AI Lab for Wireless Communications

Week2 - Syndrome decoding & maximum likelihood decoding

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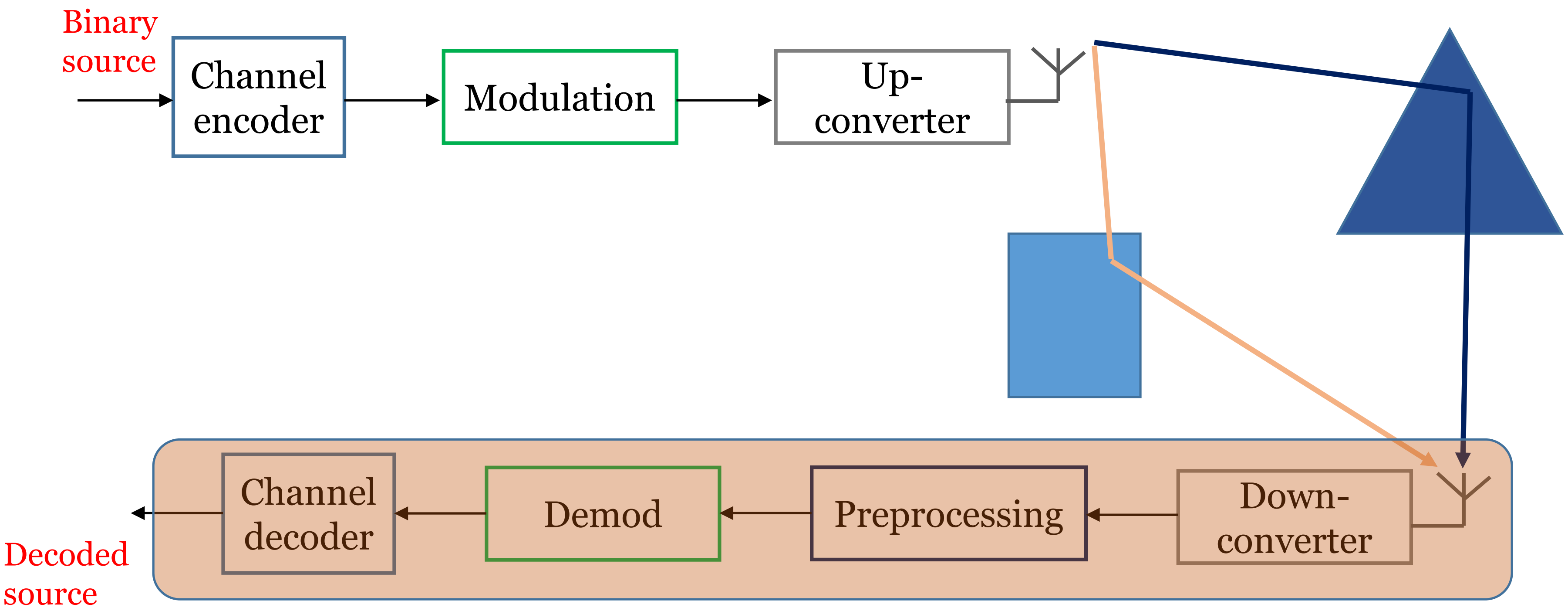
A digital communication system over wireless channel



- Channel enc/dec allows error correction by adding redundancy
- Channel decoding is the part we focus on in the following courses

Recall: (7,4)-Hamming Coded

- *4*-bit message $\boldsymbol{m}$, *7*-bit codeword $\boldsymbol{c}$
- Relationship: $\boldsymbol{c} = \boldsymbol{mG}, \boldsymbol{rH^T} = \boldsymbol{0}$

- Generator matrix

$$\mathbf{G} := \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

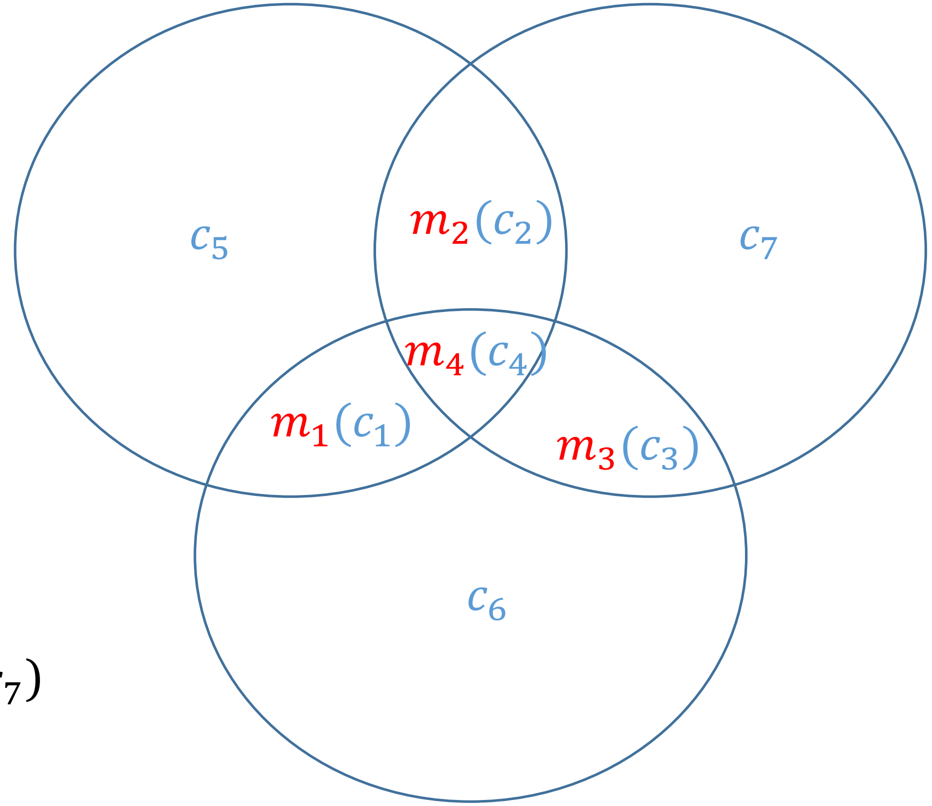
- Parity check matrix

$$\mathbf{H} := \begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Recall: Idea of Hamming code

- The relationship of each bits in the codeword can be illustrated by the following figure
- $\boldsymbol{c} = \boldsymbol{mG}$

$$\mathbf{G} := \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$



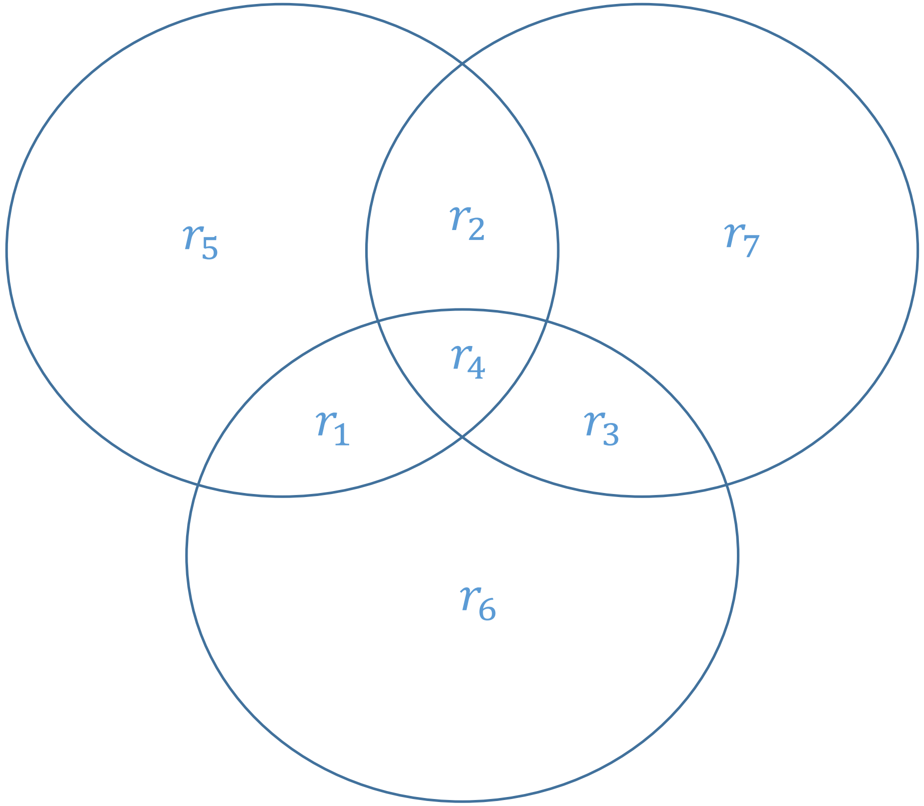
where $\boldsymbol{m} = (m_1, m_2, m_3, m_4)$ and $\boldsymbol{c} = (c_1, c_2, c_3, c_4, c_5, c_6, c_7)$

