第五节 量子力学

1. The wave function $\psi(x) = A\exp\{-\frac{b^2x^2}{2}\}$, where A and b are real constants, is a normalized eigenfunction of the Schrödinger equation for a particle of mass M and energy E in a one dimensional potential V(x) such that V(x) = 0 at x = 0. Which of the following is correct?

(A) $V = \frac{\hbar^2 b^4}{2M}$

(B) $V = \frac{\hbar^2 b^4 x^2}{2M}$

(C) $V = \frac{\hbar^2 b^6 x^4}{2M}$

(D) $E = \hbar^2 b^2 \left(1 - b^2 x^2\right)$

(E) $E = \frac{\hbar^2 b^4}{2M}$

解：Schrödinger 方程

$$-\frac{\hbar^2}{2M}\frac{d^2}{dx^2}\psi = (E-V)\psi\text{，}$$

$$-\frac{\hbar^2}{2M}b^2(b^2x^2-1)\exp\left(-\frac{b^2x^2}{2}\right) = (E-V)\exp\left(-\frac{b^2x^2}{2}\right)\text{。}$$

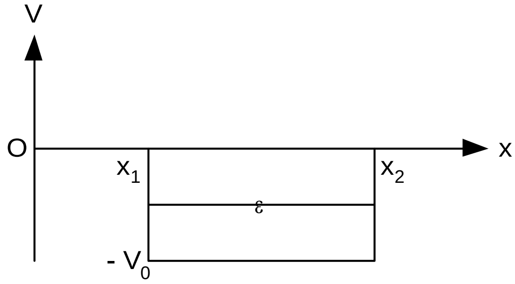
在上式中令 x=0，得

$$E = \frac{\hbar^2 b^2}{2M}\text{。}$$

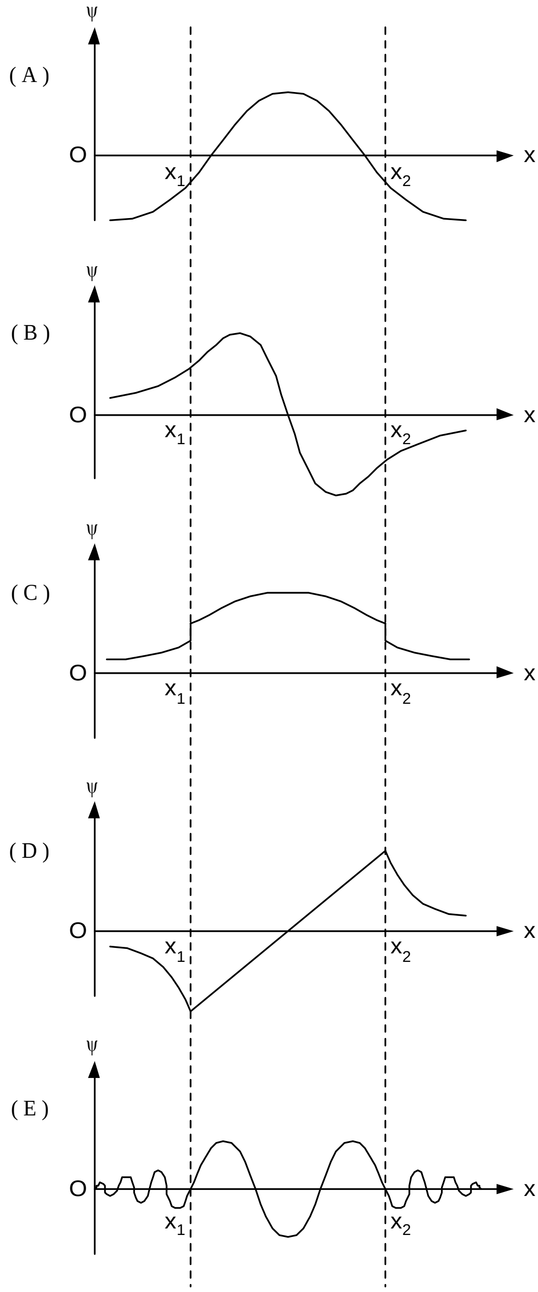
将 E 的表达式代回 Schrödinger 方程，得

$$V = \frac{\hbar^2 b^4}{2M}x^2\text{。}$$

选（B)。



2. An attractive one-dimensional square well has depth V_0 as shown above. If there is a bound state at an energy ε<0, which of the following best shows a possible wave function for this state?



解：(A) 不对，当 $x\to\infty$ 时，波函数ψ没有趋于 0。(C) 不对，波函数ψ在 x_1 和 x_2 处不连续。(D)不对，波函数ψ在 x_1 和 x_2 处导数不连续（以上两条对波函数的要求可参阅曾谨言《 量子力学导论》第二版，pp. 53，北京大学出版社）。在 $x > x_2$ 和 $x < x_1$ 处，$V(x) = 0$ 。由 Schrödinger 方程

$$\psi''(x) = -\frac{2m}{\hbar^2}[E - V(x)]\psi(x) = \frac{2m|\varepsilon|}{\hbar^2}\psi(x)$$

所以 $\psi(x)$ 大于零处，$\psi''(x)$ 大于零；$\psi(x)$ 小于零处，$\psi''(x)$ 小于零。其行为正如(B)中所描述，而不应像(E)那样振荡。

3. If an electron were confined to nuclear dimensions, the uncertainty in its momentum would be most nearly

(A) 0.2 eV/c

(B) 200 eV/c

(C) 200 KeV/c

(D) 200 MeV/c

(E) 200 GeV/c

解：测不准关系

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}。$$

取 $\Delta x \cdot \Delta p = \hbar$，原子核的尺度为 $\Delta x \sim 10^{-15}$ m，动量不确定度为

$$\Delta p = \frac{\hbar}{\Delta x} = \frac{1.054\times10^{-34}}{10^{-15}}$$
$$= \frac{1.054\times10^{-19}\times3\times10^{8}}{1.6\times10^{-19}}\ \text{eV/c} = 219\ \text{MeV/c}$$

