Problem Chosen	2020 MCM/ICM Summary Sheet	Team Control Number
A		2010504

Migrating fish, Migrating fishery

--- A Prediction Model for Fish Migration in the Context of Global Warming

Global warming is one of the major challenges today. It triggers a series of global problems, including the rising temperature of ocean. Temperature is a decisive factor for the survival of marine animals. Currently, the increasing temperature is making many species of fish migrate. Such dramatic changes arrest our attention. Our team is hired by a small fishing company in Scotland to cope with various problems caused by the massive migration of their major prey, herring and mackerel.

First, we consult a large amount of data, collecting the recent temperature near Scotland and the corresponding population distribution of two species of fish (**Fig.14,15**). Utilizing the statistics, we build the temperature distribution in position (**Fig.4**) and time (**Fig.3**) respectively with physical reasoning and data fitting; and combine them to obtain the temperature distribution with respect to time and position. After that, we reveal the relationship between the likelihood of inhabiting and temperature with Gaussian function. Then, we integrate these results to build the distribution prediction model which uncovers the distribution of the fish population in regard to time and positions. Using this model, we situate the most likely migrating destinations in latitude and longitude every five years from 2020 to 2070 (**Fig.5**).

Second, based upon the first model, the sustainable development model for small companies is established. According to this model, we can obtain the maximum fishing radius based on the company's own attributes such as the type of the fishing vessel. Then, with the variation range of significant parameters in the first model, the best, worst and most likely case are considered and described by the longest profitable operation time. Next, among the two solutions given, we select the relocation solution by weighing the pros and cons and analyze the selected one in detail. We analyze how daily profit vary with different relocation time and solve the most cost-efficient time (**Fig.8**). At last, it is stressed that hunting the fish outside the nation is illegal; if herring and mackerel enter another country, companies may seek international cooperation to continue fishing them.

Last but not the least, two models are looked into emphatically in the sensibility analysis. The changes in the outcome are observed when vital parameters are adjusted.

We also write an article about our work to the magazine *Hook Line and Sinker*, aiming to help the fishermen benefit from the findings we acquire.

Key words: Migration, Ocean Temperature

CONTENTS

1 Introduction

1.1 Restatement of the Problem

Ocean is home to a wide and diverse variety of species. These species' thriving is dependent on the quality of their habitats. Therefore, if the ocean changes to be unsuitable for a species to continue living, it is a common practice that they abandon present habitats in seek of new and better ones. Such migration may be what the herring and mackerel in Scottish North Atlantic are about to undergo if and when ocean temperatures grow too high for them to live. Currently, because these two species play a vital role in the Scottish fishing industry and the migration hinders small fishing companies to catch them, we are to build a mathematical model to predict the migrating process so as to assist companies in adjusting their operations to the situation.

Specifically, we aim to provide help to the Scottish North Atlantic fishery managers by offering:

- A model to predict the tracks of herring and mackerel over the next 50 years;
- The best and worst situations as well as the most possible time lapsed before small companies have to take action;
- Practical measures that small companies can adopt to continue operating;
- Effects of certain portions of fish's crossing the national borders.

1.2 Background

Since the 19th century, the increasing amount of greenhouse gas has caused the ocean to absorb more than 90% of the anthropogenic heat released in the climate system.[1] Scientists who worked to predict the future climate generally came to the conclusion that the temperature of our planet is going to increase.

As the Earth warms up, the ocean temperatures rise accordingly, which affects the natural habitats of ocean-dwelling fish. Herring and mackerel are two major species that Scottish hunt for food. They both live in shallow water with the temperature of 10℃ approximately. Hence, if the surrounding temperature changes too much, they will migrate to another suitable habitat.

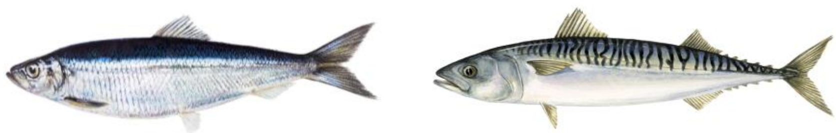

Figure 1. Image of herring (left) and mackerel (right)

Such shift in the population distribution exerts an influence upon the fishing companies' business. Because frozen fish is usually less valuable than fresh fish and not every fishing vessel can afford on-board refrigeration, some companies cannot fish at the spots which are too far from the selling market and thus facing the urgent need for changing.

1.3 Our Approach

To locate the most possible future habitat for the fish in the next 50 years, we need to consider how ocean temperatures will increase and how the habitat changes with the varying temperatures. Hence, there are technically two models. The first one predicts ocean temperatures at different spots over 50 years. The second one generates possible migrating destinations with the temperatures predicted by the first one.

In **3.1**, we obtain the distribution of temperature in time through physical analysis and distribution of temperature in position through data fitting. With two distribution functions and the relationship between them, we combine them into an integrated distribution model. In **3.2**, we firstly obtain the likelihood of inhabiting with respect to temperature. Then, using a rating model we develop, we rate the possible locations and compare them to find the location with the highest score, which is the most likely location. The first problem is solved hitherto.

To solve the second problem, we establish the company sustainable development model (**4.1.1**). We adjust the parameters temperature prediction model, adjusting the concentration of greenhouse gas to the values in the best, worst and most likely cases. According to our assumptions, when the concentration of global carbon dioxide remains steady, the time that company can operate profitably is longest, which is the best case. It is similar in the worst and most possible case.

To solve the third problem, the pros and cons of both options are weighed and the proposal to update the vessel is excluded. Then, the feasibility of first proposal is justified by judging whether relocation helps sustain the company on the long run (**4.2**).

As for the last question, we state that the company can no longer directly capture fish in the territorial waters of other countries due to legal reasons. Companies may continue fishing through foreign trade (**4.3**).

2 Assumptions

2.1 Assumptions with Justifications

To solve the problem more efficiently, we have simplified the situation by making the following assumptions.

1. All the temperature mentioned in the essay is the surface temperature of ocean. We unify the depth because herring and mackerel dwell on the shallow layer of the ocean.
2. The temperature distribution in time is the same in all the positions we consider. We make this assumption because the temperature changes in time vary with different positions on a scale of merely 10^{-2}, which is a neglectable variance.
3. The probability of occurrence for both species follows the Gaussian distribution with the water temperature being the only variable.

This assumption is based on the central limit theorem. The probabilities of occurrence can be considered as the mean of independent identically distributed random variables with finite variance; according to the theorem, the mean approaches a normally distributed random variable as the sample enlarges.

4. The distribution described in assumption 3 does not vary within the time that we take into consideration.
   The time consistency is justified by the fact that it is relatively stable at different periods of time and in various regions, thus possibly the probability distribution is an internal property of a species.
5. The migration of fish is affected only by temperature.
   We do not take other factors into consideration because temperature is the major factor and no other factors are mentioned in the problem.
6. The location of the small company is assumed to be at Aberdeen（57.1706° N,-2.1179° W）. There are also some constants concerning the properties of small fishing company that are set by consulting the data online. The assumed constants are marked with * in **Table2**.
7. The size and the population density of the 2 kinds of fish population remain unchanged in the 50 years.

2.2 Notations

All the variables and constants used in the essay are listed as follows.

Table 1. Variables and its definitions and units

Variable	Definition	Unit
E	Internal energy	J
Q	Heat absorbed by the object	J
W	Work done on the object	J
Q_{net}	Net thermal flux acquired by the sea	$W \cdot m^{-2}$
Q_{atm}	Thermal flux transferred from the atmosphere to sea	$W \cdot m^{-2}$
Q_{sun}	Radiation emitted by the sun	$W \cdot m^{-2}$
Q_{extra}	Extra atmospheric reverse radiation caused by the accumulation of greenhouse gas	$W \cdot m^{-2}$
Q_{lw}	Long-wave radiation emitted by the sea in total	$W \cdot m^{-2}$
Q_{lw_0}	Long-wave radiation emitted by the sea naturally	$W \cdot m^{-2}$
$\widetilde{Q_{lw}}$	Man-made long-wave radiation emitted by the sea	$W \cdot m^{-2}$

H	Sensible heat of the ocean	$W \cdot m^{-2}$
LE	Latent heat of the sea	$W \cdot m^{-2}$
t	Time	year
T	Temperature of the ocean	℃ or K
x	Latitude	degree
y	Longitude	degree
$P(T)$	Probability of inhabiting or occurring	
X, Y	Matrix	
$r(x, y)$	Rating function to evaluate the likelihood to inhabit	
$L(t), L_{1,2}(t)$	Maximum fishing distance/radius with respect to time	km

