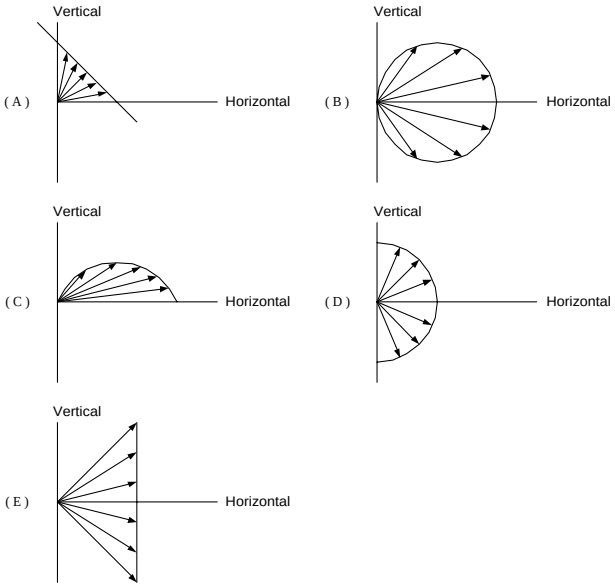
第一节 经典力学

1. A satellite orbits the Earth in a circular orbit. An astronaut on board perturbs the orbit slightly by briefly firing a control jet aimed toward the Earth's center. Afterward, which of the following is true of the satellite's path?

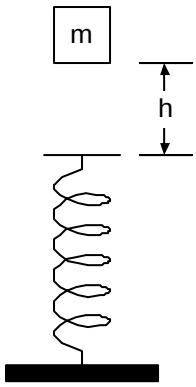
(A) It is a ellipse.
(B) It is a hyperbola.
(C) It is a circle with larger radius.
(D) It is a spiral with increasing radius.
(E) It exhibits many radial oscillations per revolution.

解：圆轨道受到小扰动后仍是闭合轨道，因此是椭圆。选（A）。

2. A person standing on the surface of the Earth throws a ball. The ball leaves the thrower's hand with initial velocity v_i and has final velocity v_f just before it is caught. If air resistance is negligible, which of the following diagrams correctly represents a possible sequence of velocity vectors for the ball?



解：因为球在水平方向不受力，所以其水平动量不变，即速度的水平分量不变，只有（E）符合。选(E)。



3. As shown above, a block of mass m is released from rest at a distance h above a vertical massless spring with spring constant k, what is the maximum kinetic energy of the block?

(A) mgh

(B) $mgh+\frac{1}{4}\frac{m^2g^2}{k}$

(C) $mgh+\frac{1}{2}\frac{m^2g^2}{k}$

(D) $mgh+\frac{m^2g^2}{k}$

(E) $2mgh$

解：物体速度最大时$\frac{dv}{dt}=0$，即加速度为零，此时物体受力平衡。设速度最大时弹簧压缩量为 x，则

$$mg=kx,$$

$$x=\frac{mg}{k}。$$

由机械能守恒

$$mg(h+x)=\frac{1}{2}mv^2+\frac{1}{2}kx^2。$$

将 x 的表达式代入，

$$E_k=\frac{1}{2}mv^2=mg(h+x)-\frac{1}{2}kx^2=mgh+\frac{m^2g^2}{2k}$$

。

选（C）。

4. A bullet of mass m traveling at speed v strikes a block of mass M, initially at rest, and is embedded in it as shown above. How far will the block with the bullet embedded in it slide on a rough horizontal surface of coefficient of kinetic friction μ_k before it comes to rest?

(A) $(\frac{m+M}{m})(\frac{v^2}{2\mu_k g})$

(B) $(\frac{m+M}{M})(\frac{v^2}{2\mu_k g})$

(C) $(\frac{m+M}{M})^2(\frac{v^2}{2\mu_k g})$

(D) $(\frac{m}{m+M})(\frac{v^2}{2\mu_k g})$

(E) $(\frac{m}{m+M})^2(\frac{v^2}{2\mu_k g})$

解：子弹射入过程很短，时间及此过程中所走过距离可忽略不计。由动量守恒，

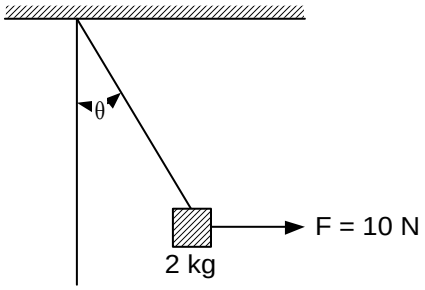
$$mv = (m+M)V。$$

之后由于摩擦做匀减速运动，机械能全部转换为摩擦热，则

$$\frac{1}{2}(m+M)V^2 = \mu_k (m+M)gs，$$

$$s = \frac{V^2}{2\mu_k g} = (\frac{v^2}{2\mu_k g})(\frac{m}{m+M})^2。$$

选（E)。



5. A 2-kilogrom box hangs by a massless rope from a ceiling. a force slowly pulls the box horizontally to the side until the horizontal force is 10 newtons. The box is then equilibrium as shown above. The angle that the rope makes with the vertical is closest to

(A) arctan 0.5
(B) arcsin 0.5
(C) arctan 2.0
(D) arcsin 2.0
(E) 45°

解：物体重 20N，角度

$$\theta = \arctan\frac{10}{20} = \arctan 0.5。$$

选（A)。

6. A 5-kilogram stone is dropped on a nail and drives the nail 0.025 meter into a piece of wood. If the stone is moving at 10 meters per second when it hits the nail, the average force exerted into the wood is most nearly

(A) 10 N
(B) 100 N
(C) 1000 N
(D) 10,000 N
(E) 100,000 N

解：力对石头做功等于其动能减少，

$$F = \frac{1}{2}mv^2 / h = \frac{1}{2}\times 5\times 10^2 / 0.025 = 10000\text{N}。$$

选（D)。

7. A machine gun fires bullets of mass 20 grams each at a rate of 1200 bullets per minute. The bullets hit a thick wooden target at a speed of 600 meters per second and are stopped in the target. The average force exerted on the target by the bullets striking it is

(A) 144N
(B) 240N

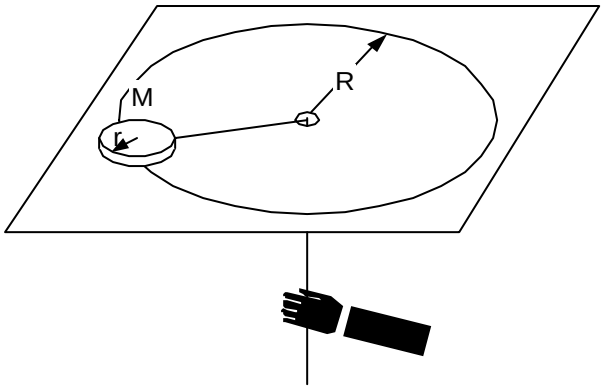
(C) 14.4×10^3 N
(D) 24.0×10^3 N
(E) 14.4×10^3 N
解：由冲量定理

$$\int F dt = \Delta P ,$$

$$F = \frac{\Delta mV}{\Delta t} = \frac{20\times10^{-3}\times1200\times600}{60} = 240\ \text{N}。$$

选(B)。

Questions 8-9



A uniform cylindrical puck of radius r and mass M is attached to the end of a cord that passes through a hole in a fixed horizontal frictionless table as shown above. The center of the puck moves in a circle of radius R with angular speed ω

8. The magnitude of the angular momentum of the puck about the hole is

(A) $\frac{3}{2}M\omega R^2$

(B) $M(R^2+r^2)\omega$

(C) $M(R^2+r^2)^{\frac{1}{2}}\omega^2$

(D) $M(R^2+\frac{1}{2}r^2)\omega$

(E) $M(R^2+\frac{1}{2}r^2)^{\frac{1}{2}}\omega^2$

解：首先，圆盘绕自身圆心的转动惯量为

$$I = \frac{1}{2}Mr^2 ,$$

由平行轴定理，圆盘绕桌子中心的转动惯量为

$$I = I_0 + MR^2 = \frac{1}{2}Mr^2 + MR^2 ,$$

所以它相对于洞的角动量为

$$L = I\omega = M(R^2+\frac{1}{2}r^2)^{\frac{1}{2}}\omega^2 。$$

选（E)。

9. If the string is pulled slowly downward so that the center of the puck moves in a circle of smaller radius, quantities that are conserved include which of the following?

I. Angular momentum
II. Linear momentum
III. Kinetic energy

(A) I only
(B) III only
(C) I and II only
(D) II and III only
(E) I, II, and III

解：由于是有心力，所以冰球相对洞的角动量守恒，I 正确。在圆运动中动量本身就不是一个守恒量，变化中冰球又显然受力，所以动量守恒无从谈起，II 不对。变化中冰球受到了指向洞的力，沿此方向又有位移，所以绳子对其做功，动能不守恒。或者由动能为 $\frac{1}{2}I\omega^2$，而角动量守恒，$I\omega = \text{constant}$，动能显然不能守恒，III 不对。选（A)。

10. An expression for the potential energy of two ions is

$$P.E. = \frac{-kq^2}{r} + \frac{b}{r^9} .$$

What is the constant b as a function of the equilibium spacing r_0?

(A) $\frac{kq^2}{r_0^{10}}$

(B) $\frac{kq^2}{r_0}$

(C) $\frac{8kq^2}{9r_0}$

(D) $\frac{kq^2r_0}{10}$

(E) $\frac{kq^2r_0^8}{9}$

解：因为 r_0 处势能最小，由

$$\frac{dU_{P.E.}}{dr}=\frac{kq^2}{r^2}-\frac{9b}{r^{10}}=0$$

解出

$$r_0^{\ 8}=\frac{9b}{kq^2},$$

$$b=\frac{kq^2r_0^{\ 8}}{9}。$$

选（E)。

11. A rigid cylinder rolls at constant speed without slipping on top of a horizontal plane surface. The acceleration of a point on the circumference of the cylinder at moment when the point touches the plane is

(A) directed forward
(B) directed backward
(C) directed up
(D) directed down
(E) zero

解：因为触地点相对于圆心做匀速圆周运动，所以相对于圆心的加速度为竖直向上方向，而圆心做匀速直线运动，加速度为零，所以触地点相对于圆心的加速度就是其对地的加速度。选(C)。

Questions 12-13

A cylinder with moment of inertia 4 kg·m^2 about a fixed axis initially rotates at 80 radians per second about this axis. A constant torque as applied to slow it down to 40 radius per second.

12. The kinetic energy lost by the cylinder is

(A) 80 J
(B) 800 J
(C) 4000 J
(D) 9600 J
(E) 19,200 J

解：刚体定轴转动的动能为

$$T=\frac{1}{2}I_\omega\omega^2,$$

动能变化为

$$\Delta T=\frac{1}{2}I\left(\omega_0^{\ 2}-\omega'^2\right)=\frac{1}{2}\times4\times\left(80^2-40^2\right)=9600\text{J}$$

。

选(D)。

13. If the cylinder takes 10 seconds to reach 40 radius per second, the magnitude of the applied torque is

(A) 80 N·m
(B) 40 N·m
(C) 32 N·m
(D) 16 N·m
(E) 8 N·m

解：由刚体定轴转动的动力学方程

$$M=I\beta,$$

其中 M 为外力矩，β为刚体的角加速度。对于本题

$$M=I\beta=I\frac{\omega_0-\omega'}{t}=4\times\frac{80-40}{10}=16\text{N}\cdot\text{m}。$$

选（D)。

Questions 14-15

A nonrelativistic particle of mass m moves in a plane. Its position is described by the polar coordinates r and θ, with time derivatives $\dot r$ and $\dot\theta$. There exists a potential energy U = kr^2, where k is a constant.