。

选(D)。

4. The total energy in quantum mechanics corresponds to the differential operator given by

(A) $i\hbar\frac{\partial}{\partial t}$

(B) $i\hbar^2\frac{\partial^2}{\partial t^2}$

(C) $-i\hbar\nabla$

(D) $i\hbar\nabla$

(E) $\hbar\nabla\cdot\nabla$

解：Schrödinger 方程

$$i\hbar\frac{\partial}{\partial t}\psi = \hat{H}\psi = E\psi。$$

选 (A)。

5. The third lowest energy level of a one-dimensional quantum mechanical harmonic oscillator of frequency f has an energy of

(A) 0

(B) $\frac{3}{2}hf$

(C) 2hf

(D) $\frac{5}{2}hf$

(E) 3hf

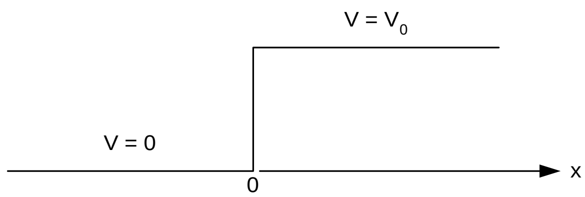
解：由一维谐振子能级公式

$$E_n = \left(n+\frac{1}{2}\right)\hbar\omega = \left(n+\frac{1}{2}\right)hf$$

而第三个最低能级对应 n = 2，能量

$$E_2 = \frac{5}{2}hf。$$

选 (D)。



$$V(x) = \begin{cases} 0 & \text{for } x<0 \\ V_0 & \text{for } x>0 \end{cases}$$

6. The figure above represents a step function in potential energy for electrons moving along the x-direction in a one-dimensional problem. A monochromatic beam of electrons of energy E is incident on the barrier from the left. If $E>V_0$, then which of the following is correct?

(A) At x=0, the sum of the amplitudes of the incident wave and the reflected wave is equal to the amplitude of the transmitted wave.

(B) At x=0, the amplitude of the transmitted wave is zero.
(C) The wave number of the reflected wave is less than that of the incident wave.
(D) The wave number of the reflected wave is equal to that of the transmitted wave.
(E) Electrons pass over the potential barrier without reflection but with reduced speed.

解：设入射平面波为 e^{ikx}，其中$k=\sqrt{2mE}/\hbar$。

在 $x<0$ 处有入射和反射波，在 $x>0$ 处仅有透射波。波函数为

$$\psi(x)=\begin{cases} e^{ikx}+\mathrm{R}e^{-ikx}, & x<0; \\ \mathrm{T}e^{\mathrm{i}k'x}, & x>0 \end{cases}$$

其中$k'=\sqrt{2m(E-V_0)}/\hbar$。

在 $x=0$ 处有波函数连续条件

$$1+R=T。$$

选（A）。

7. If f is frequency and h is Planck's constant, the zero-point energy of a one-dimensional quantum mechanical harmonic oscillator is
(A) 0
(B) $\frac{1}{3}hf$
(C) $\frac{1}{2}hf$
(D) hf
(E) $\frac{3}{2}hf$

解：由一维谐振子能级公式

$$E_n=\left(n+\frac{1}{2}\right)\hbar\omega=\left(n+\frac{1}{2}\right)hf，$$

而零点能对应于 n=0，所以

$$E_0=\frac{1}{2}hf。$$

答案选（C）。

Questions 8-9

Two spin-$\frac{1}{2}$ particles, 1 and 2, have spins in a singlet state with spin wave function

$$\Psi(1,2)=\frac{1}{\sqrt{2}}[\alpha(1)\beta(2)-\alpha(2)\beta(1)],$$

where α and β refer to up and down spins, respectively, along any chosen axis. The spin of particle 1 is measured along the z-axis and found to be up.

8. A simultaneous measurement of the spin of particle 2 along the z-axis would yield which of the following results?
(A) Up with 100% probability
(B) Down with 100% probability
(C) Up with 25% probability, down with 75% probability
(D) Up with 50% probability, down with 50% probability
(E) Up with 75% probability, down with 25% probability

解：对于粒子 2，波函数为$\alpha(2)$（自旋向上）和 $\beta(2)$（自旋向下）的等概率叠加。选（D）。还可以这样看问题。令总自旋 $S=S_1+S_2$，$\psi(1,2)$是（s^2，s_z）的本征态，本征值为（0，0），自旋单态，即总角动量 S 及 z 分量 S_z 为零。$S_z=s_{1z}+s_{2z}$，所以 s_{2z} 平均值也为零，所以自旋向上向下几率相同。选（D）。

9. A simultaneous measurement of the spin of particle 2 along the x-axis would yield which of the following results
(A) Up with 100% probability
(B) Down with 100% probability
(C) Up with 25% probability, down with 75% probability
(D) Up with 50% probability, down with 50% probability
(E) Up with 75% probability, down with 25% probability

解：由上一问讨论，总角动量为零，则要求角动量的各分量均为零。对于本题，要求

$$S_x = s_{1x} + s_{2x} = 0,$$

所以 s_{2x} 平均值也应为零，所以自旋 x 分量向上向下的几率应该相同。选（D）。

10. The wave function of a particle is $e^{i(kx-\omega t)}$, where x is position, t is time, and k and ω are positive real numbers. The wave function represents a simultaneous eigenstate of

(A) position and momentum
(B) energy and time
(C) energy and momentum
(D) position and time
(E) energy, momentum, position, and time

解：此为一维平面波，代表一维自由运动粒子。所以能量和动量为守恒量。选(C)。

11. The three operators (L_x, L_y, L_z) for the components of angular momentum commute with the Hamiltonian. Therefore the angular momentum is

(A) equal to zero
(B) equal to energy in magnitude
(C) a unit vector
(D) proportional to sinθ
(E) a constant of motion

解：因为 $[l_\alpha, H] = 0$，其中 α = x, y, z，所以

$$[l_\alpha{}^2, H] = l_\alpha[l_\alpha, H] + [l_\alpha, H]l_\alpha = 0,$$

$$[l^2, H] = \sum_\alpha [l_\alpha{}^2, H] = 0,$$

角动量为守恒量，或称运动常数。选（E）。

12. Under an exchange of both coordinates and spins, the complete wave function for a system of two electrons must be

(A) antisymmetric
(B) symmetric
(C) additive
(D) incoherent
(E) orthogonal to either independent wave function

解：电子为 Fermion，而 Fermion 要求波函数交换反对称。选(A)。

13. A spinless particle is confined in a cubical box of side L for which the potential is

V = 0, for 0 ≤ x, y, z ≤ L
V = ∞, otherwise.