Table 2. Constants and its definitions and values

Constants	Definition	Unit	Value
σ	Stefan-Boltzmann constant	$W \cdot m^2 \cdot k^{-4}$	5.67×10^{-8}
c, c_k	Concentration of greenhouse gas (in the year k)	ppm	To be determined
c_b	Concentration of greenhouse gas in 1750, also serves as the balanced concentration	ppm	278
γ_1	Coefficient of the thermal flux intercepted by greenhouse gas		To be determined
γ_2	Coefficient of the long-wave radiation on the surface of the sea		To be determined
ω	Coefficient of temperature growth rate		To be determined
T_0	Initial temperature of the ocean	℃	To be determined
μ	Mean of Gaussian function	℃	To be determined
S	Standard variance of Gaussian function		To be determined
a	Peak of Gaussian function		To be determined
k, k_i	Growth rate of temperature over time (in the year k)		To be determined

$*M_{herring}$	Least mass of herring the company needs to catch to sustain	ton	100
$*M_{mackerel}$	Least mass of herring the company needs to catch to sustain	ton	80
$*n$	Vessels the fishing company own		10
$*\lambda$	Times of round trip that a vessel can make from fish to the fishery market when operating 24 hours a day		To be determined
$*m$	Rated load of a fishing vessel	ton	20
$*v$	Velocity of fishing vessels	$km \cdot h^{-1}$	14.816
L	Maximum fishing distance that guarantees sustainable development	km	To be determined
α	Price of mackerel per ton	dollar/ton	800
β	Fuel expense of a vessel per km	dollar/km	3.5
t_{re}	Time of relocation	year	To be determined

3 Model Construction

3.1 The Temperature Prediction Model

To obtain the temperature prediction model, we firstly find $T(t)$ and $T(x,y)$ separately. Then, we conflate them to get the temperature distribution model.

3.1.1 Distribution of Temperature in Time

As temperature is proportional to internal energy, we try to analyze the internal energy of the ocean to help find $T(t)$. According to the first law of thermodynamics, two factors contribute to internal energy: heat absorbed by the object and work done on the object.

$$\Delta E = Q + W \tag{1}$$

In our situation, the net thermal flux on the ocean's surface Q_{net} is Q, and the work done by the internal turbulence inside the ocean is W. Because the latter factor is incapable of warming up the ocean, only the former factor is taken into consideration. Hence, according to the air-sea-thermal flux equation.[3]

$$Q_{net} = Q_{atm} - Q_{lw} - H - LE \tag{2}$$

In the equation,

$$Q_{atm} = Q_{sun} + Q_{extra} \tag{3}$$

$$Q_{lw} = Q_{lw_0} + \widetilde{Q_{lw}} \tag{4}$$

Substitute (3)(4) into (2), we get

$$Q_{net} = Q_{sun} + Q_{extra} - Q_{lw_0} - \widetilde{Q_{lw}} - H - LE \tag{5}$$

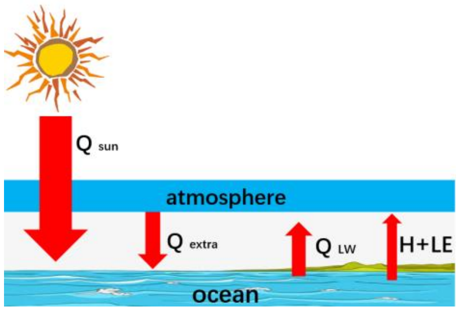

Figure 2. The schematic diagram of thermal flux

Because the temperature and Q_{net} are considered on a yearly scale, we can leave out the variables that fluctuate during the year but cancel out in the whole year. Generally, the net thermal flux stays steady on a yearly basis. In this case, only two variables in (5) are results of contrived activities: Q_{extra} and $\widetilde{Q_{lw}}$. Therefore, all the other four variables cancel out when added up in a year.

$$\int_0^{1yr} (Q_{sun} + Q_{extra} - Q_{lw_0} - \widetilde{Q_{lw}} - H - LE)dt = \int_0^{1yr} (Q_{extra} + \widetilde{Q_{lw}})dt \tag{6}$$

Then, we adopt the Stefan-Boltzmann law with the simplified equation,

$$Q_{net} = Q_{extra} - \widetilde{Q_{Lw}} = \gamma_1 c\sigma T^4 - \gamma_2 \sigma T^4 \tag{7}$$

Based on thermodynamics, temperature is in direct proportion to internal energy.

$$T(t) \propto E(t) = \int_0^t Q_{net}\, dt \tag{8}$$

Derive both T(t) and the integral with respect to time, we get

$$\frac{dT}{dt} = Q_{net} = \gamma_1 \sigma T^4 \left(c - \frac{\gamma_1}{\gamma_2}\right) \tag{9}$$

Note that the scale factor is not included because the constant γ_1 is already an undetermined factor. To simplify, we assume

$$\gamma_1 \sigma = \omega, \quad \frac{\gamma_1}{\gamma_2} = c_b \tag{10}$$

Here, c_b is meaningful, representing the balanced concentration of greenhouse gas. Then the simplified equation is

$$\frac{dT}{dt} = \omega T^4 (c - c_b) \tag{11}$$

To solve this differential equation, we utilize the temperature data from 1891 to 2018 to solve the parameters.[4] However, the parameters are complex numbers. To figure out the reason, we conjecture that the ocean is obviously not an ideal blackbody, thus unable to fit into Stefan-Boltzmann's formula. Therefore, we try to adjust the exponential of T to numbers between 1 and 4, which still keeps T and t positively correlated. We find that the data fits the equation well when the exponential of T is 1. In this case, the solution of the differential equation is

$$T = T_0 e^{\omega(c-c_b)t} \tag{12}$$

The coefficient $\omega(c-c_b)$ is determined to be 0.0054, which is a small enough coefficient to approximate the exponential function to a linear function.

$$T = \omega T_0 (c-c_b)t + T_0 \tag{13}$$

Solve the parameters, we get

$$T(t) = 0.0054t - 10.8379 \tag{14}$$

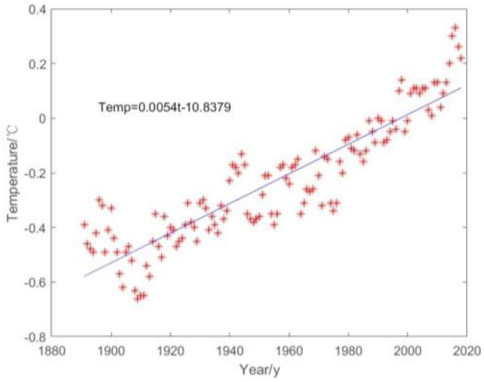

Figure 3. Fitting line of $T(t)$

3.1.2 Distribution of Temperature in Position

After consulting preview research, we find it unpractical to gain the temperature distribution in position through physical and mathematical reasoning derivation. Therefore, we choose to utilize the collected statistics for a polynomial fitting.

In getting the fitting function, we have tried adjusting the degree of two-dimensional polynomial $T(x,y)$. After several attempts, we decide to assume that latitude is quadratic and longitude is linear, because this assumption result in a relatively high coefficient of determination with a relatively low degree. Specifically, the coefficient of determination is 0.938, which is close to the perfect result 1.

Then, we complement the polynomial fitting with collected data, which gives 51 pairs of temperature with its location in latitude x and longitude y. The fitting function is

$$T(x,y) = 10.090 + 0.044x + 1.004y - 0.001x^2 - 0.013xy \tag{15}$$

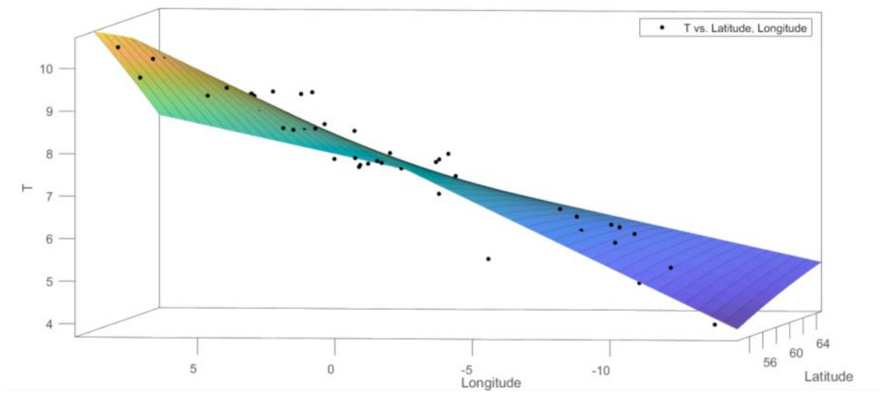

Figure 4. Fitting curve of $T(x,y)$

3.1.3 Distribution of Temperature in Time and Position

According to the assumption 2, the temperature increases by an equivalent amount in different positions. Therefore, we can acquire the temperature function in position and time by adding the temperature increment caused by time to the temperature in a certain position at the initial time.

$$T(x,y,t) = T_x(x,y) + T_t(t) - T_t(1980) \tag{16}$$

We substitute the parameters into (16) and simplify the equation. Then we obtain the distribution of temperature in time and position.

$$T(x,y,t) = -0.602 + 0.005t + 0.044x + 1.004y - 0.001x^2 - 0.013xy \tag{17}$$

3.2 The Migration Prediction Model

3.2.1 The Probability Distribution of Potential Habitats

To obtain the distribution of the probability of occurrence $P(T)$ with respect to T, we formulate their relationship with Gaussian distribution.

$$P(T) = a \cdot e^{-\frac{(T-\mu)^2}{2S^2}} \tag{18}$$

There are three parameters in the function: the curve height a, expectation μ and the half width of curve b are undetermined. To determine the parameters, we collect two sets data: the first set gives physicochemical properties of the waters near Scotland[2]; the second gives the population density distributions of mackerel and herring in the same area[3]. To clarify, the population density represents the $P(occurrence)$ due to the connection between probability and statistics.

