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Journal: 2.2
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Symbols

$\implies$ implies

$\iff$ if and only if

Notes

- A conditional sentence is false when the hypothesis is true and the conclusion is false.
- In this conditional statement, $p \implies q$, p is the **hypothesis** (or antecedent) and q is the **conclusion** (or consequent).

p	q	$p \implies q$
T	T	T
T	F	F
F	T	T
F	F	T

- A conditional statement is **vacuously true** or **true by default** if the conditional statement is true by virtue of the fact that the hypothesis is false.
- Equivalent if-then representation:

  $p \implies q \equiv \neg p \vee q$

  $\neg (p \implies q) \equiv p \wedge \neg q$ (negation of conditional)

- The **contrapositive** of $p \implies q$ is $\neg q \implies \neg p$. I.e. a conditional statement is logically equivalent to its contrapositive. The formerly mentioned equivalence is important for modus tollens (journal 2.3) and for the contrapositive method of proof (journal 4.5).
- The **converse** of the conditional, $p \implies q$, is $q \implies p$.
- The **inverse** of the conditional, $p \implies q$, is $\neg p \implies \neg q$.
- The contrapositive of the converse is the inverse. Therefore, the converse and the inverse of a conditional are logically equivalent to each other.
- The **biconditional** of p and q is "p if, and only if, q" written as $p \iff q$.

p	q	$p \iff q$
T	T	T
T	F	F
F	T	F
F	F	T

- r is a **sufficient condition** for s means "if r then s". That is, the occurence of r is *sufficient* to guarantee the occurence of s.
- r is a **neccesary condition** for s means "if not r then not s". That is, if r does not occur, then s cannot occur either.

Test Yourself

1. An if-then statement is false if, and only if, the hypothesis is <u>true</u> and the conclusion is <u>false</u>.
2. The negation of "if p then q" is <u>"p and not q"</u>.
3. The converse of "if p then q" is <u>"if q then p"</u>.
4. The contrapositive of "if p then q" is <u>"if not q then not p"</u>.

5. The inverse of "if p then q" is "if not p then not q".
6. A conditional statement and its contrapositive are logically equivalent.
7. A conditional statement and its converse are not logically equivalent.
8. "R is a sufficient condition for S" means "if R then S".
9. "R is a neccesary condition for S" means "if not R then not S".
10. "R only if S" means "if S then R".