What is the degeneracy of the third quantum level in the box?

(A) 1
(B) 3
(C) 6
(D) 9
(E) 12

解：为使波函数在盒壁上为零，各方向波矢必须满足

$$k_\alpha L = n\pi, \quad n = 1,2,3\ldots,$$

其中α=x, y, z。因此粒子的能级为

$$E_{lmn} = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2}{2m}\left(k_x{}^2 + k_y{}^2 + k_z{}^2\right) = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2\left(l^2 + m^2 + n^2\right)。$$

第一能级对应（l, m, n）取值为（1, 1, 1），第二能级对应（l, m, n）取值为（1, 1, 2）、（1, 2, 1）、（2, 1, 1），第三能级对应（l, m, n）取值为（1, 2, 2）、（2, 1, 2）、（2, 2, 1）。所以简并度为 3。选 (B)。注意这与用周期性边界条件所的情况不同，在那种情况下 n 可以取 0。

14. The hypothesis that an electron possesses spin is qualitatively significant for the explanation of all of the following topics EXCEPT the

(A) structure of the periodic table
(B) specific heat of metals
(C) anomalous Zeeman effect
(D) deflection of a moving electron by a uniform magnetic field
(E) fine structure of atomic spectra

解：（D）涉及的内容为普物甚至是高中课程，显然不需要考虑自旋就可解释。选（D）。（B）比较容易混，关键是自旋的引入使态的数目加倍。

15. Eigenfunctions for a rigid dumbbell rotating about its center have a dependence of the form $\psi(\phi) = Ae^{im\phi}$, where m is a constant. Which of the following values of A will properly normalize the eigenfunction?

(A) $\sqrt{2m}$

(B) 2π

(C) $(2\pi)^2$

(D) $\frac{1}{\sqrt{2\pi}}$

(E) $\frac{1}{2\pi}$

解：最简单的量子力学题。令

$$\int_0^{2\pi} \psi^* \psi d\phi = \int_0^{2\pi} Ae^{-im\phi} Ae^{im\phi} d\phi = A^2 2\pi = 1，$$

$$\mathrm{A} = \frac{1}{\sqrt{2\pi}}。$$

选(D)。

16. The Hamiltonian operator in the Schrödinger equation can be formed from the classical Hamiltonian by substituting

(A) wavelength and frequency for momentum and energy
(B) a differential operator for momentum
(C) transition probability for potential energy
(D) sums over discrete eigenvalues for integrals over continuous variables
(E) Gaussian distributions of observables for exact values

解：只需做如下变换：

$$p \to -i\hbar\nabla,$$

$$\frac{p^2}{2m} \to -\hbar^2\nabla^2。$$

选（B）。

17. If ψ is a normalized solution of the Schrödinger equation and Q is the operator corresponding to a physical observerble x, the quantity ψ*Qψ may be integrated to obtain the

(A) normalization constant for ψ
(B) special overlap of Q with ψ
(C) mean value of x
(D) uncertainty in x
(E) time derivative of x

解：量子力学中，可观测力学量 A 的平均值 $\overline{A} = \int \psi^* \hat{A} \psi d\tau$，其中 $\hat{A}$ 为 A 对应的算符。选（C）。

18. Which of the following is an eigenfunction of the linear momentum operator $-i\hbar\frac{\partial}{\partial x}$ with a positive eigenvalue $\hbar k$; i.e., an eigenfunction that describes a particle that is moving in free space in the direction of positive x with a precise linear momentum?

(A) $\cos kx$
(B) $\sin kx$
(C) e^{-ikx}
(D) e^{ikx}
(E) e^{-kx}

解：自由粒子本征态为量子力学基础知识，选(D)。若不敢肯定，可以现场求导：

$$-i\hbar\frac{\partial}{\partial x}e^{ikx} = -i\hbar \cdot ike^{ikx} = \hbar ke^{ikx}。$$

19. A system containing two identical particles is described by a wave function of the form

$$\psi = \frac{1}{\sqrt{2}}\left[\psi_\alpha(x_1)\psi_\beta(x_2) + \psi_\beta(x_1)\psi_\alpha(x_2)\right]$$

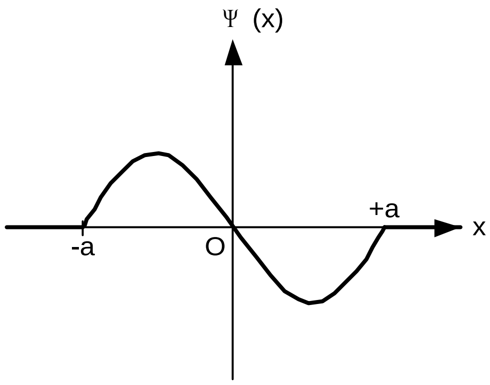
where x_1 and x_2 represent the spatial coordinates of the particles and α and β represent all the quantum numbers, including spin, of the states that they occupy. The particles might be

(A) electrons
(B) positrons
(C) protons

(D) neutrons

(E) deuterons

解：由交换对称性可知，应为 Boson。(A)、(B)、(C)、(D)分别为电子、正电子、质子、中子，均为典型的 Fermion，交换反对称。(E)为氘核，为 Boson。选（E)。



20. The figure above shows one of the possible energy eigenfunctions ψ(x) for a particle bouncing freely back and forth along the x-axis between impenetrable walls located at x=–a and x=+a. The potential energy equals zero for |x|<a. If the energy of the particle is 2 electron volts when it its in the quantum state associated with this eigenfunction, what is its energy when it is in the quantum state of lowest possible energy?

