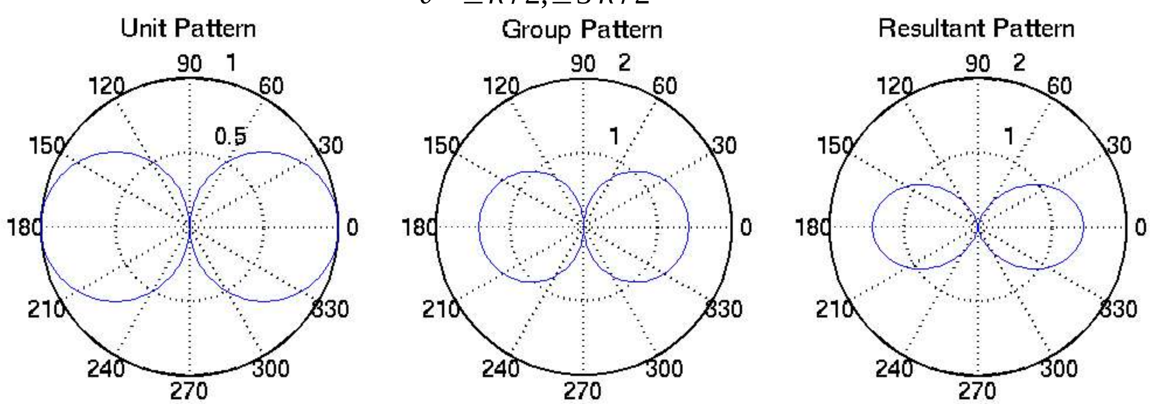
Exercise 13.6b: For the two-element antenna array of Figure 13.10, sketch the normalized field pattern when the currents are:

$$Fed\,90^o\ out\,of\,phase\,(\alpha=\pi\,,d=0.25\,\lambda)$$

$$f(\theta)=|\cos\theta|\cos\left[\frac{1}{2}(\beta\,d\cos\theta+\alpha)\right]$$

$$unit\,pattern=|\cos\theta|\,,group\,pattern=\cos\left[\frac{\pi}{4}\cos\theta+\frac{\pi}{2}\right]$$

$$group\,pattern\,nulls\,are\,at:$$
$$\theta=\pm\pi/2,\pm3\pi/2$$



Problem 13.23: For the following radiation intensities, find the directive gain and directivity:

A) $U(\theta,\phi)=\sin^2\theta\,,0<\theta<\pi\,,0<\phi<2\pi$

$$U_{ave}=\frac{1}{4\pi}\int U\ \ d\Omega=\frac{1}{4\pi}\int_0^{\pi}\int_0^{2\pi}\sin^2\theta\ \sin\theta\ \ d\phi\,d\theta=\frac{1}{2}\int_0^{\pi}\sin^3\theta\ \ d\theta$$

$$U_{ave}=\frac{1}{2}\left(\frac{\cos^3\theta}{3}-\cos\theta\right)\Bigg|_0^{\pi}=\frac{1}{2}\left(\frac{4}{3}\right)=\mathbf{\frac{2}{3}}=\boldsymbol{U_{ave}}$$

$$G_\phi=\frac{U}{U_{ave}}=\mathbf{1.5\,sin^2}\theta$$

$$D=G_{\phi,max}=\mathbf{1.5}$$

Problem 13.23 Continued:

B) $U(\theta,\phi)=4\sin^2\theta\cos^2\phi\,,0<\theta<\pi/2,0<\phi<\pi$

$$U_{ave}=\frac{1}{4\pi}\int U\ \ d\Omega=\frac{1}{4\pi}\int_0^{\pi}\int_0^{\pi}4\sin^2\theta\ \cos^2\phi\ \sin\theta\ \ d\phi\,d\theta$$

$$U_{ave}=\frac{1}{\pi}\int_0^{\pi}\cos^3\phi\ \ d\phi\int_0^{\pi}(1-\cos^2\theta)\,d(-\cos\theta)$$

$$U_{ave}=\frac{1}{\pi}\left[\frac{1}{2}\phi+\frac{\sin 2\phi}{4}\right]_0^{\pi}\left[\frac{\cos^3\theta}{3}-\cos\theta\right]_0^{\pi}=\left(\frac{1}{\pi}\right)\left(\frac{\pi}{2}\right)\left(\frac{4}{3}\right)=\mathbf{\frac{2}{3}}$$

