

1.1.3 Objective: Students will be introduced to the concept of families of functions and will further develop their understanding of what it means to investigate a function

Standard: F-IF.4

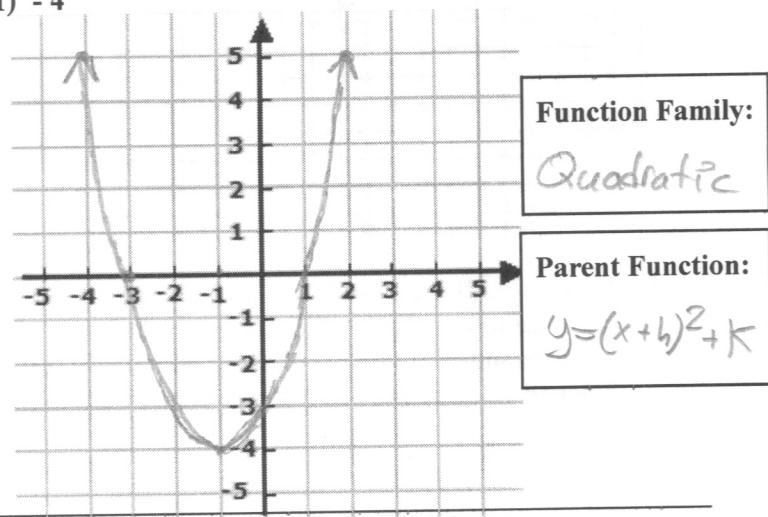
Previously, in IM2, you investigated the family of **quadratic** functions

Today, you begin to investigate the family of **rational** functions.

Warm Up Make a complete graph of the function $y = (x + 1)^2 - 4$

Which family of functions is this?

x	0	1	-1	2	-2
y	-3	0	-4	5	-3



Domain: \mathbb{R}

Range: $y \geq -4$

1-29 Your team will investigate the function of the form $y = \frac{1}{x-h}$. This function is different from others you have seen in the past. To obtain a complete graph, ensure your table includes sufficient information.

1. Make a table with integer x-values from 5 less than the value of h to 5 more than the value of h. For example, if you are working with $y = \frac{1}{x-7}$, you would begin your table at 12 and end it at 2. What do you notice about all of your x-values?

Two
Different
Both Decreasing
Separated by
the h value:

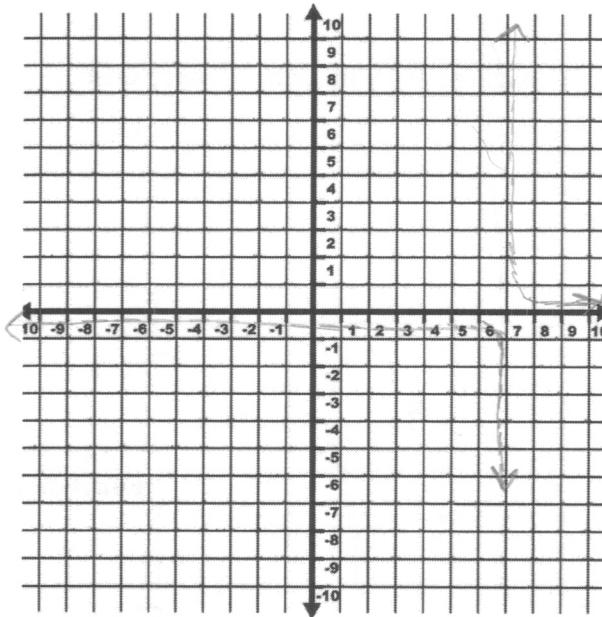
- What do you notice about all of your y-values?

- Is there any x-value that has no y-value for your function? Why does this make sense?

when $x=7$ because you can't divide by zero

x	$y = \frac{1}{x-7}$	$y = f(x)$
12	$y = \frac{1}{12-7}$	$\frac{1}{5}$
11	$y = \frac{1}{11-7}$	$\frac{1}{4}$
10	$y = \frac{1}{10-7}$	$\frac{1}{3}$
9	$y = \frac{1}{9-7}$	$\frac{1}{2}$
8	$y = \frac{1}{8-7}$	1
7	$y = \frac{1}{7-7}$	undefined
6	$y = \frac{1}{6-7}$	-1
5	$y = \frac{1}{5-7}$	- $\frac{1}{2}$
4	$y = \frac{1}{4-7}$	- $\frac{1}{3}$
3	$y = \frac{1}{3-7}$	- $\frac{1}{4}$

2. Plot all of the points that you have in your table so far.



Your equation: $y = f(x) = \frac{1}{x-7}$

Parent Function : $y = f(x) = \frac{1}{x}$

Function Family
Equation: $y = f(x) = \frac{1}{x}$
Name:

