Physics GRE Sample Exam 1

Version 1.1.2: Revised September 3, 2012

TABLE OF INFORMATION

Rest mass of the electron	m_e	=	9.11×10^{-31} kg
Magnitude of the electron charge	e	=	1.60×10^{-19} C
Avogadro's number	N_A	=	6.02×10^{23}
Universal gas constant	R	=	8.31 J/mol $\cdot$ K
Boltzmann's constant	k	=	1.38×10^{-23} J/K
Speed of light	c	=	3.00×10^{8} m/s
Planck's constant	h	=	6.63×10^{-34}J $\cdot$ s $= 4.14 \times 10^{-15}$ eV $\cdot$ s
	$\hbar$	=	$h/2\pi$
	hc	=	1240 eV $\cdot$ nm
Vacuum permittivity	ϵ_0	=	8.85×10^{-12} C^2/(N $\cdot$ m^2)
Vacuum permeability	μ_0	=	$4\,\pi \times 10^{-7}$ T $\cdot$ m/A
Universal gravitational constant	G	=	6.67×10^{-11} m^3/(kg $\cdot$ s^2)
Acceleration due to gravity	g	=	9.80 m/s^2
1 atmosphere pressure	1 atm	=	1.0×10^{5} N/m^2 $= 1.0 \times 10^{5}$ Pa
1 angstrom	1 Å	=	1×10^{-10} m $= 0.1$ nm

Prefixes for Powers of 10

10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P

Moments of inertia about center of mass

Rod	$\frac{1}{12}M\ell^2$
Disc	$\frac{1}{2}MR^2$
Sphere	$\frac{2}{5}MR^2$

PHYSICS TEST

Time — 170 minutes
100 questions

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and then fill in the corresponding space on the answer sheet.

1. A centrifuge can be used to simulate large gravitational forces. Consider a centrifuge consisting of an arm of length 4 meters, rotating about a fixed pivot at constant speed. What must this speed be to simulate a gravitational acceleration of $9g$?

   (A) $2\sqrt{g}$ m/s
   (B) $3\sqrt{g}$ m/s
   (C) $6\sqrt{g}$ m/s
   (D) $18\sqrt{g}$ m/s
   (E) $36\sqrt{g}$ m/s

2. A block of mass m moving with velocity v collides with a heavier block of mass $3m$, initially at rest. If the blocks stick together after collision, they move with velocity V. If the collision is perfectly elastic and the lighter block bounces back in the opposite direction, the heavier block moves with velocity V'. What is V'/V?

   (A) 1/4
   (B) 1/2
   (C) 1
   (D) 3/2
   (E) 2

3. An LC circuit, consisting of a solenoid and a parallel-plate capacitor, has resonant frequency ω. If the linear dimensions of all circuit elements are doubled, the new resonant frequency is:

   (A) $\sqrt{2}\omega$
   (B) 2ω
   (C) ω
   (D) $\omega/2$
   (E) $\omega/\sqrt{2}$

4. A point dipole with dipole moment $\mathbf{p} = p\hat{\mathbf{z}}$ is placed at the center of a thin spherical conducting shell of radius R. What is the electric field outside the shell?

   (A) $\dfrac{1}{4\pi\epsilon_0}\dfrac{p}{r^2 R}\hat{\mathbf{r}}$
   (B) 0
   (C) $\dfrac{1}{4\pi\epsilon_0}\dfrac{3(\mathbf{p}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{p}}{r^3}$
   (D) $-\dfrac{1}{4\pi\epsilon_0}\dfrac{p}{r^2 R}\hat{\mathbf{r}}$
   (E) $-\dfrac{1}{4\pi\epsilon_0}\dfrac{3(\mathbf{p}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{p}}{r^3}$

5. The ground-state energy of helium is 79 eV. If the ground-state wavefunction of helium were a simple product of $1s$ wavefunctions, $\Psi_{100}(\mathbf{r}_1)\Psi_{100}(\mathbf{r}_2)$, the predicted ground-state energy would be 108 eV. What is the MAIN factor that accounts for this discrepancy?

   (A) electron-electron Coulomb repulsion

   (B) nonzero orbital angular momentum in the ground state

   (C) spin-spin coupling between the orbital electrons

   (D) spin-spin coupling between the nucleons

   (E) none of these

6. The energy of gamma rays from a transition of a nucleus from the first excited state to its ground state is measured. Which of the following is true of the measurement?

   (A) Gamma rays from this transition are part of a continuum of gamma rays from the de-excitation of low-lying states.

   (B) The measured mean energy must correspond to the energy of a vibrational state of the nucleus.

   (C) The measured width of the spectral peak must be $\hbar/(2\tau)$, where τ is the lifetime of the excited state.

   (D) The measured mean energy is greater than the true transition energy.

   (E) The measured mean energy is less than the true transition energy.

7. A system of electrons is in a box of fixed volume. If the number of electrons in the box is doubled, the Fermi energy is multiplied by a factor of

   (A) $2^{-1/2}$

   (B) $2^{1/2}$

   (C) $2^{2/3}$

   (D) 2

   (E) $2^{3/2}$

8. A gas of electrons is confined to a 2-dimensional surface at $z = 0$ but is otherwise free to move in the x and y directions. An external magnetic field is applied so that the electrons feel a harmonic oscillator potential, $U = \frac{1}{2}m\omega^2(x^2 + y^2)$. The temperature of the system is well above the Fermi temperature. What is the specific heat per particle of the electron gas?

   (A) $\frac{1}{2}k$

   (B) k

   (C) $\frac{3}{2}k$

   (D) $2k$

   (E) $\frac{5}{2}k$

9. A particle with mass m and angular momentum l moves in a constant central potential $U(r) = -k/r$, with $k > 0$. What, if any, is the radius of its stable circular orbit?

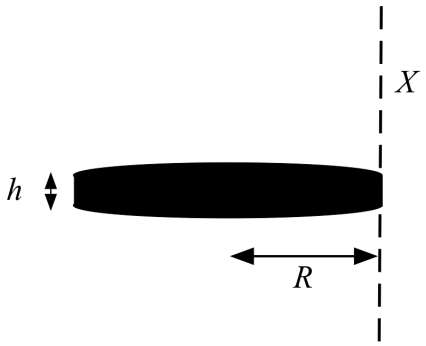
   (A) The particle has no allowed stable circular orbit

   (B) $\dfrac{l^2}{mk}$

   (C) $\dfrac{l^2}{2mk}$

   (D) $\dfrac{2l^2}{mk}$

   (E) $\dfrac{2l^2}{3mk}$



10. A disk (shown edge-on in the figure) has thickness h, density $\rho(r) = Ar^3$, mass M, and radius R. What is its moment of inertia about axis X, which is perpendicular to the axis of the disk and passes through the edge of the disk?

    (A) MR^2
    (B) $\frac{1}{2}MR^2$
    (C) $\frac{12}{7}MR^2$
    (D) $\frac{3}{2}MR^2$
    (E) $\frac{5}{7}MR^2$

11. A distant galaxy is located at redshift 2. What is the observed wavelength of the 21 cm hyperfine transition line of hydrogen originating from the galaxy?

