Week Three PHY-480

Lewis

September 2025

Solutions

Question 1

Irregular dynamical behavior and chaotic dynamical behavior share certain similarities. Both can appear highly complex, non-repetitive, and sensitive giving the appearance of randomness. However, irregular dynamics may arise from stochastic or random influences, whereas chaotic dynamics are generated by deterministic equations. That is to say it's not inherently random, if you had the means you could certainly calculate it without question. They yield purely deterministic yet functionally unpredictable long-term behavior due to sensitivity to initial conditions, and they are characterized by features such as strange attractors and positive Lyapunov exponents.

Question 2

For the logistic map,

$$x_{n+1} = \lambda x_n(1 - x_n),$$

the Lyapunov exponent is calculated as

$$\nu = \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} \ln |f'(x_n)| ,$$

where

$$f'(x) = \lambda(1 - 2x).$$

The Lyapunov exponent is **positive**, indicating exponential divergence.

Question 3

As the logistic map parameter $\lambda \in [1, 4]$ increases, the system undergoes a period-doubling route to chaos:

- For $1 < \lambda < 3$, trajectories converge to a stable fixed point ($\lambda < 0$).
- At $\lambda \approx 3$, the fixed point loses stability and a period-2 cycle appears.

- Further increases in λ lead to successive period doublings.
- At $\lambda \approx 3.57$, infinitely many bifurcations accumulate and chaos begins ($\lambda > 0$).
- For $3.57 < \lambda < 4$, the system is chaotic, with periodic "windows" (e.g., period-3) embedded within chaos.

Question 4

An orbit diagram shows what values a system settles into when you run it many times while changing a parameter. For the logistic map, this means plotting the long-term values of x as the growth rate λ changes. The Hénon map, the idea is similar but it's in 2 dimensions you're looking at the long-term points (x, y) for different values of the parameters and see the patterns or attractors that appear.

Question 5

A strange attractor is a shape in the system's long term behavior that looks irregular and never repeats exactly, but it doesn't blow up to infinity either. With the Hénon map, if you keep iterating the equations, the points end up collecting on a complicated fractal shape called the Hénon attractor.

Question 6

To find the fractal dimension of a coastline using box counting, you cover the map with a grid of boxes. You count how many boxes touch the coastline. Then you make the boxes smaller and count again. If you repeat this at different box sizes, you can see how the number of boxes grows as they get smaller. The slope of that relationship tells you the fractal dimension, which shows the complexity of the coastline.