

1)

1.

Abs.	Good	Medium	Strong delay	
Without alcohol	120	60	20	200 (1)
With alcohol	60	100	40	200 (2)
	180 (3)	160 (4)	60 (5)	400 (6)

- (1) 200 subjects were in the group that received no alcohol.
- (2) 200 subjects were in the group that received a standardized amount of alcohol.
- (3) 180 subjects had a good reaction time.
- (4) 160 subjects had a medium reaction time.
- (5) 60 subjects had a strong reaction delay.
- (6) 400 subjects were tested.

2.

$f(a_i b_j)$	Good	Medium	Strong delay	
Without alcohol	2/3 (1)	3/8	1/3	
With alcohol	1/3	5/8	2/3	
	1	1	1	

- (1) 2/3 of subjects who had a good reaction time didn't receive alcohol.

$f(b_j a_i)$	Good	Medium	Strong delay	
Without alcohol	3/5 (1)	3/10	1/10	1
With alcohol	3/10	1/2	2/20	1

- (1) 3/5 of subjects who didn't receive alcohol had a good reaction time.

3.

H_0 : Reaction time doesn't correlate with alcohol intake

H_1 : Reaction time correlates with alcohol intake

$$\chi^2 = \frac{\left(120 - \frac{180 \cdot 200}{400}\right)^2}{\frac{180 \cdot 200}{400}} + \frac{\left(60 - \frac{160 \cdot 200}{400}\right)^2}{\frac{160 \cdot 200}{400}} + \frac{\left(20 - \frac{60 \cdot 200}{400}\right)^2}{\frac{60 \cdot 200}{400}} + \frac{\left(60 - \frac{180 \cdot 200}{400}\right)^2}{\frac{180 \cdot 200}{400}} + \frac{\left(100 - \frac{160 \cdot 200}{400}\right)^2}{\frac{160 \cdot 200}{400}} + \frac{\left(40 - \frac{60 \cdot 200}{400}\right)^2}{\frac{60 \cdot 200}{400}}$$

$$\approx 36.78 > 10.597$$

→ i.e. we reject H_0 at significance level $\alpha = 0.005$.

2)

1.

Temperature (F): Intervall-scale

Socio-Economic Status: Ordinal

Temperature (K): Ratio-scale

Distance: Intervall-scale

Grades in School: Ordinal

Description: Nominal

Date: Intervall-scale

2.

Intervall-scale: temperature (Celsius), time (e.g. 00:00)

Ratio-scale: number of sales, weight

3)

1. **Original L1 and L ∞ Distances:**

- $\sum |A_i - B_i|$ for each time point i .

$$L1 = |-1.66-0.29| + |0.30-0.89| + |-0.08-0.82| + |0.10-0.97| + |-1.17-0.53| + |-0.05-0.83| + |0.84-1.06| + |-0.66-0.67| + |0.42-0.86| + |-0.99-0.51|$$

- $|-1.66-0.29| = |-1.95| = 1.95$ $|-1.66-0.29| = |-1.95| = 1.95$
- $|0.30-0.89| = |-0.59| = 0.59$ $|0.30-0.89| = |-0.59| = 0.59$
- $|-0.08-0.82| = |-0.90| = 0.90$ $|-0.08-0.82| = |-0.90| = 0.90$
- $|0.10-0.97| = |-0.87| = 0.87$ $|0.10-0.97| = |-0.87| = 0.87$
- $|-1.17-0.53| = |-1.70| = 1.70$ $|-1.17-0.53| = |-1.70| = 1.70$
- $|-0.05-0.83| = |-0.88| = 0.88$ $|-0.05-0.83| = |-0.88| = 0.88$
- $|0.84-1.06| = |-0.22| = 0.22$ $|0.84-1.06| = |-0.22| = 0.22$
- $|-0.66-0.67| = |-1.33| = 1.33$ $|-0.66-0.67| = |-1.33| = 1.33$
- $|0.42-0.86| = |-0.44| = 0.44$ $|0.42-0.86| = |-0.44| = 0.44$
- $|-0.99-0.51| = |-1.50| = 1.50$ $|-0.99-0.51| = |-1.50| = 1.50$

$$L1 = 1.95 + 0.59 + 0.90 + 0.87 + 1.70 + 0.88 + 0.22 + 1.33 + 0.44 + 1.50 = 10.38$$

- $L\infty = \max(1.95, 0.59, 0.90, 0.87, 1.70, 0.88, 0.22, 1.33, 0.44, 1.50) = 1.95$

1. Offset Translation (Mean Subtraction):

- $|A1'-B1'|=0.912$
- $|A2'-B2'|=0.448$
- $|A3'-B3'|=0.138$
- $|A4'-B4'|=0.168$
- $|A5'-B5'|=0.662$
- $|A6'-B6'|=0.158$
- $|A7'-B7'|=0.818$
- $|A8'-B8'|=0.292$
- $|A9'-B9'|=0.598$
- $|A10'-B10'|=0.462$

$$L1_{\text{offset}} = 0.912 + 0.448 + 0.138 + 0.168 + 0.662 + 0.158 + 0.818 + 0.292 + 0.598 + 0.462 = 4.656$$

$$L_{\infty \text{offset}} = \max(0.912, 0.448, 0.138, 0.168, 0.662, 0.158, 0.818, 0.292, 0.598, 0.462) = 0.912$$

