Rectangular Patch Antenna Array Design

Lewis Collum

CONTENTS

***Abstract*—We re-create an example design for a patch antenna, given by Balanis [1], to simulate an array of patches using an RT/duroid 5880 substrate. We also include illustrations for the far-zone radiation pattern of both a single patch and array of patches.**

I. SINGLE PATCH ANTENNA: PHYSICAL DESIGN

A. Design Context

We start by designing a single patch, which we will use to create an antenna array (in section III).

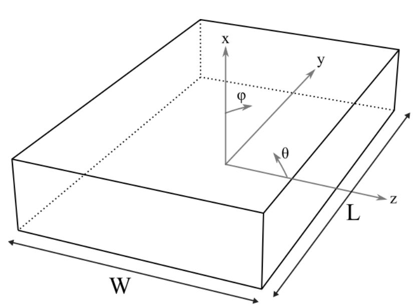

Fig. 1. Patch Model with spherical (θ and ϕ) and cartesian coordinates.

For the patch substrate, we are using RT/duroid 5880, which has a relative permittivty of $\epsilon_r = 2.2$. The height of the material is designed to be $h = 0.1588\,\mathrm{cm}$. Our patch is intended to resonate at $f_r = 10\,\mathrm{GHz}$.

With these three parameters, we can find the patch width and height. Then, we can illustrate the radiation pattern for the patch.

B. Fringing Effects

Fringing fields at the lengths of the patch makes the patch appear to have a greater length than it actually does. This is important since the effective dimensions of the patch affect the resonant frequency. If the physical length of the patch is L, then the effective length, L_{eff}, can be written as

$$L_{eff} = L + 2 \cdot \Delta L, \tag{1}$$

where ΔL is the additional length on one end of the patch.

The additional length can be related to the width of the patch, W and the effective relative permittivity of the dieletric substrate, ϵ_{eff}, as [1]

$$\frac{\Delta L}{h} = 0.412 \frac{(\epsilon_{eff} + 0.3)\left(\frac{W}{h} + 0.264\right)}{(\epsilon_{eff} - 0.258)\left(\frac{W}{h} + 0.8\right)}. \tag{2}$$

The effective relative permittivity, ϵ_{eff}, is given as [1]

$$\epsilon_{eff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2}\left(1 + 12 \cdot \frac{h}{W}\right)^{-1/2}. \tag{3}$$

C. Physical Width

A model for patch width is [1]

$$W = \frac{\lambda_r}{2}\sqrt{\frac{2}{\epsilon_r + 1}} \tag{4}$$

where λ is the wavelength at the resonance frequency, f_r.

$$\lambda = \frac{c}{10\,\mathrm{GHz}} = 3.00\,\mathrm{cm}$$

The patch's physical width, from eq. 4, is

$$\boxed{W = 1.185\,\text{cm}}$$

D. Effective Length

A model for patch length (without accounting for fringing effects) is, [1]

$$L_{eff} = \frac{\lambda}{2\sqrt{\epsilon_{eff}}} \tag{5}$$

Using eq. 3, we find the effective relative permittivity to be

$$\begin{aligned}\epsilon_{eff} &= \frac{2.2+1}{2} + \frac{2.2-1}{2}\left(1 + 12 \cdot \frac{0.1588\,\text{cm}}{1.185\,\text{cm}}\right) \\ &= 1.97.\end{aligned}$$

Thus,

$$\boxed{L_{eff} = 1.07\,\text{cm}}$$

E. Physical Length

The physical length is the effective length minus the extensions lengths caused by fringing fields, as per eq. 1.

By eq. 2, the extension length on one side, ΔL, is

$$\Delta L = 0.0825\,\text{cm}$$

So,

$$\begin{aligned}L &= L_{eff} - 2\Delta L \\ &= 1.07\,\text{cm} - 2 \cdot 0.0825\,\text{cm} \\ &= \boxed{0.905\,\text{cm}}\end{aligned}$$

F. Summary

The width, height, and effective length, are used in following calculations as the effective dimensions of the patch. The actual length of the patch is used for building the antenna and impedance matching, but it is not used to analyze the radiation pattern of the patch.