    (A) 7 cm
    (B) 10.5 cm
    (C) 21 cm
    (D) 42 cm
    (E) 63 cm

12. A free particle incident from $x = -\infty$ encounters a potential barrier $V(x) = A\delta(x)$ at $x = 0$. The correct boundary conditions for the wavefunction near $x = 0$ are:

    (A) $$\psi(0_+) = \psi(0_-)$$ $$\frac{d\psi}{dx}|_{x=0_+} - \frac{d\psi}{dx}|_{x=0_-} = \frac{2mA}{\hbar^2}\psi(0)$$

    (B) $$\psi(0_+) = \psi(0_-)$$ $$\frac{d\psi}{dx}|_{x=0_+} - \frac{d\psi}{dx}|_{x=0_-} = \frac{2mA}{\hbar^2}$$

    (C) $$\psi(0_+) = \psi(0_-)$$ $$\frac{d\psi}{dx}|_{x=0_+} - \frac{d\psi}{dx}|_{x=0_-} = -\frac{2mA}{\hbar^2}\psi(0)$$

    (D) $$\psi(0_+) - \psi(0_-) = \frac{2mA}{\hbar^2}$$ $$\frac{d\psi}{dx}|_{x=0_+} = \frac{d\psi}{dx}|_{x=0_-}$$

    (E) $$\psi(0_+) = \psi(0_-)$$ $$\frac{d\psi}{dx}|_{x=0_+} = \frac{d\psi}{dx}|_{x=0_-}$$

13. What is the numerical value of the Planck mass (*not* the reduced Planck mass)?

    (A) 1.22×10^{-54} kg
    (B) 6.63×10^{-48} kg
    (C) 1.06×10^{-34} kg
    (D) 4.34×10^{-16} kg
    (E) 2.18×10^{-8} kg

14. A spaceship traveling at $0.6c$ toward a planet transmits a signal at 1 GHz to the planet's inhabitants. What frequency is the signal when it is received on the planet?

    (A) 1GHz
    (B) 2 GHz
    (C) 2.5 GHz
    (D) 4 GHz
    (E) 8 GHz

15. A point charge of mass m and charge q is held a distance d above an infinite conducting plate. When the charge is released, it will be attracted towards the plate. What is its speed when it reaches a distance $d/2$ above the plate? (Ignore all relativistic effects.)

(A) $q/\sqrt{8\pi m\epsilon_0 d}$
(B) $q/\sqrt{4\pi m\epsilon_0 d}$
(C) $2q/\sqrt{\pi m\epsilon_0 d}$
(D) $4q/\sqrt{\pi m\epsilon_0 d}$
(E) $q^2/\sqrt{16\pi m\epsilon_0 d^2}$

16. An initially uncharged 10-μF parallel-plate capacitor is charged with a constant current of 1 mA. What is the potential difference between the plates after one second?

(A) 0.01 V
(B) 1 V
(C) 10 V
(D) 100 V
(E) 1000 V

17. A particle in a 1-dimensional infinite square well between $x = 0$ and $x = L$ is subject to the following perturbation

$$\delta V(x) = \begin{cases} V_0, & x < L/2 \\ 0, & \text{otherwise.} \end{cases}$$

What is the leading-order shift in the energy of the first excited state? Recall that the wavefunction for the first excited state is

$$\psi(x) = \sqrt{\frac{2}{L}} \sin \frac{2\pi x}{L}.$$

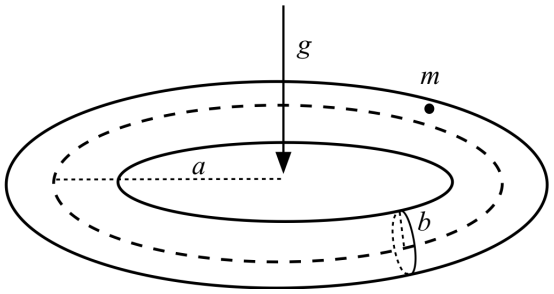
(A) $-V_0$
(B) V_0
(C) $V_0/4$
(D) 0
(E) $V_0/2$

18. Which of the following is NOT true about the isothermal expansion phase of a Carnot cycle?

(A) The free energy of the gas increases.
(B) The entropy of the gas increases.
(C) The isothermal expansion phase is reversible.
(D) The expansion takes place at the temperature of the "hot" reservoir.
(E) The gas does work on its surroundings.

19. Monochromatic blue light of wavelength 450 nm is shined on a slit of width a. A diffraction pattern is observed on a screen 10 m away. What must a be such that the width of the central diffraction maximum is 100 times the width of the slit?

(A) 45 nm
(B) 450 nm
(C) 0.045 mm
(D) 0.21 mm
(E) 0.30 mm



Questions 20-21 refer to a particle of mass m, confined to the surface of a torus with central radius a and cross-sectional radius b, oriented such that the Earth's gravitational field points perpendicular to the plane of the circle of radius a. Letting ϕ and θ be the angular coordinates on the circles of radii a and b, respectively, a Lagrangian for this system is

$$L = \frac{1}{2}m(a+b\cos\theta)^2\dot{\phi}^2+\frac{1}{2}mb^2\dot{\theta}^2-mgb\sin\theta.$$

20. What is the conjugate momentum to ϕ?

(A) $\frac{1}{2}m\dot{\phi}(a+b\cos\theta)^2$

(B) $m\dot{\phi}(a+b\cos\theta)^2$

(C) $\frac{1}{2}mb^2\dot{\theta}$

(D) $mb^2\dot{\theta}$

(E) $mgb\cos\theta$

21. Which of the following quantities represents the total energy?

(A) L

(B) $L+mgb\sin\theta$

(C) $L-mgb\sin\theta$

(D) $L+2mgb\sin\theta$

(E) $L-2mgb\sin\theta$

22. A resistor with resistance R and an inductor with inductance L are in series with a voltage source. For $t<0$, the voltage is 0. For $t>0$, the voltage source is V. At what time t does it take for the voltage across the inductor to drop to half of its initial level?

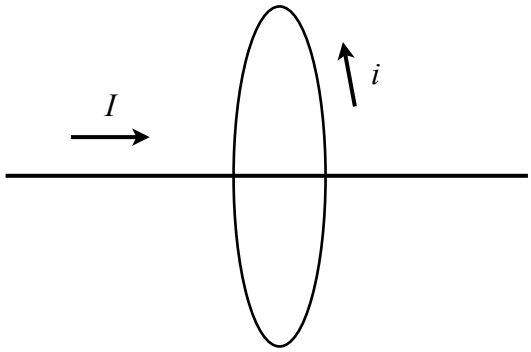
(A) $\dfrac{L\ln 2}{R}$

(B) $\dfrac{L}{R}$

(C) $\dfrac{L}{R\ln 2}$

(D) $\dfrac{2L}{R}$

(E) 0



23. A straight wire carrying current I passes through the center of a circular wire carrying current i. If the circular loop of wire has radius R, what is the tension on the circular wire due to the field produced by the straight wire?

