合。v的分布应该有一个最可几值，反映在图线上应该有一个极大峰。选（D）。

45. Which of the following describes a liquid of ^{3}He atoms?

(A) Maxwell-Boltzmann statistics only
(B) Bose-Einstein statistics only
(C) Fermi-Dirac statistics only
(D) Bose-Einstein statistics above some critical temperature T_C and Fermi-Dirac statistics below T_C
(E) None of the above, because it is in the liquid state

解：液体原子间相互作用很强，不能应用近独立系统统计。选（E）。