From the statistics, we extract 40 pairs of population density and its matched temperature. We choose the data from the first half year in 1980, because the data in 1980 is the latest available ones we can collect and spring and summer are the period when fishermen usually fish.

To estimate the parameters, we adopt the least square estimation method, which is a mathematical optimization technique that can minimize errors and seek the best functional collocation of data.

Firstly, we take the natural log of both sides of equation (1):

$$\ln P(T) = \ln a - \frac{(T-\mu)^2}{2S^2} = \left(\ln a - \frac{\mu^2}{2S^2}\right) + \frac{T\mu}{S^2} - \frac{T^2}{2S^2} \tag{19}$$

Suppose our data $(T_i, P(T_i))(i = 1,2,3,\ldots,20)$ can be described by the Gaussian function and $P(T_i)$ represents the probability of inhabiting. Assume:

$$\ln P(T_i) = \mathrm{y_i} \tag{20}$$

$$ln\, a - \frac{\mu^2}{2S^2} = x_0 \tag{21}$$

$$\frac{\mu}{S^2} = x_1 \tag{22}$$

$$-\frac{1}{2S^2} = x_2 \tag{23}$$

When taking all the data in equation(20)(21)(22) into consideration, equation (20)(21)(22) can be expressed in matrix form.

$$\begin{bmatrix} y_1 \\ \vdots \\ y_{20} \end{bmatrix} = \begin{bmatrix} 1 & T_1 & {T_1}^2 \\ \vdots & \vdots & \vdots \\ 1 & T_{20} & {T_{20}}^2 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} \tag{24}$$

Abbreviated as

$$Y = tX \tag{25}$$

According to least square estimation, the generalized least squares solution of the matrix Y is:

$$X = (t^T t)^{-1} t^T Y \tag{26}$$

Ultimately, we acquire the parameters through calculation.

Table 2. Parameters in Gaussian distribution

		μ/°C	S	a/(°C^2)
Herring	Jan. ~ Mar.	8.873	0.235	0.6483
	Apr. ~ Jun.	9.164	0.05289	0.4145
Mackerel	Jan. ~ Mar.	8.418	0.2546	0.2913

The probability distribution curve can be plotted.(Figure 5,6,7)

3.2.2 The Rating Mechanism of Potential Habitats

To find the most likely locations for the two fish species for the next 50 year, we develop a mechanism to rate quantificationally how likely the locations are for the fish to inhabit.

First, to narrow down the locations we need to rate and compare, we only take the locations with the optimum temperature into consideration, which are the spots on an isotherm. The optimum temperature is the expectation μ in Gaussian distribution in **3.2.1**. To obtain the isotherms over 50 years, we set temperature $T(x, y, t)$ to be the optimum temperature and time set t to be the year we are to investigate.

$$T(x, y, t_0) = \mu \quad (t_0 \text{ is a constant here}) \tag{27}$$

Equation (27) is a quadratic function with respect to x and y, and is the isotherm of the optimum temperature. We calculate 11 isotherms from 2020 to 2070 every 5 years.

Second, from the isotherm, we take a spot every degree in latitude and take a total of 10 spots. These spots are rated by the rating function we build.

$$r(x) = \oiint_D T(x, y, t_0) P\big(T(x, y, t_0)\big)\, dx\, dy \quad D: (x-1)^2 + (y-1)^2 \leq 1 \tag{28}$$

We build our rating function using a double integral because the fish population is located in a range of area instead of one infinitely small point. Hence, the rating function actually evaluates the collective likelihood of an area to the habitat. The integration area is the medium size of the fish population. The integrand describes how much an infinitesimal area contributes to the likelihood, because $T(x, y, t_0)$ is the temperature at the infinitesimal area and $P\big(T(x, y, t_0)\big)$ is the probability of occurrence when temperature equals $T(x, y, t_0)$. The integral operation sums up the likelihood of all infinitesimal areas in the integration area, thus being the likelihood of the integration area.

Afterwards, we complement the rating method to obtain the most likely locations over the 50 years.

Table 3. The most likely positions for herring

Position / Time/year	Latitude	Longitude
2020	58	-0.2465
2025	59	0.8811
2030	59.5	1.2144
2035	59.5	1.4091
2040	59.5	1.53325
2045	59.5	1.52975
2050	59	1.68105
2055	59	1.92655
2060	59.5	2.12545
2065	59.5	2.432
2070	59.5	3.00625

Figure 5. The most likely positions for herring(left) and mackerel(right)

4 Model Applications

4.1 Predictions of Special Cases

In order to predict the best, worst, and most likely scenario, we first formulate the critical condition at which companies can continue operating, expressing the outcome with the farthest distance small companies can travel for fishing (**4.1.1**). Second, we estimate how rapidly ocean temperatures rise based on how rapidly the concentration greenhouse gas rises (**4.1.2**). Finally, we complement the two models and give the three special cases in terms of the longest sustainable operation time (**4.1.3**).

4.1.1 Sustainable Development Model for Small Companies

Suppose that a small company has m medium-sized fishing vessels. Under normal weather, the fishing vessels can carry n tons of fish and travel at maximum speed of v. And, the fishery market is L km away from the fish. We do not take the time of mooring and refueling into consideration; then the fishing vessel can go λ times of round trip from the fish to the market in Scottish fishing ports when operating 24 hours a day.

$$\lambda = \frac{24v}{2L} = \frac{12v}{L} \tag{29}$$

Assume that during the fishing season, the company needs to catch at least $M_{herring}$ tons of herring or $M_{mackerel}$ tons of mackerel every day in order to maintain profitability, which means this is the least amount of harvest companies require to be sustainable. Hence, the critical condition for sustainable operation is

$$n\lambda m \geq min\{M_{herring}, M_{mackerel}\} \tag{30}$$

From equation (29)(30), we obtain the maximum operating radius of sustainable development L.

$$L = \frac{12nmv}{min\{M_{herring}, M_{mackerel}\}} \tag{31}$$

4.1.2 Temperature Growth Rate Model

According to equation (13), the growth rate of temperature over time k is proportional to the concentration increment.

$$k = \omega T_0(c - c_b) \propto (c - c_b) \tag{32}$$

We can tell from equation (32) that the growth rate k increases as the concentration of greenhouse gas c increases. However, this increase in 50 years is actually very small. Hence, we take the mean of the growth rate in 2020 and 2070 as the growth rate in 50 years.

$$\bar{k} = \frac{1}{2}(k_{2020} + k_{2070}) \tag{33}$$

From the fitting function (14), we have determined a set of parameters k_0, c_0, c_b, which can be utilized to calculate $\bar{k}$. Based on the proportional relationship described in formula (32),

$$\bar{k} = \frac{1}{2}k_0\left(\frac{c_{2020} - c_b}{c_{2018} - c_b} + \frac{c_{2070} - c_b}{c_{2018} - c_b}\right) \tag{34}$$

- **The best case.**

In the best case, effective approaches are taken to stop the growth rate from gaining; so the growth rate remains unchanged.

$$\overline{k_{best}} = k_0 \tag{35}$$

This means the original fitting function $T(x, y, t)$ is the temperature distribution for the best case.