14. Which of the following is the Lagrangian of the particle?

(A) $L=\frac{1}{2}m\left(\dot r^2+r^2\dot\theta^2\right)-kr^2$

(B) $L=\frac{1}{2}m\left(\dot r^2+\theta^2\right)+kr^2$

(C) $L=\frac{1}{2}m\left(\theta^2\dot{r}^2+r^2\dot{\theta}^2\right)-kr^2$

(D) $L=\frac{1}{2}m\left(\dot{r}^2+r\dot{r}\dot{\theta}+r^2\dot{\theta}^2\right)-kr^2$

(E) $L=\frac{1}{2}m\left(\dot{r}^2+2r\dot{r}\dot{\theta}+r^2\dot{\theta}^2\right)-kr^2$

解：对于有势力系，Lagrange 函数定义为

$$L=T-V\text{。}$$

本题中，由于极坐标系下的速度表达式为

$$\mathbf{v}=\dot{r}\mathbf{e}_r+r\dot{\theta}\,\mathbf{e}_\theta\text{。}$$

而两分量之间垂直，所以

$$L=T-V=\frac{1}{2}m\left(v^2{}_r+v^2{}_\theta\right)-V$$

$$=\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2\right)-kr^2$$

选(A)。

15. Which of the following quantities remains constant?

(A) $m\left(\dot{r}^2+r^2\dot{\theta}^2\right)$

(B) $mr^2\dot{\theta}^2$

(C) kr^2

(D) $mr\dot{\theta}$

(E) $mr^2\dot{\theta}$

解：$\mathbf{F}=\frac{dU}{dr}=2kr\hat{\mathbf{r}}$，或者由于$U=kr^2$与θ无关，为有心力场，角动量守恒。利用上题中速度的表达式，径向速度为$r\dot{\theta}$，所以角动量表达式为$mr^2\dot{\theta}$。答案选(E)。

16. A ball dropped from a height h. As it bounces off the floor, its speed is 80 percent of what it was just before it hit the floor. The ball will then rise to a height of most nearly

(A) 0.94 h

(B) 0.80 h

(C) 0.75 h

(D) 0.64 h

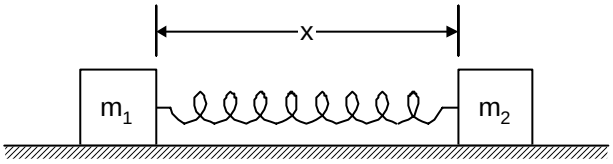
(E) 0.50 h

解：由机械能守恒

$$\frac{1}{2}mv^2=mgh\text{，}$$

$$\frac{h'}{h}=\left(\frac{v'}{v}\right)^2=0.8^2=0.64\text{。}$$

选(D)。



17. Two masses, m_1 and m_2 , are joined by a massless spring of force constant k and placed on a horizontal frictionless surface as shown above. The system is released from rest when the separation between the masses is x. If the unstretched length of the spring is x_0, the speed of mass m_1 when the two masses are a distance x_0 apart is

(A) $\sqrt{\frac{k}{m_1}(x-x_0)^2}$

(B) $\sqrt{\frac{k}{m_2}(x-x_0)^2}$

(C) $\sqrt{\frac{k}{m_1+m_2}(x-x_0)^2}$

(D) $\sqrt{\frac{k}{m_1}\cdot\frac{m_2}{m_1+m_2}(x-x_0)^2}$

(E) $\sqrt{\frac{k}{m_2}\cdot\frac{m_1}{m_1+m_2}(x-x_0)^2}$

解：设恢复原长时两物体速度为 v_1、v_2，由动量守恒

$$m_1v_1=m_2v_2\text{。}$$

由机械能守恒

$$\frac{1}{2}k(x-x_0)^2=\frac{1}{2}m_1{v_1}^2+\frac{1}{2}m_2{v_2}^2。$$

联立以上二式，解得

$$v_1=\sqrt{\frac{k}{m_1}\cdot\frac{m_2}{m_1+m_2}(x-x_0)^2}。$$

选(D)。

17. What is the number of degrees of freedom for 6 particles moving freely in one plane?

(A) 6
(B) 8
(C) 10
(D) 12
(E) 18

解：二维运动，每个粒子的自由度为 2，而且相互之间无关联，没有限制条件，所以总数为 2×6=12。答案选（D)。

18. A mass m_1 at the end of a spring executes simple harmonic motion with a period T_1. The period of oscillation of a different mass m_2 on the same spring is

(A) $T_1\sqrt{\frac{m_2}{m_1}}$

(B) $T_1\sqrt{\frac{m_1}{m_2}}$

(C) T_1

(D) $2\pi\sqrt{\frac{m_1}{m_2}}$

(E) $2\pi\sqrt{\frac{m_2}{m_1}}$

解：一维谐振子公式，$T=2\pi\sqrt{\frac{m}{k}}$。答案选（A)。

19. Under the influence of a mutual interaction, an object orbits another object that is fixed. The orbit lies in a plane and the areas swept out by the radius vector in equal times are equal. What can be correctly concluded about the force between the objects?

(A) It is central.
(B) It is inverse-square.
(C) It is conservative.
(D) It is gravitational.
(E) None of these conclusions is justified.

解：角动量

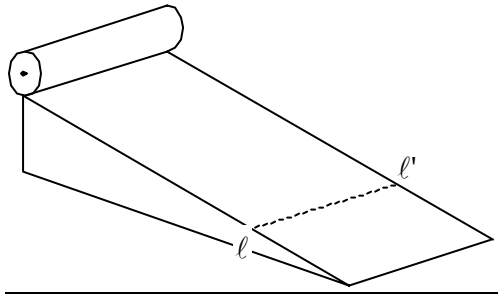
$$|\mathbf{L}|=|\mathbf{r}\times m\mathbf{v}|=m\left|\mathbf{r}\times\frac{d\mathbf{r}}{dt}\right|=2m\frac{d\sigma}{dt},$$

其中

$$d\sigma=\frac{1}{2}|\mathbf{r}\times d\mathbf{r}|$$

为矢径 **r** 在 dt 时间内扫过的面积。由本题所给条件，$\frac{d\sigma}{dt}$ 为常量，故角动量守恒。有心力可保证角动量守恒。选（A)。

Questions 20-21



In two experiments, two cylinders, X and Y, are released from rest at the top of the same inclined ramp and roll down without slipping. Let t_X and t_Y be the respective times taken for the cylinders to reach a particular line ll' on the ramp shown above.

20. In the first experiment, both cylinders are solid, uniform, and of identical dimensions; but they are made of different materials so that the mass of X is twice that of Y, $M_X = M_Y$. Which of the following relationships between t_X and t_Y is correct?

(A) $t_X \geq 2t_Y$
(B) $2t_Y > t_X > t_Y$
(C) $t_X = t_Y$
(D) $2t_X > t_Y > t_X$
(E) $t_Y \geq 2t_X$

解：設斜面傾角爲θ，圓柱體受摩擦力爲 f，則

$$mg\sin\theta - f = ma \text{ 。}$$

因爲摩擦力 f 提供了力矩，所以

$$fr = I\beta \text{ 。}$$

無滑捲軸件

$$\beta r = a \text{ 。}$$

由以上三式解得

$$a = \frac{mg\sin\theta}{m + \frac{I}{r^2}} \text{ 。}$$

圓柱體轉動慣量 $I = \frac{1}{2}mr^2$ ，代入上式得

$$a = \frac{2}{3}g\sin\theta \text{ 。}$$

與 m、r 無關。選（C）。

21. In the second experiment, both cylinders are solid, uniform, of identical length, and of the same density; but their radii are different so that the mass of X is twice that of Y, $M_X = 2M_Y$. Which of the following relationships between t_X and t_Y is correct?

(A) $t_X \geq 2t_Y$
(B) $2t_Y > t_X > t_Y$
(C) $t_X = t_Y$
(D) $2t_X > t_Y > t_X$
(E) $t_Y \geq 2t_X$

解：利用上一問的結論。選(C)。

22. A golf ball is hit from ground level with an initial velocity v_0 at an angle θ with respect to the ground. If air resistance is negligible and the magnitude of the gravitational acceleration is g, the ball hits the ground at what distance from the point at which it was hit?

(A) $\frac{{v_0}^2}{g}$

(B) $\frac{{v_0}^2\sin\theta}{g}$

(C) $\frac{{v_0}^2\cos\theta}{g}$

(D) $\frac{2{v_0}^2\tan\theta}{g}$

(E) $\frac{2{v_0}^2\sin\theta\cos\theta}{g}$

解：飞行时间为

$$t = \frac{2v_0\sin\theta}{g},$$

水平位移为

$$S = v_0\cos\theta\, t = \frac{2{v_0}^2\sin\theta\cos\theta}{g} \text{。}$$

选（E)。

23. A spherical neutron star has a uniform mass density ρ. What is the period of rotation below which material will fly off the equator? (Use nonrelativistic mechanics and let G be the universal gravitational constant.)

(A) $\frac{3}{4\pi G}$

(B) $\frac{4}{3\pi G}$

(C) $\left(\frac{3}{8\pi\rho G}\right)^{\frac{1}{2}}$

(D) $\left(\frac{\pi}{\rho G}\right)^{\frac{1}{2}}$

(E) $\left(\frac{3\pi}{\rho G}\right)^{\frac{1}{2}}$

解：临界状态质点所受万有引力全部用来提供所需向心力：

$$\frac{GMm}{r^2} = m\omega^2 r,$$

$$\omega = \sqrt{\frac{GM}{r^3}} = \sqrt{\frac{4\pi G\rho}{3}},$$

所以

$$T=\frac{2\pi}{\omega}=\left(\frac{3\pi}{\rho G}\right)^{\frac{1}{2}}。$$

选（E)。

24. A particle of mass M moves along the x-axis under the influence of a conservative field with the potential energy $V=\frac{b}{2}x^2$. If the particle starts from rest at x = 1, its maximum velocity is

(A) $\sqrt{\frac{Mb}{2}}$

(B) $\sqrt{Mb}$

(C) $\sqrt{2Mb}$

(D) $\sqrt{\frac{b}{2M}}$

(E) $\sqrt{\frac{b}{M}}$

解：因为所受力为保守力，机械能守恒。势能最小时速度最大：

$$\frac{b^2}{2}x_{\max}=\frac{1}{2}mv_{\max}{}^2,$$

解得

$$v_{\max}=\sqrt{\frac{b}{M}}。$$

选(E)。

25. A particle moves in a force field given by **F** = r^2 **r**, where **r** is the position vector. If there are no other forces, quantities that remain constant include which of the following?

I. Total energy

II. Torque about the origin

III. Angular momentum about the origin

(A) I only

(B) III only

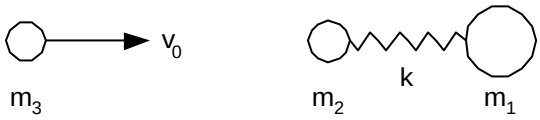
(C) I and II only

(D) II and III only

(E) I, II, and III

解：力场为保守场，能量守恒，I 正确。又为有心力场，且 **F** 指向原点，相对于原点的力矩为零，II 正确。显然相对于原点的角动量守恒，III 正确。

选（E)。



26. Particle 1 of mass m_1 and particle 2 of mass $m_2=\frac{1}{2}m_1$ are coupled by a massless spring of force constant k and lie at rest on a horizontal frictionless surface as shown above. A third particle of mass $m_3=m_2=\frac{1}{2}m_1$ and speed v_0 strikes particle 2 along the axis of the spring and sticks to particle 2. The speed of the center of mass of the system after the collision is

(A) $\frac{v_0}{4}$

(B) $\frac{v_0}{3}$

(C) $\frac{v_0}{2}$

(D) v_0

(E) $v_0+\sqrt{\frac{8k}{3m_1}}$

解：质心动量等于各部分动量之和。碰撞前后动量守恒，

$$m_3v_0=(m_1+m_2+m_3)V_{MC},$$

$$V_{MC}=\frac{m_3}{m_1+m_2+m_3}v_0=\frac{v_0}{4}。$$

选(A)。

27. A planet of mass m moves about the Sun of mass M. G is Newton's constant, r is the planet's distance from the Sun, and v is the planet's speed. Except for

an additive constant the planet's potential energy is

(A) $\frac{1}{2}mv^2+\frac{GMm}{r}$

(B) $\frac{1}{2}mv^2-\frac{GMm}{r}$

(C) $-\frac{GMm}{r}$

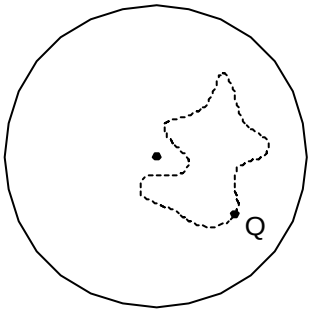
(D) $-\frac{GMm}{r^2}$

(E) mgr

解：取无穷远为势能零点，则万有引力势为

$$U=\int_r^\infty -\frac{GM}{r^2}dr=-\frac{GM}{r}。$$

选（C）。



28. The diagram above show a top view of a phonograph turntable mounted on a frictionless bearing. Initially, the turntable is at rest and a massive bug is asleep at point Q. The bug wakes up, takes a walk such as the one indicated by the dotted line, returns to Q, and goes back to sleep. Afterward, the turntable is

(A) again at rest in the same position

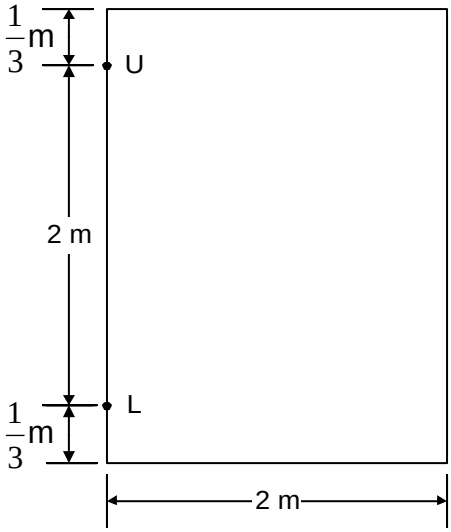
(B) at rest, but in the same position only if the walk did not encircle the pivot point

(C) at rest, but not necessarily in the same position whether or not the walk encircled the pivot point

(D) at rest only if the walk did not encircle the pivot point

(E) not necessarily at rest, whether or not the pivot point was encircled

解：因为系统没受到相对圆心的外力矩，所以相对于圆心角动量守恒，最后转桌必处于静止状态。小虫和转桌间有相对运动，转桌可有任意的角位移。答案选（C）。



29. The weight of a door is entirely supported by the upper hinge U which is 2 meters from the lower hinge L as shown above. Assume the mass per unit area of the door is constant and the hinges have negligible size. If the door weighs 200 newtons and is 2 meters wide, of what magnitude is the horizontal force exerted by the lower hinge L?

