第七节 狭义相对论

1. A positive kaon (K^+) has a rest mass of 494 MeV/c^2, whereas a proton has a rest mass of 938 MeV/c^2. If a kaon has a total energy that is equal to the proton rest energy, the speed of the kaon is most nearly

(A) 0.25c
(B) 0.40c
(C) 0.55c
(D) 0.70c
(E) 0.85c

解：相对论质量变化公式

$$m=\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}},$$

$$v=\left[1-\left(\frac{m_0}{m}\right)^2\right]c=\left[1-\left(\frac{494}{938}\right)^2\right]c\approx 0.85c。$$

选（E)。

Questions 2-3

In an inertial frame S, a particle has a momentum (P_x, P_y, P_z) = (5, 3, $\sqrt{2}$) MeV/c and a total energy = 10 MeV.

2. The speed u of the particle as measured in a frame S is most nearly

(A) $\frac{3}{8}c$
(B) $\frac{2}{5}c$
(C) $\frac{1}{2}c$
(D) $\frac{3}{5}c$
(E) $\frac{4}{5}c$

解：由相对论公式

$$E^2=m_0{}^2c^4+p^2c^2,$$

静质量为

$$m_0=\frac{\sqrt{E^2/c^2-p^2}}{c}=\sqrt{100-(25+9+2)}/c,$$
$$=8\,\mathrm{MeV/c^2}$$

而其运动质量为

$$m=\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}=10\,\mathrm{MeV/c^2},$$

由此解得$v=\frac{3}{5}c$。选(D)。

3. Which of the following combinations of momentm p' and energy E' could represent the motion of the particle described above as observed in another inertial frame S' moving with an unspecified velocity v relative to s?

(A) $p'=(0,0,8)\mathrm{MeV/c}$,
$E'=\sqrt{128}\mathrm{MeV}$

(B) $p'=(8,0,\sqrt{2})\mathrm{MeV/c}$,
$E'=10\mathrm{MeV}$

(C) $p'=(31,4,6)\mathrm{MeV/c}$,
$E'=\sqrt{949}\mathrm{MeV}$

(D) $p'=(50,-30,\sqrt{200})\mathrm{MeV/c}$,
$E'=100\mathrm{MeV}$

(E) $p'=(100,10,0)\mathrm{MeV/c}$,
$E'=10{,}000\mathrm{MeV}$

解：间隔 $E^2-p^2c^2=m_0{}^2c^4$ 为守恒量，与所选参考系无关，可通过这一条验证只有选项（A）

$E'^2-p'^2c^2=64\,\mathrm{MeV}^2$。选(A)。

4. The percentage increase in the energy of a particle

whose speed is changed from rest to 0.8c (where c is the speed of light) is closest to

(A) 50%

(B) 67%

(C) 75%

(D) 80%

(E) 88%

解：相对论质量变换公式

$$m=\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}=\frac{5}{3}m_0,$$

质能关系

$$E=mc^2=\frac{5}{3}m_0c^2。$$

选（B)。

5. Two spaceships, each measuring 100 meters in length in its own rest frame, pass by each other traveling in opposite directions. Instruments on spaceship I determine that the front end of spaceship I requires $\frac{5}{3}\times10^{-7}$ second to traverse the full length of spaceship II. Given c = 3×10^8 meters per second, what is the magnitude of the relative velocity of the two spaceships?

(A) $\frac{1}{\sqrt{6}}c$

(B) $\frac{1}{2}c$

(C) $\frac{1}{\sqrt{2}}c$

(D) $\frac{2}{\sqrt{5}}c$

(E) $\frac{2}{\sqrt{3}}c$

解：由相对论公式

$$l=l_0\sqrt{1-\frac{v^2}{c^2}}$$

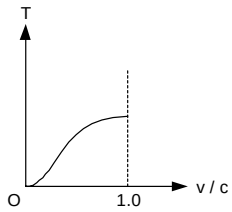
则

$$\frac{l}{v}=\frac{l_0\sqrt{1-\frac{v^2}{c^2}}}{v}=\frac{100\sqrt{1-\frac{v^2}{c^2}}}{v}=t=\frac{5}{3}\times10^{-7}$$

解的 $v=\frac{2}{\sqrt{5}}c$。答案选(D)。

6. An electron has speed v. A plot of the kinetic energy T of the electron versus the ratio $\frac{v}{c}$ would look most like which of the following graphs?

