EE381 HW 7
Lewis Collum
Updated: April 6, 2020

1 - Biot-Savart Law

A - Compared to Coloumb's Law

Similarities:

- Proportional to $1/R^2$
- Work on the principle of superposition

Differences:

- Coulomb's
  - Point charge produces electric field
  - Direction of E is radial to point charge
- Biot-Savart's
  - Current element produces magnetic field
  - Direction of B is perpependicular to $\hat{r}$

B - Magnetic Field Intensity (H) from a line at a point

$$\vec{H} = \frac{I}{4\pi}\int_l \frac{d\vec{l} \times \hat{R}}{r^2}$$

$$d\vec{l} \times R = dz(\hat{z} \times \hat{R})$$

$$= dz \cdot \hat{\phi} \sin\hat{\theta}$$

$$= \frac{I\hat{\phi}}{4\pi}\int_a^b \frac{\sin\theta \, dz}{r^2}$$

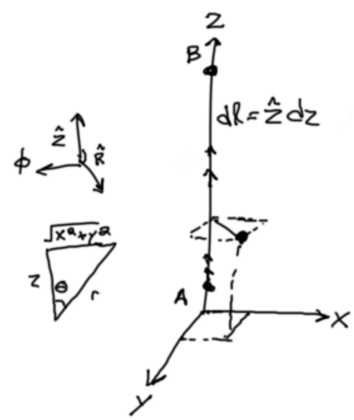
$$\sin\theta = \frac{z}{r}$$

$$r = x^2 + y^2 + z^2$$

$$= \frac{I\hat{\phi}}{4\pi}\int_a^b \frac{\sqrt{x^2+y^2}\, dz}{(x^2+y^2+z^2)^{3/2}}$$

IDENTITY $$\int_a^b \frac{dz}{(x^2+y^2+z^2)^{3/2}} = \frac{1}{(\sqrt{x^2+y^2})^2}\left[\frac{z}{(x^2+y^2+z^2)^{1/2}}\right]_a^b$$

$$= \frac{I\hat{\phi}}{4\pi\sqrt{x^2+y^2}} \cdot \left[\frac{b}{\sqrt{x^2+y^2+b^2}} - \frac{a}{\sqrt{x^2+y^2+a^2}}\right] \checkmark$$



2 - Solenoid

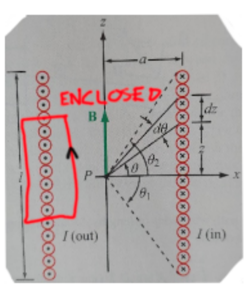
$$\oint \vec{B}\, d\vec{s} = \mu_0 I_{enc}$$

$$\underbrace{\int \vec{B} d\vec{s}}_{\text{TOP}} + \underbrace{\int \vec{B} d\vec{s}}_{\text{BOTTOM}} + \underbrace{\int \vec{B} d\vec{s}}_{\text{LEFT}} + \underbrace{\int \vec{B} d\vec{s}}_{\text{RIGHT}} = \mu_0 I_{enc}$$

$$0 + 0 + 0 + \vec{B}\ell = \mu_0 I_{enc}$$

$$\vec{B} = \frac{\mu_0 I_{enc}}{\ell}$$

$$\vec{B} = \frac{\mu_0 I N}{\ell}\hat{z} \checkmark$$



3 - Tokamak

A - without E

$$F = F_e + F_m$$

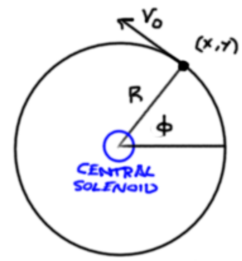
$$= \cancel{e\vec{E}} + e\vec{v_0} \times \vec{B}$$