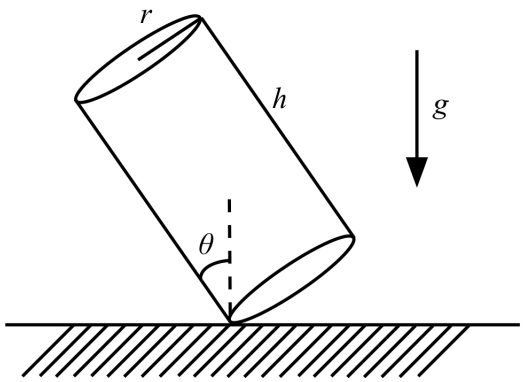
(A) $\dfrac{\mu_0 iI}{2\pi R^2}$

(B) $\dfrac{\mu_0 I^2}{2\pi R}$

(C) $\dfrac{\mu_0 i^2}{2\pi R}$

(D) $\dfrac{\mu_0 iI}{2\pi R}$

(E) 0



24. A uniform cylinder of height h and radius r is placed on a flat surface and tipped at an angle θ from the vertical. Find θ_0 such that when the cylinder is released from $\theta > \theta_0$, it falls over.

(A) $\arctan(2r/h)$

(B) $\arctan(r/h)$

(C) $\arctan(r/2h)$

(D) $\arccos(2r/h)$

(E) $\arccos(r/h)$

25. Consider a beam of muons produced in the upper atmosphere (100 km above the Earth's surface) with energy 3 GeV. The muon's mass is approximately 100 MeV, and its lifetime at rest is 2×10^{-6} s. The fraction of muons detected at ground level is most nearly

(A) $e^{-0.5}$

(B) e^{-1}

(C) e^{-5}

(D) e^{-10}

(E) e^{-15}

26. Graphene, a 2-dimensional allotrope of carbon, displays unusual electronic properties. In particular, the dispersion relation for electrons in graphene is:

(A) $\omega \propto \sqrt{|k|}$

(B) $\omega \propto |k|$

(C) $\omega \propto |k|^2$

(D) $\omega \propto |k|^3$

(E) $\omega \propto |k|^4$

27. Consider an electron subject to a Coulomb potential $V(r) = -\dfrac{e^2}{4\pi\epsilon_0}\dfrac{1}{r}$ and a uniform electric field $\mathbf{E} = E_0\hat{\mathbf{z}}$. For small E_0, the leading-order nonzero correction to the $n = 2$ states is:

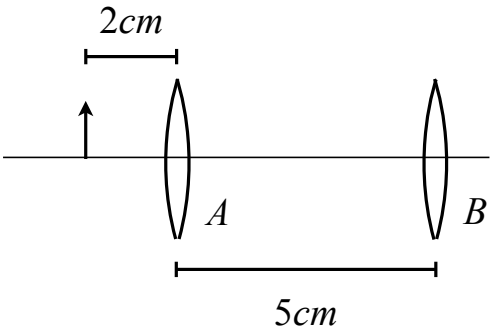
(A) independent of E_0

(B) proportional to E_0

(C) proportional to E_0^2

(D) proportional to E_0^3

(E) proportional to E_0^4



28. In the optical arrangement shown above, converging lenses A and B both have focal length 5 cm. An object is placed 2 cm to the left of lens A. Where is the image of the object located?

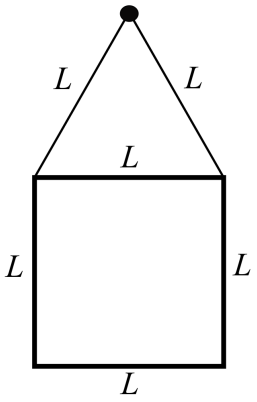
(A) 5 cm to the right of B

(B) 6.25cm to the right B

(C) 12.5 cm to the left of B

(D) 12.5 cm to the right of B

(E) No image is formed.



29. A square picture frame is made of four identical rods of mass M, length L, and uniform mass density. The frame is hung from the wall by two massless wires, also of of length L, attached to a nail, as shown in the figure. What is the frame's moment of inertia with respect to rotations about the axis through the nail?

(A) $\frac{1}{4}ML^2$

(B) $\left(\frac{8}{3}+\sqrt{3}\right)ML^2$

(C) $\left(\frac{16}{3}+2\sqrt{3}\right)ML^2$

(D) $\left(\frac{19}{3}+2\sqrt{3}\right)ML^2$

(E) $\left(\frac{41}{6}+3\sqrt{3}\right)ML^2$

30. Which values of spin quantum numbers are NOT possible for system consisting of spin-1 particle and a spin-2 particle?

(A) $l = 3$, $m = 3$

(B) $l = 1$, $m = 0$

(C) $l = 2$, $m = 1$

(D) $l = 2$, $m = 0$

(E) $l = 0$, $m = 0$

31. The radial wavefunction of the $2p$ state of hydrogen is

$$R_{21}(r) = \frac{1}{\sqrt{24}} a_0^{-5/2} r \exp(-r/2a_0),$$

where a_0 is the Bohr radius. What is the most probable value of r in this state?

(A) $a_0/2$

(B) a_0

(C) $2a_0$

(D) $4a_0$

(E) $6a_0$

32. Mass spectrometry uses which of the following physical properties of ions to determine the chemical makeup of a substance?

(A) Dipole moment

(B) Nuclear spin

(C) Charge-to-mass ratio

(D) Atomic number

(E) Electronegativity

33. A musician tuning a violin to a tuning fork at 440 Hz hears a beat frequency of 3 Hz. What is the frequency of the note produced by the violin?

(A) 428 Hz

(B) 434 Hz

(C) 437 Hz

(D) 443 Hz

(E) It is impossible to tell from the given information

34. The USS Enterprise, moving at speed $0.5c$ with respect to a nearby planet, fires a photon torpedo of speed c at a Romulan warship, initially 6000 km away, which is retreating away from the Enterprise at constant velocity. According to the Enterprise's clock, the torpedo made contact with the warship 0.1 seconds after firing. How fast was the warship traveling, in the frame of the planet?

(A) $\frac{13}{28}c$

(B) $\frac{13}{16}c$

(C) $\frac{13}{14}c$

(D) c

(E) $\frac{13}{10}c$

35. An ice skater is spinning with arms extended at an angular velocity of 5.0 radians/sec. After drawing her arms in, her new angular velocity is 8.0 radians/sec. If the skater's moment of inertia with arms extended was I, her moment of inertia with arms drawn in is:

(A) I

(B) $3I$

(C) $\frac{8}{5}I$

(D) $\frac{5}{8}I$

(E) $\sqrt{\frac{5}{8}}I$

36. Suppose that a particle in one-dimensional system has a Lagrangian L with a potential that is constant in time and such that

$$\begin{aligned} \frac{\partial L}{\partial t} &= 0 \\ \frac{\partial L}{\partial x} &= 0. \end{aligned}$$

Which of the following must be true?