3. Now you will need to add more values to your table to see what is happening to your function as your input values get close to your value of h . Choose eight input values that are very close to your value of h and on either side of h . For example, if you are working with $h = 7$, you might choose input values such as 6.5, 6.7, 6.9, 6.99, 7.01, 7.1, 7.3, and 7.5. **For each new input value, calculate the corresponding output and add the new point to your graph.**

- **Table Example:** If $h = 7$ then one close by value is $x = 6.5$.

$$x = 6.5 \Rightarrow y = \frac{1}{6.5 - 7} = \frac{1}{-0.5} = -2$$

x	$y = \frac{1}{x-7}$	$y = f(x)$
6.5	$y = \frac{1}{6.5-7}$	-2
6.7	$y = \frac{1}{6.7-7}$	-3.3
6.9	$y = \frac{1}{6.9-7}$	-10
6.99	$y = \frac{1}{6.99-7}$	-100
7.01	$y = \frac{1}{7.01-7}$	100
7.1	$y = \frac{1}{7.1-7}$	10
7.3	$y = \frac{1}{7.3-7}$	3.3
7.5	$y = \frac{1}{7.5-7}$	2

4. Fully describe the graph you completed in #2:

- Shape: Hyperbolic

- x- and y-intercepts: $y\text{-int: } (0, \approx -0.14, 3)$

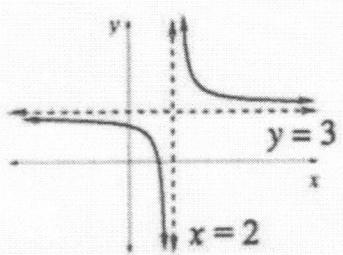
- Domain & range: $D: \mathbb{R}; X \neq 7, R: \mathbb{R}; Y \neq 0$

- Increasing or decreasing: Decreasing

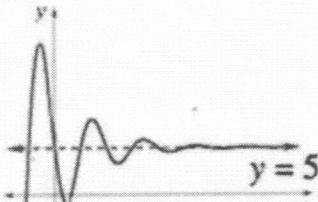
- Asymptotes(complete after reading math notes below)

$x = 7, y = 0$

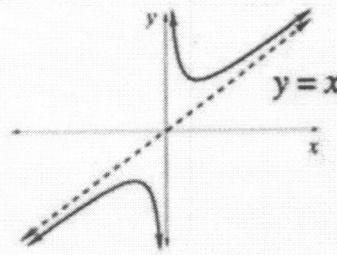
Ex: A graph with a horizontal asymptote at $y = 3$ and a vertical asymptote at $x = 2$



Ex: A graph with a horizontal asymptote at $y = 5$.



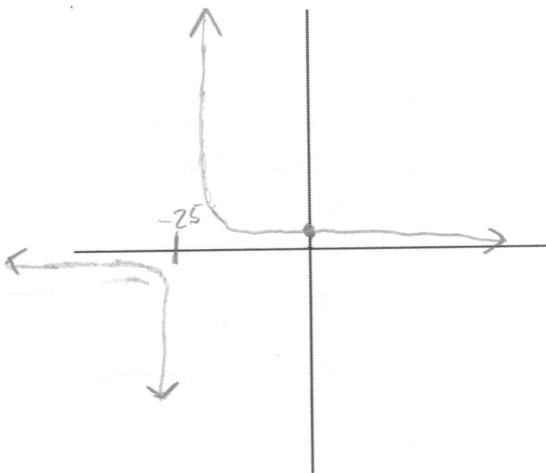
Ex: A graph with a diagonal asymptote at $y = x$. Note that this course focuses on vertical and horizontal asymptotes.



- In this course, asymptotes will almost always be **horizontal** or **vertical** lines
- What are asymptotes? An asymptote is a line that a graph approaches but never touches.
 - Example: population might approach a certain limit or decline towards zero as an asymptote.
 - Demonstrate how performance on a task might improve quickly at first and then slow down as it approaches a maximum potential
- Vertical Asymptotes:
 - Vertical asymptotes occur in Rational functions
 - Vertical asymptotes occur at values of x where the denominator of the function becomes 0, making the function undefined
 - Vertical asymptotes are vertical lines with the equation $x = c$ (c being the x value that makes the denominator equal 0)
 - Vertical asymptotes always happen in the middle of the graphs.
 - Graphs never cross the vertical asymptote, because the function is undefined at that x -value
 - The x -value of the asymptote is not in the Domain of the function.
 - You become familiar with recognizing vertical asymptotes **graphically**
 - To find the vertical asymptote **algebraically**, we set the denominator of the rational function equal to zero and solve for X.
- Horizontal asymptotes
 - Horizontal asymptotes are horizontal lines with equations $y = c$
 - Horizontal asymptotes describe how a function behaves as **x approaches positive or negative infinity**.
 - Graphs never cross the horizontal asymptotes, so the $y = c$ may be part of the Range of the function
 - Horizontal asymptotes represent the y value that the function is approaching as the x approaches ∞ or $-\infty$.
 - Right now, you are becoming familiar with how to recognize horizontal asymptotes **graphically**
 - Later on, we will learn how to find the horizontal asymptote **algebraically**.