(A) 80 N

(B) 100 N

(C) 120 N

(D) 140 N

(E) 200 N

解：因为门的重量完全由 U 点支撑，所以 L 点无竖直方向的力，L 处受力沿水平方向。系统对 U 点力矩平衡，

$$G\cdot 1\text{m}=F_L\cdot 2\text{m}，$$

$$F_L=100\text{N}。$$

选（B）。

30. A thin uniform rod of mass m and length l is hinged at one end to a level floor and stands vertically. If allowed to fall, the rod will strike the floor with an angular speed ω. If the same rod were cut in half to length l/2 and the initial conditions remained unchanged, it would strike the floor with an angular speed most nearly equal to

(A) 2ω

(B) $\omega\sqrt{2}$

(C) ω

(D) $\omega/\sqrt{2}$

(E) ω/2

解：由能量守恒，势能转换为转动能量

$$mg\frac{l}{2}=\frac{1}{2}I\omega^2。$$

对于质量为 m 长度为 l 的均匀杆，相对一端的转动惯量为

$$I=\int_0^l x^2\frac{m}{l}dx=\frac{1}{3}ml^2。$$

解得

$$\omega=\sqrt{\frac{3g}{l}}，$$

可见只与杆长有关。第二次杆长变为一半，$\omega'=\sqrt{2}\omega$。选（B)。

31. An asteroid has a radius of 5×10^5 meters and an acceleration due to gravity of $\frac{1}{4}$ that on Earth. The velocity of escape for an object starting on the surface of the asteroid is most nearly

(A) 150 m/s

(B) 720 m/s

(C) 1560 m/s

(D) 5550 m/s

(E) 11,200 m/s

解：所谓逃逸速度，是恰好使物体的机械能为 0。所以

$$-\frac{GM_am}{r}+\frac{1}{2}mv_e^2=0，$$

$$v_e=\sqrt{\frac{2GM_a}{r}}。$$

而由重力加速度

$$\frac{GM_a}{r^2}=\frac{1}{4}g，$$

$$\mathrm{GM_a}=\frac{1}{4}gr^2，$$

所以

$$v_e=\sqrt{\frac{gr}{2}}=\sqrt{\frac{9.8\times5\times10^5}{2}}=1565。$$

选(C)。

32. A solid ball weighs 5.0 newtons in air and 3.0 newtons submerged in water. If the ball weighs 2.0 newtons submerged in an unknown liquid, the specific gravity of the unknown liquid is most nearly

(A) 0.66

(B) 1.00

(C) 1.25

(D) 1.50

(E) 1.75

解：由重力与浮力和拉力平衡

$$G-F_{水}=3.0\mathrm{N}，$$

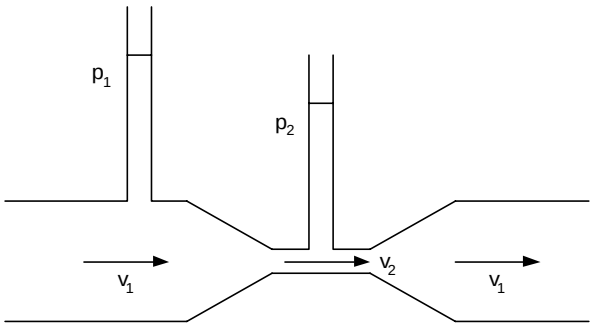
$$G-F_{液}=2.0\mathrm{N}。$$

由阿基米德定律 $F_{浮}=\rho gV$，得

$$\frac{F_{液}}{F_{水}}=\frac{\rho_{液}}{\rho_{水}}，$$

$$\rho_{液}=\frac{F_{液}}{F_{水}}\rho_{水}=\frac{G-2.0}{G-3.0}\rho_{水}=1.5。$$

选（D)。



33. Water flows through a Venturi tube as shown in the diagram above. The radius of the large cross section of the pipe is 2 centimeters and the raduis of the constricted portion of the pipe is 1 centimeter. If the speed of the water in the large cross section is 1 meter per second, the pressure difference (p_1-p_2) is most nearly

(A) 0.6×10^2 N/m
(B) 3×10^2 N/m
(C) 1.5×10^3 N/m
(D) 7.5×10^3 N/m
(E) 37.5×10^3 N/m
解：由流量守恒

$$v_1S_1 = v_2S_2\,,$$

$$v_2 = 4v_1\,。$$

由 Bernoulli 方程，

$$\frac{1}{2}\rho v_1^{\,2} + P_1 + \rho g h_1 = \frac{1}{2}\rho v_2^{\,2} + P_2 + \rho g h_2\,,$$

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^{\,2} - v_1^{\,2}) + \rho g(h_2 - h_1)\,。$$

代入数字计算，选（B）。考试现场有可能不记得 Bernoulli 方程，其实它是机械能守恒在流体力学中的一种体现。再结合量纲分析，很容易推出这个形式。

34. A rock is thrown vertically upward with initial speed v_0. Assume a friction force proportional to –**v**, where **v** is the velocity of the rock, and neglect the buoyant force exerted by air. Which of the following is correct?
(A) The acceleration of the rock is always equal to **g**.
(B) The acceleration of the rock is equal to **g** only at the top of the flight.
(C) The acceleration of the rock is always less than **g**.
(D) The speed of the rock upon return to its starting point is v_0.
(E) The rock can attain a terminal speed greater than v_0 before it returns to its starting point.
解：在顶点，石块的速度为零，此时空气摩擦力为零，故加速度为 **g**，选（B）。对于选项（E），石块有可能获得收尾速度，但由于有阻力，必小于 v_0。

Question 35-37
The potential energy of a body constrained to move on a straight line is kx^4, where k is a constant. The position of the body is x, its speed v, its linear momentum p, and its mass m.

35. The force on the body is
(A) $\frac{1}{2}mv^2$
(B) $-4kx^3$
(C) kx^4
(D) $-\frac{kx^5}{5}$
(E) mg
解：直接对势能求导得

$$F = -\frac{dU}{dx} = -4kx^3\,。$$

选(B)。从量纲上看，（A）、（C）为能量，（D）为能量乘以距离，均不是力的单位。

36. The Hamiltonian function for this system is
(A) $\frac{p^2}{2m} + kx^4$
(B) $\frac{p^2}{2m} - kx^4$
(C) kx^4
(D) $\frac{1}{2}mv^2 - kx^4$
(E) $\frac{1}{2}mv^2$
解：对于变换方程不显含 t 的有势力系（这一条件在 SUB 考试中几乎必然满足），H=T+V。选（A）。

37. The body moves from x_1 at time t_1 to x_2 at time t_2. Which of the following quantities is an extremum for the x–t curve corresponding to this motion, if end points are fixed?
(A) $\int_{t_1}^{t_2}\left(\frac{1}{2}mv^2 - kx^4\right)dt$
(B) $\int_{t_1}^{t_2}\left(\frac{1}{2}mv^2\right)dt$
(C) $\int_{t_1}^{t_2}(mxv)dt$
(D) $\int_{x_1}^{x_2}\left(\frac{1}{2}mv^2 + kx^4\right)dx$

(E) $\int_{x_1}^{x_2}(mv)dx$

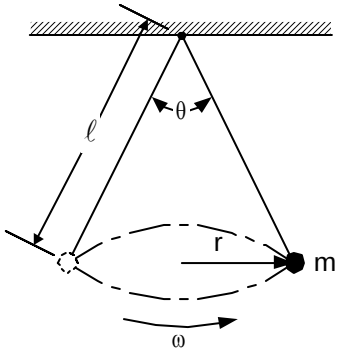
解：本题涉及分析力学的变分原理部分。由 Hamilton 原理，定义 Hamilton 作用量

$$S=\int_{t_1}^{t_2}Ldt\ ,$$

则真实运动使 S 取极值，其中 L 为拉格朗日量，

$$L=T-V\ 。$$

选（A）。



38. The figure above represents a point mass m attached to the ceiling b a cord of fixed length l. If the point mass moves in a horizontal circle of radius r with uniform angular velocity ω, the tension in the cord is

(A) $mg\left(\frac{r}{l}\right)$

(B) $mg\cos\left(\frac{\theta}{2}\right)$

(C) $\frac{m\omega r}{\sin\left(\frac{\theta}{2}\right)}$

(D) $m\left(\omega^2r^2+g^2\right)^{\frac{1}{2}}$

(E) $m\left(\omega^4r^2+g^2\right)^{\frac{1}{2}}$

解：竖直方向受力平衡

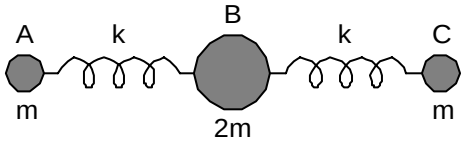
$$F\cos\theta/2=mg\ ,$$

$$F=\frac{mg}{\cos\theta/2}=\frac{mgl}{\sqrt{l^2-r^2}}。$$

从另一个角度看 F 等于重力和离心力的合力

$$F=m\left(\omega^4r^2+g^2\right)^{\frac{1}{2}}。$$

选（E）。



39. Three masses are connected by two springs as shown above. A longitudinal normal mode with frequency $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ is exhibited by

(A) A, B, C all moving in the same direction with equal amplitude

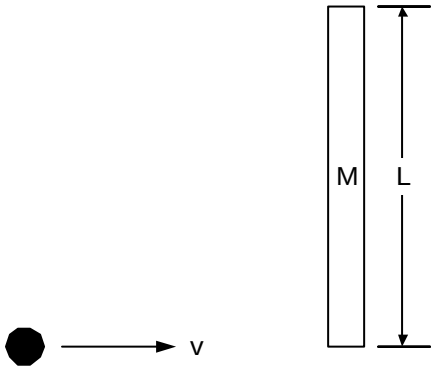
(B) A and C moving in opposite directions with equal amplitude, and B at rest

(C) A and C moving in the same direction with equal amplitude, and B moving in the opposite direction with the same amplitude

(D) A and C moving in the same direction with equal amplitude, and B moving in the opposite direction with twice the amplitude

(E) none of the above

解：（B）所属情况是可实现的，A、C 对 B 的作用力始终大小相等方向相反，B 保持静止，A、C 与普通弹簧振子完全相等。选（B）。



View from above

40. A uniform stick of length L and mass M lies on a frictionless horizontal surface. A point particle of mass m approaches the stick with speed v an a straight line perpendicular to the stick that intersects

the stick at one end, as shown above. After the collision, which is elastic, the particle is at rest. The speed V of the center of mass of the stick after the collision is

(A) $\frac{m}{M}v$

(B) $\frac{m}{M+m}v$

(C) $\sqrt{\frac{m}{M}}v$

(D) $\sqrt{\frac{m}{M+m}}v$

(E) $\frac{3m}{M}v$

解：由动量守恒，碰撞后杆的动量为 mv。由于质点组相对质心动量为零，所以质点组的动量等于质心速度乘以总质量：

$$mv=(M+m)V_c,$$

$$V_c=\frac{m}{M+m}v。$$

选（B)。

41. A particle of mass m that move s along the x-axis has potential energy $V(x)=a+bx^2$，where a and b are positive constants. Its initial velocity is v_0 at x=0. It will execute simple harmonic motion with a frequency determined by the value of

(A) b alone
(B) b and a alone
(C) b and m alone
(D) b, a, and m alone
(E) b, a, m, and v_0

解：由一维谐振子公式，$\omega=\sqrt{\frac{2b}{m}}$。选（C)。ω 显然与 a 无关，因为势能的零点选取有任意性，或者说质点受力 $F=-\frac{dV}{dx}=-2bx$ 与 a 无关。

Questions 42-43

The equation of motion of a rocket in free space can be written

$$m\frac{dv}{dt}+u\frac{dm}{dt}=0$$

where m is the rocket's mass, v is its velocity, t is time, and u is a constant.