I. Energy is conserved

II. Linear momentum is conserved

III. The potential is nonzero

IV. The Euler-Lagrange equations are not satisfied.

(A) I. only

(B) II only

(C) I and II

(D) I, II and III

(E) I, III and IV

37. A beam of particles with luminosity 10^{22} cm^{-2}s^{-1} is incident upon a target with scattering cross section 10^{-20} cm^2. Assuming a detector has an efficiency of 0.5 for detecting products of the scattering process, how many events will the detector see if the experiment runs for 1 day?

(A) $4.20 \cdot 10^2$ events

(B) $4.00 \cdot 10^4$ events

(C) $7.20 \cdot 10^5$ events

(D) $4.32 \cdot 10^6$ events

(E) $8.64 \cdot 10^6$ events

38. An electron in a cyclotron moves in a circular orbit at a fixed radius in the presence of a constant magnetic field $\mathbf{B}$. If the strength of the magnetic field is tripled, by what factor must the electron's momentum change to keep it orbiting at the same radius?

(A) $\sqrt{3}$

(B) 3

(C) $1/\sqrt{3}$

(D) $1/3$

(E) $3/2$

39. Two circular loops (radius b) of current, with centers at $(0, 0, a/2)$ and $(0, 0, -a/2)$, are oriented parallel to the xy-plane. One loop carries a current I in the $\hat{\phi}$ direction and the other carries current I in the $-\hat{\phi}$ direction. What is the magnitude of the magnetic field on the z-axis for $z \gg a, b$?

(A) $\dfrac{3\mu_0 Iab}{2z^3}$

(B) $\dfrac{3\mu_0 Iab^2}{2z^4}$

(C) $\dfrac{3\mu_0 Ia^2 b}{2z^4}$

(D) $\dfrac{3\mu_0 Ia^2 b^2}{2z^5}$

(E) 0

40. Which of the following is true about the total orbital angular momentum operator, L^2, of a particle subjected to an arbitrary force?

I. Always commutes with L_x, L_y, L_z

II. Always commutes with the total angular momentum J^2

III. Always commutes with the Hamiltonian

(A) I only

(B) II only

(C) III only

(D) I and II

(E) I, II, and III

41. A quantum system has a Hamiltonian given by

$$H = \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & -ib \\ 0 & ib & 0 \end{pmatrix},$$

where a, b, c are real positive constants. What are the possible results of a measurement of the energy of the system?

(A) b, $\pm a$

(B) a, $\pm b$

(C) a, b, $a + b$

(D) a, $\pm\sqrt{ab}$

(E) a, $\pm b^2$.

42. If magnetic monopoles were discovered, which of the following would no longer be a quantity defined on all of space?

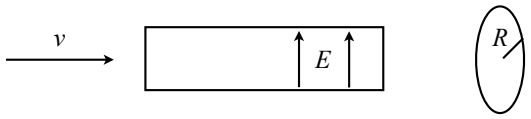
(A) $\nabla \cdot \mathbf{B}$

(B) The magnetic vector potential $\mathbf{A}(\mathbf{r})$

(C) A magnetic dipole moment

(D) The stress-energy tensor

(E) $\nabla \cdot \mathbf{E}$



43. A beam of nonrelativistic protons (mass m, charge q) and neutrons of velocity v enters a region of length L with an electric field E perpendicular to the direction of the beam. At the end of the region of length L, the beam strikes a circular target of radius R. Assuming that the diameter of the beam is much smaller than R, what is the minimum electric field E needed to deflect all protons before they strike the target?

(A) $\dfrac{mLv^2}{2qR^2}$

(B) $\dfrac{2mLv^2}{qR^2}$

(C) $\dfrac{mRv^2}{2qL^2}$

(D) $\dfrac{2mRv^2}{qL^2}$

(E) $\dfrac{4mLv^2}{qR^2}$

44. Put the following in chronological order, starting with the earliest.

I. Epoch of reionization

II. Nucleosynthesis

III. Big bang

IV. Lepton epoch

(A) I, II, III, IV

(B) III, I, II, IV

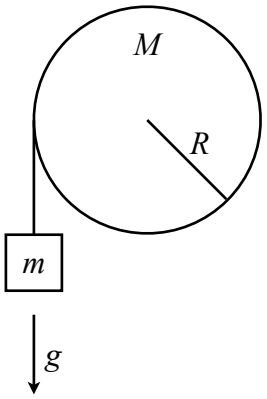
(C) III, II, IV, I

(D) III, I, IV, II

(E) III, IV, II, I

45. For a *monoatomic* ideal gas, which of the following is constant during adiabatic changes of state?

(A) $PV^{1/2}$
(B) PV
(C) $PV^{5/3}$
(D) $PV^{7/5}$
(E) $PV^{9/7}$



46. A string of length L and negligible mass is completely wound around a solid cylinder of uniform density, of mass M and radius R, and it has a small weight of mass m attached to its end. If the weight is released from rest under the influence of gravity, what is its velocity when the string is entirely unwound?

(A) $\sqrt{\dfrac{4mgL}{M+2m}}$
(B) $\sqrt{\dfrac{2mgL-MR^2}{2m}}$
(C) $\sqrt{2gL}$
(D) $\sqrt{\dfrac{2(m+M)gL}{m}}$
(E) $\sqrt{\dfrac{2mgL-2MR^2}{m}}$

47. An object is placed at rest in a potential field $U(x,y,z) = x + y^2 - \cos z$. What is the force on the object?

(A) $\mathbf{F}(x,y,z) = -\hat{\mathbf{x}} - 2y\hat{\mathbf{y}} - \sin z\hat{\mathbf{z}}$
(B) $\mathbf{F}(x,y,z) = x\hat{\mathbf{x}} + 2y\hat{\mathbf{y}} - \cos z\hat{\mathbf{z}}$
(C) $\mathbf{F}(x,y,z) = -x\hat{\mathbf{x}} - 2y\hat{\mathbf{y}} + \cos z\hat{\mathbf{z}}$
(D) $\mathbf{F}(x,y,z) = -\hat{\mathbf{x}} - 2y\hat{\mathbf{y}} + \cos z\hat{\mathbf{z}}$
(E) $\mathbf{F}(x,y,z) = \hat{\mathbf{x}} + 2y\hat{\mathbf{y}} + \sin z\hat{\mathbf{z}}$

48. Consider a system with three energy levels $-\epsilon$, 0, ϵ, and degeneracies $d(-\epsilon) = 2$, $d(0) = 1$, $d(\epsilon) = 3$. What is the energy of the system as $T \to \infty$?

(A) $\epsilon/5$
(B) $\epsilon/6$
(C) $5\epsilon/6$
(D) 0
(E) ϵ

49. The heat capacity per particle at constant volume of a relativistic ideal gas is:

(A) $3k/2$
(B) $k/2$
(C) k
(D) $2k$
(E) $3k$

50. An electromagnetic wave propagates in vacuum with electric field $E_0\cos(kx - \omega t)\hat{\mathbf{z}}$. What is the average magnitude of the Poynting vector in SI units, where the average is taken over one period of oscillation?