- **The worst case.**

In the worst case, the concentration of greenhouse gas c grows with a steady rate of 0.484ppm, which is the concentration growth rate from 1780 to 2018.[6]

$$c_k = c_0 + 0.484(k - 2018) \tag{36}$$

Using the growth rate of concentration, we put equation (33)(34)(36) to solve $\overline{k_{worst}}$.

$$\overline{k_{worst}} = 1.11k_0 = 0.0060 \tag{37}$$

- **The most likely case.**

In the most likely scenario, we take into consideration the policy that countries may adopt against global warming. UNEP(United Nations Environment Programme) has regulated that the growth rate of concentration of greenhouse gas c should reduce by more than 7.6% in every country.[7] Hence, we assume the growth rate of c is 92.4% of the rate last year over the 50 years.

$$\begin{cases} c_{k+1} = c_k + 0.484 \cdot 0.924^{k-2018} \\ c_k = c_{k-1} + 0.484 \cdot 0.924^{k-2017} \\ \quad \dots \dots \\ c_{2019} = c_{2018} + 0.484 \cdot 0.924^0 \end{cases} \tag{38}$$

Put equation (33)(34)(38) to solve $\overline{k_{likely}}$.

$$\overline{k_{likely}} = 1.06k_0 = 0.0057 \tag{39}$$

4.1.3 Analysis of Three Special Cases

First, we complement the development model for small companies to obtain the maximum fishing distance for two species of fish.

- **Herring.**

  If the company only hunt for herring, maximum operating radius of sustainable development is $L_{herring}$. The result can also be expressed by degrees in latitude and longitude.

$$L_{herring} = 365km \Leftrightarrow 3.187^{\circ} \tag{40}$$

  In this case, the largest range for fishing herring is a circular area whose center is the migrating destinations we predict in 3.2 and whose radius is $L_{herring}$. The equation for this circular area is

$$(x - 57.171)^2 + (y + 2.118)^2 \leq 3.187^2 \tag{41}$$

  After substituting the predicted locations (**3.2**) into inequation (41), we find that herring is within the area of fishing in 2020 and is beyond the companies' reach after 2025. To speculate, the actual turning point is between the year 2020 and 2025.

- **Mackerel.**

  If the company only hunt for mackerel, the radius $L_{mackerel}$ and the circular area equation can be calculated similarly.

$$L_{mackerel} = 444km \Leftrightarrow 3.975^{\circ} \tag{42}$$

$$(x - 57.171)^2 + (y + 2.118)^2 \leq 3.975^2 \tag{43}$$

  Again, we substitute the predicted locations (**3.2**) into inequation (41). We discover that mackerel is close enough for fishing during 2020~2050 and is too far to hunt for during 2055~2070. Hence, the turning point is between 2050 and 2055.

Because we have assumed that companies can operate when they can hunt for two or one species of fish, the turning point of operating is the turning point of mackerel, which migrates beyond the fishing zone later.

To find the exact year after which small companies cannot fish, we adopt the model in **3.2** to find the migrating destinations in the four years from 2051 to 2054. And, we discover that 2053 is the year when mackerel just migrate beyond the fishing zone.

Second, we combine the information about the temperature growth rate and the fishing area to get the time when companies can no longer fish in the best, worst and most likely cases.

- **The best case.**

According to equation (35), the growth rate of temperature $\overline{k_{best}}$ is the same as the original parameter k_0 solved in **3.1.1**. Therefore, the time we calculate with the original parameters is the time in the best case, that is to say: 2053 is the year after which small companies cannot fish.

- **The worst case.**

According to equation (37), the growth rate of temperature $\overline{k_{worst}}$ is 1.11 times as much as k_0 in the worst case. Hence, the isotherms are moving 1.11 times faster than those in the best case. As the isotherms move in a parallel direction, the time that mackerel needs to migrate far enough is proportional.

$$2020 + \frac{k_0}{\overline{k_{worst}}}(2053 - 2020) = 2050 \tag{44}$$

- **The most likely case.**

Likewise, the year that is most likely to be the transition is calculated to be 2051.

$$2020 + \frac{k_0}{\overline{k_{likely}}}(2053 - 2020) = 2051 \tag{45}$$

4.2 Strategies for Small Fishing Companies

To propose a solution to the possible consequence of closing down, we suggest the small fishing companies to relocate their assets from the present location to closer to where both herring and mackerel are moving. To justify our proposal, we first illustrate why the other option should not be selected (**4.2.1**). Second, we demonstrate that relocating is an advantageous option with analysis (**4.2.2**).

4.2.1 Analysis for not Updating the Fishing Vessels

In the option of updating vessels, the updated vessels have two advantages: they can operate without land-based support and ensure the freshness and high quality of the hunted fish for a period time. Although this kind of fishing vessels seem greatly useful to most companies, updating vessels cannot fundamentally solve the problem of sustainable development in our model.

The main reason for abandoning this option is that small vessels are not suitable for fishing in remote areas of the ocean based upon two factors. On the one hand, small vessels travel too slowly to fish in distant regions and ensure sufficient fuel at the same time. On the other hand, according to sustainable development model for small companies (**4.1.1**), the operating radius is set to a constant value by the properties of the small company. In other words, even if the vessels could travel beyond that radius, the company still cannot sustain. According to the results in **4.1.3**, the fish will migrate out of the company's maximum operating radius around 2050, after which this measure will be ineffective in sustaining the company.

Hence, we do not suggest this proposal.

4.2.2 Analysis for Relocating

To assess the feasibility of relocating, we quantitatively determine whether relocation can increase the income of fishing companies. Specifically, we choose a harbor city Wick (58.44° N, -3.10° W) in the northeast of Scotland as the relocation position for consideration of being close to where fish migrate.

After inspecting the probable migration positions we predict, we find that mackerel is always closer to the fishing company than herring. Moreover, the market price of the mackerel is about twice that of the herring.[8] Therefore, we suppose that the fishing company only hunt for mackerel to maximize profit in this case.

Based on the analysis in **4.1.1**, the daily fish harvested by the fishing company during the fishing season $\varphi(t)$ can be determined.

$$daily\ harvest = \frac{12nmv}{L(t)} \tag{46}$$

Then, we can get the daily profit by subtracting daily fuel expense from the daily harvest. The daily profit of the fishing company during 2025~2070 can be described with a piecewise function, where $L_1(t)$ (or $L_2(t)$) represents the distance between the fish and Aberdeen (or Wick) starting from 2025 (the company is very close to fish before 2025 thus not being considered).

$$\varphi(t) = \begin{cases} \alpha \cdot \dfrac{12nmv}{L_1(t)} - 2\beta L_1(t), & t \in [0, t_{re} - 1] \\ \alpha \cdot \dfrac{12nmv}{L_2(t)} - 2\beta L_2(t), & t \in [t_{re} + 1, 45] \end{cases} \tag{47}$$

In this function, $L_1(t)$ and $L_2(t)$ are obtained by fitting a polynomial equation with the predicted data in **3.2.2**.

$$L_1(t) = 0.1151t + 0.9309 \tag{48}$$

$$L_2(t) = 0.0021t^2 - 0.028t + 2.9916 \tag{49}$$

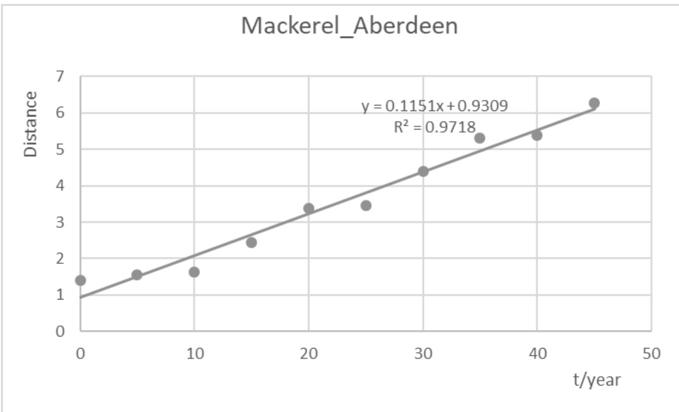

Figure 6.The time function of the distance between mackerel and Aberdeen

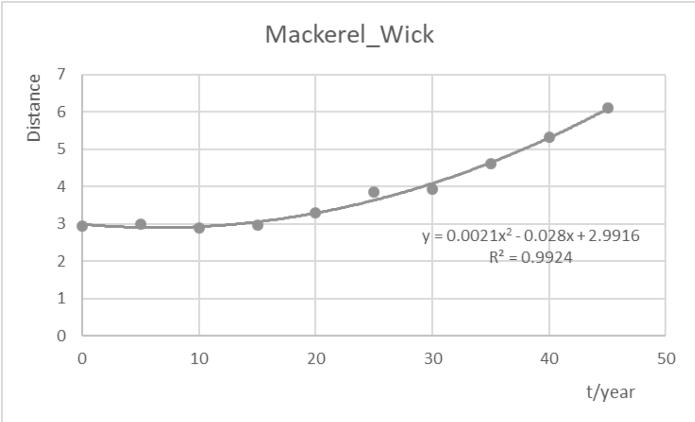

Figure 7. The time function of the distance between mackerel and Wick

Finally, we can calculate the daily average profit φ_{aver} during 2025~2070.

$$\varphi_{aver} = \frac{1}{46}\sum_{0}^{45}\varphi(t) \tag{50}$$

To determine whether it is worthwhile to relocate, the daily average profit φ_{aver} is compared with the relocation expense spread to each day.

