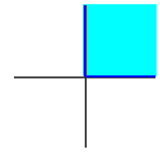
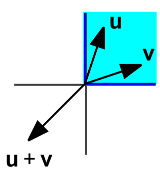
Student: Lewis C **Date:** 05/11/20	**Instructor:** Scott Fulton **Course:** MA339 Applied Linear Algebra	**Assignment:** Section 2.8 Homework

1. A set in $\mathbb{R}^2$ is displayed to the right. Assume the set includes the bounding lines. Give a specific reason why the set H is not a subspace of $\mathbb{R}^2$. (For instance, find two vectors in H whose sum is not in H, or find a vector in H with a scalar multiple that is not in H. Draw a picture.)

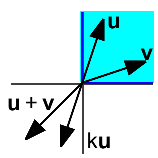


Let **u** and **v** be vectors and let k be a scalar. Select the correct choice below and, if necessary, fill in the answer box within your choice.

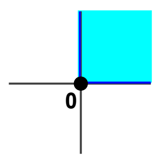
A. The set is not a subspace because it is closed under scalar multiplication, but not under sums. For example, the sum of (3,1) and (1,3) is not in the set.



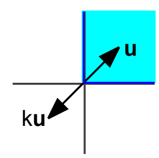
B. The set is not a subspace because it is not closed under either scalar multiplication or sums. For example, ____ multiplied by (1,3) is not in the set, and the sum of (3,1) and (1,3) is not in the set.



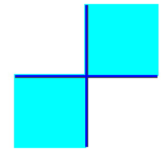
C. The set is not a subspace because it does not include the zero vector.



D. The set is not a subspace because it is closed under sums, but not under scalar multiplication. For example, -1 multiplied by (1,1) is not in the set.



2. A set in $\mathbb{R}^2$ is displayed to the right. Assume the set includes the bounding lines. Give a specific reason why the set H is not a subspace of $\mathbb{R}^2$. (For instance, find two vectors in H whose sum is not in H, or find a vector in H with a scalar multiple that is not in H. Draw a picture.)



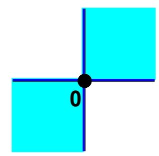
Let **u** and **v** be vectors and let k be a scalar. Select the correct choice below and, if necessary, fill in the answer box within your choice.

A. The set is not a subspace because it is not closed under either scalar multiplication or sums. For example, _____ multiplied by (0,1) is not in the set, and the sum of (2,2) and $(-1,-3)$ is not in the set.

B. The set is not a subspace because it is closed under scalar multiplication, but not under sums. For example, the sum of (2,2) and $(-1,-3)$ is not in the set.

C. The set is not a subspace because it is closed under sums, but not under scalar multiplication. For example, _____ multiplied by (0,1) is not in the set.

D. The set is not a subspace because it does not include the zero vector.



3. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 5 \\ -7 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -2 \\ -8 \\ 13 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} -3 \\ -11 \\ 19 \end{bmatrix}$. Determine if **w** is in the subspace of $\mathbb{R}^3$ generated by $\mathbf{v}_1$ and $\mathbf{v}_2$.

Is **w** is in the subspace of $\mathbb{R}^3$ generated by $\mathbf{v}_1$ and $\mathbf{v}_2$?

- No
- Yes

4. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 6 \\ -6 \\ 24 \end{bmatrix}$, $\mathbf{p} = \begin{bmatrix} 4 \\ -1 \\ 28 \end{bmatrix}$, and $A = [\mathbf{v}_1\ \mathbf{v}_2\ \mathbf{v}_3]$.

a. How many vectors are in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

b. How many vectors are in Col A?

c. Is **p** in Col A? Why or why not?

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a. How many vectors are in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$? Select the correct choice below and, if necessary, fill in the answer box within your choice.

- **A.** 3 (Type a whole number.)
- **B.** There are infinitely many vectors in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

b. How many vectors are in Col A? Select the correct choice below and, if necessary, fill in the answer box within your choice.

- **A.** (Type a whole number.)
- **B.** There are infinitely many vectors in Col A.

c. Is **p** in Col A? Why or why not?