(A) 0 eV

(B) $\frac{1}{\sqrt{2}}$ eV

(C) $\frac{1}{2}$ eV

(D) 1 eV

(E) 2 eV

解：只有一个节点，说明处于第一激发态，n = 2。由一维无限深势阱的能级公式

$$E_n = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2}{2m}\left(\frac{\pi}{a}\right)^2 n^2,$$

知基态的能量

$$E_0 = \frac{E_1}{4} = \frac{1}{2} eV。$$

选（C)。

21. A particle of mass M is in an infinitely deep square well potential V where

$$V = 0 \text{ for } -a \le x \le a, \text{ and}$$
$$V = \infty \text{ for } x < -a, a < x.$$

A very small perturbing potential V' is superimposed on V such that

$$V' = \varepsilon\left(\frac{a}{2} - |x|\right) \text{ for } \frac{-a}{2} \le x \le \frac{a}{2}, \text{ and}$$
$$V' = 0 \text{ for } x < \frac{-a}{2}, \frac{a}{2} < x.$$

If $\psi_0, \psi_1, \psi_2, \psi_3, \ldots$ are the energy eigenfunctions for a particle in the infinitely deep square well potential, with ψ_0 being the ground state, which of the following statements is correct about the eigenfunction ψ_0' of a particle in the perturbed potential V+V'?

(A) $\psi_0' = a_{00}\psi_0,\ a_{00} \ne 0$

(B) $\psi_0' = \sum_{n=0}^{\infty} a_{0n}\psi_n$ with $a_{0n} = 0$ for all odd values of n

(C) $\psi_0' = \sum_{n=0}^{\infty} a_{0n}\psi_n$ with $a_{0n} = 0$ for all even values of n

(D) $\psi_0' = \sum_{n=0}^{\infty} a_{0n}\psi_n$ with $a_{0n} \ne 0$ for all values of n

(E) None of the above

解：V 和 V'均为偶宇称态，则 ψ_0' 必为偶宇称态。因为一维无限深势阱的全体本征态构成正交完备组，所以 ψ_0' 可被它们展开。由于第 n 个本征态的宇称为（–）^{n}，所以不应有 n 为奇数的态的成分。选（B)。

Questions 22-23

A particle of charge e and mass m is trapped in a one-dimentional square well of width 2a with impenetrable walls. The walls are located at x = –a and x = a.

22. If the quantum mechanical states are labled by n (n = 1 is the ground state), what is the expectation value of x^2 for very large n ?

(A) 0

(B) $\frac{1}{6}a^2$

(C) $\frac{1}{4}a^2$

(D) $\frac{1}{3}a^2$

(E) $\frac{1}{2}a^2$

解：一维无限深势阱的波函数为 sin 或 cos 函数。由于 n 表示波函数的节点数，当 n 很大时，sin 或 cos 函数的周期很小，以至于各点 ψ 的大小近似相等，即趋于经典情况。则

$$\overline{x^2} = \frac{1}{2a}\int_{-a}^{a} x^2 dx = \frac{1}{3}a^2 \text{。}$$

选（D）。也可进行严格计算，再令 n → ∞，结果相同。

23. A small electric field of strength E_0 is applied in the x direction. What is the change of the energy of the first excited state of this particle due to this electric field? (Assume that the total potential, due to the walls and the electric field, is 0 at x=0.)

(A) $\frac{-eE_0a}{2}$

(B) $\frac{-eE_0a}{3}$

(C) 0

(D) $\frac{eE_0a}{3}$

(E) $\frac{eE_0a}{2}$

解：第一激发态，波函数为

$$\psi_1(x) = \sqrt{\frac{1}{a}}\sin\left(\frac{\pi}{a}x\right) \text{。}$$

电场所产生的微扰可表示为 $H' = -E_0 ex$。由于微扰 Hamilton 函数为奇函数，

$$E^1 = \langle \psi_1 | H' | \psi_1 \rangle = -E_0 e\int_{-a}^{a} |\psi_1(x)|^2 x dx = 0 \text{。}$$

一级微扰为 0。选（C）。

25. A particle of mass M is infinitely deep quare well potential V where

V = ∞ for x<–a, a<x

V = 0 for –a≤x≤a

What is the eigenfunctions for the two lowest eigenstates?

(A) $\frac{1}{\sqrt{a}}\sin\frac{\pi x}{2a}, \frac{1}{\sqrt{a}}\cos\frac{\pi x}{2a}$

(B) $\frac{1}{\sqrt{a}}\sin\frac{\pi x}{a}, \frac{1}{\sqrt{a}}\cos\frac{2\pi x}{a}$

(C) $\frac{1}{\sqrt{a}}\cos\frac{\pi x}{2a}, \frac{1}{\sqrt{a}}\sin\frac{\pi x}{a}$

(D) $\frac{1}{\sqrt{a}}\cos\frac{\pi x}{a}, \frac{1}{\sqrt{a}}\sin\frac{2\pi x}{a}$

(E) $\frac{1}{\sqrt{a}}\cos\frac{2\pi x}{a}, \frac{1}{\sqrt{a}}\sin\frac{4\pi x}{a}$

解：由 Schrödinger 方程

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi \text{，}$$

由于–a≤x≤a 时，V = 0。

$$\nabla^2\psi + \frac{2mE}{\hbar^2}\psi = 0 \text{。}$$

解为

$$\psi = \sin kx \text{，} \cos kx \text{，其中} k = \sqrt{\frac{2mE}{\hbar^2}} \text{。}$$