42. The constant u represents the speed of the

(A) rocket at t=0
(B) rocket after its fuel spent
(C) rocket in its instantaneous rest frame
(D) rocket's exhaust in a stationary frame
(E) rocket's exhaust relative to the rocket

解：由动量守恒

$$(m+dm)v=m(v+dv)+Vdm,$$

其中 V 为燃料被抛出后的速度。

$$mdv+(V-v)dm=0,$$

令 $u=V-v$ 为燃料的相对抛出速度，则

$$m\frac{dv}{dt}+u\frac{dm}{dt}=0。$$

选(E)。

43. The equation can be solved to give b as a function of m. If the rocket has $m=m_0$ and v=0 when it starts, what is the solution?

(A) um_0/m

(B) $u\exp(m_0/m)$

(C) $u\sin(m_0/m)$

(D) $u\tan(m_0/m)$

(E) None of the above.

解：由前题结论

$$m\frac{dv}{dt}+u\frac{dm}{dt}=0$$

得

$$\frac{dm}{m}=-\frac{dv}{u}。$$

两边积分得

$$m(t)=m_0e^{-\frac{v(t)-v_0}{u}},$$

因此

$$v(t)=v_0+u\ln\frac{m_0}{m(t)}=u\ln\frac{m_0}{m(t)}。$$

选（E）。

44. The period of a hypothetical Earth satellite orbiting at sea level would be 80 minutes. In terms of the Earth's radius R_e, the radius of a synchronous satellite orbit (period 27 hours) is most nearly

(A) 3 R_e

(B) 7 R_e

(C) 18 R_e

(D) 320 R_e

(E) 5800 R_e

解：由向心力公式

$$m\omega^2 r=\frac{GMm}{r^2},$$

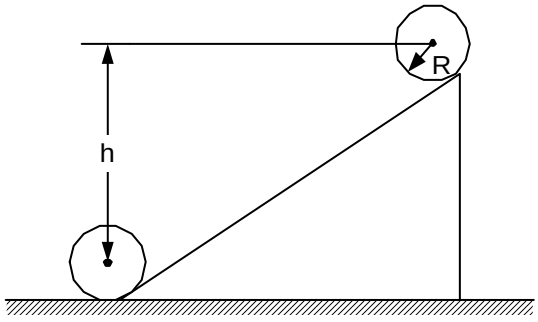
其中 $\omega=\frac{2\pi}{T}$，得

$$T^2=\frac{4\pi^2}{GM}r^3,$$

周期比为

$$\frac{27\times 60}{80}=\frac{81}{4},$$

由此得半径之比约为 7.4。选(B)。或者直接用 Kepler 第三定律，周期的平方与半径的立方之比为常数。但是要切忌，这只是对绕同一天体运行的星体适用。



45. A hoop of mass M and radius R is at rest at the top of an inclined plane as shown above. The hoop rolls down the plane without slipping. When the hoop reaches the bottom, its angular momentum around its center of mass is

(A) $MR\sqrt{gh}$

(B) $\frac{1}{2}MR\sqrt{gh}$

(C) $M\sqrt{2gh}$

(D) Mgh

(E) $\frac{1}{2}Mgh$

解：圆环的转动惯量为 $I=MR^2$。由能量守恒，

$$\frac{1}{2}Mv^2+\frac{1}{2}I\omega^2=Mgh,$$

由无滑滚动，

$$\omega R=v,$$

联立上面两式，解得

$$\omega=\frac{\sqrt{gh}}{R}。$$

所以角动量

$$L=I\omega=MR^2\cdot\frac{\sqrt{gh}}{R}=MR\sqrt{gh}。$$

选（A）。

46. A particle is constrained to move along the x-axis under the influence of the net force **F**=–k**x** with amplitude A and frequency f, where k is a positive constant. When x=A/2, the particle's speed is

(A) $2\pi fA$

(B) $\sqrt{3}\pi fA$

(C) $\sqrt{2}\pi fA$

(D) πfA

(E) $\frac{1}{3}\pi fA$

解：质点显然做简谐振动，势能为 $\frac{1}{2}kx^2$。由能量守恒，

$$\frac{1}{2}kA^2=\frac{1}{2}k(\frac{A}{2})^2+\frac{1}{2}mv^2,$$

解得

$$v=\sqrt{\frac{3k}{4m}}A=\frac{\sqrt{3}}{2}\omega A。$$

因为

$$f=\frac{1}{T}=\frac{\omega}{2\pi},$$

所以

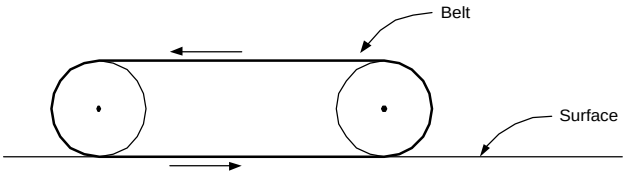
$$v=\sqrt{3}\pi fA。$$

选(B)。

47. A system consists of two charged particles of equal mass. Initially the particles are far apart, have zero potential energy, and one particle has nonzero speed. If radiation is neglected, which of the following is true of the total energy of the system?

(A) It is zero and remains zero.
(B) It is negative and constant.
(C) It is positive and constant.
(D) It is constant, but the sign cannot be determined unless the initial velocities of both particles are known.
(E) It cannot be a constant of the motion because the particles exert force on each other.

解：开始时系统势能为 0，动能不为 0，从而动能为正值，总能量为正。不考虑辐射，则系统与外界无相互作用，能量守恒。选(C)。



48. An electric sander has a continuous belt that rubs against a wood surface as shown schematically above. The sander is 100 percent efficient and draws a current of 9 amperes from a 120-volt line. The belt speed is 10 meter per second. If the sander is pushing against the wood with a normal force of 100 newtons, the coefficient of friction is most nearly

(A) 0.02
(B) 0.2
(C) 0.4
(D) 1.1
(E) 10

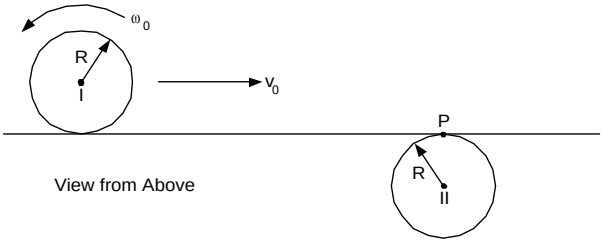
解：功率全消耗在克服摩擦力上，所以

$$f=\frac{P}{v}=\frac{UI}{v}=\frac{120\times 9}{10}=108\ \mathrm{N},$$

摩擦系数为

$$\mu=\frac{f}{N}=\frac{108}{100}=1.08\approx 1.1。$$

选(D)。



49. Two uniform cylindrical disks of identical mass M, radius R, and moment of inertia $\frac{1}{2}MR^2$, as shown above, collide on a frictionless, horizontal surface. Disk I, having an initial counterclockwise angular velocity ω_0 and a center-of-mass velocity $v_0=\frac{1}{2}\omega_0 R$ to the right, makes a grazing collision the two disks stick together, the magnitude of the total angular momentum about the point P is

(A) zero
(B) $\frac{1}{2}MR^2\omega_0$
(C) $\frac{1}{2}MRv_0$
(D) MRv_0
(E) dependent on the time of the collision

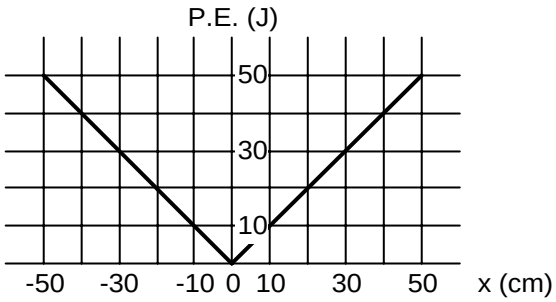
解：碰撞前对于盘 I，质心角动量为

$$MV_0R=\frac{1}{2}M\omega_0R^2,$$

方向垂直纸面向里；而转动部分角动量为

$$I\omega_0=\frac{1}{2}M\omega_0R^2。$$

方向垂直纸面向外。两部分角动量大小相等方向相反，故总角动量为 0。因为碰撞前后相对 P 点的角动量守恒，所以碰撞后仍为 0。选（A)。注意仔细看图，盘 I 不是做无滑滚动。



50. The graph above shows the potential energy (P.E.) in joules of an object of mass m moving horizontally in a conservative one-dimensional force field. If the initial total energy of the object is 30 joules, which of the following statements is NOT correct?

(A) The magnitude of the force acting on the object when it is at x = –15 cm is 100 N
(B) The kinetic energy of the object when it is at x = 5 cm is 25 J.
(C) The speed of the object is a maximum at x = 0.
(D) The motion of the object is periodic in time and has an amplitude of 30 cm.
(E) The period of the motion is independent of the mass of the object.

解：$F=\dfrac{dU}{dx}=100\ \mathrm{N}$，为恒力，且为恢复力，方向与位移相反，故（A）正确。（B）、（C）、（D）显然正确。因为质点受力大小保持不变，质量不同时，加速度不同，而振幅 A 相同，均为 30 cm，所以周期将不同。选（E）。

51. A thin uniform steel chain is 10 meters long with a mass density of 2 kilograms per meter. One end of the chain is atteched to a horizontal axle having a radius that is small compared to the length of the chain. If the chain initially hangs vertically, the work required to slowly wind it up on to the axle is closest to

(A) 100 J
(B) 200 J
(C) 1,000 J
(D) 2,000 J
(E) 10,000 J

解：铁链从垂吊状态变到缠卷状态，重力势能改变为

$$\Delta E_P=mg\Delta h=l\rho gl/2=2\times10\times10^2/2=1000\ \mathrm{J}$$

。

外界对铁链做功 $W\geq E_P$。选（C）。

52. Two circular hoops, X and Y, are hanging on nails in a wall. The mass of X is four-times shat of Y, and the diameter of X is also four times that of Y. If the period of small oscilations of X is T, the period of small oscillations of Y is

(A) T
(B) T/2
(C) T/4
(D) T/8
(E) T/16

解：圆环相对于其圆心的转动惯量为 $I_0=MR^2$。

由平行轴定理，相对环上一点的转动惯量为

$$I=I_0+MR^2=2MR^2。$$

所以 X 、Y 相对于悬挂点的转动惯量之比为

$$\frac{I_X}{I_Y}=\frac{2M_XR_X^{\ 2}}{2M_YR_Y^{\ 2}}=64。$$

由复摆周期公式

$$T=2\pi\sqrt{\frac{I}{Gl}},$$

其中 l 为质心到转动轴的距离，X、Y 周期之比为

$$\frac{T_X}{T_Y}=\sqrt{\frac{I_X}{I_Y}\frac{G_Yl_Y}{G_Xl_X}}=\sqrt{\frac{64}{1}\frac{1}{4}\frac{1}{4}}=2。$$

所以

$$T_Y=\frac{T_X}{2}。$$

选（B）。

53. A certain 100-meter length of piano wire has a mass of 1 kilogram. If transverse waves move on the wire with a speed of 300 meters per second, the tension in the wire is closest to

(A) 10^1 N
(B) 10^2 N
(C) 10^3 N
(D) 10^4 N

(E) 10^5 N

解：由连续介质中的波速公式

$$v=\sqrt{\frac{T}{\rho}},$$

$$T=\rho v^2=\frac{1}{100}\times 300^2=900\text{N},$$

选（C）。这个公式即使考试时记不清，也很容易由量纲分析得到。速度的量纲为米·秒$^{-1}$，线密度的量纲为千克·米$^{-1}$，张力量纲为千克·米·秒$^{-2}$，张力和线密度之比的量纲为米^{2}·秒$^{-2}$，为速度量纲的平方，于是可得公式的形式。

54. Which of the following is true of an automobile that is accelerated from rest along a straight level road by an engine that supplies constant power?

(A) Its acceleration is constant.

(B) Its acceleration decreases with time.

(C) Its acceleration is directly proportional to its velocity.

(D) Its velocity is proportional to the distance travelled.

(E) Its kinetic energy is proportional to the distance travelled.

