Week Three Assignments PHY - 480

Lewis

September 2025

1 Introduction

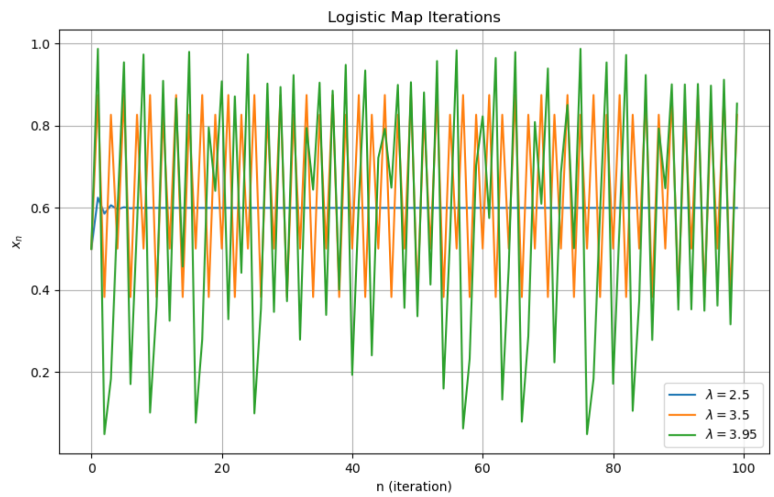

Figure 1: Iterations of the logistic map $x_{n+1} = \lambda x_n(1 - x_n)$ for initial condition $x_0 = 0.5$ and $\lambda = 2.5, 3.5, 3.95$. For $\lambda = 2.5$, the sequence converges to a fixed point near $x \approx 0.6$ (steady state). For $\lambda = 3.5$, the system enters a period-2 oscillation, alternating between two values. For $\lambda = 3.95$, the system exhibits chaotic behavior with no steady state.

2 Hénon Map Analysis

The Hénon map is defined as:

$$x_{n+1} = 1 - ax_n^2 + y_n, \qquad y_{n+1} = bx_n$$

For $a = 1.4$ and $b = 0.3$, the map shows chaotic behavior and a fractal attractor.

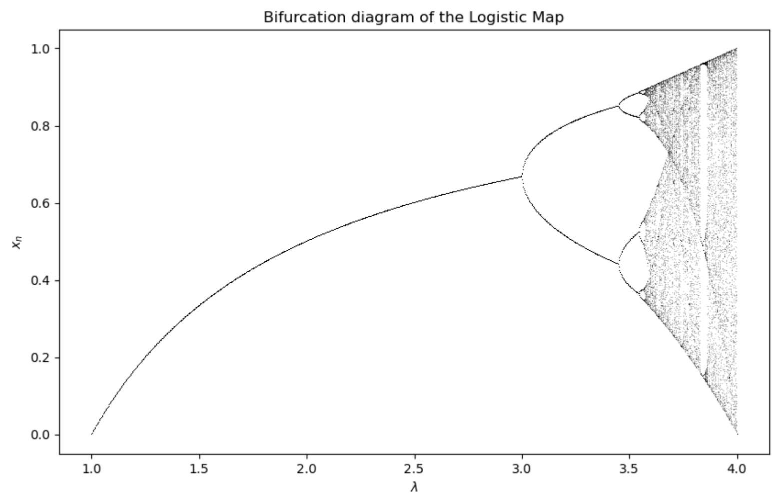

Figure 2: Bifurcation diagram of the logistic map. For small λ, the system settles to a fixed point and the Lyapunov exponent ν is negative. As λ increases, period doubling occurs and ν approaches zero at each bifurcation. For $\lambda 3.57$, ν becomes positive and the system shows chaotic behavior, with small periodic windows.

2.1 Orbit Diagram

2.2 Fractal Dimension

A box counting analysis was done on one million points of the Hénon attractor. The slope of the log log plot of $N(\varepsilon)$ vs. ε gives:

$$D_f \approx 1.23$$

2.3 Comparison

Both the logistic and Hénon maps show period doubling routes to chaos. The Hénon map is two dimensional and produces a fractal attractor with $D_f \approx 1.23$.

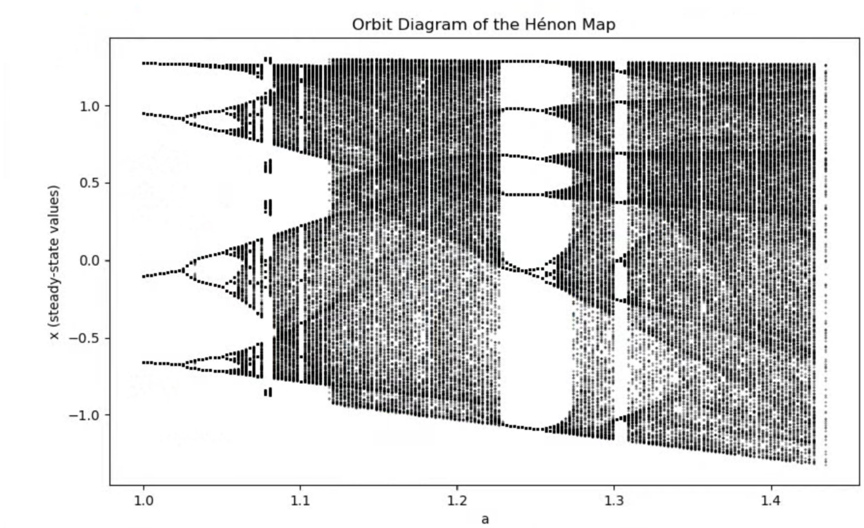

Figure 3: Orbit diagram of the Hénon map for $a \in [1.0, 1.5]$ and $b = 0.3$. As a increases, the system transitions from a fixed point to period doubling and chaos.

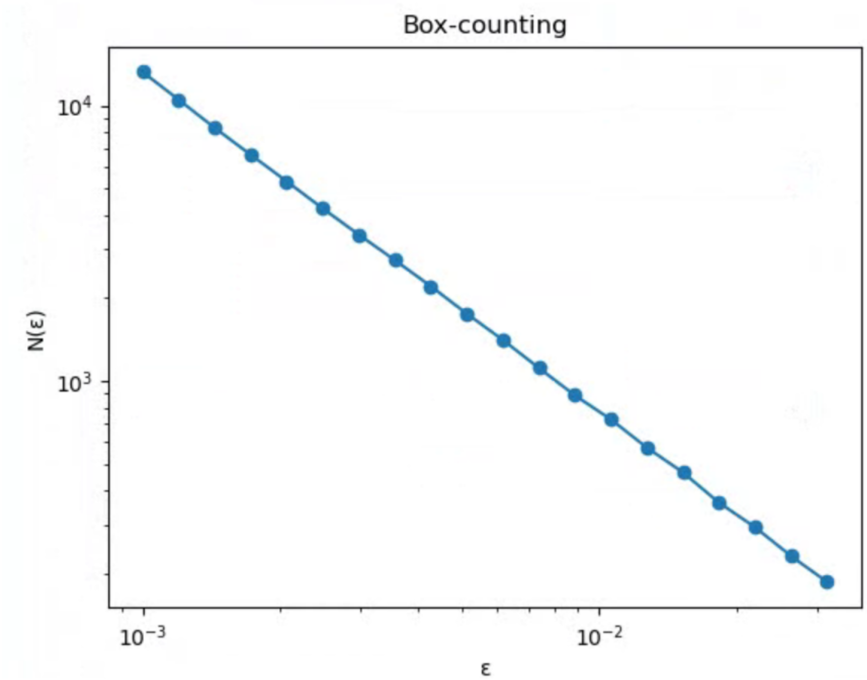

Figure 4: Box counting plot used to estimate the fractal dimension of the Hénon attractor.