(A)



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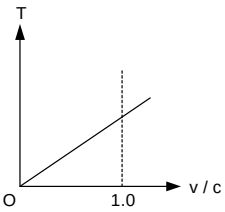

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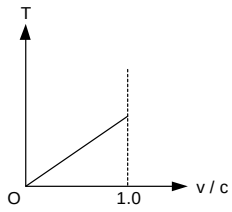
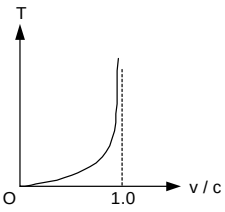
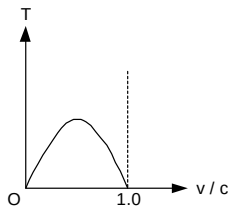


(D)



(E)



解：由相对论质量变换公式

$$m=\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

和质能关系

$$E_{total}=mc^2,$$

得动能为

$$T = E_{total} - E_s = m_0c^2\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}-1\right)。$$

v→∞ 时，$\frac{v}{c}\to 1$，而电子的静质量显然不为 0，所以动能 T→∞。选（D）。

7. Two events are each observed by two observers, S and S', who are in uniform motion with respect to each other. Observer S notes that the two events are separated by 3 seconds, and their locations are 10 meters apart. Observer S' notes that the two events are separated by 5 seconds. The distance between their locations, as seen by S', is closest to

(A) 0 m

(B) 2 m

(C) 10 m

(D) 17 m

(E) 10^9 m

解：二事件的时空间隔

$$\Delta S^2 = c^2\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$$

为协变量，不因参照系的变换而改变。本题中由两事件在 S 和 S'内时空间隔相等得

$$c^2 3^2 - 10^2 = c^2 5^2 - x^2，$$

$$x = 4c = 1.2\times 10^9\,\mathrm{m}。$$

选（E）。

8. The lifetime of a particular meson at rest is 10^{-8} second and its mass is 10^{-25} gram. If its velocity in the laboratory is $2*10^8$ meters per second, how far will it travel in one lifetime, if both distance and lifetime are measured in the laboratory frame?

(A) 10^{-3} m

(B) 2 m

(C) $\sqrt{5}$ m

(D) $\frac{6}{\sqrt{5}}$ m

(E) $\frac{9}{\sqrt{5}}$ m

解：由相对论公式，在实验室系看这个介子的寿命延长为

$$\tau = \frac{\tau_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{3}{\sqrt{5}}\times 10^{-8}\,\mathrm{s}，$$

所以它可通过的距离为

$$l = v\tau = \frac{6}{\sqrt{5}}\,\mathrm{m}。$$

选（D）。

9. A proton has kinetic energy of 500 GeV (1 GeV = 10^9 eV). The momentum of this proton is most nearly

(A) 22 GeV/c

(B) 30 GeV/c

(C) 250 GeV/c

(D) 500 GeV/c

(E) 707 GeV/c

解：质子的静质量为 938MeV/c，静能约为 1GeV。所以题目中质子的动能远大于静能，总能量近似等于动能。由相对论公式

$$E = \sqrt{m_0{}^2c^4 + P^2c^2} \approx Pc，$$

$$P \approx \frac{E}{c} = 500\mathrm{GeV/c}。$$

选（D）。

10. Two observers O and O' observe two events a and B. the observers have a constant relative speed of 0.8c. In units such that the speed of light is 1, observer O obtained the following coordinates:

Event A: x=3, y=3, z=3, t=3

Event B: x=5, y=3, z=1, t=5

What is the length of the space-time interval between these two events, as measured by O'?

(A) 1

(B) $\sqrt{2}$

(C) 2

(D) 3

(E) $2\sqrt{3}$

解：由狭义相对论，事件的时空距离

$$\Delta S^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c^2 \Delta t^2$$

为守恒量，与参考系无关。所以只需要在 O 的参考系中计算即可。选（C）。

11. Which of the following statements most accurately describes how an electromagnetic field behaves under a Lorentz transformation?

(A) The electric field transforms completely into a magnetic field.
(B) If initially there is only an electric field, after the transformation there may be both an electric and a magnetic field.
(C) The electric field is unaltered.
(D) The magnetic field is unaltered.
(E) It cannot be determined unless a gauge transformation is also specified.