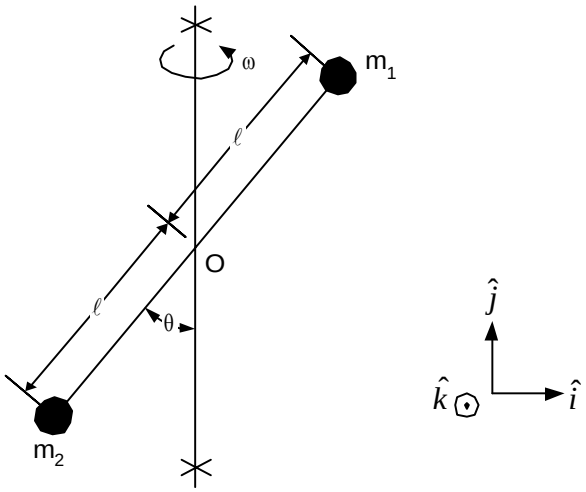
解：由功率的表达式

$$P=F\cdot v\text{。}$$

功率恒定，则速度从 0 开始变大时，牵引力将减小。而摩擦力 f 不变，所以加速度

$$a=\frac{F-f}{m}$$

随速度的增加变小，直到减小为零。此后汽车作匀速直线运动。选（B）。



55. A rigid dumbbell consists of point masses m_1 and m_2 joined together by a massless rod of length 2l.The dumbbell is fixed at an angle to a shaft about which it rotates with angular velocity $\omega\,\hat{j}$ in the coordinate system shown above. At the instant shown, the velocity of m_1 is directed into the paper, and the velocity of m_2 is directed out of the paper. The vector angular momentum L of the system about the point O is

(A) $(m_1+m_2)l^2\omega\hat{j}$

(B) $(m_1+m_2)(l\cos\theta)^2\omega\hat{j}$

(C) $(m_1+m_2)(l\sin\theta)^2\omega\hat{j}$

(D) $(m_1+m_2)l^2\omega\sin\theta(\sin\theta\,\hat{j}-\cos\theta\,\hat{i})$

(E) $(m_1+m_2)l^2\omega\sin\theta(\cos\theta\,\hat{j}+\sin\theta\,\hat{i})$

解：m_1 相对于 O 点的角动量为

$$\mathbf{L}_1=\mathbf{r}\times m_1\mathbf{v}=lm_1v_1\left(-\cos\theta\,\mathbf{i}+\sin\theta\,\mathbf{j}\right)=m_1l^2\omega\sin\theta\left(-\cos\theta\right.$$

。

同理 m_2 相对于 O 点的角动量为

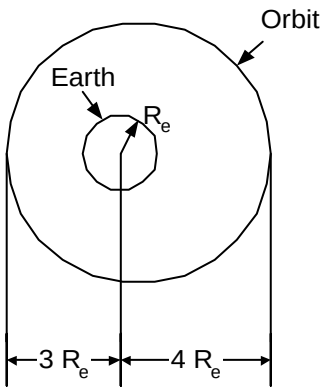
$$\mathbf{L}_1=\mathbf{r}\times m_2\mathbf{v}=m_2l^2\omega\sin\theta\left(-\cos\theta\,\mathbf{i}+\sin\theta\,\mathbf{j}\right)$$

刚体角动量是各质点角动量之和，

$$\mathbf{L}=\mathbf{L}_1+\mathbf{L}_2=\left(m_1+m_2\right)l^2\omega\sin\theta\left(-\cos\theta\,\mathbf{i}+\sin\theta\,\mathbf{j}\right)$$

。

选（D）。



56. A satellite is in elliptical orbit about the Earth as shown above with maximum distance from the center of the Earth of $4R_e$ and minimum distance of $3R_e$, where R_e is the radius of the Earth. If g is the acceleration due to gravity at the Earth's surface, the maximum speed of the satellite is

(A) $\sqrt{\frac{8}{21}gR_e}$

(B) $\sqrt{\frac{4}{7}gR_e}$

(C) $\sqrt{\frac{3}{4}gR_e}$

(D) $\sqrt{gR_e}$

(E) $\sqrt{\frac{4}{3}gR_e}$

解：在近地点和远地点，径向速度为零，所以速度 V_a、V_b 沿角向。由角动量守恒

$$V_a \cdot 3R_e = V_b \cdot 4R_e \text{；}$$

由机械能守恒

$$\frac{1}{2}mv_a^{\ 2} - \frac{GMm}{3R_e} = \frac{1}{2}mv_b^{\ 2} - \frac{GMm}{4R_e}\text{。}$$

联立以上两式，解得

$$v_a = \sqrt{\frac{8}{21}gR_e}\text{ 。}$$

因为在近地点时引力势能最小（绝对值最大），所以动能最大。所以近地点速度就是速度的最大值。选（A）。

57. A metal sphere of mass m falls from rest in a tall vertical tube containing a viscous fluid. The magnitude of the retarding force on the sphere is given by βv where v is the speed of the sphere and β is a constant. If the buoyant force on the sphere is negligible, the maximum (terminal) speed of the sphere as it falls through the fluid is most nearly

(A) $\frac{1}{2}mg\beta$

(B) mgβ

(C) $\frac{1}{2}\frac{mg}{\beta}$

(D) $\frac{mg}{\beta}$

(E) $2\frac{mg}{\beta}$

解：达到最大（收尾）速度时，小球所受的合力为零，做匀速直线下滑运动。根据题意，忽略浮力有

$$mg = \beta v\text{，}$$

由此解得收尾速度

$$v = \frac{mg}{\beta}\text{。}$$

选（D）。

58. Under the influence of a mutual interaction, an object orbits another object that is fixed. The orbits lies in a plane and the areas swept out by the radius vector in equal times are equal. What can be correctly concluded about the force between the objects?

(A) It is central.
(B) It is inverse-square.
(C) It is conservative.
(D) It is gravitational.
(E) None of these conclusions is justified.

解：面积速度

$$\frac{ds}{dt}=\left|\frac{1}{2}\mathbf{v}\times\mathbf{r}\right|=\frac{J}{2m}=const.,$$

又由于题中指明是平面运动，因此角动量 **J** 的方向也固定，所以角动量 **J** 守恒。是中心力场。选（A）。

59. A machine gun fires bullets of mass 20 grams each at rate of 1200 bullets per minute. The bullets hit thick wooden target at a speed of 600 meters per second and are stopped in the target. The average force exerted on the target by the bullets striking it is

(A) 144N
(B) 240N
(C) 14.4×10^{3} N
(D) 24.0×10^{4} N
(E) 14.4×10^{6} N

解：直接利用冲量原理即可写出靶子的受力

$$F=\frac{dp}{dt}=\frac{1200\times600\times0.020}{60}=240\text{N}。$$

选（B）。

Questions 60-61 relate to a particle of mass M that is moving in an attractive central force field. The potential energy function representing the attractive central force field can be written as V(r) = –k/r. At a certain time the particle has angular momentum **L** and total energy E.

60. At some later time, which of the following statements will be true of the angular momentum **L** and total energy E of the particle?

(A) **L** will have changed, but E will not.
(B) E will have changed, but **L** will not.
(C) Neither **L** nor E will have changed.
(D) Both **L** and E will have changed.
(E) It is not possible to say what will happen to **L** and E.

解：由于为有心力，所以角动量 **L** 守恒；由于有势函数存在，说明其为保守力场，能量 E 守恒。选（C）。

61. For a given nonzero angular momentum, there is a minimum energy for which it is possible to find a solution to the equations of motion. At this minimum energy, the particle is moving in a

(A) circular orbit
(B) noncircular elliptical orbit
(C) parabolic orbit
(D) hyperbolic orbit
(E) straight line

解：题目中所给势场与万有引力相同，很多结论可以借用。由于物体运动轨道为抛物线时，机械能为 0；轨道为双曲线时，机械能为正。均比椭圆轨道能量高。（C）、（D）不对。当物体做圆运动时，在不改变其角动量情况下，给它一个径向速度，增加它的机械能，物体将变为做椭圆运动，而角动量并没有改变。同理通过减少径向速度为零，不改变角动量，也可将一物体从椭圆轨道变为圆轨道，机械能降低。或者说对任一椭圆轨道，总有一圆轨道，二者角动量相同，单元轨道能量低。选（A）。

62. Two particles, each with mass m, are attached to each other by a spring having a spring constant k. The composite molecule like object is traveling freely in space and, at the same time, it is vibrating. The frequency of vibration is

(A) $\frac{1}{4\pi}\sqrt{\frac{k}{m}}$

(B) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$

(C) $\frac{1}{\pi}\sqrt{\frac{k}{m}}$

(D) $\frac{1}{2\pi}\sqrt{\frac{k}{2m}}$ t

(E) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

解：两个质点的质心不受外力，因此质心系是惯性参考系，在质心系中

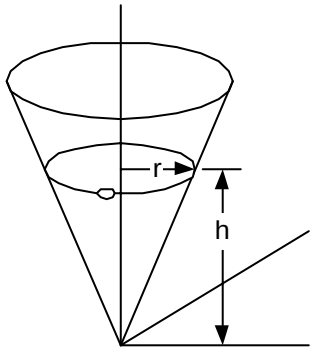
$$\begin{aligned}m\dot{x}_1+m\dot{x}_2&=0\\ m\ddot{x}_1&=-k(x_1+x_2-x_0)\end{aligned},$$

消元得标准波动方程形式

$$m\ddot{x}_1' + 2kx_1' = 0,$$

x_1'与 x_1 仅相差某个平移的常数。选（E）。

另一种看法：质心固定的话，每个质点相当于在半根弹簧的作用下独立振动，两根串联弹簧的等效弹性系数均为 2k。



63. A particle of mass m moves in a horizontal circle of radius r on the inside of a frictionless cone with a vertical axis as shown above. If the plane of the particle is a distance h above the apex of the cone, what is the speed of the particle?

(A) $\sqrt{gr}$

(B) $\frac{1}{2}\sqrt{gr}$

(C) $\sqrt{gh}$

(D) $\frac{1}{2}\sqrt{gh}$

(E) $r\sqrt{\frac{g}{h}}$

解：由于内壁光滑，因此根据受力分析易得重力与法向支持力的合力提供向心力。

$$mgctg\theta = \frac{mv^2}{r},$$

$$ctg\theta = \frac{h}{r}。$$

解以上方程，选（C）。

64. A ball rises after one bounce to 80 percent of the height from which it is released from rest. If air resistance is negligible and the ball is released from rest from a height H_0, what is its speed just before it hits the floor for its Nth bounce?

(A) $(0.8)^{2N}\sqrt{2gH_0}$

(B) $(0.8)^{N}\sqrt{2gH_0}$

(C) $(0.8)^{N-\frac{1}{2}}\sqrt{2gH_0}$

(D) $(0.8)^{\left(\frac{N-1}{2}\right)}\sqrt{2gH_0}$

(E) $(0.8)^{N-1}\sqrt{2gH_0}$

解：每次碰撞前后小球损失的能量为 80%，因此每次碰后返回的速度值是碰前的 $0.8^{1/2}$。第一次着地前小球的速度大小又机械能守恒不难算出

$$v_0 = \sqrt{2gH_0}。$$

由等比级数的通项公式可以写出第 N 次着地前小球的速度

$$v_N = (0.8)^{\left(\frac{N-1}{2}\right)}\sqrt{2gH_0}。$$

选（D）。

65. A particle of mass m is located at $x_0 = \pi/a$, a minimum of the periodic potential $U(x) = U_0\cos \alpha x$. If the particle is displaced slightly from x_0 and released, what is the approximate angular frequency of its oscillations?

(A) $\alpha\sqrt{\frac{U_0}{m}}$

(B) $\alpha\sqrt{\frac{U_0}{2m}}$

(C) $\frac{\alpha}{2}\sqrt{\frac{U_0}{m}}$

(D) $\sqrt{\frac{\alpha U_0}{m}}$

(E) $\sqrt{\frac{\alpha U_0}{2m}}$

解：将势能在 x_0 附近作 Taylor 展开

$$U(x_0+\Delta x)=U_0\left(1-\frac{1}{2}(\alpha\Delta x)^2\right)+o(\Delta x^2),$$

由其中二次项的系数求得等效弹性系数为

$$k=U_0\alpha,$$

因此振动圆频率

$$\omega=\sqrt{\frac{k}{m}}。$$

选（A）。

66. An alpha particle with nonrelativistic kinetic energy K collides head-on with a massive gold nucleus with charge Ze. The distance of slowest approach is

(A) $\frac{2Ze^2}{K}$

(B) $\frac{Z^2e^4}{K^2}$

(C) $\frac{2K}{Ze^2}$

(D) $\frac{Ze^2}{137K}$

(E) $2Ze^2K$

解：直接利用经典力学中能量守恒的观点求解，由题意，可忽略 Au 原子的动能，当初始时α粒子的动能完全转换为两者之间的电势能时，两者最接近，且接近速度减小并反向。

$$K=\frac{2Ze^2}{R_{\min}}。$$

选（A）。

Question 67-68

A nonrelativistic particle of mass m moves in a plane. Its position is described by the polar coordinates r and θ, with time derivatives $\dot{r}$ and $\dot{\theta}$. There exists a potential energy U = kr^2, where k is a constant.