由波函数在 x = ± a 处的连续条件，

$$\psi(\pm a) = 0 \text{，}$$

得当|x| < a 时波函数为

$$\psi(x)=\begin{cases}\sqrt{\frac{1}{a}}\cos\left(\frac{n\pi x}{2a}\right), & n=1,3,5,\dots\\ \sqrt{\frac{1}{a}}\sin\left(\frac{n\pi x}{2a}\right), & n=2,4,6,\dots\end{cases}$$ 。

选（C）。

26. A particle of energy E moving in one dimension is scattering by a potential barrier of height V_0 (V_0 > E) and width L. If $q=\sqrt{8\pi^2 m|E-y_0|/h^2}$, A is constant, and aL >> 1, the transmission coefficient is best approximated by

(A) Ae^{-2qL}

(B) Ae^{2qL}

(C) A sinh qL

(D) A sin qL

(E) A/[1+(qL)^2]

解：详细的推导可参阅曾谨言《量子力学导论》，第二版，63 页，北京大学出版社。考试现场显然没时间去推公式，可定性考虑。显然势垒宽度越大，L 越大，透射系数越小。而（B）、（C）随 L 增大，（D）随 L 成振荡关系，均被排除。对于（A）、（E），联想到 α衰变中也发生势垒贯穿，而半衰期随能量减小而迅速变长，透射系数与粒子能量（由 q 代表）的关系应为指数形式。选（A）。

27. A one-dimensional beam of particles each of kinetic energy E travels along the x-axis from left to right. It encounters a potential energy step of height E_0, with E > E_0.What is the reflection probability?

(A) $(\frac{E}{E_0})^2$

(B) $\dfrac{E_0-\sqrt{E(E-E_0)}}{E_0+\sqrt{E(E-E_0)}}$

(C) $\dfrac{2E-E_0-\sqrt{E(E-E_0)}}{2E-E_0+\sqrt{E(E-E_0)}}$

(D) $\dfrac{2E+E_0-\sqrt{E(E-E_0)}}{2E+E_0+\sqrt{E(E-E_0)}}$

(E) $\dfrac{2E-E_0-2\sqrt{E(E-E_0)}}{2E-E_0+2\sqrt{E(E-E_0)}}$

解：详细的推导可参阅《量子力学导论》，第二版，63 页，北京大学出版社。考试现场可用极限法挑出正确答案。当 $E_0\to 0$ 时，相当于没势垒，反射几率为 0。几个选项中只有（E）符合此极限。选（E）。

28. A free particle moving in one dimension line has following wave function at time = 0:

$$\psi(X)=d^{-\frac{1}{2}}e^{-|x|/d}$$, where d is a constant.

What is the probability that a measurement of the position of the particle at time = 0 will yield a result between x_1 and x_2 ($x_2 > x_1 > 0$)?

(A) 0

(B) $(x_2-x_1)/d$

(C) $(x_2-x_1)^2/d^2$

(D) $\frac{1}{2}(e^{-x_1/d}-e^{-x_2/d})$

(E) $\frac{1}{2}(e^{-2x_1/d}-e^{-2x_2/d})$

解：几率积分为

$$p(x_2>x>x_1)=\int_{x_1}^{x_2}|\Psi(x)|^2dx=\frac{1}{2}\left(-e^{-2x/d}\right)\Big|_{x_1}^{x_2}$$

。

选（E）。

29. Two particles have angular-momentum quantum numbers $l_1 = l_2 = 4$, $m_1 = 3$, and $m_2 = 2$. Which of the following is an allowed value for l corresponding to $L = L_1 + L_2$?

(A) 0

(B) 1

(C) 2

(D) 4

(E) 6

解：根据矢量合成的原理，合角动量的投影

$$m=m_1+m_2=5,$$

而合角动量的大小显然大于它的投影值，l > m = 5。选（E）。

30. A particle of energy E moving in one dimension is scattered by a potential barriers of height V_0(V_0 > E) and width L. If $q=\sqrt{8\pi^2 m|E-V_0|/h^2}$. A is a constant and qL >> 1, the transmission coefficient is best approximated by

(A) Ae^{-2qL}
(B) Ae^{2qL}
(C) A sinh qL
(D) A sin qL
(E) A / [1+$(qL)^2$]

解：一维势垒穿透的隧道效应。在 qL >> 1 的条件下，shqL >> 1，透射系数

$$T\approx\frac{16E(V_0-E)}{V_0}e^{-2qL}\propto e^{-2qL}。$$

选（A）。

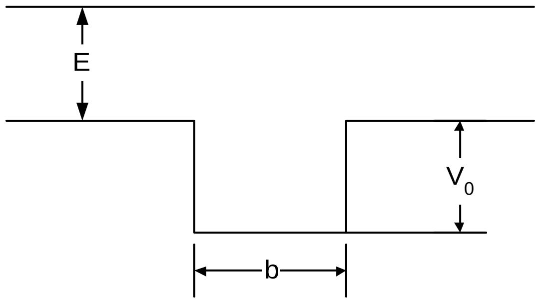
31. The wave function for the lowest energy level of hydrogen is $\Psi=\frac{e^{-r/a_0}}{\sqrt{\pi a_o^{3/2}}}$, where a_0 is the Bohr radius. The expectation value $\left\langle\frac{1}{r}\right\rangle$ of reciprocal distance of the electron from the nucleus is

(A) 1 / a_0
(B) 1.5 / a_0
(C) 2 / a_0
(D) a_0
(E) ∞

解：直接积分求平均值

$$\left\langle\frac{1}{r}\right\rangle=\int_0^\infty\frac{1}{r}\Psi^2\cdot 4\pi r^2 dr=\frac{1}{a_0}。$$

选（A）。



32. A wave packet of electrons having a mean energy E is incident from the left on the one-dimensional square well of depth V_0 and width b shown above. Unusually high transmission of the electrons (analogous to the Ramsauer-Townsend effect) will take place when the energy of the electrons most nearly satisfies which of the following conditions?

