

Symbols

c	Speed of light (3×10^8 m/s).
ω	Angular velocity (m/s).
u	Phase velocity (m/s).
γ	(Complex) propogati on constant.
β	Phase constant, also called “wavenum- ber” (rad/m).
α	Attenuation constant.
ϕ	Phase.
ϕ_0	Phase offset. If positive, wave leads (left-shifted). If negative, wave lags (right-shifted).
$e^{-\alpha x}$	Attenuation factor + x direction (in a lossy medium).
Z_0	Characteristic impedance (Ω).
Z_L	Load impedance.
z_L	Normalized load impedance [eq (10)].
Γ	Voltage reflection coefficient. The am- plitudes of the reflected and incident voltage waves at the load.
L'	The combined inductance of both con- ductors per unit length, in H/m.
G'	The conductance of the insulation medium between the two conductors per unit length, in S/m.
C'	The capacitance of the two conductors per unit length in, F/m.
R'	The combined resistance of both conduc- tors per unit length, in Ω /m.
ϵ	Permittivity (dielectric insulator).
ϵ_0	Permittivity of free space (8.854×10^{-12} F/m).
ϵ_r	Relative permittivity.
ϵ_{eff}	Effective relative permittivity.
μ	Permeability (dielectric insulator).
μ_0	Permeability of free space ($4\pi \times 10^{-7}$ H/m).
μ_c	Permeability of conducting strip.
σ	Conductivity (dielectric insulator).
σ_c	Conductivity of conducting strip.

Equations

$$\omega = 2\pi f = \frac{2\pi}{T}$$
$$\beta = \frac{2\pi}{\lambda}$$
$$u_p = \frac{\lambda}{T} = f\lambda = \frac{\omega}{\beta}$$
$$\phi(x,t) = \omega t - \beta x + \phi_0$$
$$y(x,t) = A \cos(\phi(x,t))$$
$$e^{j\theta} = \cos(\theta) + j \sin(\theta)$$

Euler's Identity

$$\frac{d\tilde{V}(z)}{dz} = -(R' + j\omega L')\tilde{I}(z)$$
$$\frac{d\tilde{I}(z)}{dz} = -(G' + j\omega C')\tilde{V}(z)$$
$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')} = \alpha + j\beta$$
$$\alpha = \Re(\gamma); \beta = \Im(\gamma)$$
$$\frac{d^2\tilde{I}(z)}{dz^2} - \gamma^2\tilde{I}(z) = 0$$
$$\frac{d^2\tilde{V}(z)}{dz^2} - \gamma^2\tilde{V}(z) = 0$$
$$Z_0 = \frac{R' + j\omega L'}{\gamma} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} = \frac{V_0^+}{I_0^+} = \frac{-V_0^-}{I_0^-}$$

Telegrapher's equations

Wave equations

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1}$$
$$= \frac{z_L - 1}{z_L + 1}$$
$$z_L = \frac{Z_L}{Z_0}$$

$$V_0^+ = |V_0^+|e^{j\phi^+}; \quad V_0^- = |V_0^-|e^{j\phi^-}$$

$$v(x,t) = \Re(\tilde{V}(x)e^{j\omega t})$$
$$= |V_0^+|e^{-\alpha x} \cos(\omega t - \beta x + \phi^+)$$
$$+ |V_0^-|e^{\alpha x} \cos(\omega t + \beta x + \phi^-)$$

$$\epsilon_r = \frac{\epsilon}{\epsilon_0}$$

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

For coaxial, two-wire, parallel-plate:

$$u_p = \frac{c}{\sqrt{\epsilon_r}}$$

For microstrip:

$$u_p = \frac{c}{\sqrt{\epsilon_{\text{eff}}}}$$

$$R' = 0 \quad (\because \sigma_c = \infty);$$

$$G' = 0 \quad (\because \sigma = 0);$$

$$C' = \frac{\sqrt{\epsilon_{\text{eff}}}}{Z_0 c};$$

$$L' = Z_0^2 C';$$

$$\alpha = 0 \quad (\because R' = G' = 0);$$

$$\beta = \frac{\omega}{c} \sqrt{\epsilon_{\text{eff}}}.$$

For all TEM lines:

$$L' C' = \mu\epsilon$$

$$\frac{G'}{C'} = \frac{\sigma}{\epsilon}$$

Lossless Line

$$R' \ll \omega L'; \quad G' \ll \omega C' \rightarrow R' = G' \approx 0 \rightarrow$$

$$\gamma = \alpha + j\beta = j\omega\sqrt{L'C'}$$

From equation (8),

$$Z_0 = \sqrt{\frac{L'}{C'}}$$

From equations (2)/(3) and (26)

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{L'C'}}$$

$$u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{L'C'}}$$

From equation (23), (26) and (29)

$$\beta = \omega\sqrt{\mu\epsilon}$$

$$u_p = \frac{1}{\sqrt{\mu\epsilon}}$$

Notes

- Fundamental Properties of EM waves:
- A *monochromatic* (single frequency) EM wave consists of electric and magnetic fields that oscil-
late at the same frequency f .
 - The phase velocity of an EM wave in a vacuum is
the speed of light, c .
 - In vacuum, the wavelength of an EM wave is re-
lated to its oscillation frequency f by $\lambda = \frac{c}{f}$.
 - For passive transmission lines, α is either zero or
positive. The gain region of a laser is an example
of an active transmission line with a negative α .
 - Microstrip line is considered a *quasi-TEM* because
E and **F** are not everywhere perfectly orthogonal.
 - From equation (12), or a similar equation, the term
 $\dots e^{-j\beta x}$ is the **incident wave** (travelling from
source to load, or in the positive x direction). The
term $\dots e^{j\beta x}$ is the **reflected wave** (travelling from
load to source, or in the negative x direction).