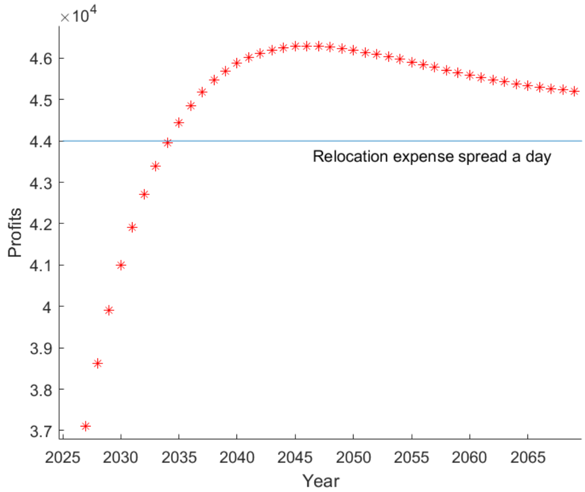

Figure 8. The profits of different relocation time

If the profit is less than relocation expense, the company will not relocate for the cost cannot be recovered. Otherwise, relocation is more cost-efficient. To be more specific, the most economical time for relocation is 2046.

4.3 Effects of the National Border

If herring or mackerel cross the national and enter another country, Scottish fishing companies are not to hunt for the fish outside Scotland but can seek international cooperation opportunities to continue profiting.

- The United Nations Convention on the Law of the Sea stipulates that a country have the right to explore, develop, use, maintain, manage the seabed and subsoil and the natural resources overlying it, and the right to utilize it for artificial facilities, scientific research, and environmental protection in its exclusive economic zone.[9]

According to our predictions in **3.2**, it is very likely that herring and mackerel will enter Norwegian waters around 2070 and 2060 respectively. At that time, it is illegal to fish in other countries.

- Although it is not possible to directly fish, because these two species have a large domestic market, I suggest that the company can cooperate with other fisheries companies to import fresh fish to ensure that the company continues to benefit.

5 Sensitivity Analysis

In reality, the distribution of fish population is affected by many factors, such as temperature, salinity, pressure, food, predators, etc. In this problem, we only consider the effects of their surrounding temperature, predicting the most likely positions for migration mainly with the optimal temperature for inhabiting (**3.2**). Also, in the profit evaluation model (**4.2.2**), the company's profit is closely related to the unit price of the fish among the other factors. Therefore, we conduct the sensitivity analysis studying the impact of different optimal temperature on the migrating locations of a species and the impact of unit price on the profit model.

5.1 Impact of Optimum Temperature

According to **3.2.2**, we can draw a conclusion that fish is most likely to be on an isotherm of its optimum temperature. When different fish species with different optimal temperature are considered, they are probably distributed on approximately parallel isotherms.

When the optimal temperature changes from 7.618℃ to 9.218℃, longitude varies by 6 degrees or so and latitude ranges from 4 to 10 degrees, which shows the most likely positions for migrating, which locate on the isotherms, have a strong correlation with the species' optimum temperature.

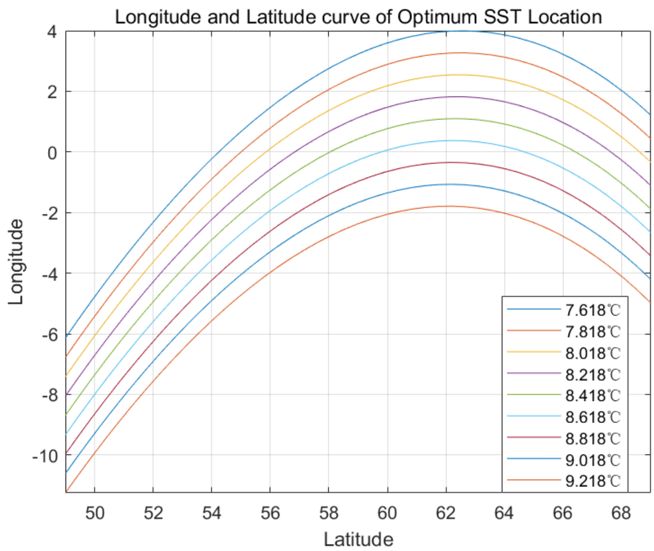

Figure 9. Sensitivity analysis on the optimum temperature of mackerel

5.2 Impact of the Unit Price

We assume that the company makes profit only by selling fish, and the amount of fish the company can catch depends on the distance between the fish and the port. Then, the profit in time is calculated while the unit price takes different values. The scatter diagram of profit is plotted and shown in **Figure 8**.

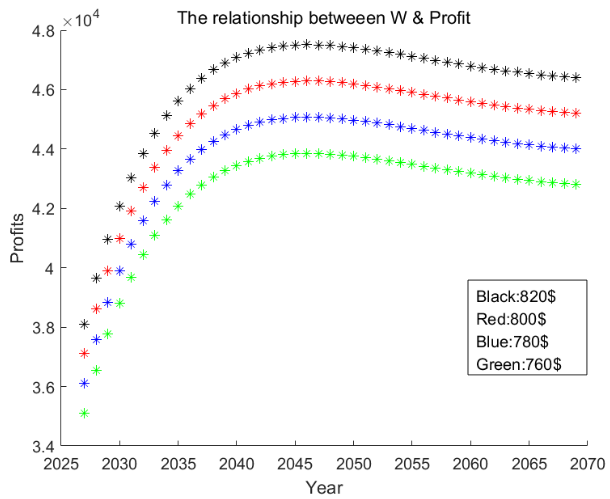

Figure 10. Sensitivity analysis on the unit price of mackerel

It can be seen that as the unit price increases, the profit of company increases greatly and rapidly. And we can also conclude that when price increases, the time when company get maximum profit is put off. It can be summarized that the change of unit price can not only affect the size of profit, but also affect the trend of profit with the year.

6 Conclusions

6.1 Strengths and Weaknesses

6.1.1 Strengths

- All the three problems are solved with detailed analysis, either by statistical methods or by theoretical reasoning.
- The temperature distribution model in time is based on extensive data from Japan Meteorological Agency. This model can predict temperature changes over time not only in the ocean around Scotland, but in any sea area in theory.
- Our models include a large number of parameters. These parameters include the global concentration of greenhouse gas, fishing companies' operating radius, the price of mackerel and herring, speed and load of medium size fishing vessels and the price of fuel for fishing boats. All of the parameters have an actual meaning in reality and have a wide range of application other than this problem.

6.1.2 Weaknesses

- The migration of fish is affected by much more than one factor. But we do not take any factor other than temperature into account.
- Due to the limitations of the algorithm and the computing power, we cannot utilize

large amount of data to compute more accurate results. Especially in 3.1, the solution needs a powerful computer to make complicated analysis and calculation. If granted better conditions, we can find the migrating positions with higher accuracy in coordinates and time.

- A portion of our formulas are complex and may be simplified to a neater form.

6.2 Future work

- We can improve the model by collecting more data to make the prediction more accurate.
- The algorithm we use can be improved to deal with larger quantities of data with less computing power.

7 Article for *Hook Line and Sinker*

Migrating Fish, Migrating Fishery

Global warming does harm to human beings in terms of triggering natural catastrophes, melting downing the glaciers, threatening species with extinction and so on. Beyond these widely known hazards, global warming also influences we fishermen directly by causing our major prey to migrate. To face the upcoming changes, the fishery industry is supposed to “migrate” in accordance with the migration of fish. Our team has stimulated the situation and come up with some solutions.

Nowadays, there are fishing companies’ complaint that they harvest less than before with the same fishing operation. To explain the underlying mechanism, we are enlightened by the phenomena that the lobster in Maine, USA is gradually migrating north to Canada, because it keeps seeking a new habitat with suitable water temperatures when the current habitat warms up excessively. To identify the future migrating track of herring and mackerel, we build a mathematical model to predict the temperature in the fishing region and anticipate the fish’s most likely location over 50 years. First of all, through the investigation of regional geographical differences, we described a law of ocean temperature changing with latitude and longitude, and explained the current trend of ocean warming based on the law of thermal radiation. Secondly, considering the enormous impact of the greenhouse effect on global warming, we introduced the concentrations of carbon dioxide into the model and analyzed three possibilities for future ocean temperature changes. Then we used the data from previous years to determine the pattern of movement of fish in response to changes in ocean temperature, and predicted where they would most likely be in the next 50 years. Finally, by analyzing the input-output problem, we provided a solution to the dilemma that small fishing companies may face in the future.

