EE381 HW 2

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2.2

(a)

$$G' = \frac{\pi\sigma}{\ln\left[D/d + \sqrt{(D/d)^2 - 1}\right]}$$

$$= \frac{2E{-}6\,\frac{S}{m} \times \pi}{\ln\left[\frac{3cm}{2mm} + \sqrt{\left(\frac{3cm}{2mm}\right)^2 - 1}\right]}$$

$$\boxed{= 1.85\ \mu S/m}$$

$$C' = \frac{G'\epsilon}{\sigma} = \frac{G'\epsilon_r\epsilon_0}{\sigma}$$

$$= \frac{G' \times 2.6 \times 8.85\times10^{-12}\,F/m}{2\times10^{-6}\,\frac{S}{m}}$$

$$\boxed{= 21.3\ pF/m}$$

$$L' = \frac{\mu\epsilon}{C'} = \frac{\mu_0\epsilon_r\epsilon_0}{C'}$$

$$= \frac{4\pi\times10^{-7}\frac{H}{m} \cdot 2.6\epsilon_0}{C'}$$

$$\boxed{= 1.36\ \mu H/m}$$

$$\sigma_c = 5.8\times10^7$$

$$R' = \frac{2R_s}{\pi d}$$

$$R_s = \sqrt{\frac{\pi f \mu_0}{\sigma_c}}$$

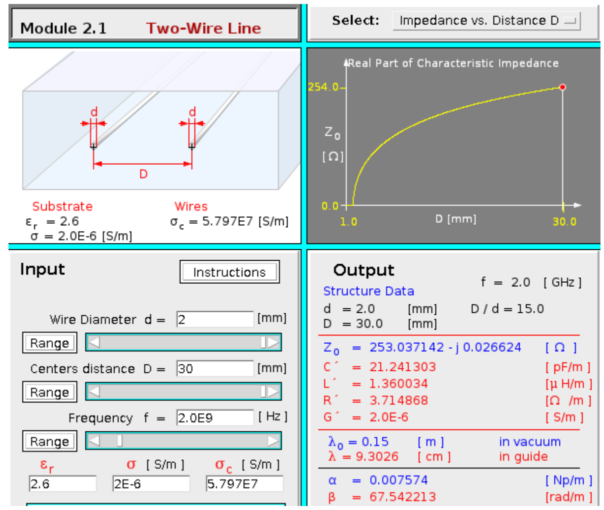
$$= \sqrt{\frac{\pi \cdot 2\times10^9 \cdot 4\pi\times10^{-7}}{5.8\times10^7}}$$

$$\boxed{= 11.7\ m\Omega}$$

$$= \frac{2 \cdot 11.7\cdot10^{-3}}{\pi \cdot 2\times10^{-3}}$$

$$\boxed{= 3.72\ \Omega/m}$$

(b)



2.6

a	0.5cm
b	1cm
ϵ_r	4.5
σ	$10^{-3}\mathrm{S\,m^{-1}}$
f	1GHz

(a)

$$G' = \frac{2\pi\sigma}{\ln(b/a)}$$
$$= \boxed{0.091\mathrm{S\,m^{-1}}}$$

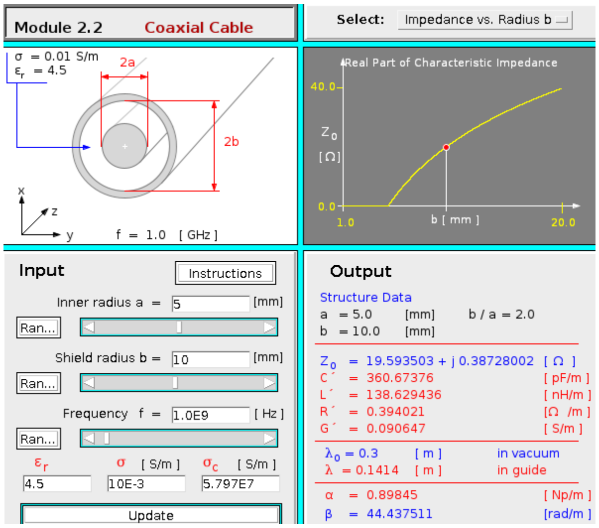
$$C' = \frac{G'\epsilon}{\sigma}$$
$$= \frac{G' \cdot 4.5\epsilon_0}{10^{-3}}$$
$$= \boxed{361\mathrm{pF\,m^{-1}}}$$

$$L' = \frac{\epsilon\mu}{C'}$$
$$= \frac{4.5\epsilon_0\mu_0}{C'}$$
$$= \boxed{139\mathrm{nH\,m^{-1}}}$$

$$R_s = \sqrt{\frac{\pi \cdot 1\mathrm{GHz} \cdot \mu_0}{5.8\times10^7}}$$
$$= 0.00825\Omega$$

$$R' = \frac{R_s}{2\pi}\left(\frac{1}{a} + \frac{1}{b}\right)$$
$$= \boxed{0.394\Omega\,\mathrm{m^{-1}}}$$