- **A.** **p** is in Col A, because the system [A **p**] is consistent.
- **B.** **p** is not in Col A, because A has too few pivot positions.
- **C.** **p** is not in Col A, because the system [A **p**] is not consistent.
- **D.** **p** is in Col A, because A has pivot positions in every row.

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5. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -4 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} -7 \\ -2 \\ 2 \end{bmatrix}$, and $\mathbf{p} = \begin{bmatrix} 3 \\ 1 \\ -3 \end{bmatrix}$. Determine if **p** is in Col A, where $A = [\mathbf{v}_1\ \mathbf{v}_2\ \mathbf{v}_3]$.

Is **p** in Col A?

- **A.** No, because the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ are linearly dependent.
- **B.** Yes, because the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ are linearly independent.
- **C.** Yes, because the augmented matrix [A **p**] is consistent.
- **D.** No, because the augmented matrix [A **p**] is not consistent.

6. Let $\mathbf{v}_1 = \begin{bmatrix} -3 \\ 0 \\ 6 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -2 \\ 2 \\ 3 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 0 \\ -6 \\ 3 \end{bmatrix}$, and $\mathbf{p} = \begin{bmatrix} 5 \\ 10 \\ -3 \end{bmatrix}$. Determine if $\mathbf{p}$ is in Nul A, where $A = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 \end{bmatrix}$.

Is **p** in Nul A?

- **A.** No, because A**p** is not equal to the zero vector.
- **B.** No, because the augmented matrix [A **p**] is not consistent.
- **C.** Yes, because the augmented matrix [A **p**] is consistent.
- **D.** Yes, because A**p** is equal to the zero vector.

7. Give integers p and q such that Nul A is a subspace of $\mathbb{R}^p$ and Col A is a subspace of $\mathbb{R}^q$.

$$A = \begin{bmatrix} 1 & -3 & -7 & 4 \\ -6 & 5 & 2 & 9 \end{bmatrix}$$

Nul A is a subspace of $\mathbb{R}^p$ for p = 4 and Col A is a subspace of $\mathbb{R}^q$ for q = 2 .

8. Determine if the set is a basis for $\mathbb{R}^3$. Justify your answer.

$$\begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} -6 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 8 \\ 3 \\ 2 \end{bmatrix}$$

Is the given set a basis for $\mathbb{R}^3$?

- **A.** Yes, because these three vectors form the columns of a 3×3 matrix that is not invertible.
- **B.** Yes, because these three vectors form the columns of an invertible 3×3 matrix.
- **C.** No, because these three vectors form the columns of a 3×3 matrix that is not invertible.
- **D.** No, because these three vectors form the columns of an invertible 3×3 matrix.

9. Determine if the set is a basis for $\mathbb{R}^3$. Justify your answer.

$$\begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ 1 \\ -3 \end{bmatrix}$$

Is the given set a basis for $\mathbb{R}^3$?

- **A.** Yes, because these two vectors form the columns of an invertible matrix.
- **B.** Yes, because these two vectors are linearly independent.
- **C.** No, because these two vectors are linearly dependent.
- **D.** No, because these two vectors form a matrix which cannot possibly have a pivot in each row.

10. Mark each statement as true or false. Justify each answer. Complete parts (a) through (e) below.

a. A subspace of $\mathbb{R}^n$ is any set H such that (i) the zero vector is in H, (ii) $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{u} + \mathbf{v}$ are in H, and (iii) c is a scalar and $c\mathbf{u}$ is in H.

A. The statement is true. Any set of elements of $\mathbb{R}^n$ that satisfies conditions (i), (ii), and (iii) is a subspace by definition.

B. The statement is false. It must also be separately specified that $a\mathbf{u} + b\mathbf{v}$ are in H when $\mathbf{u}$ and $\mathbf{v}$ are in H and a and b are scalars.

C. The statement is false. Conditions (ii) and (iii) must be satisfied for each $\mathbf{u}$ and $\mathbf{v}$ in H, which is not specified in the given statement.

D. The statement is false. It must also be separately specified that $\mathbf{u} - \mathbf{v}$ are in H when $\mathbf{u}$ and $\mathbf{v}$ are in H.