(A) $\dfrac{4E_0^2}{c\mu_0}$
(B) 0
(C) $\dfrac{E_0^2}{c\mu_0}$
(D) $\dfrac{E_0^2}{2c\mu_0}$
(E) $-\dfrac{E_0^2}{2c\mu_0}$

51. An observation of the reaction $e^+e^- \to \gamma$ would violate which of the following conservation laws?

(A) lepton number
(B) photon number
(C) angular momentum
(D) energy-momentum
(E) baryon number

52. The nucleus can be considered as a degenerate Fermi gas. A neutron scattering experiment determines that the Fermi momentum of nucleons in the carbon nucleus is about 40 MeV/c. Which of the following is approximately an absolute lower bound on the nuclear radius?

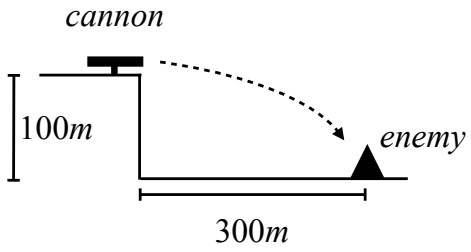
(A) 976 fm
(B) 87.4 fm
(C) 1.23 fm
(D) 0.132 fm
(E) 0.00510 fm

53. Which of the following does NOT obey Bose-Einstein statistics?

(A) neutrinos
(B) photons
(C) ^{4}He nuclei
(D) ^{4}He atoms
(E) pions

54. The observation of a sharp line of gamma rays of energy 511 keV from the center of our galaxy is most naturally explained by which of the following processes?

(A) hydrogen hyperfine transitions
(B) Hawking radiation
(C) ammonia maser transitions
(D) electron-positron annihilation
(E) supernovae

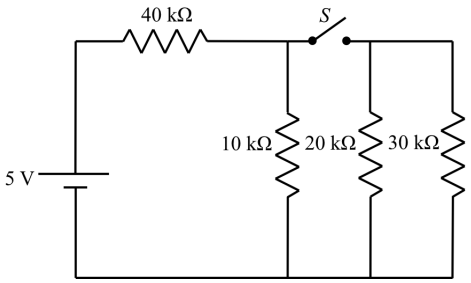


55. A soldier can fire a cannon horizontally from the top of a 100 m cliff. He wants to hit an enemy encampment at 300 m horizontal distance from the cliff. What must the initial velocity of his cannonball be in order to strike the encampment, neglecting air resistance?

(A) 22.4 m/s
(B) 47.4 m/s
(C) 67.1 m/s
(D) 94.9 m/s
(E) 134.2 m/s

56. In empty space, a spaceship of mass M can accelerate by expelling matter at velocity v_0 relative to the spaceship from its exhaust. If the spaceship starts at rest, how much matter must be exhausted in order to achieve a final velocity v?

(A) $M(1 - \exp(-v_f/v_0))$
(B) $M(1 - \exp(-v_f/2v_0))$
(C) $M \exp(v_f/2v_0)$
(D) $M \exp(v_f/v_0)$
(E) $M(1 - \exp(-2v_f/v_0))$



57. Consider the circuit shown in the diagram. When switch S is open, the current through the 10 kΩ resistor is I_1. After switch S is closed, the current through the same resistor is I_2. What is I_2/I_1?

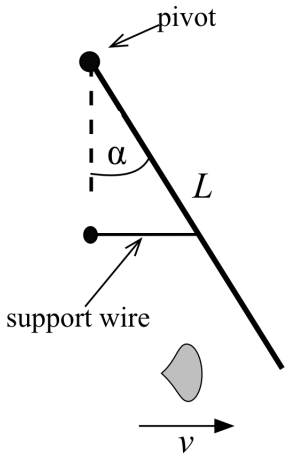
(A) 3/5

(B) 11/6

(C) 1

(D) 4/5

(E) 1/6



58. A rod of length L and mass M is attached to a pivot and suspended at an angle α from the vertical using a support wire, as shown in the diagram. A lump of clay of mass m is fired at the end of the rod with a velocity v. Just before the clay makes contact with the rod, the wire is cut. Assuming the clay and rod stick together after collision, what is the angular velocity in radians of the rod-clay system? (You may treat the lump of clay as a point mass.)

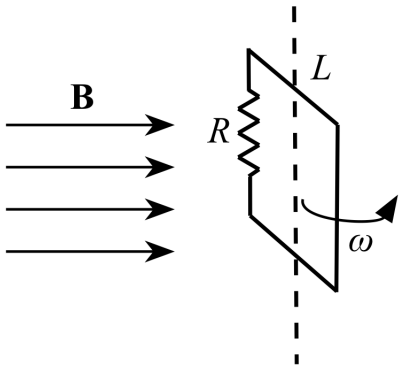
(A) $\dfrac{mv\cos\alpha}{(M+m)L}$

(B) $\dfrac{3mv\cos\alpha}{(M+m)L}$

(C) $\dfrac{3mv\cos\alpha}{(M+3m)L}$

(D) $\dfrac{3mv}{(M+3m)L}$

(E) $\dfrac{3mv}{ML}$



59. A square loop of wire of side length L, containing a load resistor R, is oriented perpendicular to the xy-plane and rotates about the z-axis at angular frequency ω in the presence of a uniform magnetic field $\mathbf{B} = B_0\hat{\mathbf{x}}$, as shown in the diagram. If $L = 10$ cm, $B_0 = 2$ tesla, and $R = 100.0\ \Omega$, what must ω be so that the average power dissipated in the resistor is 1.0 W?

(A) 25 rad/sec
(B) 50 rad/sec
(C) 314 rad/sec
(D) 500 rad/sec
(E) 707 rad/sec

60. In calculating the entropy of a micro-canonical ensemble, the inverse temperature $\beta = 1/kT$ can be viewed as a Lagrange multiplier enforcing the constraint of fixed total energy. Similarly, the chemical potential μ is related to the Lagrange multiplier for:

(A) fermion number
(B) particle number
(C) pressure
(D) volume
(E) magnetization

61. A spin-1/2 particle interacts with a magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$ through a Hamiltonian $H = (-\mu_B g B/2\hbar)\sigma_z$, where μ_B is the Bohr magneton and g is the particle's gyromagnetic ratio. Consider a system of these spin-1/2 particles in equilibrium at temperature T. Let A be the ratio of the number of spin-up particles to spin-down particles. If the strength of the magnetic field is doubled, the new ratio of spin-up to spin-down particles is:

(A) A^{-2}
(B) A
(C) A^2
(D) e^A
(E) $A\exp(\mu_B g B/\hbar k T)$

62. Which of the following is equivalent to $\nabla^2(1/r)$?

(A) $-4\pi\delta^3(\mathbf{r})$
(B) $4\pi\delta^3(\mathbf{r})$
(C) 0
(D) 4π
(E) -4π

63. Which of the following is closest to the Compton wavelength of the proton?

(A) 10^{-15} m
(B) 10^{-13} m
(C) 10^{-12} m
(D) 10^{-10} m
(E) 10^{-9} m

Questions 64-65 refer to the following scenario. A K^0 of mass m_K and energy E in the lab frame decays to a π^+ and a π^-, both of mass m_π. The π^- is observed to be emitted antiparallel to the K^0 momentum.