67. Which of the following is the Lagrangian L of the particle?

(A) $L=\frac{1}{2}m(\dot{r}^2+r^2\dot{\theta}^2)-kr^2$

(B) $L=\frac{1}{2}m(\dot{r}^2+\theta^2)+kr^2$

(C) $L=\frac{1}{2}m(\theta^2\dot{r}^2+r^2\dot{\theta}^2)-kr^2$

(D) $L=\frac{1}{2}m(\dot{r}^2+r\dot{r}\dot{\theta}+r^2\dot{\theta}^2)-kr^2$

(E) $L=\frac{1}{2}m(\dot{r}^2+2r\dot{r}\dot{\theta}-r^2\dot{\theta}^2)-kr^2$

解：粒子的 Lagrange 函数

$$L=T-V=T_n+T_\tau-V=\frac{1}{2}m(\dot{r}^2+r^2\dot{\theta}^2)-kr^2$$

，

选（A）。

68. Which of the following quantities remain constants?

(A) $m(\dot{r}^2+r^2\dot{\theta}^2)$

(B) $mr^2\dot{\theta}^2$

(C) kr^2

(D) $mr\dot{\theta}$

(E) $mr^2\dot{\theta}$

解：接上题，Lagrange 函数 L 中不显含广义坐标θ，因此根据 Lagrange 方程

$$\frac{\partial L}{\partial\dot{\theta}}=mr^2\dot{\theta}=p_\theta=const。$$

选（E）。

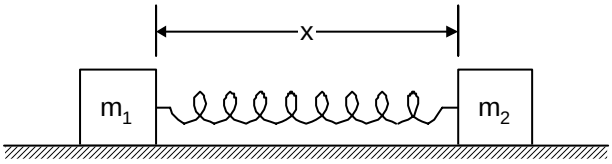
69. A particle with mass 10 kg is moving in a straight line under the influence of a force applied opposite to the motion along the same straight line. The equation $F = 2v^2$ represents the magnitude of the force F in newtons as a function of the instantaneous speed of the object at time t = 0 is 10 meters per second. At time t = –2 seconds, the speed of the object is most nearly

(A) 2.0 m/s
(B) 2.5 m/s
(C) 3.3 m/s
(D) 4.0 m/s
(E) 5.0 m/s

解：根据 Newton 运动定律 **F** = m**a**，直接可对时间反演积分求解初速度

$$\frac{dv}{dt}=\frac{F}{m}=\frac{2v^2}{10}\Rightarrow\int_{10}^{v_{-2}}\frac{dv}{v^2}=\int_0^{-2}\frac{1}{5}dt\ ,$$

积分可求得结果。选（A）。



70. Two masses, m_1 and m_2, are joined by a massless spring of force constant k and placed on a horizontal frictionless surface as shown above. The system is released from rest when the separation between the masses is x. If the unstretched length of the spring is x_0, the speed of mass m_1 when the two masses are at the distance x_0 apart is

(A) $\sqrt{\frac{k}{m_1}(x-x_0)^2}$

(B) $\sqrt{\frac{k}{m_2}(x-x_0)^2}$

(C) $\sqrt{\frac{k}{m_1-m_2}(x-x_0)^2}$

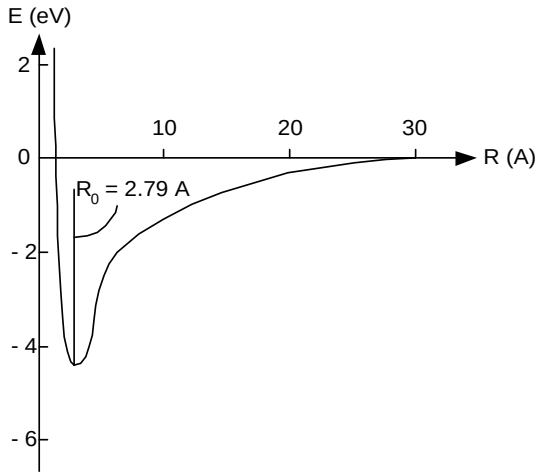
(D) $\sqrt{\frac{k}{m_1}\cdot\frac{m_2}{m_1+m_2}(x-x_0)^2}$

(E) $\sqrt{\frac{k}{m_2}\cdot\frac{m_1}{m_1+m_2}(x-x_0)^2}$

解：直接利用动量守恒定律和机械能守恒定律可以联合求解

$$m_1v_1+m_2v_2=0$$
$$\frac{1}{2}k(x-x_0)^2=\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2\ ,$$

可求得 v_1。选（D）。



71. The diagram above shows the energy E of a KCl molecule as a function of the internuclear distance R. True statements include which of the following?

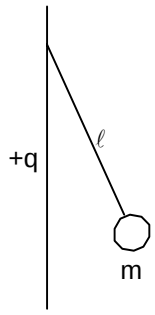
I. The energy required to dissociate a KCl molecule is about 4.4 eV.

II. R_0 refers to the equilibrium separation of the atoms in the molecule when it is in its ground state.

III. A KCl molecule in all of its bound electronic states would have the same E vs. R curve.

(A) I only
(B) III only
(C) I and II only
(D) II and III only
(E) I, II, and III

解：在势能曲线的最低点，对应的能量负值即将 KCl 原子分离所需要的最小能量，相应的距离是振动的平衡位置，即处于基态的平衡距离。I、II 都对，III 不对。选（C）。



72. A small sphere of mass m and charge q (cgs units) is attached by a string of length l to an infinite vertical plane that has a surface charge density σ. If q

> 0 and σ > 0, what is the tension in the string?

(A) mg

(B) 2πσq

(C) $(mg)^2/2\pi\sigma q$

(D) $(2\pi\sigma q)^2/mg$

(E) $\sqrt{(mg)^2+(2\pi\sigma q)^2}$

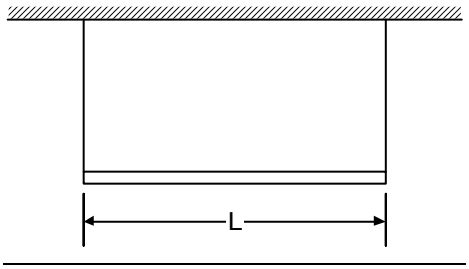
解：拉力与电场斥力和重力的合力平衡，无穷大带电平面附近的电场强度公式应掌握。

$$T^2=(mg)^2+(Eq)^2,$$

$$E=2\pi\sigma。$$

选（E）。

Questions 73-74



A thin uniform stick of length L and mass M is supported in a horizontal position by vertical strings on its ends, as shown above. One of the strings is cut at time t. The following questions pertain to the time (t + δ) where $\delta\to 0$.

73. The angular acceleration of the stick is

(A) 0

(B) $\frac{g}{2L}$

(C) $\frac{g}{L}$

(D) $\frac{3g}{2L}$

(E) $\frac{2g}{L}$

解： 在(t + δ) 时刻杆刚开始运动，相对于悬点的转动仅受重力矩的影响。

$$Mg\frac{L}{2}=I\beta=\frac{1}{3}ML^2\beta\Rightarrow\beta=\frac{3g}{2L}。$$

选（D）。

74. The tension in the remaining string is

(A) 0

(B) $\frac{1}{4}Mg$

(C) $\frac{1}{2}Mg$

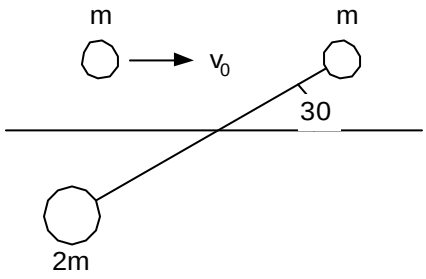
(D) $\frac{3}{4}Mg$

(E) Mg

解：考察质心的运动，根据 Newton 方程可得

$$Ma_c=M\beta\frac{L}{2}=Mg-T,$$

解得 T = Mg / 4。选（B）。



75. A puck of mass m slides with velocity v_0 on a horizontal frictionless surface as shown in the top view above. Lying on the surface at rest is a dumbbell formed by two other pucks, one of mass m and one of mass 2m, joined by a massless rod of length l. The axis of the dumbbell is inclined at a angle of 30° with respect to v_0 as shown. The moving puck collides and sticks to the lighter end of the dumbbell. The kinetic energy (translation and rotation) of the system after the collision is

(A) $\frac{5}{8}mv_0^2$

(B) $\frac{1}{2}mv_0^2$

(C) $\frac{5}{32}mv_0^2$

(D) $\frac{1}{8}mv_0^2$

(E) $\frac{1}{64}mv_0^2$

解：此题-计算稍繁，比较简单一点是选择在质心系里计算。

$$v_c = \frac{v_0}{4},$$

角动量守恒，

$$\left(m\frac{3}{4}v_0 - m\frac{1}{4}v_0 + 2m\frac{1}{4}v_0\right)\frac{L}{2}\cdot\sin 30 = 2\left(2m\left(\frac{L}{2}\right)^2\right)\omega$$

，

从而解得

$$\omega = \frac{v_0}{4l}。$$

碰后系统的总能量

$$E = \frac{1}{2}4mv_c^2 + \frac{1}{2}I\omega^2 = \frac{5}{32}mv_0^2。$$

选（C）。

76. An asteroid has a radius of 5×10^5 meters and an acceleration due to gravity of 1/4 that on Earth. The velocity of escape for an object starting on the surface of the asteroid is most nearly

(A) 150 m/s
(B) 720 m/s
(C) 1560 m/s
(D) 5550 m/s
(E) 11,200 m/s

解：星球的逃逸速度（第一宇宙速度）为

$$v_1 = \sqrt{Rg'} = \sqrt{\frac{1}{4}Rg'} \approx 5550\text{m/s}。$$

选（D）。

77. An object of mass m is released from rest far above the Earth, at a distance r from the center of the Earth. The Earth's radius is R and the acceleration due to gravity at the Earth's surface is g. If air resistance is neglected and nonrelativistic motion is assumed, what is the speed of the object when it strikes the Earth's surface?

(A) $\sqrt{2gr^2/R}$

(B) $\sqrt{2g(r^2-R^2)/r}$

(C) $\sqrt{g(r-R)}$

(D) $\sqrt{2g(r-R)}$

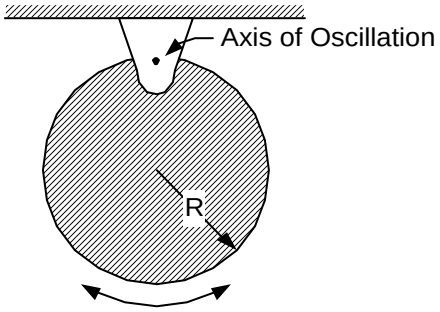
(E) $\sqrt{2gR(r-R)/r}$

解：物体的机械能守恒

$$-\frac{GMm}{r} = -\frac{GMm}{R} + \frac{1}{2}mv^2,$$

$$-g\frac{R}{r} = -g + \frac{1}{2}v^2。$$

由此可解得速度 v。选（E）。



78. A uniform disk of radius R is suspended at its edge as shown above. It is free to swing back and forth in the plane of the disk. What is the frequency of small oscillations of this disk?

(A) $\frac{1}{2\pi}\sqrt{\frac{2g}{R}}$

(B) $\frac{1}{2\pi}\sqrt{\frac{2g}{3R}}$

(C) $\frac{1}{2\pi}\sqrt{\frac{3g}{4R}}$

(D) $\frac{1}{2\pi}\sqrt{\frac{g}{R}}$

(E) $\frac{1}{2\pi}\sqrt{\frac{3g}{2R}}$

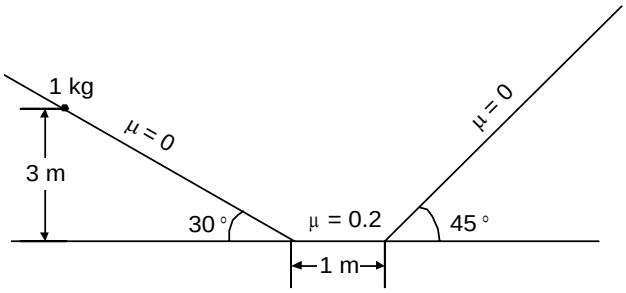
解：相当于一个复摆。圆盘相对于悬点的转动惯量

$$I = mR^2 + \frac{1}{2}mR^2 = \frac{3}{2}mR^2。$$

小振动的频率为

$$f=\frac{\omega}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{mgR}{I}}=\frac{1}{2\pi}\sqrt{\frac{2g}{3R}}。$$

选（B）。



79. A 1-kilogram mass is initially held at rest at a height of 3 meters from the bottom of a 30 frictionless inclined plane, as shown above. The mass is released, slides down the plane, and comes to a 1-meter horizontal table with coefficient of friction μ = 0.2. After crossing this rough table, the mass slides up a 45° frictionless inclined plane. How many times does the mass cross the table before stopping?

