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1. a)

$$\begin{array}{r|ccccc}
 & \overset{\text{rick}}{B} & \overset{\text{ood}}{Wo} & \overset{\text{heep}}{S} & \overset{\text{heat}}{Wh} & \overset{\text{re}}{O} \\
\text{Road} & 1 & 1 & 0 & 0 & 0 \\
\text{Settlement} & 1 & 1 & 1 & 1 & 0 \\
\text{city} & 0 & 0 & 0 & 2 & 3 \\
\text{Dev card} & 0 & 0 & 1 & 1 & 1
\end{array}$$

rows represent buildings

Columns represent Materials

b) $w = \begin{bmatrix} 1 \\ 2 \\ 6 \\ 3 \\ 8 \end{bmatrix}$

total = XW

where X = Matrix above (original 4x5)

$XW = \begin{bmatrix} 3 \\ 12 \\ 30 \\ 17 \end{bmatrix}$ $\rightarrow$ Road = 3, settlement = 12, City = 30, dev card = 17

1. c) $X = [6\ 2\ 2\ 0]$

W = original matrix (4x5)

$XW = [\underset{\text{Brick}}{8}\ \underset{\text{wood}}{8}\ \underset{\text{sheep}}{2}\ \underset{\text{wheat}}{6}\ \underset{\text{ore}}{6}]$

d) $[8\ 8\ 2\ 6\ 6] \times \begin{bmatrix} 1 \\ 2 \\ 6 \\ 3 \\ 8 \end{bmatrix} = 102$ total cost

PSet1_LA.py

```python
import numpy as np

#a)
x = [[1,1,0,0,0],[1,1,1,1,0],[0,0,0,2,3],[0,0,1,1,1]]
#b)
y = [1,2,6,3,8]
#c)
z = [[6,2,2,0]]

mx = np.matrix(x)
my = np.matrix(y)
mz = np.matrix(z)

#b)
xy = mx @ my.T
#c)
xz = mz @ mx
#d)
final = xz @ my.T

print(xy)
print(xz)
print(final)
```

```
lewiss-mbp:desktop lewisarnsten$ python3 PSet1_LA.py
[[ 3]
 [12]
 [30]
 [17]]
[[8 8 2 6 6]]
[[102]]
```

2. a) $X^T = [x_1\ x_2 \cdots x_n]$ $\quad X = \begin{bmatrix} -x_1^T- \\ -x_2^T- \\ -x_3^T- \\ \vdots \\ -x_n^T- \end{bmatrix}$ $\quad C = \dfrac{X^T X}{n}$

outer product representation

$$C_n = x_1 * [\text{——} x_1^T \text{——}] + x_2 * [\text{——} x_2^T \text{——}]$$
$$+\, x_3 * [\text{——} x_3^T \text{——}] + \dots + x_n * [\text{——} x_n^T \text{——}]$$

b) Since the rows of X and the columns of X^T are linearly independent the rank of c is p. ($X^T X = p \times p$, $X^T = p \times n$, $X = 1 \times p$)

3. let $X = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 \end{bmatrix}$

a) The rank of X is 3

b) $XX^T = \begin{bmatrix} 4 & 0 & 0 & 4 \\ 0 & 3 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 4 & -1 & 0 & 6 \end{bmatrix}$ which has a rank of 3

c) The largest set of linearly independent columns is 3. The first 3 columns are linearly independent.

4. a) $X = \begin{bmatrix} 0.95 & 0.95 \\ 0.95 & -0.95 \\ 0.91 & -0.95 \\ -0.92 & -0.92 \end{bmatrix}$ $a\begin{bmatrix} 0.95 \\ 0.95 \\ 0.91 \\ -0.92 \end{bmatrix} + b\begin{bmatrix} 0.95 \\ -0.95 \\ -0.95 \\ 0.92 \end{bmatrix} = 0$, only $a=b=0$ is a solution

Thus the columns of the matrix are linearly independent as neither is a scalar multiple of the other.

b) $X = \begin{bmatrix} 0 & 1 & 4 \\ 1 & 2 & -2 \\ 5 & 8 & 0 \end{bmatrix}$ $a\begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix} + b\begin{bmatrix} 1 \\ 2 \\ 8 \end{bmatrix} + c\begin{bmatrix} 4 \\ -2 \\ 0 \end{bmatrix} = 0$

$a=b=c=0$, so the columns are linearly independent

only solution

4 cont.

c) $X = \begin{bmatrix} 2 & 2 & 3 \\ 9 & 4 & 11 \\ 10 & 6 & 13 \end{bmatrix}$ $\quad a\begin{bmatrix} 2 \\ 9 \\ 10 \end{bmatrix} + b\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} + c\begin{bmatrix} 3 \\ 11 \\ 13 \end{bmatrix} = 0$

$a = b = c = 0$, $a = 2$ $b = 1$ $c = -2$

So the columns are not linearly independent

d) $X = \begin{bmatrix} 1 & 2 \\ -1 & 2 \\ 1 & -2 \end{bmatrix}$ $\quad a\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + b\begin{bmatrix} 2 \\ 2 \\ -2 \end{bmatrix} = 0$

$a = b = c = 0$, so the columns are linearly independent. The rank is 2, as it is full rank.