Week Three Assignments PHY - 480
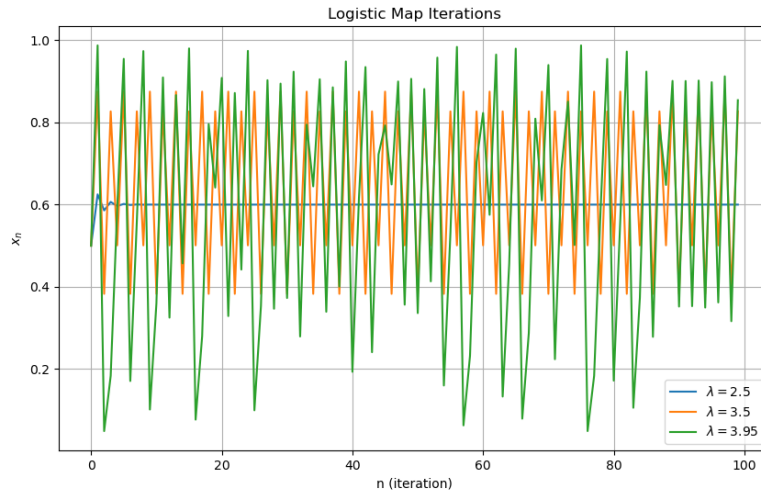
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September 2025

1 Introduction

Figure 1: Iterations of the logistic map $x_{n+1} = \lambda x_n (1 - x_n)$ for initial condition $x_0 = 0.5$ and $\lambda = 2.5, 3.5, 3.95$. For $\lambda = 2.5$, the sequence converges to a fixed point near $x \approx 0.6$ (steady state). For $\lambda = 3.5$, the system enters a period-2 oscillation, alternating between two values. For $\lambda = 3.95$, the system exhibits chaotic behavior with no steady state.
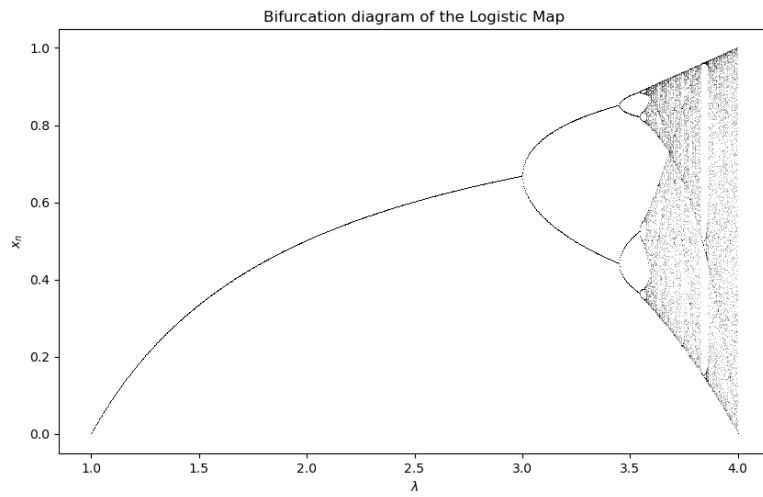
Figure 2: Bifurcation diagram of the logistic map. For small λ, the system settles to a fixed point and the Lyapunov exponent ν is negative. As λ increases, period-doubling occurs and ν approaches zero at each bifurcation. For $\lambda 3.57$, ν becomes positive and the system shows chaotic behavior, with small periodic windows.