EE381 **Homework 5**

Due Date: 03/6 **At the beginning of class**

Problem 1

A ring placed along $y^2 + z^2 = 4$, $x = 0$ carries a uniform charge of $5\mu C/m$.

a. Find $\vec{D}$ at $(x, y, z) = (3,0,0)$
b. If two identical point charges Q are placed at $(0, -3,0)$ and $(0,3,0)$ in addition to the ring, find the value of Q such that $\vec{D} = 0$ at P.

Problem 2

a. Show that the electric field at point $(x, y, z) = (0,0, h)$ due to the rectangle described by $-a \leq x \leq a$, $-b \leq y \leq b$, $z = 0$ carrying uniform charge ρ_s is:

$$\vec{E} = \frac{\rho_s}{\pi\varepsilon_o} tan^{-1}\left[\frac{ab}{h(a^2 + b^2 + h^2)^{1/2}}\right]\hat{z}$$

b. If $a = 2$, $b = 5$, $\rho_s = 10^{-5} C/m^2$, find the total charge on the plate and the electric field intensity at $(0,0,10)$

c. Show that very far from the plate (i.e., $h \gg a, b$), the electric field converges to that generated by a point charge:

$$\vec{E} = \frac{Q_{plate}}{4\pi\varepsilon_o h^2}\hat{z}$$

d. Show that very near the plate (i.e., $h \ll a, b$), the electric field converges to that generated by an infinite sheet of charge:

$$\vec{E} = \frac{\rho_s}{2\varepsilon_o}\hat{z}$$

The following information may be useful (using https://www.integral-calculator.com/):

$$\int \frac{1}{(y^2 + h^2)\sqrt{y^2 + g^2}} dy = -\frac{1}{h\sqrt{g^2 - h^2}} tan^{-1}\left(\frac{gh\sqrt{\frac{y^2}{g^2} + 1}}{\sqrt{g^2 - h^2 x}}\right) + Constant$$

where g is any constant (when performing the y integral). Also, you may find the following relation useful:

$$tan^{-1}\left(\frac{x}{y}\right) = \frac{\pi}{2} - tan^{-1}\left(\frac{y}{x}\right)$$

Problem 3

The Thomson model of a hydrogen atom is a sphere of uniformly distributed positive charge with an electron (a point charge) at its center. The total positive charge equals the electronic charge e. Prove that when the electron is at a distance r from the center of the sphere of positive charge, it is attracted with a force:

$$F = \frac{e^2 r}{4\pi\varepsilon_o R^3}$$

where R is the radius of the sphere.

Problem 4: Classical Radius of the Electron

Assume that the electron is a spherical *shell* with a uniform charge density over the shell (and of course, a total charge e). Argue (from an energy perspective (hint: $E = mc^2$)) why, and show with calculations, that the radius of the electron should be:

$$R_{electron} = \frac{e^2}{8\pi\varepsilon_o mc^2}$$

Problem 5

To verify that $\vec{E} = yz\hat{x} + xz\hat{y} + xy\hat{z}$ *V/m* is truly an electric field (and a conservative field), show that:

a. $\nabla \times \vec{E} = 0$
b. $\oint_L \vec{E} \cdot d\vec{l} = 0$, where L is the edge of the square defined by $0 < x, y < 2, z = 1$.

Problem 6

A spherical charge distribution is given by

$$\rho_v = \begin{cases} \rho_o \dfrac{r}{a}, & r < a \\ 0, & r > a \end{cases}$$

Find V everywhere.