Week Eleven PHY-480

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1 Pre-Quiz 20 Answers

Q20.1

For the Hamiltonian

$$H = -\sum_{i=1}^{N} J Z_i Z_{i+1} - \lambda \sum_{i=1}^{N} X_i,$$

the $Z_i Z_{i+1}$ term is diagonal, while each X_i flips one spin. Thus, each state connects to N others differing by one spin flip. Each row of the $2^N \times 2^N$ Hamiltonian therefore has N **non-zero off-diagonal elements.**

Q20.2

If

$$H|\psi_l\rangle = E_l|\psi_l\rangle,$$

and

$$|\psi(0)\rangle = \sum_l c_l|\psi_l\rangle,$$

then the time evolution is

$$|\psi(t)\rangle = \sum_l c_l e^{-iE_l t/\hbar}|\psi_l\rangle.$$

Each eigenstate picks up a phase factor $e^{-iE_l t/\hbar}$.

Q20.3

Goal: Build and diagonalize H to find ground state energy E_G and magnetization m_G.

```
import numpy as np
from scipy.sparse import kron, identity, csr_matrix
from scipy.sparse.linalg import eigsh
```

```
sx = csr_matrix(np.array([[0,1],[1,0]]))
sz = csr_matrix(np.array([[1,0],[0,-1]]))
I  = csr_matrix(np.eye(2))

def build_H(N,J,lam):
    H = csr_matrix((2**N,2**N),dtype=float)
    for i in range(N):
        ZZ = 1
        for j in range(N):
            ZZ = kron(ZZ, sz if j in [i,(i+1)%N] else I)
        H -= J*ZZ
        X = 1
        for j in range(N):
            X = kron(X, sx if j==i else I)
        H -= lam*X
    return H

E, psi = eigsh(build_H(6,1,1), k=1, which='SA')
```

This uses sparse matrices and finds the lowest eigenvalue (ground state).

Q20.4

Near $\lambda_c = 1$, magnetization follows

$$m \propto (\lambda_c - \lambda)^x.$$

From the 2D Ising model correspondence,

$$x = \frac{1}{8}.$$

Thus, $m \sim (\lambda_c - \lambda)^{1/8}$ as $N \to \infty$ and $\lambda \to \lambda_c$ from below.

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2 Pre-Quiz 21 Answers

Q21.1

T_1 is the **energy relaxation time**, the time for a qubit to lose energy and return to its ground state. T_2 is the **dephasing time**, the time over which a qubit maintains quantum phase coherence. T_G is the **gate time**, the duration of a single quantum gate operation. A large ratio T_2/T_G indicates many coherent gate operations can occur before decoherence.

Q21.2

Rabi oscillations describe the periodic transfer of population between the two energy levels of a qubit when driven by a resonant oscillating field. They measure how coherently the qubit responds to external control and indicate phase stability and gate fidelity.

Q21.3

The time evolution operator for a time-dependent Hamiltonian is

$$U(t) = Te^{-i\int_0^t H(s)\,ds}.$$

Using the Trotter approximation, it becomes

$$U(t) \approx \prod_{j=0}^{n-1} e^{-iH(t_j)\delta t},$$

where $\delta t = t/n$. This decomposes the evolution into short, sequential unitary steps that can be computed independently.

Q21.4

Algorithm to compute magnetization dynamics of the transverse Ising model:

1. Discretize total time t into n steps of size δt.
2. For each step t_j, build
$$H(t_j) = -J\sum_i Z_i Z_{i+1} - \lambda(t_j)\sum_i X_i, \quad \lambda(t_j) = \lambda_0 \cos(\omega t_j).$$
3. Compute
$$\psi(t_{j+1}) = e^{-iH(t_j)\delta t}\psi(t_j).$$
4. At each step, calculate magnetization
$$m(t_j) = \frac{1}{N}\sum_i \langle \psi(t_j)|Z_i|\psi(t_j)\rangle.$$
5. Plot $m(t)$ versus t for different λ_0 and ω values.