Recall: Idea of Hamming code

- The relationship of each bits in the codeword can be illustrated by the following figure
- $\boldsymbol{r}\boldsymbol{H}^T = \mathbf{0}$

$$\mathbf{H} := \begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

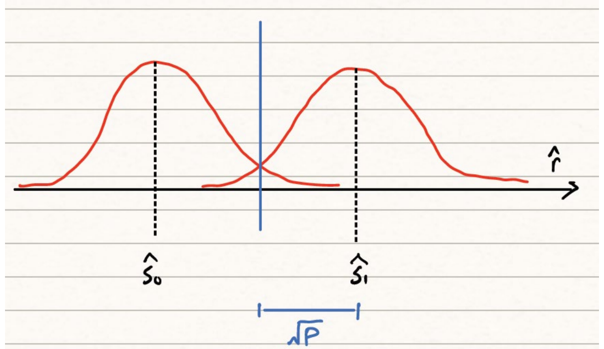
where $\boldsymbol{r} = (r_1, r_2, r_3, r_4, r_5, r_6, r_7)$



Recall Channel Model

- When transmit, we map coded bits to baseband signal
- Binary phase shift keying (BPSK)

$$x_i = \sqrt{P}(2c_i - 1), \qquad \boldsymbol{x} = [x_1, \dots, x_n]$$



- Additive white Gaussian noise (AWGN) channel

$$y_i = x_i + w_i,\ \ w_i \sim N(0, \frac{N_0}{2}),\ \ \boldsymbol{y} = [y_1, \dots, y_n]$$

Syndrome decoding

Concept of syndrome decoding (1/2)

- We first do hard decision

$$\hat{z}_i = sign\,(y_i) \quad \text{and} \quad \hat{d}_i = (\hat{z}_i + 1)/2$$

- Assume $\boldsymbol{e}$ is the error, we have

$$\begin{aligned} \widehat{\boldsymbol{d}}\boldsymbol{H}^T &= \boldsymbol{s} \\ \Rightarrow \quad (\boldsymbol{x}+\boldsymbol{e})\boldsymbol{H}^T &= \boldsymbol{s} \\ \Rightarrow \quad \boldsymbol{x}\boldsymbol{H}^T + \boldsymbol{e}\boldsymbol{H}^T &= \boldsymbol{s} \\ \Rightarrow \quad \boldsymbol{0} + \boldsymbol{e}\boldsymbol{H}^T &= \boldsymbol{s} \end{aligned}$$

- We call **s** is the syndrome, if $\boldsymbol{s} \neq \boldsymbol{0}$, then there must exist error

Concept of syndrome decoding (2/2)

- How to recover the signal?