(A) 1

(B) 5

(C) 15

(D) 10

(E) 100

解：最初的机械能完全被摩擦所损耗，每滑过一次水平的桌面，克服摩擦所损耗的能量

$$\Delta E=\mu mgs=0.2g\ ,$$

故滑块静止前滑过桌面的次数

$$n=\frac{mgH}{\Delta E}=\frac{3g}{0.2g}=15。$$

选（C）。

80. The speed v of a particle moving in a straight line is given by $v=\frac{a}{bt+c}$, where t is time and a, b, c are constants. The magnitude of the resultant force on the particle is directly proportional to

(A) b^{-2}

(B) b^{-1}

(C) b^{0}

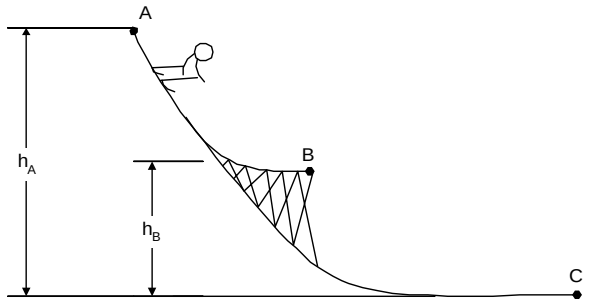
(D) b^{1}

(E) b^{2}

解：直接根据 Newton 第二定律可以求出质点所受的阻力

$$f=m\frac{dv}{dt}=-\frac{mab}{(bt+c)^2}。$$

选（D）。



81. The skier shown above leaves point A from rest, skies down a frictionless ski jump, leaves the jump at point B, and hits the ground at point C. The height at A and B are h_A and h_B. What is the skier's speed just before hitting the ground?

(A) $\sqrt{2gh_B}$

(B) $\sqrt{2gh_A}$

(C) $\sqrt{2g(h_A-h_B)}$

(D) $\sqrt{2gh_Ah_B/(h_A+h_B)}$

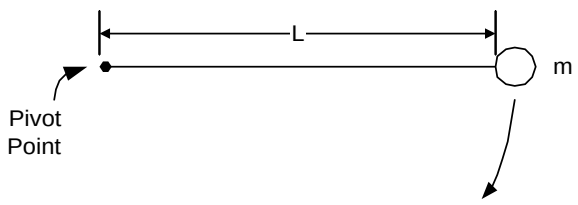
(E) $\sqrt{2g(h_A+h_B)}$

解：整个下滑、跳跃过程是满足机械能守恒的

$$mgh_A=\frac{1}{2}mv^2\ ,$$

$$v=\sqrt{2gh_A}。$$

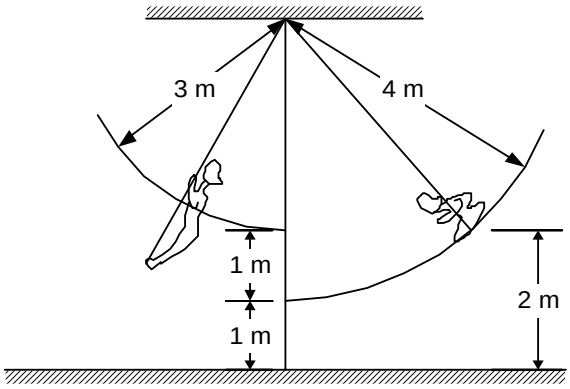
选（B）。



82. A massless rod of length L is pivoted about a horizontal axis through one end. A small object of mass is attached to the other end. The rod is released from rest in a horizontal position, as shown above, and swings as a pendulum. Assume there is no friction and no air resistance. Which of the following quantities remains constant throughout the motion of the pendulum?

(A) Linear momentum of m
(B) Angular momentum of m relative to the pivot point
(C) Gravitational potential energy
(D) Kinetic energy
(E) None of the above

解：小球的机械能守恒。而下落过程中小球所受的合力、合力矩都不为零，动量和角动量都不守恒。选（E）。



83. A girl is initially crouching on a massless swing that is held at rest so that her center of mass is 2 meters above the ground. Assuming that the girl weighs 500 newtons, her center of mass is 4 meters from the pivot of the swing. The swing is released from rest. At the bottom of the arc the girl stands up, instantaneously raising her cernter of mass by 1 meter. The dotted line in the figure above represents the path of her center of mass. What is the ratio of her velocity just after she stands up to her velocity just before she stands up?

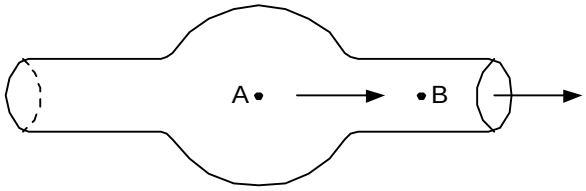
(A) 6/5
(B) 5/4
(C) 4/3
(D) 3/2
(E) 2

解：在女孩的下蹲过程中，由于重力沿竖直方向，整个系统没有受到外力矩，相对悬点的角动量守恒。

$$ml_0v_0 = mlv\,,$$

$$\frac{v}{v_0} = \frac{l_0}{l} = \frac{4}{3}。$$

选（C）。



84. Water flows through a horizontal pipe of varying cross section, as shown in the figure above. The pressure is p_A at A and p_B at B. Which of the following statements is true?

(A) p_A is greater than p_B because the velocity is greater at B.
(B) p_A is greater than p_B because A is upstream from B.
(C) p_A is less than p_B because A is upstream from B.
(D) p_A equals p_B according to Pascal's theorem.
(E) p_A equals p_B according to Bernoulli's theorem.

解：首先根据流体的连续性原理

$$v_As_A = v_Bs_B\,,$$

故 $v_A < v_B$。然后由 Bernoulli 方程

$$p_A + \frac{1}{2}\rho v_A^2 + \rho gh = p_B + \frac{1}{2}\rho v_B^2 + \rho gh\,,$$

得 $p_A > p_B$。选（A）。

85. A satellite of mass m circles the Earth at rasius r. Its period T is proportional to

(A) r^3/m

(B) $(r^3/m)^{1/2}$

(C) $r^{-3/2}m^0$

(D) $r^{3/2}m^0$

(E) $(m/r^3)^{1/2}$

解：地球与卫星间的万有引力提供卫星的向心加速度，

$$\frac{GMm}{r^2}=m\omega^2 r\ ,$$

$$T=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{r^3}{GM}}\propto r^{3/2}\ 。$$

选（D）。

86. The figure above shows two objects of mass m and 4m placed on a frictionless track at heights of 4h and h, respectively. They start from rest, slide along the track, collide at the bottom, and stick together. The speed of the objects just after the collision is

(A) 0

(B) $\frac{2}{5}\sqrt{2gh}$

(C) $\sqrt{gh}$

(D) $\sqrt{2gh}$

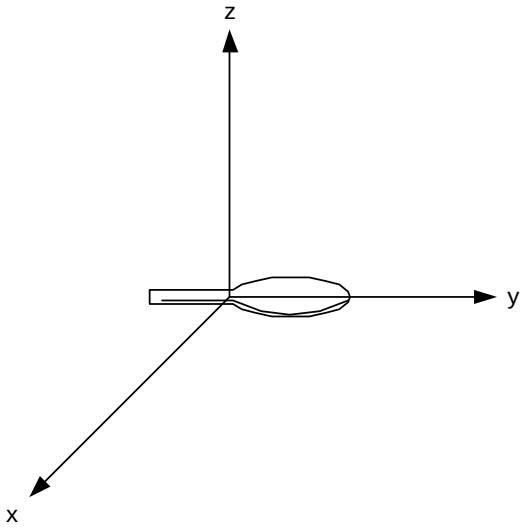
(E) $2\sqrt{2gh}$

解：是二体的完全非弹性碰撞，利用动量守恒，

$$4m\sqrt{2gh}-m\sqrt{2g\cdot 4h}=5mv\ ,$$

$$v=\frac{2}{5}\sqrt{2gh}\ 。$$

选（B）。



87. A table tennis paddle lies in the x-y plane, as shown above. Its center of mass is at the origin of the coordinate system. If I_x, I_y,and I_z are the moments of inertia about the tthree axes, which of the following is true?

(A) $I_x=I_y=I_z$

(B) $I_x<I_y<I_z$

(C) $I_x<I_z<I_y$

(D) $I_y<I_x<I_z$

(E) $I_z<I_y<I_x$

解：根据转动惯量的正交轴定理，$I_z=I_x+I_Y$ 最大。而圆盘和把两部分的质心都在 Y 轴上，而不在 X 轴上，说明质量的分布相对 X 轴更分散，因此 $I_X>I_Y$。选（D）。

88. A spaceship of mass m is initially in a circular orbit around an isolated star of mass M. The radius of the orbit is R. If the kinetic energy of the spaceship is doubled by a short burst of its engine, what is the final total energy of the spacship? (The potential energu is taken to be zero when the spaceship is at infinite distance from the star. G is the gravitational constant.)

(A) $-\frac{1}{2}\frac{GMm}{R}$

(B) $-\frac{1}{4}\frac{GMm}{R}$

(C) 0

(D) $\frac{1}{4}\frac{GMm}{R}$

(E) $\frac{1}{2}\frac{GMm}{R}$

解：初始时，宇宙飞船的动能

$$T_0=\frac{1}{2}mv_0^2=\frac{1}{2}\frac{GMm}{R}=-\frac{1}{2}V(R),$$

因此，加速后的总能量

$$E=2T_0+V(R)=0。$$

选（C）。

89. The mass of the Moon is 1.2 percent of the mass of the Earth. The radius of the Moon is 27 percent of the radius of the Earth. Approximately how long does it take for a rock released from rest from a height of 0.5 meter above the Moon's surface to strike the surface of the Moon?

(A) 0.25 s

(B) 0.8 s

(C) 2.5 s

(D) 8 s

(E) 25 s

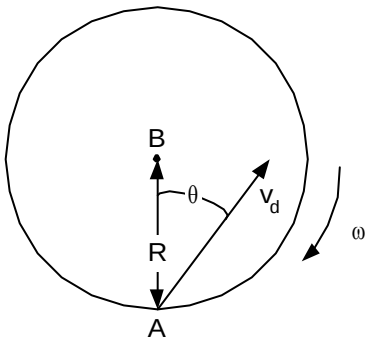
解：月球上的重力加速度

$$g'=\frac{GM'}{R'^2}=\frac{G\cdot 0.012M}{(0.27R)^2}\approx\frac{1}{6}g,$$

故自由落体的下落时间

$$t=\sqrt{\frac{2h}{g'}}=\sqrt{\frac{12\times 0.5}{10}}\approx 0.8\text{s}。$$

选（B）。



90. A person at A is rotating at a radius R and angular speed ω about a vertical axis B, as shown above. The person throws a ball with horizontal speed v_0 with respect to the person. If the ball is to hit B, at what angle θ must the person aim?

(A) 0

(B) $\omega R/v_0$

(C) $\tan^{-1}(\omega R/v_0)$

(D) $\tan^{-1}(v_0/\omega R)$

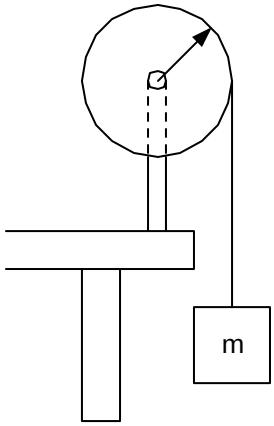
(E) $\sin^{-1}(\omega R/v_0)$

解：小球抛出的速度在静止系中看来应该是沿径向的，根据相对速度合成的原理

$$\sin\theta=\frac{\omega R}{v_0},$$

$$\theta=\sin^{-1}(\omega R/v_0)。$$

选（E）。



91. A mass m is tied to a light string wound around a pulley of moment of inertia I and radius R, as shown above. The pulley bearing is frictionless. The tension in the string as the mass falls is

(A) mg

(B) $(I/R^2)g$

(C) $(I/R^2+m)g$

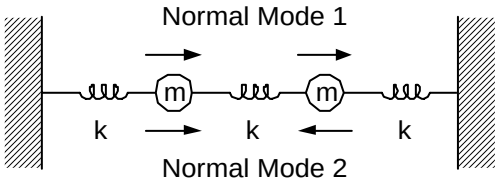
(D) $(m-I/R^2)g$

(E) $\dfrac{I/R^2}{m+I/R^2}mg$

解：假设绳中张力为 T，对滑轮和滑块的运动写出受力方程为

$$\mathrm{TR}=\mathrm{I}\beta=I\frac{a}{R},$$
$$mg-T=ma$$

以上两式联立消去 a 可解得张力 T。选（E)。



92. Two objects of mass m are connected by three identical springs of negligible mass, as shown above. Two normal modes of oscillation are shown by the arrows. The ratio ω_1/ω_2 of the frequencies of the two normal modes is

(A) $\sqrt{1/3}$

(B) $\sqrt{2/3}$

(C) 1

(D) $\sqrt{3/2}$

(E) $\sqrt{3}$

解：振动模式一中两个物体同相振动，中间的那根弹簧没有作用。振动模式二中两个物体反向振动，它们的质心静止，等效的弹性系数

$$k'=k+2k=3k\text{。}$$

$$\omega_1=\sqrt{\frac{k}{m}},$$

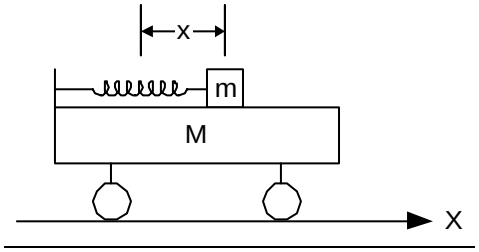
$$\omega_2=\sqrt{\frac{k'}{m}}=\sqrt{\frac{3k}{m}},$$

相比得

$$\frac{\omega_1}{\omega_2}=\frac{1}{\sqrt{3}}\text{。}$$

选（A)。

Questions 93-94



A car of mass M moves horizontally along the X-axis, as shown above. A mass m is attached to the car by a spring having spring constant k. The x-coordinate measures of the position of m; x = 0 corresponds to the position of m in which the spring in neither stretched nor compressed. The axes x and X are parallel.