解：在相对论下，电磁场被完全统一起来。电磁势φ和 **A** 是同一四维矢量的不同分量，

$$A_\mu = (\mathbf{A}, \frac{i}{c}\varphi),$$

在不同参考系间变换时，可相互转换。选（B）。

12. If a newly discovered particle X moves with a speed equal to the speed of light in vacuum, then which of the following must be true?

(A) The rest mass of X is zero.
(B) The spin of X equals the spin of a photon.
(C) The charge of X is carried on its surface.
(D) X does not spin.
(E) X cannot be detected.

解：由相对论公式

$$E = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} c^2,$$

要使以光速运动的粒子的能量为有限值，则其静质量 m_0 必须为零。选（A）。

Question 13-14

A car of rest length 5 meters passes through a garage of rest length 4 meters. Due to the relativistic Lorentz contraction, the car is only 3 meters long in the garage's rest frame. There are doors on both ends of the garage, which open automatically when the front of the car reaches them and close automatically when the rear passes them. The opening or closing of each doorrequires a negligible amount of time.

13. The velocity of the car in the garage's rest frame is

(A) 0.4c
(B) 0.6c
(C) 0.8c
(D) greater than c
(E) not determinable from the data given

解：由大家熟知的公式，

$$l = l_0 \sqrt{1-\frac{v^2}{c^2}},$$

$$\sqrt{1-\frac{v^2}{c^2}} = \frac{l}{l_0} = \frac{3}{5},$$

解得

$$v = 0.8c。$$

选（C）。

14. The length of the garage in the car's rest frame is

(A) 2.4 m
(B) 4.0 m
(C) 5.0 m
(D) 8.3 m
(E) not determinable from the data given

解：利用上面的公式，且在车库参照系相对汽车参照系速度也为 0.8c，计算得 2.4m。选（A）。

15. Which of the following statements is the best response to the question:

"Was the car ever inside a closed garage?"

(A) No, because the car is longer than the garage in all reference frames.
(B) No, because the Lorentz contraction is not a "real" effect.
(C) Yes, because the car is shorter than the garage in all reference frames.
(D) Yes, because the answer to the question in the

garage's rest frame must apply in all reference frames.

(E) There is no unique answer to the question, as the order of door openings and closings depends on the reference frame.

解：可能有人不敢选择（E），似乎一件事必须有个定论。其实不然，“公说公有理，婆说婆有理”也有可能。在车库参照系中，设后门坐标为 x = 0，且 t = 0 时刻后门开启，则后门关闭的时空坐标为（0，$\frac{3}{0.8c}$），前门开启的时空坐标为（4，$\frac{4}{0.8c}$）。两事件间的时空间隔为

$$\Delta s^2 = c^2\Delta t^2 - \Delta x^2 < 0\,,$$

为类空事件，即二者之间没有因果联系。因此不同参考系之间说法不一并不矛盾。

16. The measured index of refraction of x-rays in rock salt is less than one. This is consistent with the theory of relativity because

(A) relativity deals with light waves traveling in a vacuum only

(B) x-rays cannot transmit signals

(C) x-ray photons have imaginary mass

(D) the theory of relativity predates the development of solid-state physics

(E) the phase velocity and group velocity are different

解：我们常用的公式 $v = \frac{c}{n}$ 中所指速度为相速度 V_p。而相速度超光速是有可能的，因为它不伴随着能量的传播。而群速度 V_g 才表示能量传播的速度，它不能超光速。再记一下两个常用公式

$$V_p = \frac{\omega}{k},\ V_g = \frac{d\omega}{dk}。$$

17. A Σ^0 particle (mass M_1) decays at rest in the laboratory into a Λ^0 particle (mass M_2) and a massless photon. The energy of the Λ^0 particle is

(A) $\frac{M_1}{2}c^2$

(B) $\frac{M_1^2 + M_2^2}{2M_1}c^2$

(C) $\frac{(M_1 + M_2)^2}{2M_1}c^2$

(D) $\frac{M_1^2 - M_2^2}{2M_1}c^2$

(E) $\frac{M_1 - M_2}{2}c^2$

解：设 Λ^0 粒子能量为 E。则光子能量为 $M_1c^2 - E$，动量为 $(M_1c^2 - E)/c$。因为反应前系统动量为零，所以 Λ^0 粒子动量与光子动量大小相等，也为 $(M_1c^2 - E)/c$。代入

$$E = \sqrt{M_2{}^2c^4 + p^2c^2}\,,$$

$$E^2 = M_2{}^2c^4 + \left(M_1c^2 - E\right)^2\,,$$

解得

$$E = \frac{M_2{}^2 + M_1{}^2}{2M_1}c^2。$$

选（B）。

Questions 18-19

A monoenergetic beam consists of unstable particles with energies 100 times their rest energy. The mean life of the particle is 10^{-10} second in their rest frame. IN the laboratory frame, the length of the beam from the point of production to the detector is 6 meters.