(A) $\frac{h}{\sqrt{2M(E+V_0)}}=2b$

(B) $\frac{h}{\sqrt{2M(E+V_0)}}=4b$

(C) $\frac{h}{\sqrt{2M(E+V_0)}}=8b$

(D) $\frac{h}{\sqrt{2ME}}=4b$

(E) $\frac{h}{\sqrt{2ME}}=2b$

解：形成共振透射（T = 1）的条件是

$$k'b=n\pi,$$

$$k'=\sqrt{\frac{2m(E+V_0)}{\hbar^2}}。$$

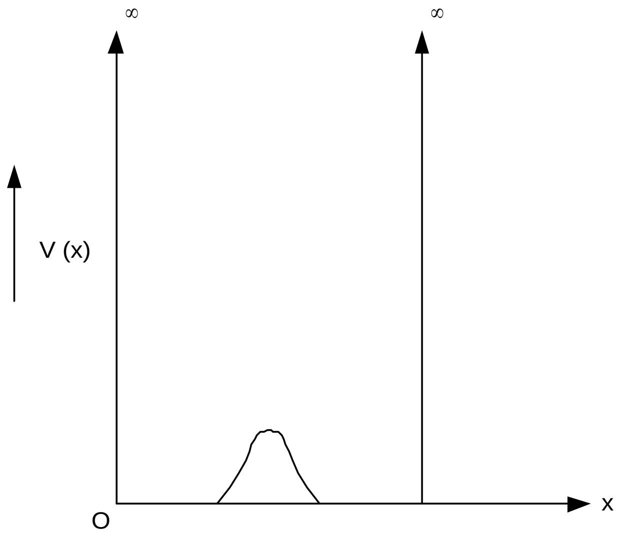
上式中当 n = 1 时即是选项（A)。选（A)。

33. The Pauli exclusion principle results from the quantum mechanical fact that no two electrons in an atom can

(A) have the same set of quantum numbers
(B) have the same spatial wave function
(C) have the same spin
(D) interact with each other

(E) be in an excited state simultaneously

解：Pauli 不相容原理：不可能由两个全同的 Fermi 子处于同一个单粒子态。而电子属于 Fermion。选（B）。



34. A one-dimensional square well potential with infinitely high sides is shown above. In the lowest energy state, the wave function is proportional to sin kx. If the potential is altered slightly by introducing a small bulge in the middle as shown, which of the following is true of the ground state?

(A) The energy of the ground state remains unchanged.
(B) The energy of the ground state is increased.
(C) The energy of the ground state is decreased.
(D) The original ground state splits into two states of lower energy.
(E) The original ground state splits into two states of higher energy.

解：微扰的 Hamilton 量始终为正，在以及近似下的修正能量<Ψ_0'|H'|Ψ>显然为正，因此基态能级上升。选（B）。

35. X and Y are two stationary states of a particle in a spherically symmetric potential. In which of the following situations will the wave functions of the two states be orthogonal?

I. X and Y correspond to different energies.
II. X and Y correspond to different total orbital angular momenta L.
III. X and Y correspond to the same L but different L_z.

(A) Not neccesarily in any of these situations.
(B) In situation I, but not neccesarily in II or III.
(C) In situation I and II, but not neccesarily in III.
(D) In situation II and III, but not neccesarily in I.
(E) In all three situations.

解：在球对称势场的问题中，不考虑自旋的情况下，完备的量子数为（n, l , l_z），解为三者的共同本征函数。对于三者中任何不完全相同的两个量子态，都是正交的。选（E）。

36. A particle of mass m moves in a thress-dimensional potential V(r) = $\frac{1}{2}$ kr^2. If k is halved, what is the ratio of the new ground-state energy to that of the old ground-state energy?

(A) $1/\sqrt{6}$
(B) 1/2
(C) $1/\sqrt{2}$
(D) $\sqrt{2}$
(E) $\sqrt{6}$

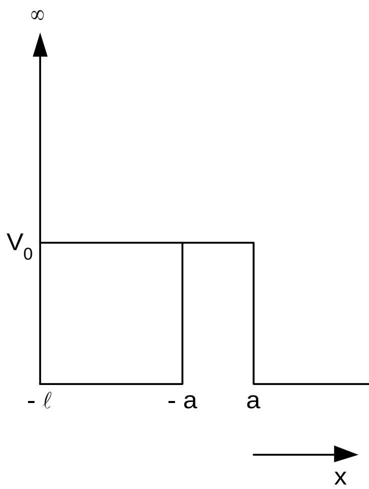
解：一维谐振子的零点能

$$E_0 = \frac{1}{2}\hbar\omega = \frac{\hbar}{2}\sqrt{\frac{k}{\mu}}。$$

因此前后零点能之比

$$\frac{E_0'}{E_o} = \sqrt{\frac{k_0'}{k_0}} = \frac{1}{\sqrt{2}}。$$

选（C）。



37. A spinless nonrelativistic particle of mass m is placed in the divided suqare well shoqn above. The potential rises to infinity at x = ± l. Which of the following is NOT true for states ψ(x) of definite energy E?

(A) If $V_0 \to \infty$, there are two states for each allowed energy.

(B) If $V_0 \to 0$, the allowed energies satisfy

$$\frac{2l}{\pi}\sqrt{\frac{2mE}{h^2}} = \text{integer.}$$

(C) For a general V_0, solutions of definite E have a definite reflection symmetry. ψ(x) = ψ(–x) or ψ(x) = – ψ(–x).

(D) ψ(l) = 0 = ψ(–l).

(E) ψ(a) = 0 = ψ(–a).

