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# 7.1

## 23

$H : \mathbf{R} \times \mathbf{R} \to \mathbf{R} \times \mathbf{R}$
$H(x, y) = (x + 1, 2 - y) \forall (x, y) \in \mathbf{R} \times \mathbf{R}$

### one-to-one

*Proof.* Let $(a, b),\ (c, d) \in \mathbf{R} \times \mathbf{R}$ such that $H(a, b) = H(c, d)$.
*[We must show $(a, b) = (c, d)$.]*
By definition of $H$,

$$(a + 1, 2 - b) = (c + 1, 2 - d).$$

This implies,

$$a + 1 = c + 1 \text{ and } 2 - b = 2 - d.$$

By basic algebra,

$$a = c \text{ and } b = d$$

Therefore,

$$(a, b) = (c, d)$$

□

### onto

*Proof.* Let $(x_1, y_1) \in \mathbf{R} \times \mathbf{R}$.
*[We must show $\exists (x_2, y_2) \in \mathbf{R} \times \mathbf{R}$ s.t. $H(x_2, y_2) = (x_1, y_1)$.]*
Suppose

$$(x_2, y_2) = (x_1 - 1, 2 - y_1).$$

Since the adding and subtracting real numbers with real numbers results in real numbers,

$$(x_2, y_2) \in \mathbf{R} \times \mathbf{R}$$

Then,

$$H(x_2, y_2) = (x_1 - 1 + 1, 2 - (2 - y_1)).$$

This implies,

$$H(x_2, y_2) = (x_1, y_1).$$

□

## 31

$f : \mathbf{R} \to \mathbf{R}$, $g : \mathbf{R} \to \mathbf{R}$ and both $f$ and $g$ are onto.
For $f$ or $g$ to be onto,

$$\forall y \in \mathbf{R},\ \exists x \in \mathbf{R} \ s.t.\ f(x) = y.$$

Counterexample:
Let $f(x) = x$ and let $g(x) = -x$ for all $x \in \mathbf{R}$. $f(x) + g(x) = 0$ for all values of $x$, and so, $f + g$ is not onto.

## 45

$$H^{-1}(x, y) = (x - 1, 2 - y)$$

# 7.3

## 3

$$G \circ F = x^3 - 1$$

$$F \circ G = (x - 1)^3$$

$$G \circ F \neq F \circ G$$

## 7

$$(K \circ H)(0) = 6(0) \bmod 4 = 0$$

## 9

$$F \circ F^{-1} = 3\left(\frac{y-2}{3}\right) + 2 = y \ \forall y \in \mathbf{R} = I_R(y)$$

## 11

## 3

## 3