$$G_{\phi}=\frac{U}{U_{ave}}=\mathbf{6\sin^2\theta\cos^2\phi}$$

$$D=G_{\phi,max}=\mathbf{6}$$

C) $U(\theta,\phi)=10\cos^2\theta\sin^2(\phi/2)\,,0<\theta<\pi\,,0<\phi<\pi/2$

$$U_{ave}=\frac{1}{4\pi}\int U\ \ d\Omega=\frac{1}{4\pi}\int_0^{\pi}\int_0^{\pi/2}10\cos^2\theta\ \sin^2\frac{\phi}{2}\ \sin\theta\ \ d\phi\,d\theta$$

$$U_{ave}=\frac{10}{4\pi}\int_0^{\pi/2}\sin^2\frac{\phi}{2}\ \ d\phi\int_0^{\pi}(\cos^2\theta)\,d(-\cos\theta)$$

$$U_{ave}=\frac{10}{4\pi}\left[\frac{1}{2}(1-\cos\phi)\right]_0^{\frac{\pi}{2}}\left[-\frac{\cos^3\theta}{3}\right]_0^{\pi}=\left(\frac{10}{4\pi}\right)\left(\frac{1}{3}\right)\left(\frac{\pi}{2}-1\right)=\mathbf{0.1514}$$

$$G_{\phi}=\frac{U}{U_{ave}}=\mathbf{66.05\cos^2\theta\sin^2\frac{\phi}{2}}$$

$$D=G_{\phi,max}=\mathbf{66.05}$$

Problem 13.24: In free space, an antenna rediates a field

$$E_{\phi s}=\frac{0.2\cos^2\theta}{4\pi r}e^{-j\beta r}kV/m$$ at far field Determine:

A) The total radiated power

$$\mathrm{P_{rad}}=\int \mathrm{P_{ave}\,dS}=\frac{1}{2\eta}\int |\mathrm{E}_{\phi s}|^2\partial \mathrm{S}$$

$$=\frac{1}{240\pi}\left(\frac{0.04\times10^6}{16\pi^2}\right)\int_0^{\pi}\int_0^{2\pi}\frac{\cos^4\theta}{\mathrm{r}^2}\mathrm{r}^2\sin\theta\;\mathrm{d}\theta\,\mathrm{d}\phi$$

$$=\frac{1}{240\pi}\left(\frac{0.04\times10^6}{16\pi^2}\right)(2\pi)\int_0^{\pi}\cos^4\theta\;\mathrm{d}(-\cos\theta)$$

$$=\frac{0.04\times10^6}{16\pi^2*120}\left(-\frac{\cos^5\theta}{5}\right)_0^{\pi}=\frac{0.04\times10^6}{16\pi^2*120}\left(\frac{2}{5}\right)=\frac{25}{3\pi^2}$$

$$\mathbf{P_{rad}=0.8443\,W}$$

B) The directive gain at $\theta=60^o$

$$\mathrm{G_d}=\frac{4\pi\,\mathrm{U}(\theta,\phi)}{\mathrm{P_{rad}}}=\frac{4\pi\,\mathrm{r^2 P_{ave}}}{\mathrm{P_{rad}}}=4\pi\,\mathrm{r}^2\left(\frac{0.04\times10^6\cos^4\theta}{16\pi^2\mathrm{r}^2(240\pi)}\right)\left(\frac{3\pi^2}{25}\right)$$

$$\mathrm{G_d}=5\cos^4\theta$$

$$\cos(60^o)=\frac{1}{2}\rightarrow\cos^4(60^o)=\frac{1}{2^4}=\frac{1}{16}$$

$$\mathrm{G_{d,60^o}}=\frac{5}{16}=\mathbf{0.3125}=\mathbf{G_{d,60^o}}$$

Problem 13.26: An array comprises two dipoles that are separated by one wavelength. If the dipoles fed by currents of the same magnitude and phase:

A) Find the array factor

$$\mathrm{AF}=2\cos\left[\frac{1}{2}(\beta\,\mathrm{d}\cos\theta+\alpha)\right],\alpha=0,\beta\,\mathrm{d}=\frac{2\pi}{\lambda}*\lambda=2\pi$$

$$\mathrm{AF}=2\cos(\pi\cos\theta)$$

B) Calculate the angles where the nulls of the pattern occur

$$\text{The nulls occur when:}$$

$$\cos(\pi\cos\theta)=0\rightarrow\pi\cos\theta=\frac{\pm\pi}{2,}\pm\frac{3\pi}{2,}\ldots$$

$$\theta=60^o,120^o=\frac{\pi}{3},\frac{2\pi}{3}$$

Problem 13.26 Continued:

C) Determine the angles where the maxima of the pattern occur

The maxima and minima occur when:

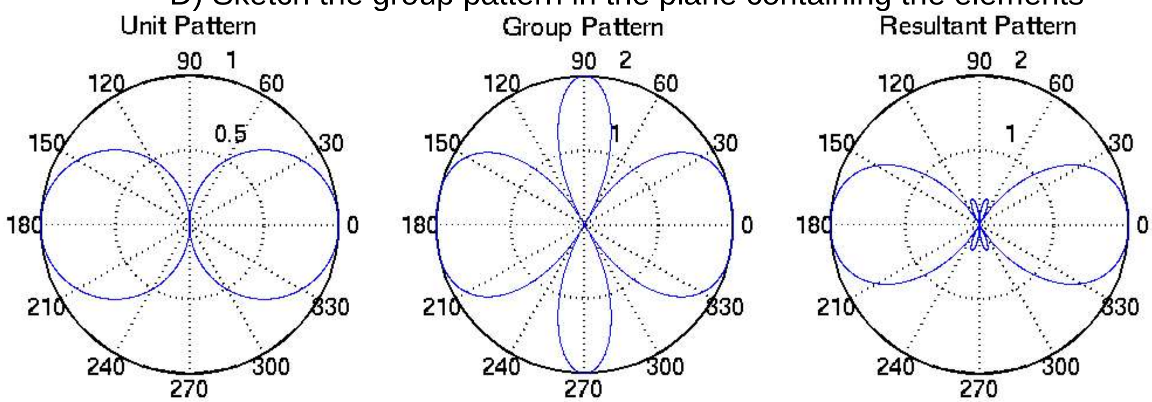
$$\frac{df}{d\theta}=0\rightarrow\sin(\pi\cos\theta)*\pi\sin\theta=0$$

$$\sin\theta=0\rightarrow\theta=0^{o},180^{o}=0,\pi$$

$$\sin(\pi\cos\theta)=0\rightarrow\pi\cos\theta=0,\pi\rightarrow\theta=0,\frac{\pi}{2},\pi,\frac{3\pi}{2}$$

$$\theta=0^{o},90^{o},180^{o},270^{o}=0,\frac{\pi}{2},\pi,\frac{3\pi}{2}=\frac{\pi}{2}*i,\forall i\in\mathbb{Z}$$

D) Sketch the group pattern in the plane containing the elements



Problem 13.27: An array of two elements that are fed by currents that are 180^{o} out of phase with each other. Plot the group pattern if the elements are separtated by:

A) $d=\lambda/4$

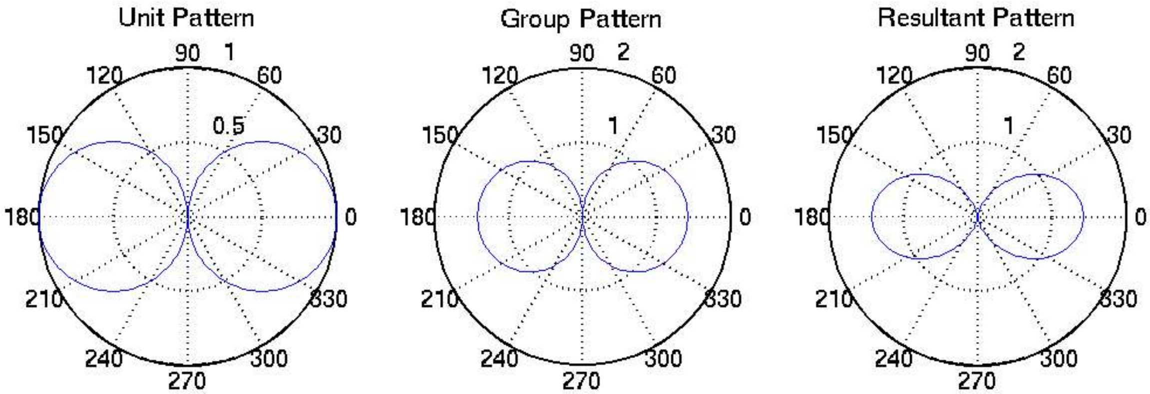
$$f(\theta)=\cos\left[\frac{1}{2}(\beta d\cos\theta+\alpha)\right]$$

$$f(\theta)=\cos\left[\frac{1}{2}\left(\frac{2\pi}{\lambda}\frac{\lambda}{4}\cos\theta+\pi\right)\right]$$

$$f(\theta)=\cos\left[\frac{\pi}{4}\cos\theta+\frac{\pi}{2}\right]$$

$$\text{Nulls exist when }\frac{\pi}{4}\cos\theta+\frac{\pi}{2}=\pm\frac{\pi}{2}\rightarrow\theta=\pm\frac{\pi}{2}$$

Problem 13.27 Continued:



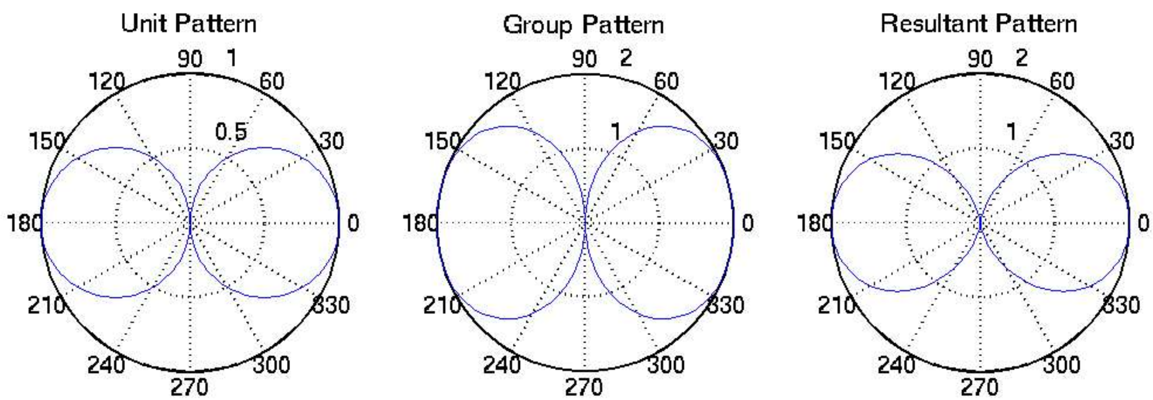
B) $d=\lambda/2$

$$f(\theta)=\cos\left[\frac{1}{2}(\beta d\cos\theta+\alpha)\right]$$

$$f(\theta)=\cos\left[\frac{1}{2}\left(\frac{2\pi}{\lambda}\frac{\lambda}{2}\cos\theta+\pi\right)\right]$$

$$f(\theta)=\cos\left(\frac{\pi}{2}[\cos\theta+1]\right)$$

$$\text{Nulls exist when }\cos\theta+1=\pm 1\rightarrow\theta=\pi$$

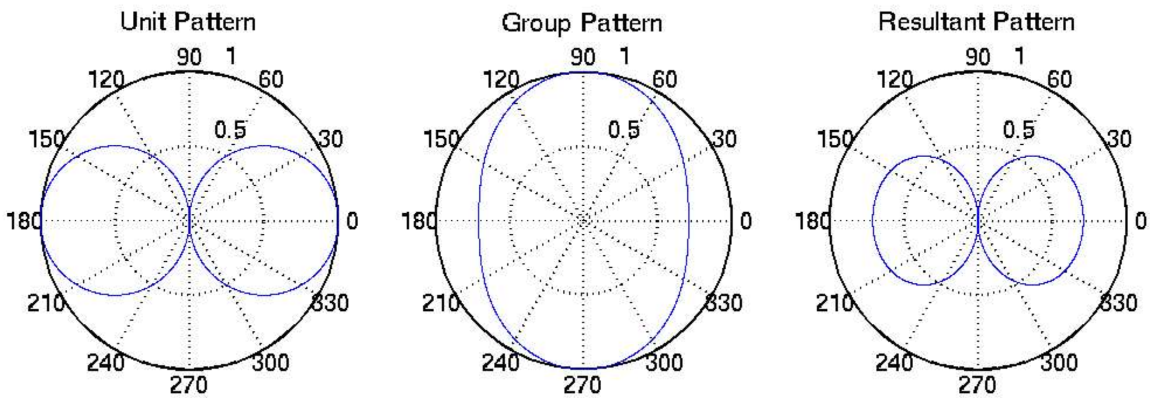


Problem 13.29: An antenna array consists of N identical Hertzian dipoles uniformly located along the z-direction. If the spacing between dipole is $\lambda/4$ sketch the group pattern when:

A) N = 2

$$f(\theta)=\cos\left[\frac{1}{2}(\beta d\cos\theta+\alpha)\right], \alpha=0, d=\frac{\lambda}{4}$$

$$f(\theta)=\cos\left(\frac{\pi}{4}\cos\theta\right)$$



B) N = 4

$$AF=\frac{\sin 2(\beta d\cos\theta)}{\sin\frac{1}{2}(\beta d\cos\theta)}$$

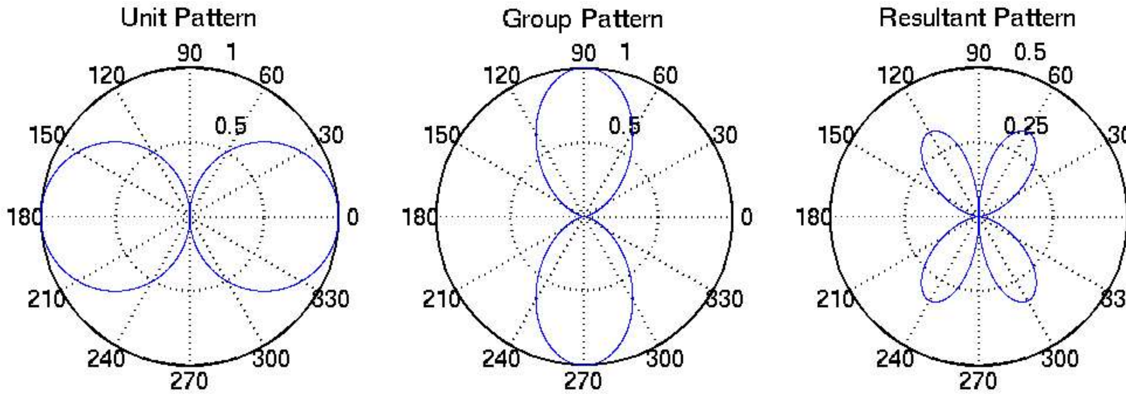
$$\text{Now}, \frac{\sin 4\theta}{\sin\theta}=\frac{2\sin 2\theta\cos 2\theta}{\sin\theta}=4\cos 2\theta\cos\theta$$

$$AF=4\cos(\beta d\cos\theta)\cos\left(\frac{1}{2}\beta d\cos\theta\right)$$

$$f(\theta)=\cos\left(\frac{2\pi}{\lambda}\frac{\lambda}{4}\cos\theta\right)\cos\left(\frac{1}{2}\frac{2\pi}{\lambda}\frac{\lambda}{4}\cos\theta\right)$$

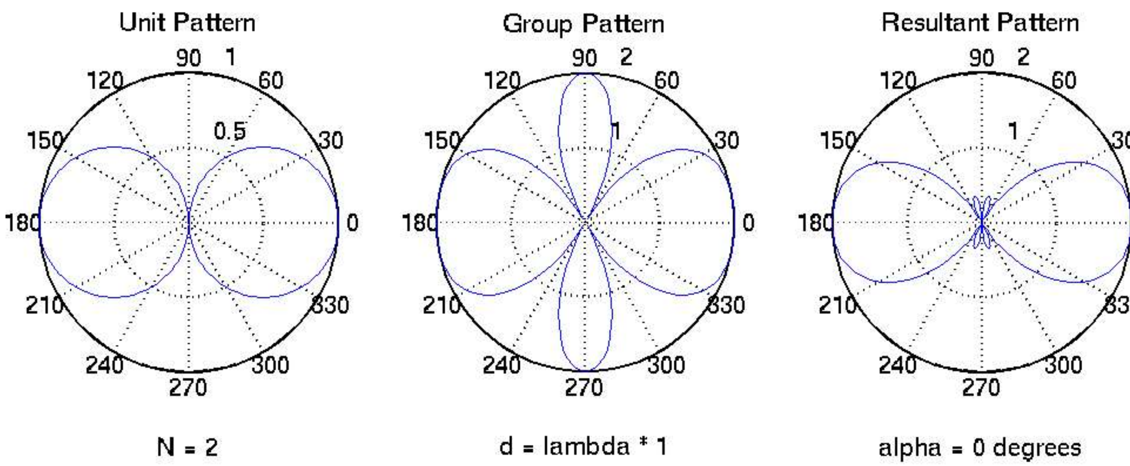
$$f(\theta)=\cos\left(\frac{\pi}{2}\cos\theta\right)\cos\left(\frac{\pi}{4}\cos\theta\right)$$

Problem 13.29 Continued:



Problem 13.30: Sketch the resultant group patterns for the four-element arrays shown in Figure 13.25.

A)



B)