93. What is the Lagrangian of this system?

(A) $\frac{1}{2}M\dot{X}^2+\frac{1}{2}m\dot{x}^2+\frac{1}{2}kx^2$

(B) $\frac{1}{2}M\dot{X}^2+\frac{1}{2}m\dot{x}^2-\frac{1}{2}kx^2$

(C) $\frac{1}{2}M\dot{X}^2+\frac{1}{2}m(\dot{X}^2+\dot{x}^2)-\frac{1}{2}kx^2$

(D) $\frac{1}{2}M\dot{X}^2+\frac{1}{2}m(\dot{X}^2+2\dot{x}\dot{X}+\dot{x}^2)-\frac{1}{2}kx^2$

(E) $\frac{1}{2}(M+m)(\dot{X}^2+\dot{x}^2)-\frac{1}{2}kx^2$

解：根据整个体系的动量守恒，

$$MX+m(X+x)=\text{const.},$$

$$v_m=-\frac{M\dot{X}}{m}=-(\dot{X}+\dot{x})\text{。}$$

系统的 Lagrange 量

$$L=\frac{1}{2}Mv_M^2+\frac{1}{2}mv_m^2-\frac{1}{2}kx^2\text{。}$$

选（D)。

94. What is ω, the angular frequency of oscillations of m, as determined by the differential equation

$$\ddot{x}+\omega^2x=0$$

(A) $\sqrt{\frac{k}{m}}$

(B) $\sqrt{\frac{k}{M}}$

(C) $\sqrt{\frac{k}{M+m}}$

(D) $\sqrt{\frac{k}{M-m}}$

(E) $\sqrt{\frac{k(M+m)}{Mm}}$

解：由上问

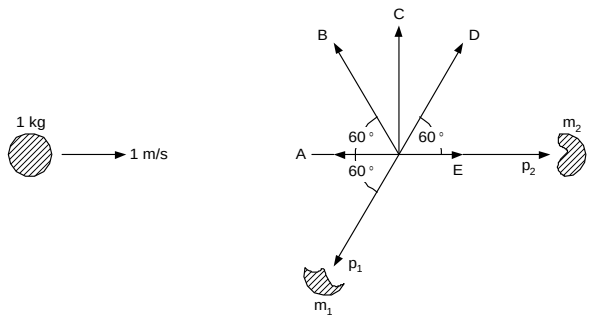
$$v_m = -\left(\dot{X} + \dot{x}\right),$$

列出 Newton 力学方程为

$$\begin{aligned} kx + m\dot{v}_m &= kx - m\left(\ddot{X} + \ddot{x}\right) \\ &= kx - m\left(\ddot{x} - \frac{m}{M+m}\ddot{x}\right) \\ &= kx - \frac{mM}{M+m}\ddot{x} = 0 \end{aligned}$$

$$\omega = \sqrt{\frac{k(M+m)}{Mm}}。$$

选（E）。



95. A body of mass 1 kilogram moves to the right with velocity 1 meter per second, as shown above on the left. The body explodes into three parts: m_1, m_2, and m_3. After the explosion, m_1 and m_2 have momenta p_1 and p_2, both of magnitude 2 kilogram·meter per second, oriented as shown above on the right. If all vectors shown start at the origin and are in the plane of the paper, the momentum of m_3 is represented by the vector

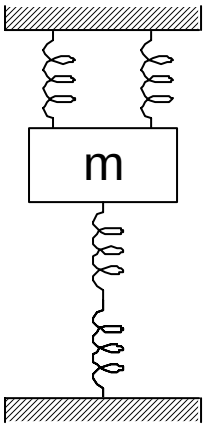
(A) A
(B) B
(C) C
(D) D
(E) E

解：整个爆炸过程前后的动量守恒，写成水平和垂直的分量形式有

$$\begin{aligned} &p_2 - p_1\cos 60° + p_{3x} = p_0 \Rightarrow p_{3x} = 0 \\ &p_{3y} = p_2\cos 60° > 0 \end{aligned}$$

。

选（C）。



96. An object of mass m is connected to two fixed surfaces by four identical springs, as shown above. The springs are of negligible mass and have spring constant k. The period of vertical oscillation is

(A) $2\pi\sqrt{2m/k}$

(B) $2\pi\sqrt{m/k}$

(C) $2\pi\sqrt{2m/5k}$

(D) $\pi\sqrt{m/k}$

(E) $\pi\sqrt{m/2k}$

解：整个体系的等效弹性系数

$$k' = 2k + \frac{k}{2} = \frac{5}{2}k\ ,$$

周期

$$T = 2\pi\sqrt{\frac{m}{k'}} = 2\pi\sqrt{\frac{2m}{5k}}。$$

选（C）。

Questions 96-97

The Lagrangian for a system with generalized coordinate q is

$$L = m\dot{q}^4 - g(q)$$

where $\dot{q} = dq/dt$ (t is the time) and g(q) is an arbitrary function of the coordinate.

96. The canonical momentum conjugate to q is

(A) $m\dot{q}$

(B) $m\dot{q}^3$

(C) $g(q)\dot{q}$

(D) $m\dot{q}^2/3$

(E) $4m\dot{q}^3$

解：广义动量

$$p_q = \frac{\partial L}{\partial \dot{q}} = 4m\dot{q}^3。$$

选（E）。

97. Which of the following is a constant of the movement for this system?

(A) $\frac{1}{2}m\dot{q}^2 + g(q)$

(B) $m\dot{q}^2 + g(q)$

(C) $m\dot{q}^4 + g(q)$

(D) $3m\dot{q}^4 + g(q)$

(E) $8m^2\dot{q}^6 + g(q)$

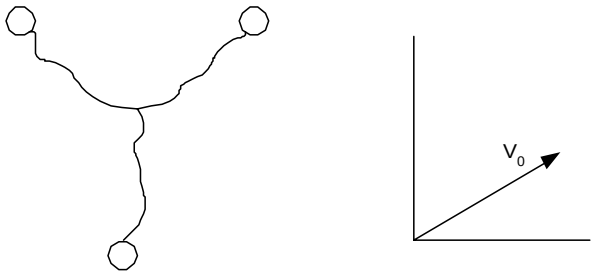
解：因为 L 不显含 t，故能量初积分

$$T + V = 2T - L$$

为常量。

$$\frac{dL}{d\dot{q}}\dot{q} - L = 3m\dot{q}^4 + g(q) = \text{const.}。$$

选（D）。



98. Three balls, each of mass m, are attached to massless rubber cords knotted together, as shown above left. The assembly is then thrown upward so that the initial velocity of the center-of-mass (CM) is as shown above right. The CM speed V has an initial value V_0. Which of the following is NOT true? (KE = kinetic energy)

(A) The CM moves like a projectile of mass 3m under the influence of gravity.

(B) $\text{Total KE} = \frac{3}{2}mV^2 + (\text{KE relative to the CM})$.

(C) The angular momentum of the system about its CM is constant.

(D) KE relative to the CM may change with time.

(E) Potential energy can be stored in the stretched rubber cords.

解：（A）是质心运动定理的结果。总动能等于质心动能加上系统相对于质心的动能，（B）正确。系统受到三个小球的重力的作用，相对于质心的合力矩为零，因此相对于质心的角动量守恒，（C）正确。系统总动能改变量等于重力做功，而重力做功等于质心动能改变量，因此系统相对于质心的动能不变，（D）错。选（D）。

99. The weight of an object on the Moon is 1/6 of its weight on the Earth. A pendulum clock that ticks once per second on the Earth is taken to the Moon. On the Moon the clock would tick once every

(A) 1/6 s

(B) $1/\sqrt{6}$ s

(C) 1 s

(D) $\sqrt{6}$ s

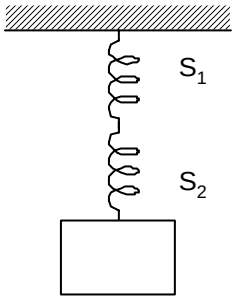
(E) 6 s

解：由单摆周期公式

$$T = 2\pi\sqrt{\frac{l}{g}},$$

$$T_M = \sqrt{\frac{g_E}{g_E}}T_E = \sqrt{6}T_E = \sqrt{6}\text{s}。$$

选（D)。



100. Two springs, S_1 and S_2, have negligible masses and the spring constant of S_1 is 1/3 that of S_2. When a block is hung from the springs as shown above and the springs come to equilibrium again, the ratio of the work done in stretching S_1 to the work done in stretching S_2 is

(A) 1/9

(B) 1/3

(C) 1

(D) 3

(E) 9

解：平衡时两弹簧上拉力均等于重物的重力，即

$$k_1\Delta x_1 = k_2\Delta x_2 = G,$$

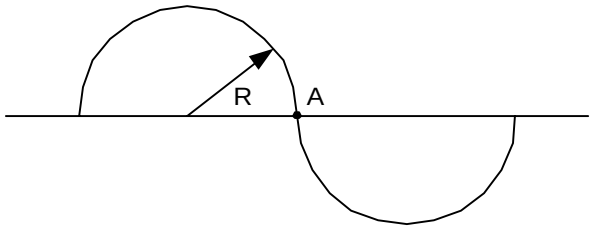
则伸长量之比为

$$\frac{\Delta x_1}{\Delta x_1} = \frac{k_2}{k_1}。$$

所以势能之比为

$$\frac{E_{1p}}{E_{2p}} = \frac{\frac{1}{2}k_1{\Delta x_1}^2}{\frac{1}{2}k_2{\Delta x_2}^2} = \frac{k_1}{k_2}\left(\frac{k_2}{k_1}\right)^2 = \frac{k_2}{k_1} = 3。$$

选（D)。



101. The S-shaped wire shown above has a mass M, and the radius of curvature of each half is R. The moment of inertia about an axis through A and perpendicular to the plane of the paper is

(A) $\frac{1}{2}MR^2$

(B) $\frac{3}{4}MR^2$

(C) MR^2

(D) $\frac{3}{2}MR^2$

(E) $2\,MR^2$

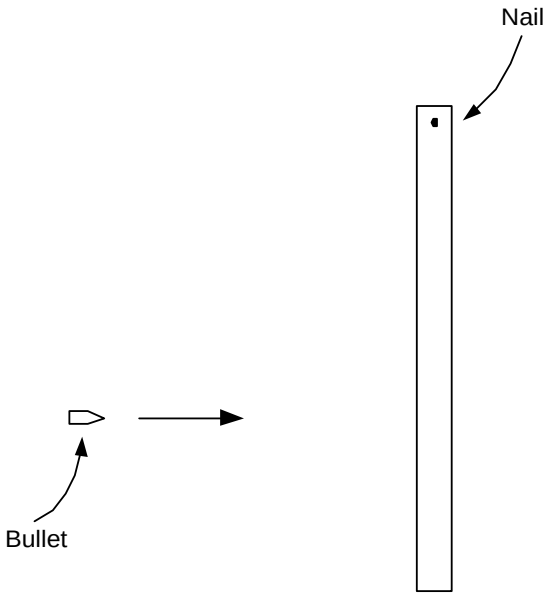
解：圆环相对其中心的转动惯量为 MR^2，由对称性每个半圆环相对其中心的转动惯量为

$$I_0 = \frac{1}{2}MR^2。$$

由平行轴定理，每个半圆环相对于 A 点的转动惯量为

$$I = I_0 + \frac{1}{2}MR^2 = MR^2。$$

所以整体相对于 A 点的转动惯量等于两个半圆环之和，$2MR^2$。选（E)。



102. The speed of rifle bullets is to be determined by using a ballistic pendulum consisting of a plank suspended from a nail, as shown above. If the friction is negligible, for any point of impact, quantities conserved during the collision for the system of the bullet and the plank include which of the following?

I. Linear momentum

II. Angular momentum about the nail

III. Angular momentum about the center of mass

(A) I only

(B) II only

(C) III only

(D) I and II

(E) I and III

解：由于钉子的固定作用，碰撞过程的动量不守恒。相对质心的合外力矩不为零，角动量也不守恒。只有相对钉子的角动量守恒。选（B）。