- 1-33. Based on what you learned about this family of functions, what will the graph $f(x) = \frac{1}{x+25}$ look like?

- a. Sketch what you predict the graph will look like.



Function Family Name: ~~Hyperbolic~~ Rational Function
 Function Family Equation: $y = f(x) = \frac{1}{x-h} + k$

- b. Use your graphing calculator to Graph $f(x) = \frac{1}{x+25}$. Do you see what you expected to see? Why or why not?

A regular Rational function that is shifted 25 to the Left

- c. Adjust the viewing window if needed. When you see the full picture of your graph, correct your sketch of the above graph, labeling all the important points.. How close was the graph to your predictions?

Completely describe your graph.

x	y
-25	undefined
undefined	0
0	0.04

Vertical Asymptote: $x = -25$
 Horizontal Asymptote: $y = 0$
 Shape: Hyperbolic
 Y-Int: $(0, 0.04)$
 Increasing / Decreasing
 Domain: $\mathbb{R}; x \neq -25$
 Range: $\mathbb{R}; y \neq 0$

1-34. FAMILIES OF FUNCTIONS

- a. You investigated several functions of the form $y = \frac{1}{x-h}$.

- a. What do all graphs $y = \frac{1}{x-h}$ have in common?

they have the same shape ~~shape~~

- b. What differences do they have?

they can be shifted $\uparrow \downarrow \leftarrow \rightarrow$

- c. Since each function of the form $y = \frac{1}{x-h}$ has the same basic relationship between x and y , this set of functions can be called a **family of functions**.

- d. The function $\frac{1}{x^h}$ is called the **parent function**.

$$ax^2 + bx + c$$

- e. In the equation, the letter h is called a **parameter**, while x and y are called **variables**. What is the difference between a parameter and a variable?
the difference is that the "independent" variable or the equation.
- f. What role do parameters and variables have in a family of functions? Use examples from this lesson to illustrate your understanding of parameters and variables.
the variables are for solutions, the
- g. What other families of functions have you studied in this course or previous courses? What is the parent function for each family?

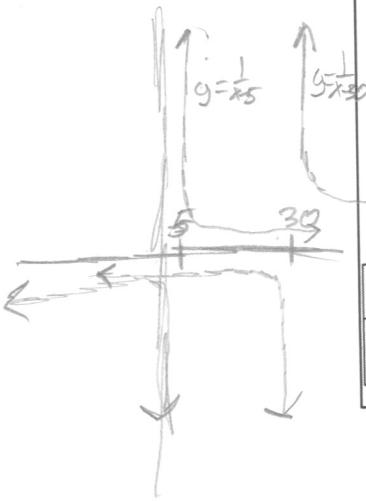
Function Family Name	Function Equation	Parent Function Equation
Linear Family	$y = ax + h$	$y = x$
Quadratic Family	$y = ax^2 + bx + c$	$y = x^2$
Exponential Family	$y = a^{x+h} + k$	$y = a^x$
Absolute Family	$y = a x-h + k$	$y = x $
Cubic Family	$y = (x-h)^3 + k$	$y = x^3$

Closure:

Explain in your own words how the graph of $f(x) = \frac{1}{x-5}$ differs from the graph of $f(x) = \frac{1}{x-30}$. Sketch a graph of each to show the differences.

the difference is that one is shifted more to the right.

Go to CANVAS->IXL and practice the skills on Exponents based on the quiz taken.



1.2.1 Math Notes: Laws of Exponents

Law	Examples
$x^m x^n = x^{m+n}$ for all x	$x^3 x^4 = x^{3+4} = x^7$ $2^5 \cdot 2^{-1} = 2^4$
$\frac{x^m}{x^n} = x^{m-n}$ for $x \neq 0$	$x^{10} \div x^4 = x^{10-4} = x^6$ $\frac{5^4}{5^7} = 5^{-3}$
$(x^m)^n = x^{mn}$ for all x	$(x^4)^3 = x^{4 \cdot 3} = x^{12}$ $(10^5)^6 = 10^{30}$
$x^0 = 1$ for $x \neq 0$	$\frac{y^2}{y^2} = y^0 = 1$ $9^0 = 1$
$x^{-1} = \frac{1}{x}$ for $x \neq 0$	$\frac{1}{x^2} = (\frac{1}{x})^2 = (x^{-1})^2 = x^{-2}$ $3^{-1} = \frac{1}{3}$

if $x^n = x^2$, then $n = 2$	$x^{2n-1} = x^5$, $2n-1 = 5$, $n=3$
$x^{\left(\frac{a}{b}\right)} = \left(\sqrt[b]{x}\right)^a$	$25^{\left(\frac{1}{2}\right)} = \sqrt[2]{25} = 5$ $8^{\left(\frac{2}{3}\right)} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$