II. Single Patch Antenna: Analysis

A. E-Plane ($\theta = 90°, -90° \leq \phi \leq 90°$)

The radiation intensity in the E-plane can be modeled as [1]

$$F_\phi = \cos(y_\phi) \cdot \frac{\sin(x_\phi)}{x_\phi} \tag{6a}$$

$$x_\phi = \frac{k_0 h}{2}\cos(\phi) \tag{6b}$$

$$y_\phi = \frac{k_0 L_{eff}}{2}\sin(\phi), \tag{6c}$$

where k_0 is the wave-number,

$$k_0 = \frac{2\pi}{\lambda} = 209.6.$$

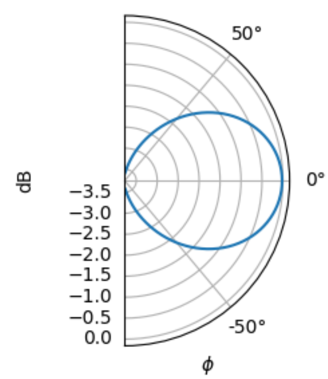

Fig. 2. E-Plane radiation pattern of the patch under design.

B. H-Plane ($\phi = 0°, 0° \leq \theta \leq 180°$)

The radiation intensity in the H-plane can be modeled as [1]

$$F_\theta = \sin(\theta) \cdot \frac{\sin(x_\theta)}{x_\theta} \cdot \frac{\sin(z_\theta)}{z_\theta} \tag{7a}$$

$$x_\theta = \frac{k_0 h}{2}\sin(\theta) \tag{7b}$$

$$z_\theta = \frac{k_0 W}{2}\cos(\theta), \tag{7c}$$

using wave-number, k_0, from section II-A.

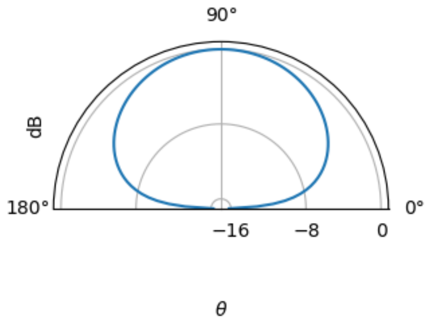

Fig. 3. H-Plane radiation pattern of the patch under design.

C. Far-Zone Total Radiation Intensity

Since the two plane models are normalized, and since the planes are orthogonal, the total radiation intensity is approximately the product of the two plane models.

That is,

$$F(\phi, \theta) = F_\phi \cdot F_\theta. \tag{8}$$

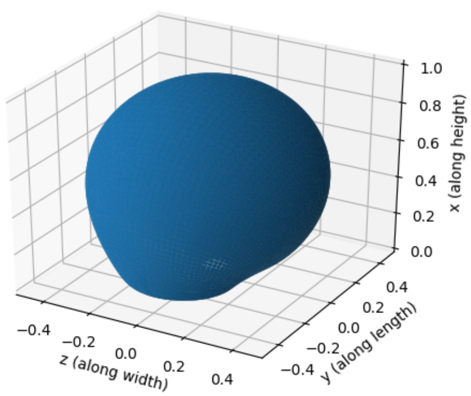

Fig. 4. Normalized radiation pattern of the patch under design using eq. 8.

D. Directivity

Directivity of the main lobe (in figure 4) is the maximum radiation intensity over the average intensity, [2]

$$D = \frac{F_{max}}{F_{average}} = \frac{1}{\frac{1}{4\pi}\Omega_p} \tag{9}$$

where Ω_p is the pattern solid angle. Note that for a *normalized* radiation intensity, the maximum, F_{max}, will be equal to 1, by definition.

The pattern solid angle is defined as [2]

$$\Omega_p = \iint_{4\pi} F(\phi, \theta)\, d\theta\, d\phi. \tag{10}$$

Since we have the electric field intensity stored as a 2-dimensional array, we use simpson's rule to calculate the double integral.

This yields a directivity of,

$$D = 3.17$$

III. Patch Grid

A. Array Factor

The array factor is a function that incorporates the positions of the patches in the array. It optionally includes weights and phase offsets. We use a simplified model which only accounts for position of elements, and assumes no element phase offset, a uniform element amplitude of one, and a uniform wave number, written as

$$AF = \sum_{i=1}^{N} e^{-jk_0|r_i|} \tag{11}$$

where $k_0|r_i|$ is the relative phase at patch `i` located at $r = (x_i, y_i, z_i)$.

The relative phase at each patch, $k_0|r_i|$, describes the phase variation for the position of the element, as

$$k_0|r_i| = \sin\theta\cos\phi z + \sin\theta\sin\phi y + \cos\theta x. \tag{12}$$

Importantly, z and x would be flipped in eq. 12, if we used z to represent the "up" axis.