E. The statement is false. The zero vector does not need to be in a set for the set to be a subspace.

b. If $\mathbf{v}_1, \ldots, \mathbf{v}_p$ are in $\mathbb{R}^n$, then $S = \text{Span}\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ is the same as the column space of the matrix $A = \begin{bmatrix} \mathbf{v}_1 & \cdots & \mathbf{v}_p \end{bmatrix}$.

A. The statement is false. S is the same as the column space of A only if the columns of A are linearly independent.

B. The statement is false. There are finitely many vectors in S but infinitely many vectors in the column space of A.

C. The statement is true. The vectors $\mathbf{v}_1, \ldots, \mathbf{v}_p$ always form a basis of both S and the column space of A.

D. The statement is true. The column space of A and S are both the set of the vectors $\mathbf{v}_1, \ldots, \mathbf{v}_p$ and the zero vector.

E. The statement is true. The column space of A and S are both the set of all linear combinations of $\mathbf{v}_1, \ldots, \mathbf{v}_p$.

F. The statement is false. There are infinitely many vectors in S but finitely many vectors in the column space of A.

c. The set of all solutions of a system of m homogeneous equations in n unknowns is a subspace of $\mathbb{R}^m$.

A. The statement is false. The described set is the column space of an $m \times n$ matrix A. This set is a subspace of $\mathbb{R}^n$.

B. The statement is false. The described set is only a subspace of $\mathbb{R}^m$ if $m < n$.

C. The statement is true. The described set is the null space of an $m \times n$ matrix A. This set is a subspace of $\mathbb{R}^m$.

D. The statement is true. The described set is the column space of an $m \times n$ matrix A. This set is a subspace of $\mathbb{R}^m$.

E. The statement is false. The described set is the null space of an $m \times n$ matrix A. This set is a subspace of $\mathbb{R}^n$.

d. The columns of an invertible $n \times n$ matrix form a basis for $\mathbb{R}^n$.

A. The statement is false. The columns of an invertible $n \times n$ matrix are not linearly independent. This means they cannot form a basis for any vector space, including $\mathbb{R}^n$.

B. The statement is true. There are n columns in an $n \times n$ matrix, each of which is a vector in $\mathbb{R}^n$. Any set of n vectors in $\mathbb{R}^n$ must form a basis for $\mathbb{R}^n$.

C. The statement is false. The columns of an invertible $n \times n$ matrix are linearly independent, but they do not span $\mathbb{R}^n$. This means they cannot form a basis for $\mathbb{R}^n$.

e. Row operations do not affect linear dependence relations among the columns of a matrix.

- **A.** The statement is true. The pivot columns of a matrix A form a basis for the column space of A. Because row operations do not affect the pivot columns of a matrix, they also cannot affect linear dependence relations among the columns of a matrix.
- **B.** The statement is true. If a series of row operations is performed on a matrix A to form B, then the equations $A\mathbf{x} = \mathbf{0}$ and $B\mathbf{x} = \mathbf{0}$ have the same set of solutions.
- **C.** The statement is false. If a series of row operations is performed on a matrix A to form B, then the equations $A\mathbf{x} = \mathbf{0}$ and $B\mathbf{x} = \mathbf{0}$ have the same set of solutions if and only if no rows are interchanged during the series of row operations. This means that it is possible for a row operation to affect linear dependence relations among the columns of a matrix.
- **D.** The statement is false. If a series of row operations is performed on a matrix A to form B, then linearly independent columns of A correspond to linearly independent columns of B. The same is true of linearly dependent columns of A. However, the exact linear dependence relations among the columns of A may not apply to the columns of B.

11. Mark each statement as true or false. Justify each answer. Complete parts (a) through (e) below.

a. A subset H of $\mathbb{R}^n$ is a subspace if the zero vector is in H.

- **A.** This statement is false. For each **u** and **v** in H and each scalar c, the sum **u + v** and the vector cu must also be in H.
- **B.** This statement is false. The subset H is a subspace if the zero vector is not in H.
- **C.** This statement is false. For each **u** and **v** in H, the product uv must also be in H.
- **D.** This statement is true. This is the definition of a subspace.

b. Given vectors $\mathbf{v}_1, \ldots, \mathbf{v}_p$ in $\mathbb{R}^n$, the set of all linear combinations of these vectors is a subspace of $\mathbb{R}^n$.

- **A.** This statement is false. This set does not contain the zero vector.
- **B.** This statement is false. This set is a subspace of $\mathbb{R}^p$.
- **C.** This statement is false. This set is a subspace of $\mathbb{R}^{n+p}$.
- **D.** This statement is true. This set satisfies all properties of a subspace.

c. The null space of an $m \times n$ matrix is a subspace of $\mathbb{R}^n$.

- **A.** This statement is false. This set is not closed under scalar multiplication.
- **B.** This statement is false. For an $m \times n$ matrix A, the solutions of $A\mathbf{x} = \mathbf{0}$ belong to $\mathbb{R}^m$.
- **C.** This statement is false. The null space of a matrix does not contain the zero vector.
- **D.** This statement is true. For an $m \times n$ matrix A, the solutions of $A\mathbf{x} = \mathbf{0}$ are vectors in $\mathbb{R}^n$ and satisfy the properties of a vector space.

d. The column space of a matrix A is the set of solutions of $A\mathbf{x} = \mathbf{b}$.

- **A.** This statement is false. The column space of A is the set of all **b** for which $A\mathbf{x} = \mathbf{b}$ has a solution.
- **B.** This statement is true. This is the definition of a column space.
- **C.** This statement is false. The column space of a matrix A is the set of solutions of $A\mathbf{x} = \mathbf{0}$.
- **D.** This statement is false. The column space of A is the set of all A for which $A\mathbf{x} = \mathbf{b}$ has a solution.

e. If B is an echelon form of a matrix A, then the pivot columns of B form a basis for Col A.

- **A.** This statement is false. The pivot columns of B form a basis for Nul A.
- **B.** This statement is true. This is the definition of a column space.
- **C.** This statement is false. The pivot columns of B form a basis for Col A only when B is in reduced row echelon form.
- **D.** This statement is false. The columns of an echelon form of a matrix are often not in the column space of the original matrix.

12. A matrix A and an echelon form of A are shown below. Find a basis for Col A and a basis for Nul A.

$$A = \begin{bmatrix} 4 & 4 & 4 & 4 \\ 5 & 2 & -7 & 29 \\ 2 & 2 & 2 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 5 & -7 \\ 0 & 1 & 4 & -8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Find a basis for Col A.

$$\left\{ \begin{bmatrix} 4 \\ 5 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 2 \end{bmatrix} \right\}$$

(Simplify your answer. Use a comma to separate answers as needed.)

Find a basis for Nul A.

$$\left\{ \begin{bmatrix} 3 \\ -4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -9 \\ 8 \\ 0 \\ 1 \end{bmatrix} \right\}$$

(Simplify your answer. Use a comma to separate answers as needed.)

13. A matrix A and an echelon form of A are shown below. Find a basis for Col A and a basis for Nul A.

$$A = \begin{bmatrix} 1 & 10 & 9 & -4 & -7 \\ -1 & 5 & 9 & 2 & -2 \\ -2 & 10 & 18 & 4 & -4 \\ 2 & 0 & -6 & -3 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & 10 & 9 & 0 & 5 \\ 0 & 5 & 6 & 0 & -1 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Find a basis for Col A.

$$\left\{ \begin{bmatrix} 1 \\ -1 \\ -2 \\ 2 \end{bmatrix}, \begin{bmatrix} 10 \\ 5 \\ 10 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ 4 \\ -3 \end{bmatrix} \right\}$$

(Use a comma to separate answers as needed. Type an integer or simplified fraction for each matrix element.)

Find a basis for Nul A.

$$\left\{ \begin{bmatrix} 3 \\ -\frac{6}{5} \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ \frac{1}{5} \\ 0 \\ -3 \\ 1 \end{bmatrix} \right\}$$

(Use a comma to separate answers as needed. Type an integer or simplified fraction for each matrix element.)