Based upon the predictions generated by our model, the migration strongly urges the smaller fishing companies to take an approach, because once the fish get too far away from the harbor, the single fishing cycle is stretched. This will directly result in the fishery company's daily income cannot support the company's going concerns. More specifically, all the small fishing companies that depend on catching herring and mackerel risk going out of business in 50 years. It seems incredible, but by our calculations it is a fact that is happening. The fish are now moving to the open sea at a rate of about 12 kilometers per year, and in 50 years the increased distance will be closer to 100 kilometers, making it impossible for small fishing boats to reach the fish.

Therefore, we have evaluated some practical changes that companies can do to adapt to migration economically. We recommend that, after 2025, all small fishing companies should consider relocating to the North-East coast of Scotland. Between 2060 and 2070, these companies could consider forming a joint company to keep their profit margin. Here we do not recommend that small fishing companies buy small vessels which are temporarily capable of operating without land-based support. The reason is that when the fish are more than a certain distance from the harbor, those fishing companies will still face the situation where they cannot make ends meet due to the prolonged fishing cycle.

Our plan will help small fishing companies continue to make money for 50 years. The fishing industry will continue to thrive in Scotland if smaller companies are willing to form coalitions. This will give fisheries companies time to adjust their business strategies, while increasing their contribution to employment in Scotland.

It is worth noting that according to the model, we found that there is an optimal solution for the time when the companies chooses to relocate. As a consequence of that, in order to get the maximize profits, companies can make the best choice based on our model

Apart from the discussion above, based on our prediction, we found that herring and mackerel could enter the territorial waters of certain European countries around 2065, so we also suggested that small fishing companies could form joint ventures to cooperate with other fishing companies from in order to obtain longer term profits.

We sincerely hope that our theoretical analysis will actually benefit our readers.

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[9] https://en.wikipedia.org/wiki/United_Nations_Convention_on_the_Law_of_the_Sea

Appendix

Some figures and tables are listed here for the consideration of space.

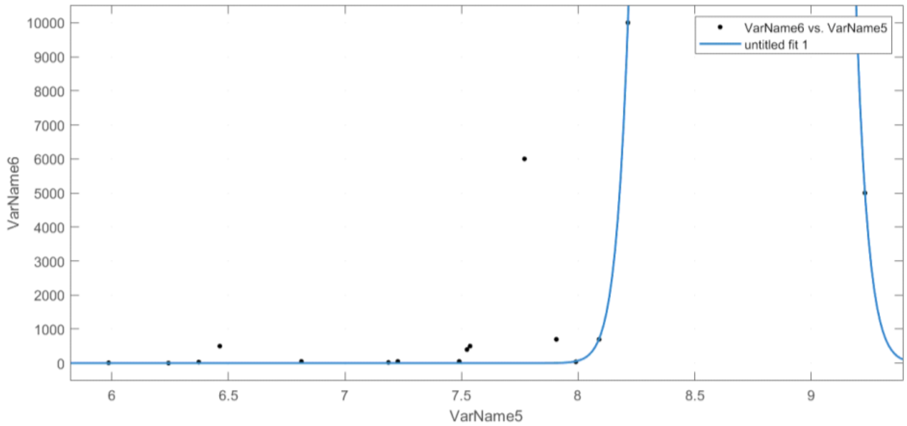

Figure 11. Gaussian distribution of herring during Jan. ~ Mar.

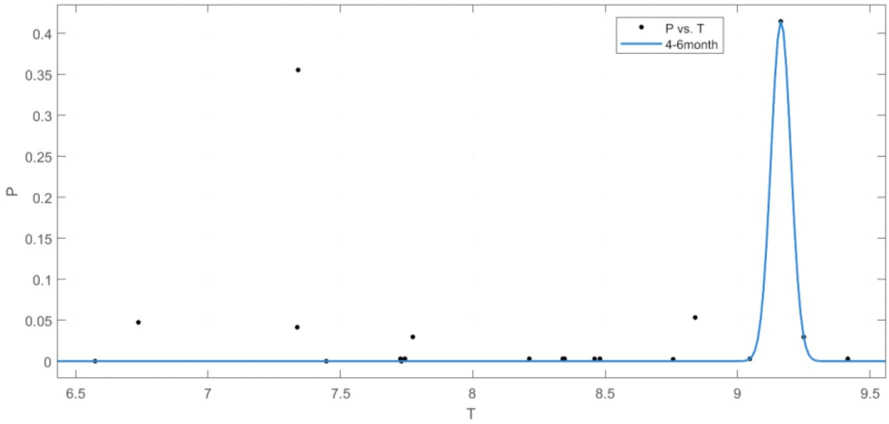

Figure 12. Gaussian distribution of herring during Apr. ~ Jun.

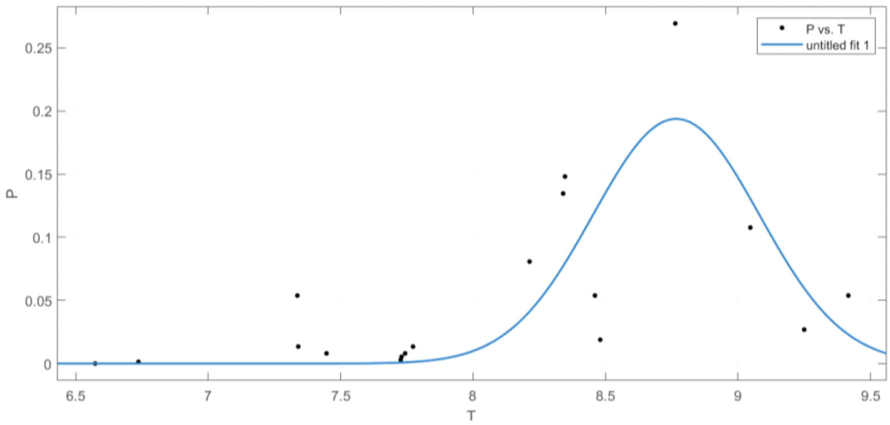

Figure 13. Gaussian distribution of mackerel during Jan. ~ Mar.