2. Amplitude Scaling (Standard Deviation Normalization):

- $|A1''-B1''|=0.1897$
- $|A2''-B2''|=0.1405$
- $|A3''-B3''|=0.0549$
- $|A4''-B4''|=0.4792$
- $|A5''-B5''|=0.2207$
- $|A6''-B6''|=0.0592$
- $|A7''-B7''|=0.1062$
- $|A8''-B8''|=0.1622$
- $|A9''-B9''|=0.4327$
- $|A10''-B10''|=0.1070$

$$L1_{\text{scaled}} = 0.1897 + 0.1405 + 0.0549 + 0.4792 + 0.2207 + 0.0592 + 0.1062 + 0.1622 + 0.4327 + 0.1070 \\ = 1.9524$$

$$L_{\infty \text{scaled}} = \max(0.1897, 0.1405, 0.0549, 0.4792, 0.2207, 0.0592, 0.1062, 0.1622, 0.4327, 0.1070) \\ = 0.4792$$

4.1

```
In [ ]: import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

data = pd.read_csv('linreg_data.csv')

x = data['X']
y = data['Y']

x_mean = np.mean(x)
y_mean = np.mean(y)

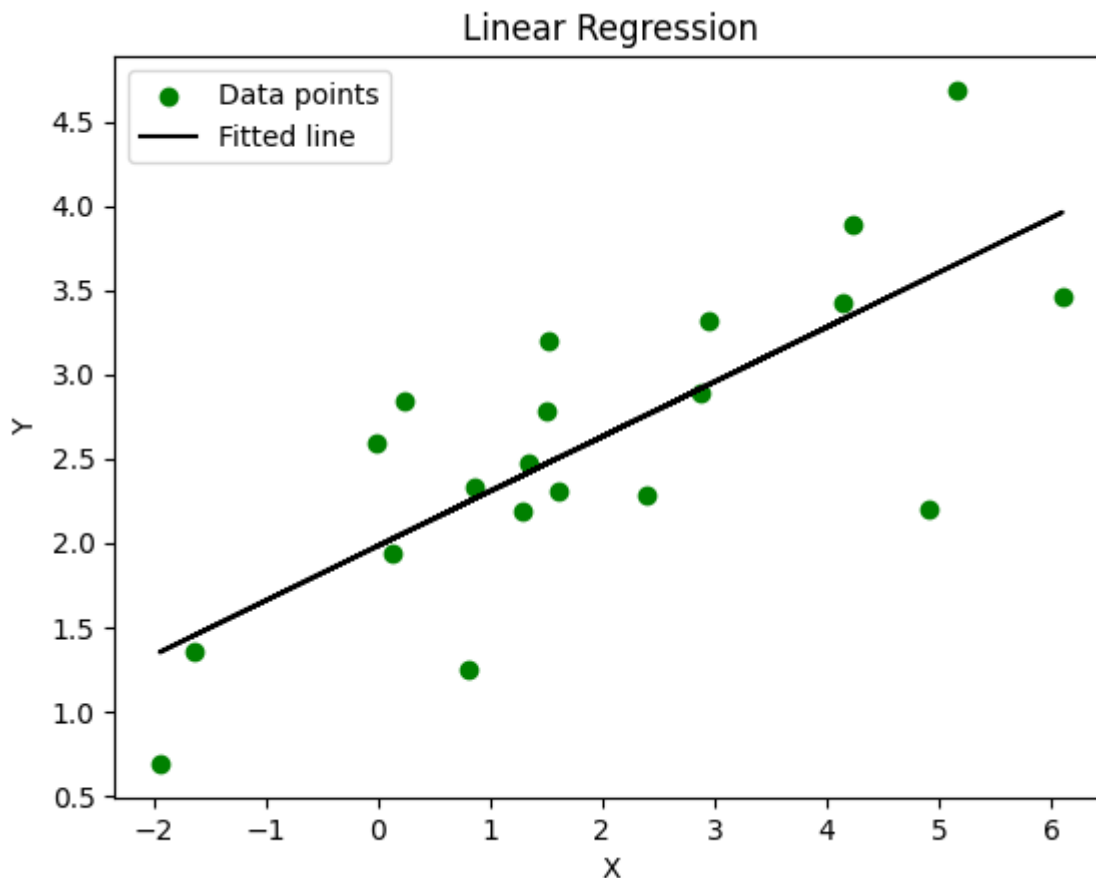
data['xy_cov'] = (data['X'] - x_mean) * (data['Y'] - y_mean)
data['x_var'] = (data['X'] - x_mean)**2

beta = data['xy_cov'].sum() / data['x_var'].sum()
alpha = y_mean - (beta * x_mean)

def predict(x):
    return alpha + beta * x

fit_line = predict(x)

plt.scatter(x, y, color='green', label='Data points')
plt.plot(x, fit_line, color='black', label='Fitted line')
plt.xlabel('X')
plt.ylabel('Y')
plt.title('Linear Regression')
plt.legend()
plt.show()
```