⇒ Define the error pattern $\hat{\boldsymbol{e}}$ that $(\hat{\boldsymbol{e}} + \boldsymbol{e})\boldsymbol{H}^T = \mathbf{0}$

- We can have

$$\widehat{\boldsymbol{x}} = \widehat{\boldsymbol{d}} + \hat{\boldsymbol{e}}$$

$$\Rightarrow \quad \widehat{\boldsymbol{x}} = (\boldsymbol{x} + \boldsymbol{e}) + \hat{\boldsymbol{e}}$$

$$\Rightarrow \quad \widehat{\boldsymbol{x}} = \boldsymbol{x}$$

Procedure of syndrome decoding

- Do hard decision
- Construct the standard array
- Compute the syndrome $\widehat{\boldsymbol{d}}\boldsymbol{H}^T = \boldsymbol{s}$
- Decide $\hat{\boldsymbol{e}} = \text{coset leader}(\boldsymbol{s})$
- Decode to $\widehat{\boldsymbol{x}} = \widehat{\boldsymbol{d}} + \hat{\boldsymbol{e}}$

Example

- We have

$$\boldsymbol{y'} \quad = \quad \boldsymbol{x'} \quad + \ (0000100)$$

$$(1101000) = (1101010) + \ (0000010)$$

- Doing syndrome decoding

$$\boldsymbol{y}\boldsymbol{H}^T = \boldsymbol{s}$$

$$\Rightarrow \quad (\boldsymbol{x} + \boldsymbol{e})\boldsymbol{H}^T = \boldsymbol{s}$$

$$\Rightarrow \quad \boldsymbol{x}\boldsymbol{H}^T + \boldsymbol{e}\boldsymbol{H}^T = \boldsymbol{s}$$

$$\Rightarrow \quad \boldsymbol{0} + \boldsymbol{e}\boldsymbol{H}^T = \boldsymbol{s}$$

Example

- Then $\boldsymbol{s} = (100)$
- By $\boldsymbol{eH}^T = \boldsymbol{s}$, there are several **e** that makes the equation holds

$$\boldsymbol{e_1} = (0000100), \boldsymbol{e_2} = (00011100)$$

- $\boldsymbol{e} = (0000100)$ is the most possible case since the probability of each bit flips is p

Standard Array

Syndrome $s = rH^T$	**Error pattern** $\hat{e}$
(0,0,0)	(0,0,0,0,0,0,0)
(0,0,1)	(0,0,0,0,0,0,1)
(0,1,0)	(0,0,0,0,0,1,0)
(0,1,1)	(0,0,1,0,0,0,0)
(1,0,0)	(0,0,0,0,1,0,0)
(1,0,1)	(0,1,0,0,0,0,0)
(1,1,0)	(1,0,0,0,0,0,0)
(1,1,1)	(0,0,0,1,0,0,0)

Performance evaluation

- Block error rate (BLER) – If the received block has one or even more bits error, we say that this Block (codeword) is error. The block error rate is the number of block errors per unit block.

- Bit error rate (BER) – If any bit of a block is wrong, we say that there is an error. The bit error rate is the number of bit errors per unit bit.

Performance evaluation (2/2)

- Signal to Noise Ratio – The ratio of signal power to the noise power. $\left(SNR = 10\log_{10}\frac{P_{signal}}{P_{noise}} \right)$
- Go through all codewords and compare whether $\widehat{\boldsymbol{m}} = \boldsymbol{m}$
- In communication system, we usually evaluate decoding scheme by observing block or bit error rate over different E_b/N_0
- Take (7,4) hamming code and SNR=η as example,
  - ➢We transmit 7 bits ($P_{signal} = 7 \cdot \eta$) per signal
  - ➢However, there only contains 4 information bits

    $\Rightarrow$ Each information bit uses $7/4 \cdot \eta$

Maximum Likelihood Decoding

ML Decoding

MLD:

$$\hat{\boldsymbol{c}} = arg\min_{\boldsymbol{c}\in C} ||\boldsymbol{y} - \boldsymbol{x}||^2$$

- This is optimal

- Complexity is very high, especially when n is large

ML Decoding Steps

- Construct all possible codeword
- Calculate the norm of received codeword and possible codeword
- Find the codeword which has the minimum norm
- Calculate the BER and BLER