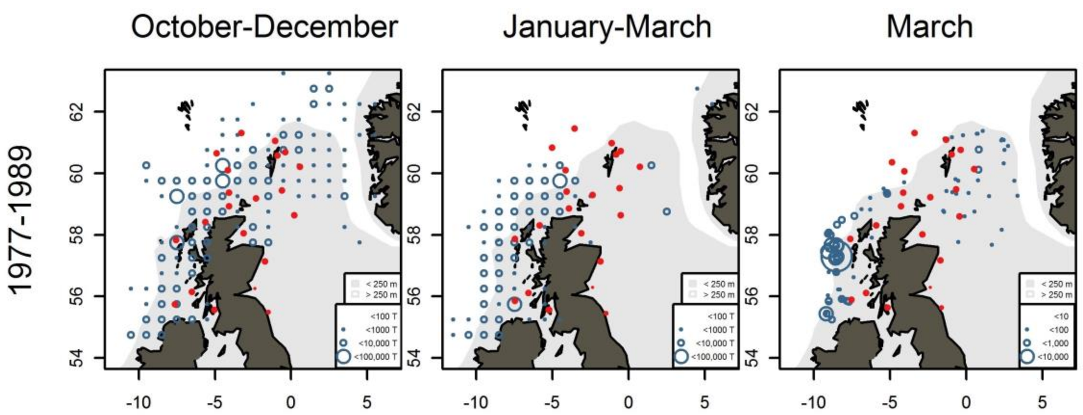

Figure 14. The distribution of Herring

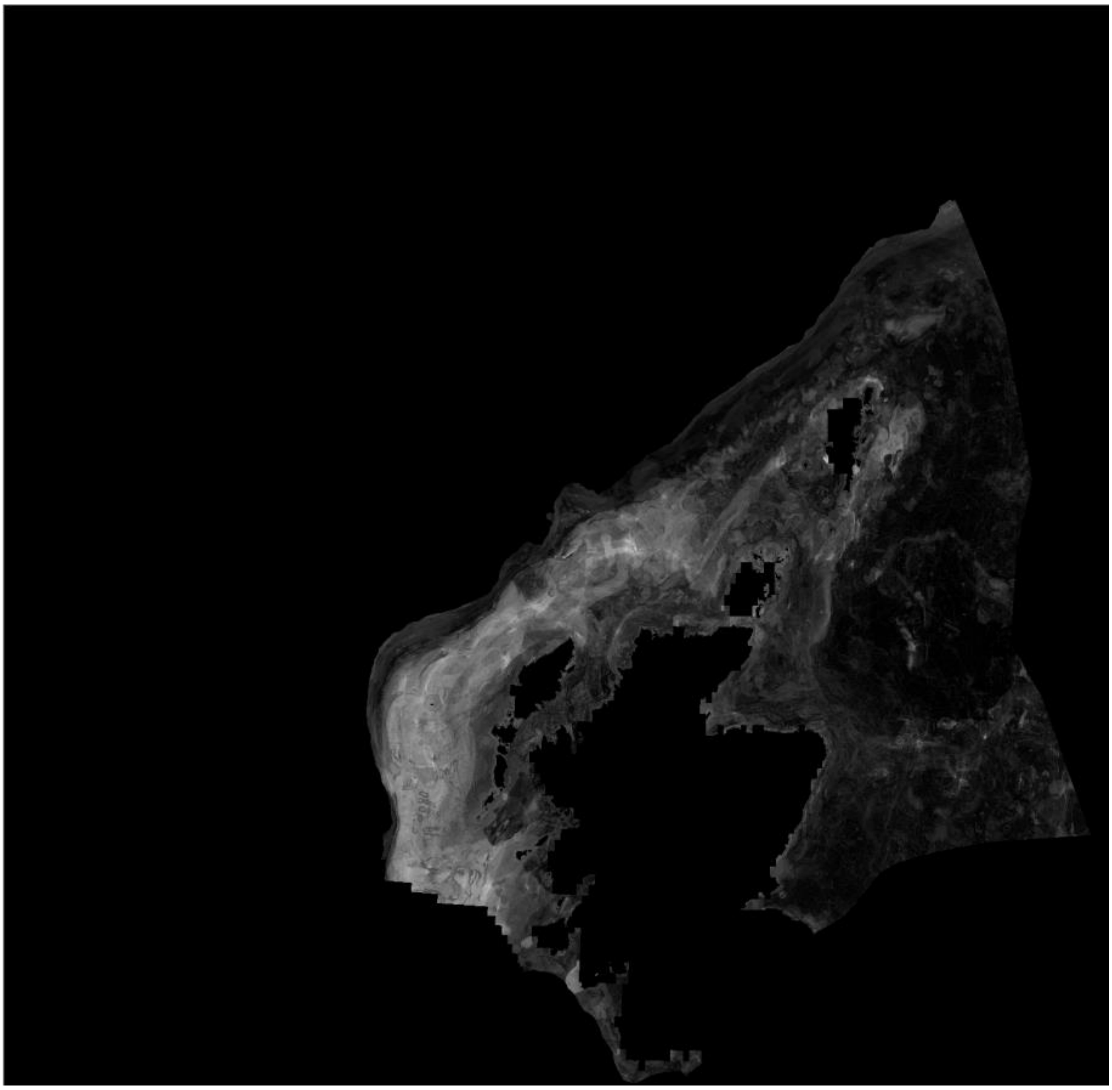

Figure 15. The distribution of Mackerel

Table 4. The data sheet of herring distribution between 2020-2070

Herring_1-3 month											
2020Y			2025Y			2030Y			2035Y		
Latitude	Longitude	Contribution	Latitude	Longitude	Contribution	Latitude	Longitude	Contribution	Latitude	Longitude	Contribution
53	-6.52	0.0128002	53	-6.43	0.0128	53	-6.34	0.0127998	53	-6.25	0.0127996
54	-5.57	0.0129218	54	-5.48	0.0129218	54	-5.39	0.0129218	54	-5.3	0.0129217
55	-4.72	0.0130446	55	-4.63	0.0130447	55	-4.54	0.0130448	55	-4.45	0.0130448
56	-3.97	0.0131735	56	-3.88	0.013174	56	-3.79	0.0131745	56	-3.7	0.013175
57	-3.33	0.01332	57	-3.24	0.013322	57	-3.14	0.0133162	57	-3.05	0.013318
58	-2.8	0.0135041	58	-2.7	0.0134926	58	-2.6	0.0134824	58	-2.51	0.0134873
59	-2.67	0.0137084	59	-2.27	0.0136927	59	-2.18	0.0137042	59	-2.08	0.0136888
60	-2.46	0.0139451	60	-1.96	0.0139278	60	-1.87	0.0139471	60	-1.77	0.0139297
61	-2.16	0.0141125	61	-1.77	0.0141235	61	-1.67	0.0141118	61	-1.57	0.0141002
62	-1.79	0.0141082	62	-1.7	0.0141297	62	-1.6	0.0141283	62	-1.5	0.0141268
2040Y			2045Y			2050Y			2055Y		
Latitude	Longitude	Contribution	Latitude	Longitude	Contribution	Latitude	Longitude	Contribution	Latitude	Longitude	Contribution
53	-6.16	0.0127994	53	-6.07	0.0128088	53	-5.98	0.0128481	53	-5.89	0.0130945
54	-5.2	0.0129209	54	-5.12	0.0129358	54	-5.03	0.0130412	54	-4.94	0.0135428
55	-4.36	0.0130449	55	-4.26	0.0130753	55	-4.17	0.0132832	55	-4.08	0.014053
56	-3.6	0.0131725	56	-3.51	0.0132546	56	-3.42	0.0136642	56	-3.33	0.014744
57	-2.96	0.0133198	57	-2.87	0.0135198	57	-2.77	0.0141126	57	-2.68	0.0152769
58	-2.42	0.0134926	58	-2.33	0.0138411	58	-2.23	0.0145938	58	-2.14	0.0155275
59	-1.99	0.0137001	59	-1.9	0.0141862	59	-1.8	0.0149123	59	-1.71	0.0153513
60	-1.68	0.0139491	60	-1.58	0.0144203	60	-1.48	0.0149106	60	-1.39	0.0148938
61	-1.49	0.0141554	61	-1.38	0.0144611	61	-1.29	0.0146257	61	-1.19	0.0144007
62	-1.4	0.0141251	62	-1.3	0.0142357	62	-1.21	0.0142158	62	-1.11	0.0140966
2060Y			2065Y			2070Y					
Latitude	Longitude	Contribution	Latitude	Longitude	Contribution	Latitude	Longitude	Contribution			
53	-5.8	0.0130894	53	-5.71	0.0130845	53	-5.62	0.0130796			
54	-4.85	0.0135418	54	-4.76	0.0135408	54	-4.67	0.0135398			
55	-4	0.0141969	55	-3.9	0.0140723	55	-3.81	0.014082			
56	-3.24	0.0147682	56	-3.14	0.0146449	56	-3.05	0.0146685			
57	-2.59	0.0153089	57	-2.49	0.0152156	57	-2.4	0.0152482			
58	-2.04	0.0153354	58	-1.95	0.0155113	58	-1.66	0.015536			
59	-1.61	0.0153555	59	-1.52	0.0153593	59	-1.42	0.0153625			
60	-1.3	0.0148768	60	-1.2	0.0149006	60	-1.1	0.0149229			
61	-1.09	0.0144185	61	-0.99	0.0144363	61	-0.9	0.0144171			
62	-1.01	0.0141027	62	-0.91	0.014109	62	-0.82	0.0141012			
Herring_4-6 month											
2020Y			2025Y			2030Y			2035Y		
Latitude	Longitude	Contribution	Latitude	Longitude	Contribution	Latitude	Longitude	Contribution	Latitude	Longitude	Contribution
53	0.9437	0.0127254	53	1.853	0.0127285	53	1.7622	0.0127316	53	1.6715	0.0127345
54	1.4239	0.0132076	54	2.3289	0.0132148	54	2.2339	0.0132217	54	2.1388	0.0132287
55	1.667	0.0138954	55	2.8623	0.0136431	55	2.7626	0.0136577	55	2.6629	0.0136719
56	2.962	0.0136281	56	3.4622	0.0139237	56	3.3573	0.0139524	56	3.2525	0.0139792
57	4.2499	0.0138704	57	4.1394	0.0139202	57	4.0288	0.0139708	57	4.3182	0.0140202
58	5.0243	0.0133727	58	4.9073	0.0134568	58	4.7904	0.0135367	58	4.6734	0.0136178
59	5.9068	0.0122353	59	5.7827	0.0123553	59	5.6586	0.012474	59	5.5344	0.0125949
60	6.9187	0.0103755	60	6.7865	0.0105309	60	6.6543	0.0106857	60	6.522	0.010844
61	8.0871	0.00791109	61	7.9456	0.00808847	61	7.8042	0.00826268	61	7.6627	0.00844183
62	9.4472	0.00521403	62	9.2952	0.00537723	62	9.1431	0.0055465	62	8.991	0.00571845
2040Y			2045Y			2050Y			2055Y		
Latitude	Longitude	Contribution	Latitude	Longitude	Contribution	Latitude	Longitude	Contribution	Latitude	Longitude	Contribution
53	1.5807	0.0127761	53	1.4899	0.0127932	53	1.3992	0.0127974	53	1.3084	0.0128011
54	2.0438	0.0133218	54	1.9488	0.0133639	54	1.8538	0.013375	54	1.7588	0.0133856
55	2.5632	0.0138668	55	2.4635	0.0139638	55	2.3639	0.0139913	55	2.2642	0.0140194
56	3.1476	0.0143579	56	3.0428	0.0145653	56	2.9379	0.0146295	56	2.8331	0.0146987
57	3.8077	0.0146955	57	3.6971	0.0151067	57	3.5865	0.0152447	57	3.476	0.0154039
58	4.5565	0.0147224	58	4.4395	0.0154694	58	4.3226	0.015742	58	4.2056	0.0160817
59	5.4103	0.0142283	59	5.2862	0.0154596	59	5.1621	0.0159526	59	5.038	0.0166144
60	6.3898	0.0129946	60	6.2576	0.0148163	60	6.1254	0.0156183	60	5.9931	0.0167885
61	7.5213	0.0109101	61	7.3798	0.0132821	61	7.2384	0.0144345	61	7.0969	0.0162725
62	8.8389	0.00812155	62	8.6869	0.0107546	62	8.5348	0.0121862	62	8.3827	0.0146935
2060Y			2065Y			2070Y					
Latitude	Longitude	Contribution	Latitude	Longitude	Contribution	Latitude	Longitude	Contribution			
53	1.2177	0.012802	53	1.1269	0.0128031	53	1.0362	0.0128031			
54	1.6638	0.0133882	54	1.5688	0.0133909	54	1.4738	0.013391			
55	2.1645	0.0140266	55	2.0648	0.014034	55	1.9651	0.0140343			
56	2.7282	0.0147179	56	2.6234	0.0147378	56	2.5185	0.0147385			
57	3.3654	0.0154518	57	3.2548	0.0155033	57	3.1442	0.0155052			
58	4.0886	0.0161926	58	3.9717	0.0163182	58	3.8547	0.0163234			
59	4.9138	0.0168511	59	4.7897	0.0171374	59	4.6656	0.0171503			
60	5.8609	0.0172459	60	5.7287	0.0178451	60	5.5965	0.0178755			
61	6.9554	0.0170688	61	6.814	0.0182	61	7.6725	0.0182665			
62	8.2306	0.0159032	62	8.0785	0.0177922	62	7.9265	0.0179163			