4.2

```
In [ ]: import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import linregress

data = pd.read_csv('linreg_data.csv')

x = data['X']
y = data['Y']

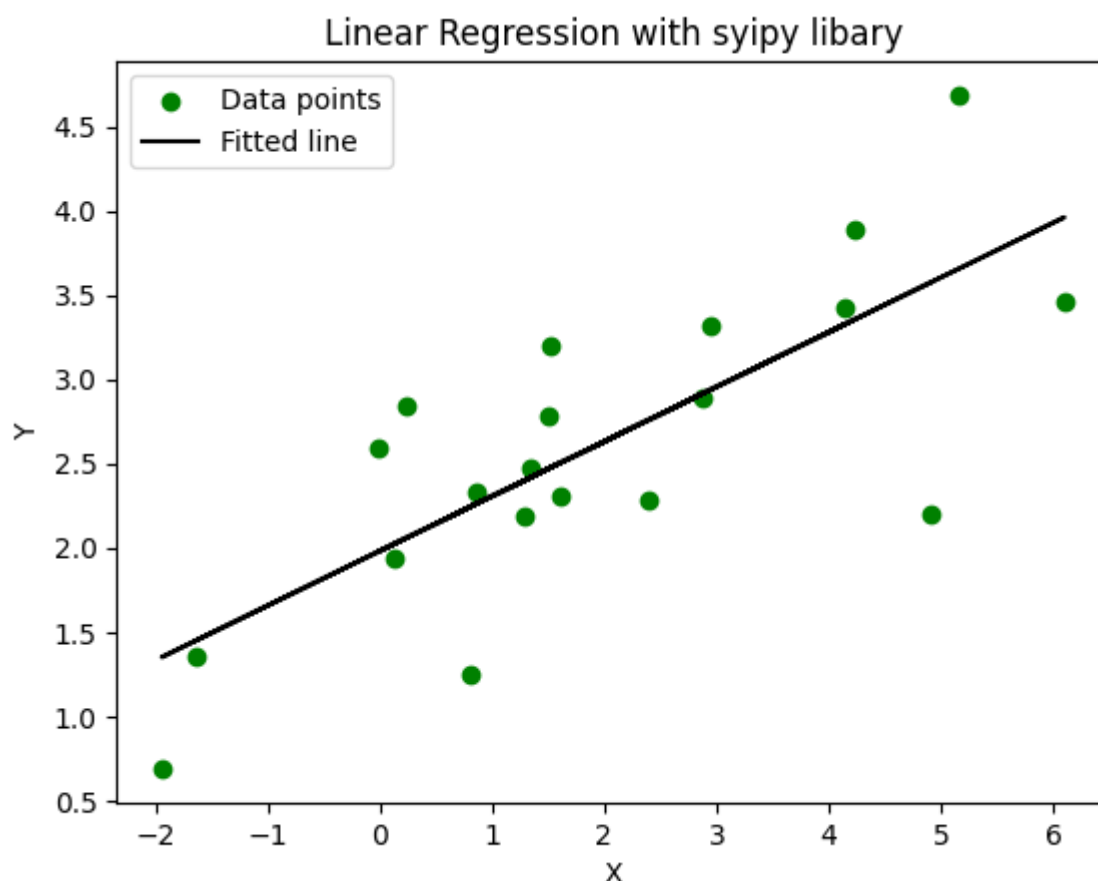
regression_result = linregress(x, y)

slope = regression_result.slope
intercept = regression_result.intercept

def predict(x):
    return intercept + slope * x

fit_line = predict(x)

plt.scatter(x, y, color='green', label='Data points')
plt.plot(x, fit_line, color='black', label='Fitted line')
plt.xlabel('X')
plt.ylabel('Y')
plt.title('Linear Regression with syipy library')
plt.legend()
plt.show()
```



4.3

```
In [ ]: import pandas as pd

data = pd.read_csv('linreg_data.csv')

x = data['X']
```

```

y = data['Y']

x_mean = x.mean()
y_mean = y.mean()

numerator = sum((x - x_mean) * (y - y_mean))

denominator = sum((x - x_mean)**2)

slope = numerator / denominator

intercept = y_mean - slope * x_mean

def predict(x):
    return intercept + slope * x

y_predicted_for_x_2 = predict(2)
print("Predicted Y value for X = 2:", y_predicted_for_x_2)

```

Predicted Y value for X = 2: 2.629747164010065

4.4

In order to smoothen the original data one can use the linear regression model to replace the former Y values with the predictions of the regression. This way its easier to see trends in the data.