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Notes

- Negation of a Universal Statement

$$\sim (\forall x \in D, Q(x)) \equiv \exists x \in D \ s.t. \ \sim Q(x)$$

- Negation of an Existential Statement

$$\sim (\exists x \in D \ s.t. \ Q(x)) \equiv \forall x \in D, \sim Q(x)$$

- Negation of a Universal Conditional Statement

$$\sim (\forall x, \ P(x) \rightarrow Q(x)) \equiv \exists x \ s.t. \ P(x) \wedge \sim Q(x)$$

- Vacuous Truth of Universal Statements

$$\forall x \in D, P(x) \rightarrow Q(x)$$

  is **vacuously true** or **true by default** iff $P(x)$ is false for every x in D.

- Neccessary and Sufficient Conditions, Only If

  sufficient condition

$$\forall x, a(x) \rightarrow b(x)$$

  neccessary condition

$$\forall x, \sim a(x) \rightarrow \sim b(x)$$

  only if

$$\forall x, \sim b(x) \rightarrow \sim a(x)$$

Test Yourself

1. A negation for "All R have property S" is
   "There is some R that does not have property S."
2. A negation for "Some R have property S" is
   "All R do not have property S."
3. A negation for "For all x, if x has property P then x has property Q" is
   "For all x, x has property P and x does not have property Q."
4. The converse of "For all x, if x has property P then x has property Q" is
   "For all x, if x has property Q then x has property P."
5. The contrapositive of "For all x, if x has property P then x has property Q" is
   "For all x, if x does not have property Q then x does not have property P."
6. The inverse of "For all x, if x has property P then x has property Q" is
   "For all x, if x does not have property P then x does not have property Q."