64. What is the speed of the π^+ in the center-of-momentum frame?

(A) $(1 - 4m_K^2/m_\pi^2)^{1/2}c$
(B) $(1 - 4m_\pi^2/m_K^2)^{1/2}c$
(C) $(1 - m_K^2/m_\pi^2)^{1/2}c$
(D) $(1 - m_\pi^2/m_K^2)^{1/2}c$
(E) $2(m_\pi^2/m_K^2)^{1/2}c$

65. What must be the initial K^0 energy such that the π^- is stationary in the lab frame?

(A) $\dfrac{m_\pi^2 c^2}{2m_K}$
(B) $\dfrac{m_K c^2}{2}$
(C) $\dfrac{m_\pi c^2}{2}$
(D) $\dfrac{(m_K^2 + m_\pi^2)c^2}{2m_\pi}$
(E) $\dfrac{m_K^2 c^2}{2m_\pi}$

66. A clarinet can be treated as a half-open pipe, where sounds are produced by standing pressure waves. For a clarinet of length 0.6 m, which of the following is a possible wavelength of a standing wave?

(A) 0.3 m
(B) 0.6 m
(C) 0.8 m
(D) 1.2 m
(E) 1.5 m

67. A sphere has a polarization of $\mathbf{P}(\mathbf{r}) = Cr^2\hat{\mathbf{r}}$. What is the electric field inside the sphere?

(A) $-\dfrac{4Cr^2}{\epsilon_0}\hat{\mathbf{r}}$
(B) $\dfrac{2Cr^2}{\epsilon_0}\hat{\mathbf{r}}$
(C) $-\dfrac{Cr^2}{\epsilon_0}\hat{\mathbf{r}}$
(D) $\dfrac{Cr^2}{4\pi\epsilon_0}\hat{\mathbf{r}}$
(E) 0

68. Suppose an electromagnetic plane wave propagating in vacuum in the $+\hat{\mathbf{z}}$-direction has a polarization with the electric field in the $+\hat{\mathbf{x}}$-direction immediately before it is reflected at normal incidence off a perfect conductor. What are the directions of the $\mathbf{E}$ and $\mathbf{B}$ vectors of the reflected wave?

(A) $\mathbf{E}$ in $+\hat{\mathbf{x}}$-direction & $\mathbf{B}$ in $+\hat{\mathbf{y}}$-direction
(B) $\mathbf{E}$ in $-\hat{\mathbf{x}}$-direction & $\mathbf{B}$ in $+\hat{\mathbf{y}}$-direction
(C) $\mathbf{E}$ in $+\hat{\mathbf{x}}$-direction & $\mathbf{B}$ in $-\hat{\mathbf{y}}$-direction
(D) $\mathbf{E}$ in $-\hat{\mathbf{x}}$-direction & $\mathbf{B}$ in $-\hat{\mathbf{y}}$-direction
(E) Electromagnetic plane waves do not reflect off perfect conductors

69. What is the value of the following commutator?

$$[[[L_x, L_y], L_x], L_x].$$

(A) $-i\hbar^3 L_z$
(B) $i\hbar^3 L_z$
(C) $-i\hbar^3 L_y$
(D) $i\hbar^3 L_y$
(E) $-i\hbar^3 L_x$

70. The vibrational frequency of diatomic oxygen is approximately 5×10^{13} Hz. The temperature at which the vibrational modes of O_2 will begin to be excited is closest to:

(A) 20 K

(B) 200 K

(C) 2000 K

(D) 20000 K

(E) 2×10^5 K

71. Which of the following does NOT represent a possible observable, written in the position basis, for a free particle in three dimensions?

(A) $-i\hbar\nabla$

(B) $x^2\partial/\partial y$

(C) $x\partial^2/\partial y^2$

(D) $x^2y^2z^2$

(E) xyz

72. The BCS theory of superconductivity explains the superconducting properties of metals at low temperature by supposing that a macroscopic number of metallic electrons all lie in the same ground state. Why does this not violate the Pauli exclusion principle?

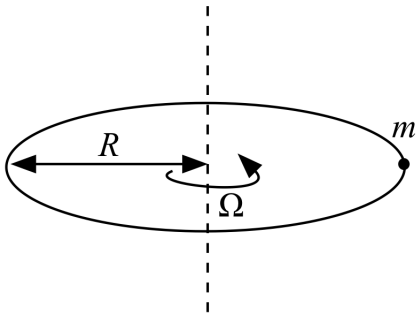
(A) BCS theory is incorrect

(B) Cooper pairs behave as bosons

(C) spin-spin coupling prevents electrons from being in the same state

(D) the Pauli exclusion principle does not apply to systems at low temperature

(E) electrons are not fermions



73. A hoop of radius R rotates at constant angular velocity Ω. A small bead of mass m is attached to the hoop, with a frictional force on the bead proportional to the difference in velocity between the bead and edge of the hoop, $F = k(R\Omega - R\omega)$, where ω is the angular velocity of the bead. If the bead begins at angular velocity ω_0, which of the following describes its subsequent motion?

(A) $\omega(t) = \omega_0 e^{-kRt/m}$

(B) $\omega(t) = \Omega - \omega_0 e^{-kRt/m}$

(C) $\omega(t) = \Omega - \omega_0 e^{-mRt/k}$

(D) $\omega(t) = \Omega(1 - e^{-kRt/m})$

(E) $\omega(t) = \Omega - (\Omega - \omega_0)e^{-kRt/m}$

74. Consider a cylinder of radius R, mass M, length z, and density $\rho(r) = Ar$ that rolls without slipping down an inclined plane of height h at an angle θ. What is the velocity of the cylinder at the bottom of the inclined plane?

(A) $\sqrt{3gh}$

(B) $\sqrt{2gh}$

(C) $\sqrt{gh}$

(D) $2\sqrt{gh}/3$

(E) $\sqrt{5gh}/2$

75. The Δ is a spin-3/2 bound state of three spin-1/2 quarks. The spin part of the wavefunction of the state with $m = +3/2$ is $|\Psi\rangle = |\uparrow\uparrow\uparrow\rangle$. What is the spin part of the wavefunction with definite spin $m = -1/2$?

(A) $|\uparrow\downarrow\downarrow\rangle$

(B) $\frac{1}{\sqrt{3}}\left(|\uparrow\downarrow\downarrow\rangle + |\downarrow\uparrow\downarrow\rangle - |\downarrow\downarrow\uparrow\rangle\right)$

(C) $\frac{1}{\sqrt{3}}\left(-|\uparrow\downarrow\downarrow\rangle + |\downarrow\uparrow\downarrow\rangle - |\downarrow\downarrow\uparrow\rangle\right)$

(D) $\frac{1}{\sqrt{3}}\left(|\uparrow\downarrow\downarrow\rangle + |\downarrow\uparrow\downarrow\rangle + |\downarrow\downarrow\uparrow\rangle\right)$

(E) $|\downarrow\downarrow\downarrow\rangle$

76. What is true of the electromagnetic field at a p-n junction at equilibrium with zero bias voltage applied?

(A) The electric field points toward the p-type semiconductor

(B) The electric field points toward the n-type semiconductor

(C) The electric field is parallel to the interface between the p-type and n-type semiconductors

(D) There is no electromagnetic field

(E) There is no electric field, but there is a magnetic field pointing toward the n-type semiconductor

77. In an inertial frame S, two events E_1 and E_2 occur at $(t, x, y, z) = (3, 4, 1, 1)$ and $(1, 3, 0, 1)$, respectively (in units where $c = 1$). In another inertial frame S', which of the following could an observer measure as the spacetime 4-vector between E_1 and E_2?

(A) $(1, 0.5, 1, 1)$

(B) $(2, 1, 0, 0)$

(C) $(3, 2, \sqrt{3}, 0)$

(D) $(2, 0, \sqrt{3}, 0)$

(E) None of these

78. A dark matter experiment takes data for a time T and observes no events. What is the 90% confidence level upper limit that one can place on the event rate in the detector?

(A) One cannot place a limit at the 90% confidence level for this experiment

(B) $-(1/T)\ln 0.9$

(C) $-(1/T)\ln 0.1$

(D) $(1/T)\ln 0.9$

(E) 0

79. An approximate Hamiltonian for the hydrogen atom is

$$H_0 = \frac{p^2}{2m_e} - \frac{e^2}{4\pi\epsilon_0}\frac{1}{r}.$$

The true Hamiltonian contains all of the following corrections to H_0. Which correction, taken *by itself*, results in the smallest change to the ground-state energy of H_0?

(A) spin-orbit coupling between the proton spin and the electron's orbital angular momentum

(B) first-order relativistic correction to the kinetic term

(C) spin-spin coupling between the electron and proton spins

(D) Lamb shift

(E) replacing electron mass with reduced mass of the electron-proton system

80. An electron neutrino emitted from the sun may be detected as a tau neutrino on Earth because:

(A) neutrinos are so difficult to detect that flavor misidentification is common

(B) electron neutrinos from the sun can annihilate and be reemitted as a pair of tau neutrinos

(C) electron neutrinos interact with the earth's magnetic field

(D) neutrino weak interaction eigenstates are not mass eigenstates

(E) scattering on ^{3}He in the sun causes flavor transmutation

81. A pair of electrons is trapped in a "quantum dot." A magnetic field is applied along the z-direction so that the singlet state has energy $-\epsilon$, and the triplet state has energies $-\epsilon/2$, $-\epsilon$, and $-3\epsilon/2$ for spins $+\hbar$, 0, and $-\hbar$ along the z-axis, respectively. What is the probability of finding the electrons in the triplet state, at temperature T?

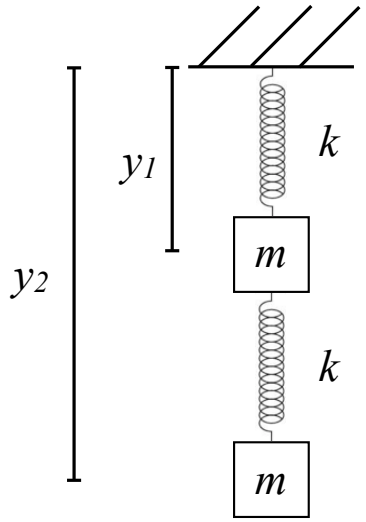
(A) 0

(B) 1

(C) $\dfrac{2}{2+e^{\epsilon/2kT}+e^{-\epsilon/2kT}}$

(D) $\dfrac{e^{\epsilon/2kT}+e^{-\epsilon/2kT}}{2+e^{\epsilon/2kT}+e^{-\epsilon/2kT}}$

(E) $\dfrac{1+e^{\epsilon/2kT}+e^{-\epsilon/2kT}}{2+e^{\epsilon/2kT}+e^{-\epsilon/2kT}}$



82. The diagram above illustrates a system consisting of a block of mass m hanging from a spring of spring constant k, with another block of mass m hanging from the first block by another spring of spring constant k. What is the Hamiltonian for this system?

(A) $\frac{1}{2}m(\dot{y_1}^2+\dot{y_2}^2)+\frac{1}{2}k(y_1^2+(y_2-y_1)^2)$ $-mg(y_1+y_2)$

(B) $\frac{1}{2}m(\dot{y_1}^2+\dot{y_2}^2)+\frac{1}{2}k(y_1^2+(y_2-y_1)^2)$ $+mg(y_1+y_2)$

(C) $\frac{1}{2}m(\dot{y_1}^2+\dot{y_2}^2)-\frac{1}{2}k(y_1^2+(y_2-y_1)^2)$ $+mg(y_1+y_2)$

(D) $\frac{1}{2}m(\dot{y_1}^2+\dot{y_2}^2)-\frac{1}{2}k(y_1^2+(y_2-y_1)^2)$ $-mg(y_1+y_2)$

(E) $-\frac{1}{2}m(\dot{y_1}^2+\dot{y_2}^2)-\frac{1}{2}k(y_1^2+(y_2-y_1)^2)$ $-mg(y_1+y_2)$

83. A particle of mass m is in the ground state of an infinite square well of size a, with energy E. The well suddenly expands to size $2a$. What is E'/E, where E' is the expectation value of the energy of the particle after this sudden expansion?

(A) 0
(B) 1
(C) $1/\sqrt{2}$
(D) $1/2$
(E) $1/4$

84. A particle of mass m and energy E is incident from the left on a delta-function barrier, $V(x) = \alpha\delta(x)$ with $\alpha > 0$. Which of the following gives the coefficient of reflection for the system?

(A) α^2
(B) $\alpha^2 E$
(C) $\dfrac{\alpha}{\hbar}\sqrt{\dfrac{m}{2E}}$
(D) $\dfrac{1}{1 + 2\hbar^2 E/m\alpha^2}$
(E) $\dfrac{1}{1 + m\alpha^2/2\hbar^2 E}$

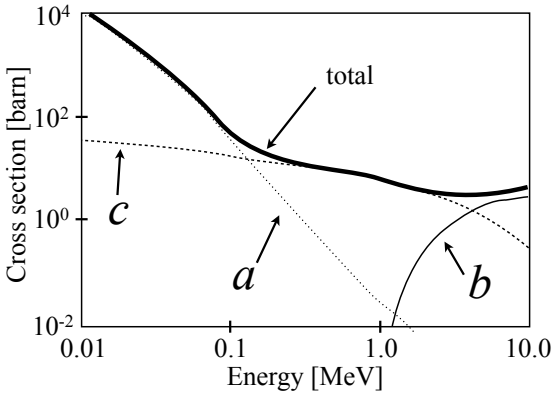
85. Which of the following is NOT true about the $2s \rightarrow 1s$ transition in the hydrogen atom?

(A) The dominant decay mode is two-photon emission
(B) It violates the selection rule $\Delta l = \pm 1$
(C) It violates the selection rule $\Delta m = \pm 1$ or 0
(D) It cannot occur in the electric dipole approximation
(E) none of these

86. Measurements of the electric dipole moment of the neutron provide sensitive tests of fundamental physics. If the neutron were found to have a nonzero electric dipole moment, one could directly conclude that which of the following symmetries is violated?

I. Parity
II. Charge conjugation
III. Time reversal

(A) I
(B) II
(C) III
(D) I and II
(E) I and III



87. The figure above shows the total cross section for photon scattering on a Pb atom as well as the cross sections for several individual process. Why does curve b drop quickly near 1 MeV?

(A) penetration depth of low-energy photons is small

(B) interactions with electrons become significant

(C) 1.022 MeV threshold for pair production

(D) Pb has no absorption lines below 1.022 MeV

(E) conservation of angular momentum

88. Let $f(x) = x$ for $x \in [-\pi, \pi]$. What is the first coefficient b_1 in the Fourier sine series for this function?

(A) 0

(B) π

(C) 1

(D) 2

(E) 4

89. Suppose that the magnetic field in a region of space is given by $\mathbf{B} = B_0(\hat{\mathbf{x}}+2x\hat{\mathbf{z}})$. Which of the following could be the vector potential?

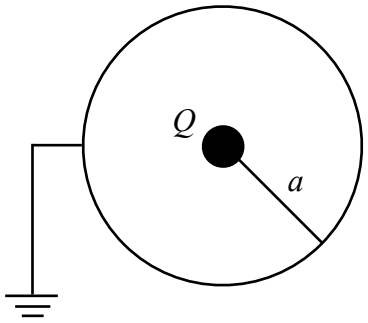
(A) $B_0(x\hat{\mathbf{y}} + x^2\hat{\mathbf{z}})$

(B) $-B_0(x\hat{\mathbf{y}} + x^2\hat{\mathbf{z}})$

(C) $-B_0(x^2\hat{\mathbf{y}} + y\hat{\mathbf{z}})$

(D) $B_0(y^2\hat{\mathbf{x}} + z\hat{\mathbf{y}})$

(E) $B_0(x^2\hat{\mathbf{y}} + y\hat{\mathbf{z}})$



90. Consider a charge configuration consisting of a ball of charge Q surrounded by a thin conducting shell of radius a. The conductor initially has no net charge, but is then connected to ground. What is the change in energy of the configuration?

(A) $-\dfrac{Q^2}{4\pi\epsilon_0 a}$

(B) $\dfrac{Q^2}{4\pi\epsilon_0 a}$

(C) $\dfrac{Q^2}{8\pi\epsilon_0^2 a^2}$

(D) $-\dfrac{Q^2}{8\pi\epsilon_0 a}$

(E) $\dfrac{Q^2}{8\pi\epsilon_0 a}$

91. A recent experiment used a measurement of the Lamb shift in muonic hydrogen, an exotic atom made up of a muon orbiting a proton, to infer which of the following properties of the proton?

(A) electric dipole moment

(B) spin

(C) charge radius

(D) mass

(E) magnetic moment

92. In tabletop atomic spectroscopy experiments using free nuclei, the difference between the frequencies of emitted and absorbed photons driven by the same electronic transition is due to:

(A) measurement error

(B) nuclear recoil

(C) gravitational redshift

(D) time dilation

(E) none of these

93. A sequence of NAND gates can create which of the following effective logic gates?

(A) AND

(B) OR

(C) NOT

(D) NOR

(E) all of the above

94. A particle moving in one dimension has the following Lagrangian

$$L = \frac{1}{2}A\dot{q}^2 - Bq^2.$$

What is the equation of motion of the particle?

(A) $\dot{q} = \dfrac{2B}{A}q$

(B) $\dot{q} = -\dfrac{2B}{A}q$

(C) $\ddot{q} = \dfrac{2B}{A}q$

(D) $\ddot{q} = \dfrac{2A}{B}q$

(E) $\ddot{q} = -\dfrac{2B}{A}q$

95. What is the energy stored in a toroidal solenoid with N winds, carrying current I, of radius R, and enclosing a volume V? You may assume that the radius r of an individual wire wind is much less than the toroid radius R.

(A) 0

(B) $\frac{\mu_0 N I^2 r^2}{4\pi R^3}$

(C) $\frac{\mu_0 N I^2 V}{8\pi^2 R^2}$

(D) $\frac{\mu_0 N^2 I^2 r^2}{4\pi R^3}$

(E) $\frac{\mu_0 N^2 I^2 V}{8\pi^2 R^2}$

96. Which of the following is true about a longitudinally polarized wave in 3 dimensions?

I. There are two linearly independent polarization vectors

II. The polarization vector(s) is/are perpendicular to the wave vector

III. The polarization vector(s) is/are parallel to the wave vector

(A) III only

(B) II only

(C) I only

(D) I and II

(E) I and III

97. Deep water waves obey the dispersion relation $\omega = A\sqrt{k}$, where A is a constant. What is the correct relationship between phase velocity and group velocity for deep water waves?

(A) $v_{phase} = \frac{1}{2} v_{group}$

(B) $v_{phase} = v_{group}$

(C) $v_{phase} = 2v_{group}$

(D) $v_{phase} v_{group} = A^4 t^2$

(E) none of these

98. When light of 5000 Å is shined on a thin film of oil ($n = 1.5$) that sits on top of a medium with $n = 2.0$, the intensity of reflected light is minimized. What is the thickness of the oil?

(A) 4×10^{-8} m

(B) 8.33×10^{-8} m

(C) 1.67×10^{-7} m

(D) 1.25×10^{-7} m

(E) 5.0×10^{-7} m

99. Suppose a particle has a wavefunction $\psi(x)$ given by

$$\psi(x) = \begin{cases} A(1-x) & : 0 < x < 1 \\ 0 & : \text{otherwise} \end{cases}$$

where A is a constant. What is the expectation value of the position of this particle?

(A) 0

(B) 1

(C) 1/12

(D) 1/4

(E) 1/2

100. What are the energy levels of a quantized system consisting of a free rigid rod of length a connecting two masses of mass m, for $n \in \mathbb{N}$?

(A) $\frac{\hbar^2 n(n+1)}{ma^2}$

(B) $\frac{\hbar^2 n(n+1)}{2ma^2}$

(C) $\frac{\hbar^2 n}{2ma^2}$

(D) $\frac{\hbar^2 n}{ma^2}$

(E) $\frac{\hbar^2 (n+1)}{ma^2}$

Answers

1. C
2. E
3. D
4. B
5. A
6. E
7. C
8. D
9. B
10. C
11. E
12. A
13. E
14. B
15. A
16. D
17. E
18. A
19. E
20. B
21. D
22. A
23. E
24. A
25. C
26. B
27. B
28. D
29. C
30. E
31. D
32. C
33. E
34. C
35. D
36. C
37. D
38. B
39. B
40. D
41. B
42. B
43. D
44. E
45. C
46. A
47. A
48. B
49. E
50. D
51. D
52. C
53. A
54. D
55. C

56. A
57. A
58. C
59. E
60. B
61. C
62. A
63. A
64. B
65. E
66. C
67. C
68. B
69. B
70. C
71. B
72. B
73. E
74. E
75. D
76. A
77. C
78. C
79. C
80. D
81. E
82. A
83. B
84. D
85. C
86. E
87. C
88. D
89. E
90. D
91. C
92. B
93. E
94. E
95. E
96. A
97. C
98. B
99. D
100. A