18. What fraction of the particles in the beam reach the detector?

(A) e^{-200}

(B) e^{-2}

(C) 0.5

(D) e^{-1}

(E) 1.0

解：由相对论公式

$$m=\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}},$$

对于本题，

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}=100,$$

v 很接近于光速 c。粒子的寿命在实验室中观测为

$$\tau=\frac{\tau}{\sqrt{1-\frac{v^2}{c^2}}}=10^{-8}\text{s}。$$

作为估算，取 v = c，则粒子束长度为

$$l=\tau c=3\text{m}。$$

选（C）。

19. If the particles have rest mass m, their momentum is most nearly

(A) mc

(B) 10 mc

(C) 70 mc

(D) 100 mc

(E) 10^4 mc

解：由于总能量远大于粒子的静止能量，则

$$E=m'c^2=\sqrt{{m_0}^2c^4+P^2c^2}\approx Pc,$$

$$P=\frac{E}{c}=100mc。$$

选（D）。

Questions 20-21

An ion which charge +q travels in the +x direction relative to a right-handed coordinates system. A magnetic field of intensity B webers per square meter is applied in the +y direction.

20. For the ion to be undeflected if its velocity is v = 0.01c, there must also be an electric field E of magnitude in volts per meter and direction equal to

(A) B, in the –y direction

(B) vB, in the –y direction

(C) B, in the +z direction

(D) vB, in the +z direction

(E) vB, in the –z direction

解：电场力应与 Lorentz 力平衡。

$$\vec{E}=-\frac{\vec{F}}{q}=-\frac{q\vec{v}\times\vec{B}}{q}=-vB\vec{k}$$

选（E）。

21. For the ion to be undeflected if its velocity is v = 0.99c, the magnitude in volts per meter of the required electric field E is equal to

(A) B

(B) vB

(C) r$\dfrac{B}{\sqrt{1-\frac{v^2}{c^2}}}$

(D) $\dfrac{vB}{\sqrt{1-\frac{v^2}{c^2}}}$

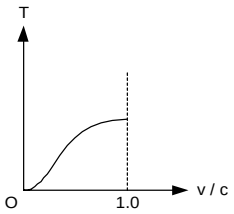
(E) $\dfrac{vB}{\left(1-\frac{v^2}{c^2}\right)}$

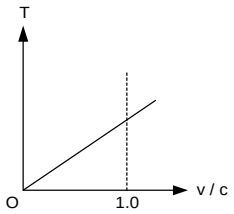
解：与上题类似，带电粒子受的总电场力

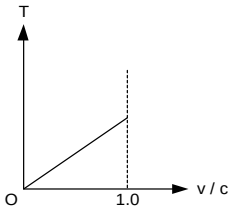
$$\mathbf{F}=q\mathbf{E}+q\mathbf{v}\times\mathbf{B}。$$

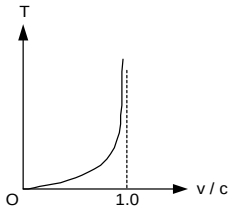
选（B）。

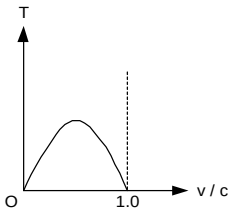
22. An electron has speed v. A plot of the kinetic energy T of the electron versus the ratio $\dfrac{v}{c}$ would look most like which of the following graphs?

(A) 

(B) 

(C) 

(D) 

(E) 

解：根据相对论的能量关系

$$T = mc^2 - m_0c^2 = m_0c^2\left(\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}-1\right),$$

定性上看，当β=0 时，T=0；当β→1 时，T→∞。选（D）。

23. Two events occur at the same place but at different times as measured in an inertial reference frame (IRF) attached to a rocket. The second event occurs t' seconds later than the first. The rocket is moving at a relativistic speed v with respect to a laboratory IRF. What is the distance between the two events, as measured in the laboratory IRF? ($\gamma = 1/\sqrt{1-v^2/c^2}$)

(A) vt'
(B) vt'γ
(C) vt' / γ
(D) v^2t' / c
(E) vt' / γ^2

解：较为简单的解法是利用这两个事件的时空间隔的相对论不变性

$$c^2t^2 - r^2 = c^2t'^2 - r'^2,$$

$$c^2(\gamma t')^2 - r^2 = c^2t'^2。$$

或者可以直接利用 Lorentz 变换式

$$x' = \gamma(x - vt)$$

进行坐标变换。选（B）。

24. A Σ^0 particle (mass M_1) decays at rest in the laboratory into a Λ^0 particle (mass M_2) and a massless photon. The total energy of the Λ^0 particle is

(A) $\frac{M_1}{2}c^2$

(B) $\frac{M_1^2 + M_2^2}{2M_1}c^2$

(C) $\frac{(M_1 + M_2)^2}{2M_1}c^2$

(D) $\frac{M_1^2 - M_2^2}{2M_1}c^2$

(E) $\frac{M_1 - M_2}{2}c^2$

解：利用核反应前后的动量-能量守恒关系

$$p_\gamma + p_2 = 0$$

$$M_1c^2 = p_\gamma c + \sqrt{M_2^2c^4 + p_2^2c^2}$$

解得

$$p_2 = \frac{m_1^2 - m_2^2}{2m_1}c,$$

从而

$$E_2 = \sqrt{M_2^2c^4 + p_2^2c^2}。$$

选（C）。

25. Our galaxy is about 10^5 light-years across. (1 light-year ≅ 10^{16} meters) The most energetic particles known have an energy of about 10^{19} eV. How long

would it take a proton ($mc^2 = 10^9$ eV) of that energy to traverse the galaxy from on the proton?

(A) $\frac{1}{3}\times10^3 s$

(B) $\frac{1}{3}\times10^6 s$

(C) $\frac{1}{3}\times10^9 s$

(D) $\frac{1}{3}\times10^{13} s$

(E) $\frac{1}{3}\times10^{23} s$

解：粒子的 $E >> mc^2$，故它的速度很接近于光速 c。其

$$\gamma = \frac{E}{E_0} = 10^{10}。$$

在粒子的静止系中看来

$$t = \frac{l'}{v} \cong \frac{l}{\gamma c} = \frac{10^5 \times 10^{16}}{10^{10} \times 3 \times 10^8} = \frac{1}{3}\times10^3(s)。$$

选（A）。

26. If the total energy of a particle is exactly twice its rest energy, the speed of the particle is

(A) $\frac{1}{2}c$

(B) $\frac{\sqrt{2}}{2}c$

(C) $\frac{3}{4}c$

(D) $\frac{\sqrt{3}}{2}c$

(E) $\sqrt{2}c$

解：利用相对论性的能量公式

$$E = \frac{E_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = 2E_0$$

不难解得

$$\frac{v}{c} = \frac{\sqrt{3}}{2}。$$

选（D）。

27. A proton has kinetic energy of 500 GeV (1GeV = 10^9 eV). The momentum of this proton is most nearly

(A) 22 GeV/c

(B) 30 GeV/c

(C) 250 GeV/c

(D) 500 GeV/c

(E) 707 GeV/c

解：在 $E >> E_0$ 的条件下，质子的动量

$$p = \frac{\sqrt{E^2 - E_0^2}}{c} \approx \frac{E}{c} = 500(GeV/c)。$$

选（D）。SUB 考试中这样的近似计算很常用，应该熟记。

Questions 28-29

In an inertial frame S, a particle has a momentum $(p_x, p_y, p_z) = (5, 3, \sqrt{2})$ MeV/c and a total energy E = 10 MeV.

28. The speed v of the particle as measured in frame S is most nearly

(A) $\frac{3}{8}c$

(B) $\frac{2}{5}c$

(C) $\frac{1}{2}c$

(D) $\frac{3}{5}c$

(E) $\frac{4}{5}c$

解：质点的总动量

$$p = \sqrt{p_x^2 + p_y^2 + p_z^2} = 6\text{MeV/c}。$$

因此

$$v = \frac{p}{m} = \frac{pc^2}{E} = \frac{3}{5}c。$$

选（D）。

29. Which of the following combinations of momentum p and energy E could represent the motion of the particle described above, as observed in another inertial frame S, moving with an unspecified velocity v, relative to S ?

(A) $p' = (0,0,8) MeV/c$

(B) $p' = (8,0,\sqrt{2}) MeV/c$

(C) $p' = (31,4,6) MeV/c$

(D) $p' = (50,-30,\sqrt{200}) MeV/c$

(E) $p' = (100,10,0) MeV/c$

解：在不同的惯性参照系中，质点的静能

$$E_0 = m_0c^2 = \sqrt{E^2 - p^2c^2}$$

应该不变，等于在 S 系中的值 8 MeV。选（B）。

30. A particle at rest in a laboratory lives for a time τ, as measured in the laboratory, before it decays. How long does this particle live according to observers in a reference frame moving at 0.6c with respect to the laboratory?

(A) 6τ

(B) 0.8τ

(C) τ

(D) $\frac{1}{0.8}\tau$

(E) $\frac{1}{0.6}\tau$

解：根据相对论中的时钟缩短效应

$$\tau' = \gamma\tau = \frac{\tau}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{\tau}{0.8}。$$

选（D）。

31. An explosion occurs at the center of a rocket shio moving at a relativistic speed v with respect to a laboratory inertial reference frame. The length of the rocket, as measured on the rocket, is L. Let t_F and t_R be the respective times at which light from the explosion reaches the front and rear of the rocket ship, where t_F and t_R are measured in the laboratory. If $\beta = v/c$ and $\gamma = \sqrt{1-\beta^2}$, what is $t_F - t_R$?

(A) 0

(B) $\frac{L\beta\gamma}{c}$

(C) $\frac{L\beta\gamma^2}{c}$

(D) $\frac{L\beta^2\gamma^2}{c}$

(E) $\frac{L\beta\gamma^3}{c}$

解：在实验室参照系中看来，火箭的长度为 L’ = L/γ，光讯号的速度为 c 不变。

$$t_F = \frac{L'}{2(c-v)},$$

$$t_R = \frac{L'}{2(c+v)},$$

故

$$t_F - t_R = \frac{L}{2\gamma}\left(\frac{1}{c-v} - \frac{1}{c+v}\right) = \frac{L\beta\gamma}{c}。$$

选（B）。

Questions 32-33

A particle with rest mass m and momentum mc/2 collides with a particle of the same rest mass that is initially at rest. After the collision, the original two particles have disappeared. Two other particles, each with rest mass m’, are observed to leave the region of the collision at equal angles of 30° with respect to the direction of the original moving particle, as shown below.

32. What is the speed of the original moving particle?

(A) $c/5$

(B) $c/3$

(C) $c/\sqrt{7}$

(D) $c/\sqrt{5}$

(E) $c/2$

解：由相对论公式

$$E=\sqrt{m_0{}^2c^4+p^2c^2}\ ,$$

$$m_v=\frac{E}{c^2}=\sqrt{m^2+\frac{p^2}{c^2}}=\frac{\sqrt{5}}{2}m\ ,$$

而

$$m_v=\frac{m}{\sqrt{1-\dfrac{v^2}{c^2}}}\ ,$$

由此解得

$$v=c/\sqrt{5}\ 。$$

选（D）。

33. What is the momentum of each of the two particles produced by the collision?

(A) $mc/5$

(B) $mc/2\sqrt{3}$

(C) $mc/\sqrt{5}$

(D) $mc/2$

(E) $mc/\sqrt{3}$

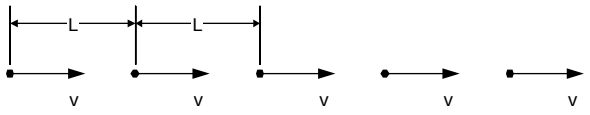
解：设新产成粒子的动量为 P'，由碰撞前后水平动量守恒，有

$$2P'\cos 30^0=mc/2\ ,$$

所以

$$P'=\frac{mc}{4\cos 30^0}=mc/2\sqrt{3}\ 。$$

选（B）。



O

34. A steady stream of identical particle, some of which are shown in the figure above, passes an observer O. As measured by O, the particles move with the same relativistic speed v and are spread along a straight line at equal intervals L. Let $\gamma=1/\sqrt{1-v^2/c^2}$. The number of particles that pass O in unit time is nearly

(A) $\dfrac{v}{L}$

(B) $\dfrac{L}{v}$

(C) $\dfrac{L}{v\gamma}$

(D) $\dfrac{v}{L\gamma}$

(E) $\dfrac{v\gamma}{L}$

解：选（A）。此题与相对论根本无关，不要上当。