解：对一般的 V_0，在 x = ±a 处波函数只需满足连续性及导数连续的边条件。当 $V_0 \to 0$ 时，相当于一个无穷高势垒中的束缚态。而当 $V_0 \to \infty$时，相当于彼此隔断的两个无穷高势垒，分别对应各自的束缚态。选（E）。

38. If S is the total spin quantum number, which of the following lists all possible spin states for three electrons?

(A) Two S = 1/2 doublets and one S = 1 triplet

(B) Two S = 0 singlets, one S = 1/2 doublet, and one S = 3/2 quartet

(C) One S = 0 singlet, one S = 1 triplet, and one S = 3/2 quartet

(D) Two S = 1/2 doublets and one S = 3/2 quartet

(E) Four S = 1/2 doublets

解：三个电子总共可能的自旋取法为：

1、两个同向，一个反向，S = 1/2，为双重态；

2、三个都同向，S = 3/2，为四重态。

选（D）。

39. A quantum system has two eigenstates : $\Psi_1(x)$ with energy E_1 and $\Psi_2(x)$ with energy E_2. These are normalized and orthogonal: that is,

$$\int_{-\infty}^{\infty}\Psi_1^*\Psi_1 dx = 1 = \int_{-\infty}^{\infty}\Psi_2^*\Psi_2 dx$$

and

$$0 = \int_{-\infty}^{\infty}\Psi_1^*\Psi_2 dx$$

If at t = 0 the system was in the state

$$\Psi(x) = \frac{1}{\sqrt{2}}[\Psi_1(x) + \Psi_2(x)],$$

the probability of finding it in this same state $\Psi(x)$ at a later time t is

(A) zero

(B) 1

(C) $\sin(E_1 - E_2)t/h$

(D) $\frac{1}{2}[e^{-iE_1t/h} + e^{-iE_2t/h}]$

(E) $\frac{1}{2}[1 + \cos(E_1 - E_2)t/h]$

解：体系的随时间演化的波函数为

$$\Psi(x,t) = \frac{1}{\sqrt{2}}\left(\Psi_1(x)e^{-iE_1t/\hbar} + \Psi_2(x)e^{-iE_2t/\hbar}\right)。$$

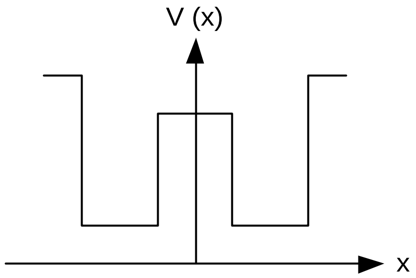
t 时刻粒子处于ψ(x, 0)态的几率是

$$p = \int_{-\infty}^{\infty}\Psi(x,0)^*\Psi(x,t) = \frac{1}{2}[e^{-iE_1t/h} + e^{-iE_2t/h}]。$$

选（D）。

Questions 40-41

The sketch below shows a one-dimensional potential for an electron. The potential is symmetric about the V-axis.



40. Which of the following statements correctly describes the ground state of the system with on electron present?

(A) A single electron must be localized in one well.
(B) The ground state will accommodate up to four electrons.
(C) The kinetic energy of the ground state will be one-half its potential energy.
(D) The wave function of the ground state will be antisymmetric with respect to the V-axis.
(E) The wave function of the ground state will be symmetric with respect to the V-axis.

解：因为势垒关于 y 轴对称，

$$V(x) = V(-x)\,,$$

所以波函数有确定的宇称。由于基态波函数与 x 轴没有节点，所以波可能是反对称性的，因为那要求

$$\psi(-x) = -\psi(x)\,,$$

从而左右两边波函数反号，必与 x 轴有节点。所以是关于 y 轴对称的。选（E）。（A）不对，电子甚至还有几率穿透势垒，不会被束缚在一边（或者说由于波函数对称）。（B）不对，由于基态波函数不变号，所以它没有简并态，否则两个简并态波函数之间不可能正交（因为要末乘积始终为正，要末始终为负）。所以只有一个基态波函数，最多能容纳两个自旋相反的电子。

41. A second electron is now added to the system. If the electrons do not interact, which of the following statements is correct?

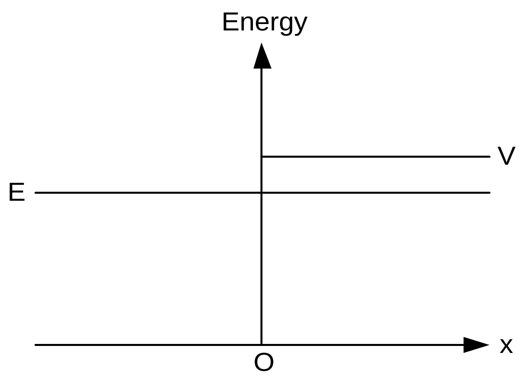
(A) The second electron must be localized in the well not previously occupied.
(B) In the ground state of the system, each of the two electrons will have the same spatial wave function.
(C) In the ground state of the system, one electron will be in a spatially symmetric state and one will be in a spatially antisymmetric state.
(D) The second electron will not be bound.
(E) Pair annihilation will occur.

解：如上一问中的分析，基态能级不简并，只有一个基态波函数，所以两个电子必须有相同的空间波函数。但只要二者自旋相反，并不违反 Pauli 不相容原理。答案选（B）。

42. An energy level of a certain isolated atom is split into three components by the hyperfine interaction coupling of the electronic and nuclear angular momenta. The quantum number j, specifying the magnitude of the total electronic angular momentum for the level, has the value j = 3/2. The quantum number i, specifying the magnitude of the nuclear angular momentum, must have the value

(A) 1/2
(B) 1
(C) 3/2
(D) 2
(E) 3

解：两个角动量耦合，设其角量子数分别为 p_1、p_2，其中 $p_1 < p_2$，则总角动量量子数 $p = p_1 + p_2$，$p_1 + p_2 - 1$，…，$p_2 - p_1$，共有 $2p_1$ 种可能取值。本题中为 3 分量，总角动量有 3 种取值可能。所以电子角动量 j 和核角动量 i 中较小的一个应为应为 1。而 j = 3/2，所以 i = 1。选（B）。



