Week Three PHY-480

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Solutions

Question 1

Irregular dynamical behavior and chaotic dynamical behavior share certain similarities. Both can appear highly complex, non-repetitive, and sensitive giving the appearance of randomness. However, irregular dynamics may arise from stochastic or random influences, whereas chaotic dynamics are generated by deterministic equations. That is to say it's not inherently random, if you had the means you could certainly calculate it without question. They yield purely deterministic yet functionally unpredictable long-term behavior due to sensitivity to initial conditions, and they are characterized by features such as strange attractors and positive Lyapunov exponents.

Question 2

For the logistic map,

$$x_{n+1} = \lambda x_n(1 - x_n),$$

the Lyapunov exponent is calculated as

$$\nu = \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} \ln |f'(x_n)| ,$$

where

$$f'(x) = \lambda(1 - 2x).$$

The Lyapunov exponent is **positive**, indicating exponential divergence.

Question 3

As the logistic map parameter $\lambda \in [1, 4]$ increases, the system undergoes a period-doubling route to chaos:

- For $1 < \lambda < 3$, trajectories converge to a stable fixed point ($\lambda < 0$).
- At $\lambda \approx 3$, the fixed point loses stability and a period-2 cycle appears.

- Further increases in λ lead to successive period doublings.
- At $\lambda \approx 3.57$, infinitely many bifurcations accumulate and chaos begins ($\lambda > 0$).
- For $3.57 < \lambda < 4$, the system is chaotic, with periodic "windows" (e.g., period-3) embedded within chaos.