We can plot the array factor for a 3x3 patch array, to visualize the characteristics of the pattern.

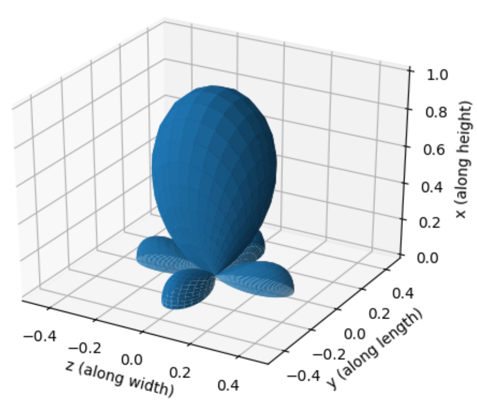

Fig. 5. 3x3 Array Factor

B. Radiation Pattern

For an array, the total radiation intensity is the normalized product of the array factor and the radiation intensity for a single patch [1]. That is,

$$F(\phi, \theta) = AF \cdot F(\phi, \theta)_0 \tag{13}$$

We use this relationship to illustrate a 2x2, 3x3, and 4x4 patch array.

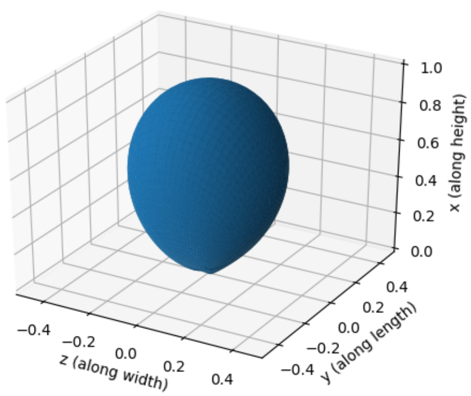

Fig. 6. 2x2 Patch Array

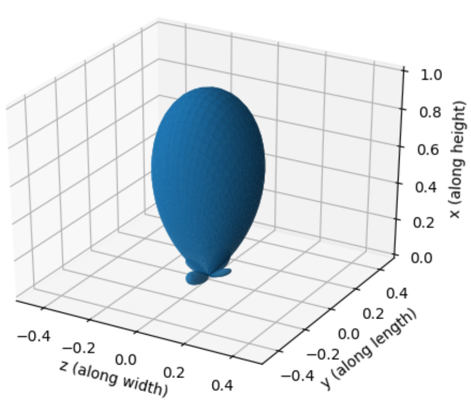

Fig. 7. 3x3 Patch Array

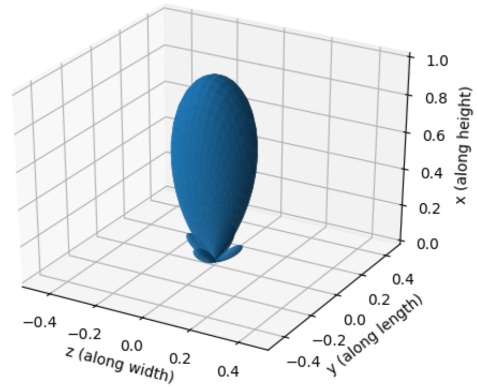

Fig. 8. 4x4 Patch Array

C. Directivity

Using eq. 13 for total array radiation and plugging the result into the solid angle equation (eq. 10), we find the directivity (eq. 9) for each patch array.

This yields patch array directivities of,

$$D_{2x2} = 7.30,$$

$$D_{3x3} = 12.53,$$

$$D_{4x4} = 19.61.$$

In theory, the more patches we add to the array, the higher the directivity (and gain) becomes. As a note, gain is

$$G = \epsilon D,$$

where ϵ is the antenna efficiency. We assume an efficiency of 100% for this analysis, so we obtain no additional by calculating gain.

IV. Conclusion

We extended the theory for designing a single patch, to begin designing an antenna array. While there are still many design factors not considered, such as efficiency and impedance matching, this design acts as a first step towards including those factors.

References

[1] C. A. Balanis, *Antenna Theory: Analysis and Design*. John Wiley Sons, second ed., 1997.

[2] F. T. Ulaby and U. Ravaioli, *Fundamentals of Applied Electromagnetics*. Pearson, seventh ed., 2015.