43. An electron with energy E and momentum $k\hbar$ is

incident from the left on a potential step of height V > E at x = 0. For x > 0, the space part of the electron's wave function has the form

(A) e^{ikx}

(B) $e^{-ik'x}$; k' < k

(C) $e^{-\alpha x}$, where α is real and positive

(D) sin kx

(E) identically zero

解：由 Schrödinger 方程

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi \text{ ,}$$

在 x > 0 处， 因为 V > E，所以

$$\frac{d^2}{dx^2}\psi - \frac{2m}{\hbar^2}(V-E)\psi = 0 \text{。}$$

方程的解为

$$\psi = Ae^{-\alpha x} \text{ ,}$$

其中

$$\alpha = \sqrt{\frac{2m(V-E)}{\hbar^2}} \text{。}$$

选（C）。

44. In a ^3S state of the helium of the possible values of the total electronic angular momentum are

(A) 0 only

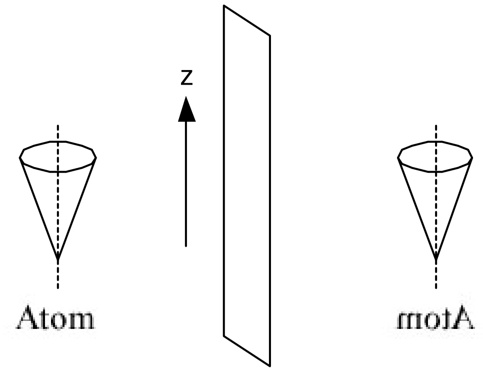
(B) 1 only

(C) 0 and 1 only

(D) 0, 1/2, and 1

(E) 0, 1, and 2

解：S=1，L=0，故 J=1。选（B）。



45. An atom is in a state for which J = 1, m_J = 1 relative to a z axis which is vertical and points upward as shown above. If the atom is observed in a vertical mirror, the image will look like an atom for which m_J

(A) has the value –1

(B) has the value 0

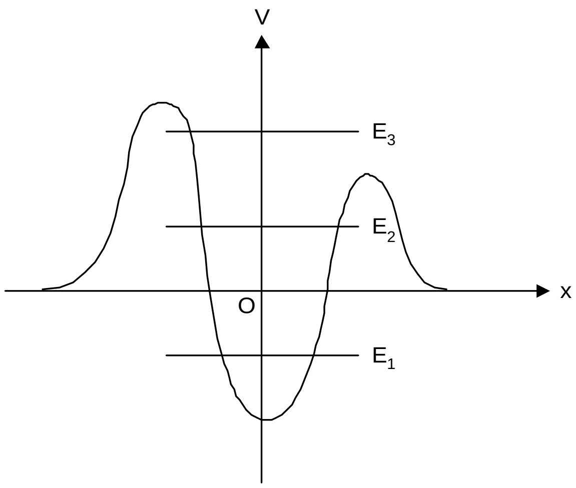
(C) has the value +1

(D) has a well-defined value but not one of the above

(E) does not have a well-defined value

解：宇称变换下角动量反号，m_J 变成–1。选（A）。

Question 46-47 refer to the one-dimensional potential energy function shown below.



46. According to quantum mechanics, which of the energies shown represents a possible permanently bound state for a particle?

(A) E_1 only

(B) E_2 only

(C) E_3 only

(D) E_1 or E_2

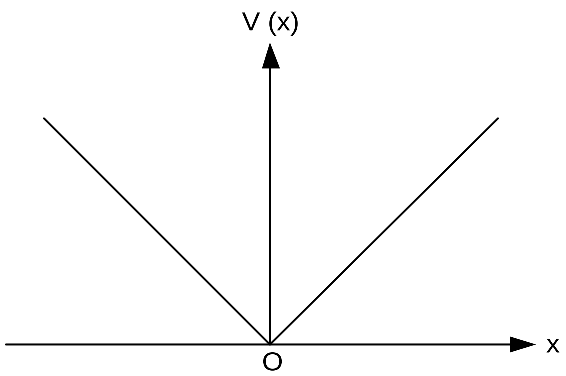
(E) E_2 or E_3

解：束缚态的总能量 $E<0$。图中只有 E_1 满足条件且恰处于势阱的底部。选（A）。

47. Which of the energies shown represents a possible metastable state from which an initially confined particle would eventually tunnel out

(A) E_1 only
(B) E_2 only
(C) E_3 only
(D) E_1 or E_2
(E) E_2 or E_3

解：E_3 显然粒子不被束缚，可以自由运动到 x 正轴的无穷远处。E_2 是亚稳态，根据量子力学的隧道效应，粒子在其中振动，最终能透射势阱自由运动。选（B）。



48. The graph above shows the potential energy as a function of position x for an infinite triangular well. Which of the following is true of the bound state energy levels of this well?

(A) They are equally spaced for all energies.
(B) They are more closely spaced at higher energies.
(C) They are less closely spaced at higher energies.
(D) They are randomly spaced.
(E) There are no bound state energy levels.

解：线性势的能级越高越密，具体解可参照曾谨言《量子力学》第二版，3.7 节。选（B）。

49. A particle moves in a potential

$V(x) = 0$, for $x < -L$.

$V(x) = -V_0$, for $-L < x < L$.

$V(x) = 0$, for $x > L$.

If the potential between $-L$ and L is shown to $-5V_0$ and the total energy of the particle is negative ($E < 0$), the number of bound states must

(A) stay the same or increase
(B) stay the same or decrease
(C) stay the same
(D) equal to zero
(E) increase

解：一维有限深势阱至少存在一个束缚态。如果 V_0 特别小，势阱中只有一个束缚态，并且深度变为 $5V_0$ 后势阱中仍只有一个束缚态，这种情况下束缚态数目不变。更一般的情况随势阱深度加大束缚态数目增多。选（A）。