The most likely locations		
	Latitude	Longitude
2020Y	58	-0.2465
2025Y	59	0.8811
2030Y	59.5	1.2144
2035Y	59.5	1.4091
2040Y	59.5	1.53325
2045Y	59.5	1.52975
2050Y	59	1.68105
2055Y	59	1.92655
2060Y	59.5	2.12545
2065Y	59.5	2.432
2070Y	59.5	3.00625

Table 5. The data sheet of mackerel distribution between 2020-2070

Mackerel_1-3 month											
2020Y			2025Y			2030Y			2035Y		
Latitude	Longitude	Contribution	Latitude	Longitude	Contribution	Latitude	Longitude	Contribution	Latitude	Longitude	Contribution
53	-4.0489	0.0477978	53	-3.9595	0.0478007	53	-3.8702	0.0476858	53	-3.7809	0.047624
54	-3.0777	0.0478637	54	-2.9875	0.0478669	54	-2.8973	0.0477716	54	-2.8072	0.0477206
55	-2.205	0.0479012	55	-2.114	0.0479047	55	-2.023	0.0478267	55	-1.932	0.047785
56	-1.4336	0.0479134	56	-1.3418	0.0479171	56	-1.2499	0.0478539	56	-1.1581	0.0478205
57	-0.7665	0.047903	57	-0.6737	0.0479067	57	-0.581	0.0478563	57	-0.4883	0.0478299
58	-0.2065	0.0478723	58	-0.1129	0.0478763	58	-0.0193	0.0478366	58	0.0743	0.0478161
59	0.2433	0.0478235	59	0.3378	0.0478276	59	0.4323	0.0477969	59	0.5268	0.0477815
60	0.5795	0.0477588	60	0.675	0.047763	60	0.7704	0.0477398	60	0.8658	0.0477285
61	0.799	0.0476797	61	0.8954	0.0476841	61	0.9917	0.047667	61	1.0881	0.0476591
62	0.8982	0.0475881	62	0.9955	0.0475925	62	1.0928	0.0475803	62	1.1901	0.0475752
2040Y			2045Y			2050Y			2055Y		
Latitude	Longitude	Contribution	Latitude	Longitude	Contribution	Latitude	Longitude	Contribution	Latitude	Longitude	Contribution
53	-3.6915	0.0474069	53	-3.6022	0.0470503	53	-3.5128	0.0468399	53	-3.4235	0.0464783
54	-2.717	0.0475387	54	-2.6268	0.0472363	54	-2.5367	0.0470576	54	-2.4465	0.0467466
55	-1.841	0.0476341	55	-1.75	0.0473805	55	-1.659	0.0472295	55	-1.568	0.0469652
56	-1.0662	0.0476964	56	-0.9744	0.0474859	56	-0.8825	0.0473596	56	-0.7907	0.0471371
57	-0.3956	0.0477292	57	-0.3029	0.047556	57	-0.2102	0.0474519	57	-0.1175	0.0472662
58	0.1679	0.0477353	58	0.2615	0.0475944	58	0.3551	0.0475094	58	0.4487	0.0473561
59	0.6213	0.0477174	59	0.7158	0.0476041	59	0.8102	0.0475357	59	0.9047	0.0474107
60	0.9612	0.0476785	60	1.0566	0.0475886	60	1.152	0.0475341	60	1.2474	0.0474333
61	1.1844	0.0476208	61	1.2808	0.0475503	61	1.3771	0.0475077	61	1.4735	0.0474274
62	1.2874	0.0475463	62	1.3847	0.047492	62	1.482	0.0474593	62	1.5793	0.0473964
2060Y			2065Y			2070Y					
Latitude	Longitude	Contribution	Latitude	Longitude	Contribution	Latitude	Longitude	Contribution			
53	-3.3341	0.0460555	53	-3.2448	0.0462048	53	-3.1555	0.0457385			
54	-2.3564	0.0463815	54	-2.2662	0.0465112	54	-2.176	0.0461048			
55	-1.477	0.0466517	55	-1.386	0.0467642	55	-1.295	0.0464139			
56	-0.6989	0.046871	56	-0.607	0.0469675	56	-0.5152	0.0466686			
57	-0.0248	0.0470423	57	0.068	0.0471245	57	0.1607	0.0468715			
58	0.5423	0.0471696	58	0.6359	0.0472395	58	0.7295	0.0470276			
59	0.9992	0.0472572	59	1.0937	0.047316	59	1.1882	0.0471405			
60	1.3429	0.0473082	60	1.4383	0.0473574	60	1.5337	0.0472136			
61	1.5698	0.0473271	61	1.6661	0.0473679	61	1.7625	0.0472515			
62	1.6766	0.0473168	62	1.7739	0.0473505	62	1.8712	0.0472576			

The most likely locations		
	Latitude	Longitude
2020Y	56	-1.4336
2025Y	56	-1.3418
2030Y	57	-0.581
2035Y	57	-0.4883
2040Y	58	0.1679
2045Y	59	0.7158
2050Y	59	0.8102
2055Y	60	1.2474
2060Y	61	1.5698
2065Y	61	1.6661
2070Y	62	1.8712

Table 6. The data sheet of mackerel distribution between 2051-2054

mackerel_1-3month								
	2051Y		2052Y		2053Y		2054Y	
Latitude	Longitude	Contribution	Longitude	Contribution	Longitude	Contribution	Longitude	Contribution
57	-0.2102	0.0475592	-0.1916	0.0475077	-0.1731	0.0473941	-0.1546	0.047332
58	0.3551	0.0475978	0.3738	0.0475558	0.3925	0.047462	0.4113	0.0474103
59	0.8102	0.0476079	0.8291	0.0475738	0.848	0.0474972	0.8669	0.0474551
60	1.152	0.0475925	1.1711	0.0475651	1.1902	0.0475031	1.2093	0.0474691