(b)



2.13

$$\gamma = \sqrt{(R'+j\omega L')(G'+j\omega C')}$$

$$= \sqrt{R'(1+j\omega L'/R')\,G(1+j\omega C'/G')}$$

$$= \sqrt{R'G'}\cdot\left(1+j\omega\frac{C'}{G'}\right)$$

$$= \sqrt{R'G'} + j\sqrt{R'G'}\,\omega\frac{C'}{G'}$$

$$\alpha = \mathrm{Re}\{\gamma\}$$

$$\boxed{=\sqrt{R'G'}} \checkmark$$

$$\leftarrow G' = \frac{R'C'}{L'}$$

$$= \sqrt{[R']^2\frac{C'}{L'}}$$

$$\boxed{= R'\sqrt{\frac{C'}{L'}}} \checkmark$$

$$\beta = \mathrm{Im}\{\gamma\}$$

$$= \omega\sqrt{R'G'\cdot\frac{[C']^2}{[G']^2}}$$

$$= \omega\sqrt{\frac{R'[C']^2}{G'}}$$

$$\leftarrow G' = \frac{R'C'}{L'}$$

$$\boxed{= \omega\sqrt{L'C'}} \checkmark$$

$$Z_0 = \sqrt{\frac{R'+j\omega L'}{G'+j\omega C'}}$$

$$= \sqrt{\frac{R'\cancel{(1+j\omega\frac{L'}{R'})}}{G'\cancel{(1+j\omega\frac{C'}{G'})}}}$$

$$= \sqrt{\frac{R'}{G'}}$$

$$\boxed{= \sqrt{\frac{L'}{C'}}} \checkmark$$

2.20

$$\Gamma = \frac{z_L - 1}{z_L + 1}$$

$$z_L = \frac{Z_L}{Z_0}$$

$$= \frac{R+j\omega L}{Z_0}$$

$$= \frac{600\Omega + j2\pi\cdot 5\,\mathrm{Mhz}\cdot 0.02\,\mathrm{mH}}{300\Omega}$$

$$\leftarrow = [2+j2.09]\Omega$$

$$= \frac{2+j2.09-1}{2+j2.09+1}$$

$$= \frac{1+j2.09}{3+j2.09}$$

$$= 0.552 + j0.313$$

$$\boxed{= 0.634\angle 29.4^\circ}$$

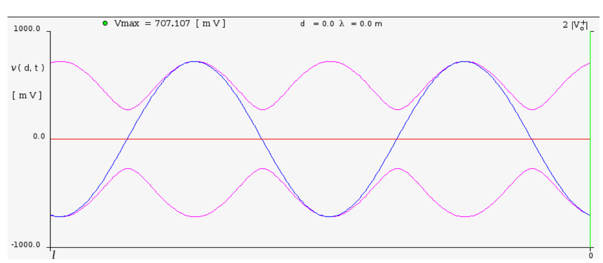
$$S = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$= \frac{1+0.634}{1-0.634}$$

$$\boxed{= 4.46}$$

S: RATIO OF $|\tilde{V}|_{max}$ to $|\tilde{V}|_{min}$, VOLTAGE STANDING-WAVE RATIO ("mismatch" BETWEEN LOAD & TRANSMISSION LINE)

