Quarter-Wavelength

$$Z_{in} = \frac{Z_0^2}{Z_L}, \qquad \text{for } l = \lambda/4 + n\lambda/2.$$

Voltage maximum	$\lvert\tilde{V}\rvert_{max} = \lvert V_0^+\rvert[1 + \lvert\Gamma\rvert]$
Voltage minimum	$\lvert\tilde{V}\rvert_{min} = \lvert V_0^+\rvert[1 - \lvert\Gamma\rvert]$
Positions of voltage maxima (also positions of current minima)	$d_{max} = \frac{\theta_r\lambda}{4\pi} + \frac{n\lambda}{2}, \quad n = 0, 1, 2, \ldots$
Position of first maximum (also position of first current minimum)	$d_{max} = \begin{cases} \frac{\theta_r\lambda}{4\pi}, & \text{if } 0 \le \theta_r \le \pi \\ \frac{\theta_r\lambda}{4\pi} + \frac{\lambda}{2}, & \text{if } -\pi \le \theta_r \le 0 \end{cases}$
Positions of voltage minima (also positions of current maxima)	$d_{min} = \frac{\theta_r\lambda}{4\pi} + \frac{(2n+1)\lambda}{4}, \quad n = 0, 1, 2, \ldots$
Position of first minimum (also position of first current maximum)	$d_{min} = \frac{\lambda}{4}\left(1 + \frac{\theta_r}{\pi}\right)$
Input impedance	$Z_{in} = Z_0\left(\frac{z_L + j\tan\beta l}{1 + jz_L\tan\beta l}\right) = Z_0\left(\frac{1+\Gamma_l}{1-\Gamma_l}\right)$
Positions at which Z_{in} is real	at voltage maxima and minima
Z_{in} at voltage maxima	$Z_{in} = Z_0\left(\frac{1+\lvert\Gamma\rvert}{1-\lvert\Gamma\rvert}\right)$
Z_{in} at voltage minima	$Z_{in} = Z_0\left(\frac{1-\lvert\Gamma\rvert}{1+\lvert\Gamma\rvert}\right)$
Z_{in} of short-circuited line	$Z_{in}^{sc} = jZ_0\tan\beta l$
Z_{in} of open-circuited line	$Z_{in}^{oc} = -jZ_0\cot\beta l$
Z_{in} of line of length $l = n\lambda/2$	$Z_{in} = Z_L, \quad n = 0, 1, 2, \ldots$
Z_{in} of line of length $l = \lambda/4 + n\lambda/2$	$Z_{in} = Z_0^2/Z_L, \quad n = 0, 1, 2, \ldots$
Z_{in} of matched line	$Z_{in} = Z_0$

$\lvert V_0^+\rvert$ = amplitude of incident wave; $\Gamma = \lvert\Gamma\rvert e^{j\theta_r}$ with $-\pi < \theta_r < \pi$; θ_r in radians; $\Gamma_l = \Gamma e^{-j2\beta l}$.

Problem 2.2 A two-wire copper transmission line is embedded in a dielectric material with $\varepsilon_r = 2.6$ and $\sigma = 2 \times 10^{-6}$ S/m. Its wires are separated by 3 cm and their radii are 1 mm each.

(a) Calculate the line parameters R', L', G', and C' at 2 GHz.

(b) Compare your results with those based on CD Module 2.1. Include a printout of the screen display.

Solution:

(a) Given:

$$\begin{aligned}
f &= 2 \times 10^9 \text{ Hz}, \\
d &= 2 \times 10^{-3} \text{ m}, \\
D &= 3 \times 10^{-2} \text{ m}, \\
\sigma_c &= 5.8 \times 10^7 \text{ S/m (copper)}, \\
\varepsilon_r &= 2.6, \\
\sigma &= 2 \times 10^{-6} \text{ S/m}, \\
\mu &= \mu_c = \mu_0.
\end{aligned}$$

From Table 2-1:

$$\begin{aligned}
R_s &= \sqrt{\pi f \mu_c/\sigma_c} \\
&= [\pi \times 2 \times 10^9 \times 4\pi \times 10^{-7}/5.8 \times 10^7]^{1/2} \\
&= 1.17 \times 10^{-2}\ \Omega, \\
R' &= \frac{2R_s}{\pi d} = \frac{2 \times 1.17 \times 10^{-2}}{2\pi \times 10^{-3}} = 3.71\ \Omega/\text{m}, \\
L' &= \frac{\mu}{\pi} \ln\left[(D/d) + \sqrt{(D/d)^2 - 1}\right] \\
&= 1.36 \times 10^{-6} \text{ H/m}, \\
G' &= \frac{\pi\sigma}{\ln[(D/d) + \sqrt{(D/d)^2 - 1}]} \\
&= 1.85 \times 10^{-6} \text{ S/m}, \\
C' &= \frac{G'\varepsilon}{\sigma} \\
&= \frac{1.85 \times 10^{-6} \times 8.85 \times 10^{-12} \times 2.6}{2 \times 10^{-6}} \\
&= 2.13 \times 10^{-11} \text{ F/m}.
\end{aligned}$$

(b) Solution via Module 2.1: