CMSC 27500: Graph Theory **Spring 2021**

Homework 2

Due: April 22, 2021

You are encouraged to discuss these problems in groups. But you must write up the solutions by yourself and also mention the names of the people you discussed with. If you referred to any books or websites, you should cite them. It's recommended you submit your homework in LaTeX but if you plan to write it by hand, please write clearly and legibly.

1. Show that every tree T has at least $\Delta(T)$ leaves. **[2 points]**
2. Prove or disprove: A connected graph is bipartite if and only if no two adjacent vertices have the same distance from any other vertex. **[3 points]**
3. Suppose F, F' are forests on the same set of vertices such that F' has strictly more edges than F. Then, show that F' must have an edge e such that $F + e$ is also a forest. **[5 points]**
4. Show that a graph is 2-edge-connected if and only if it has a strongly connected orientation, which means an orientation in which every vertex can reach any other vertex by a directed path. **[5 points]**
5. Let G be a connected graph and let r be a vertex of G. Starting from r, move along the edges of G, going whenever possible to a vertex not visited so far. If there is no such vertex, go back along the edge by which the current vertex was first reached (if the current vertex is r, then stop). Show that the edges traversed form a normal spanning tree in G with root r. **[7 points]**
6. Show that for every positive integer k, every graph of minimum degree $2k$ has a $(k+1)$-edge-connected-subgraph. **[8 points]**