2.25



$$S = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$\Gamma = \frac{z_L - 1}{z_L + 1}$$

$$z_L = \frac{Z_L}{Z_0} = \frac{100 - j50}{50}$$

$$z_L = 2 - j$$

$$= \frac{1-j}{3-j}$$

$$\Gamma = 0.4 - j0.2$$

$$= \frac{1 + |0.4 - j0.2|}{1 - |0.4 - j0.2|}$$

$$= \frac{1.45}{0.553}$$

$$\boxed{S = 2.62}$$

$$d_{max} = \frac{\theta_r \lambda}{4\pi} + \frac{\lambda}{2}$$

$$\lambda = \frac{u_p}{f}$$

$$u_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3\times10^8 \text{ m/s}}{\sqrt{2.25}}$$

$$u_p = 2\times10^8 \text{ m/s}$$

$$= \frac{2\times10^8 \text{ m/s}}{1\times10^9 \text{ 1/s}}$$

$$\lambda = 0.2\text{m}$$

$\theta_r = \theta_r$ = angle of Γ from above $= -0.46$ rad

$$= \frac{-0.46 \times 0.2\text{m}}{4\pi} + \frac{0.2}{2}$$

$$\boxed{d_{max} = 9.3\text{cm}}$$

$$d_{min} = d_{max} - \lambda/4$$

$$= 9.3\text{cm} - 20\text{cm}/4$$

$$\boxed{d_{min} = 4.3\text{cm}}$$

2.33

(a)

$$Z_{in1} = Z_0\left(\frac{z_L + j\tan\beta l}{1 + jz_L\tan\beta l}\right)$$ PG 76

$$z_L = Z_L/Z_0$$

$$= 75\Omega_{\text{antenna}}/50\Omega_{\text{line}}$$

$$z_L = 1.5\Omega$$

$$\beta l = \frac{2\pi}{\lambda}\cdot 0.2\lambda$$

$$\beta l = 0.4\pi$$

$$= 50\Omega\left(\frac{1.5\Omega + j\tan(0.4\pi)}{1 + j\,1.5\Omega\tan(0.4\pi)}\right)$$

$$\boxed{Z_{in1} = 35.2 - j8.62\ \Omega}$$

(b)

$$Z'_L = Z_{in1} \| Z_{in2}$$

$= \frac{1}{2}Z_{in1}$ since $Z_{in1} = Z_{in2}$

$$\boxed{Z'_L = 17.6 - j4.31\ \Omega}$$

(c)

$$Z_{in} = Z'_L\left(\frac{z'_L + j\tan\beta l}{1 + jz'_L\tan\beta l}\right)$$

$z'_L = z_{L1} \| z_{L2} = \frac{1}{2}z_L$ since $z_{L1} = z_{L2}$

$$= \frac{1}{2}\times 1.5\Omega$$

$$z'_L = 0.75\Omega$$

$$\beta l = \frac{2\pi}{\lambda}\cdot 0.5\lambda$$

$$\beta l = \pi$$

$$= 0.75\Omega\left(\frac{0.75\Omega + j\tan(\pi)}{1 + j\,0.75\Omega\tan(\pi)}\right)$$

$$\boxed{Z_{in} = 0.563\ \Omega}$$

Supplemental

```python
import numpy as np

microstrip = {
    'eR': 9,
    'height': 0.5, #mm
    'impedance': 50
}

s = np.arange(0.5, 1.5, 0.0001)

x = 0.56*((microstrip['eR'] - 0.9)/(microstrip['eR']+3))**0.05
y = 1 + 0.02*np.log((s**4 + 3.7e-4 * s**2)/(s**4 + 0.43)) + 0.05 * np.log(1 + 1.7e-4 * s**3)

microstrip['eEff'] = (microstrip['eR'] + 1)/2 + (microstrip['eR'] - 1)/2 * (1+10/s)**(-x*y)

t = (30.67/s)**0.75

Z0 = 60/np.sqrt(microstrip['eEff']) * np.log((6 + (2*np.pi - 6)*np.exp(-t))/s + np.sqrt(1 + 4/s**2))


sIndex = np.where(np.isclose(Z0, 50.0, rtol = 0.0001))[0][0]
microstrip['width'] = s[sIndex] * microstrip['height']

print(f"\[x = {x:.4f}\]")
print(f"\[y = {y[sIndex]:.4f}\]")
print(f"\[t = {t[sIndex]:.4f}\]")
print(f"\[\epsilon_{{eff}} = {microstrip['eEff'][sIndex]:.4f}\]")
print(f"\[Z_0 = {Z0[sIndex]:.4f}\]")
print(f"\[w = {microstrip['width']:.4f} \si{{\mm}}\]")
```

$$x = 0.5491$$

$$y = 0.9940$$

$$t = 12.5376$$

$$\epsilon_{eff} = 6.1085$$

$$Z_0 = 50.0047$$

$$w = 0.5265\,\mathrm{mm}$$