$$= e\vec{v_0}\vec{B}\sin 90$$

$$\boxed{= ev_0 B}$$

$$F_c = \frac{mv_0^2}{R} = ev_0 B \quad \text{CENTRIPETAL FORCE}$$

$$\rightarrow R = \frac{mv_0}{eB} \checkmark$$



$$\phi = \frac{\vec{v_0}t}{R} = \frac{\vec{v_0}t}{\frac{mv_0}{eB}} = \frac{eBt}{m}$$

$$x(t) = R\cos\left(\frac{\vec{v_0}t}{R}\right)\hat{x} = \frac{mv_0}{eB}\cos\left(\frac{eBt}{m}\right)$$

$$y(t) = R\sin\left(\frac{\vec{v_0}t}{R}\right)\hat{y} = \frac{mv_0}{eB}\sin\left(\frac{eBt}{m}\right)$$

$$v_x(t) = -V_0\sin\left(\frac{\vec{v_0}t}{R}\right)\hat{x} = -V_0\sin\left(\frac{eBt}{m}\right)$$

$$V_y(t) = V_0\cos\left(\frac{V_0 t}{R}\right)\hat{y} = V_0\cos\left(\frac{eBt}{m}\right)$$

B - with E

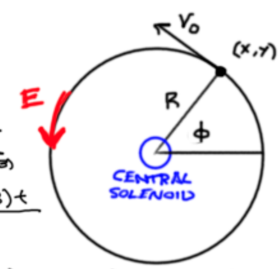
$$F = F_e + F_m$$

$$= e\vec{E} + e\vec{v_0} \times B$$

$$= e\vec{E} + e\vec{v_0}B = \frac{mv_0^2}{R}$$

$$\rightarrow R = \frac{mv_0^2}{e(\vec{E} + v_0 B)}$$

$$\phi = \frac{v_0 t}{\frac{m V_0^2}{e(\vec{E}+v_0 B)}} = \frac{e(\vec{E}+v_0B)t}{mV_0}$$



$$x(t) = R\cos\left(\frac{\vec{v_0}t}{R}\right)\hat{x} = \frac{mv_0^2}{e(\vec{E}+v_0B)}\cos\left(\frac{e(\vec{E}+v_0B)t}{mV_0}\right)\hat{x}$$

$$y(t) = R\sin\left(\frac{\vec{v_0}t}{R}\right)\hat{y} = \frac{mv_0^2}{e(\vec{E}+v_0B)}\sin\left(\frac{e(\vec{E}+v_0B)t}{mV_0}\right)\hat{y}$$

$$v_x(t) = -V_0\sin\left(\frac{\vec{v_0}t}{R}\right)\hat{x} = -V_0\sin\left(\frac{e(\vec{E}+v_0B)t}{mV_0}\right)\hat{x}$$

$$V_y(t) = V_0\cos\left(\frac{V_0 t}{R}\right)\hat{y} = V_0\cos\left(\frac{e(\vec{E}+v_0B)t}{mV_0}\right)\hat{y}$$

4 - Sliding Bar

MOTIONAL

$$V^m_{emf} = \oint_C (\vec{v} \times B) \cdot d\ell$$

$$= \int_{BAR} (\vec{v}\hat{x} \times B_0 x_0 \hat{y}) \cdot -\hat{z}\, d\ell$$

$$= v B_0 x_0 \sin 90^\circ \cdot -\ell$$

$$= -v B_0 x_0 \ell \quad \text{across BAR}$$

$$V^m_{emf} = v B_0 x_0 \ell \quad \text{across R}$$

$$x_0 = vt = 1\,m/s \cdot t = t$$

$$\ell = 1m$$

$$\boxed{= B_0 t}$$

$$I_R = \frac{V^m_{emf}}{R} = \frac{B_0 t}{10\Omega}$$

$$\boxed{= \frac{1}{10} B_0 t}$$

LENZ

$$\Phi = \int_S B\, ds$$

$$= \int_S (\hat{y} B_0 x)\, \hat{y}\, dx\, (-dz)$$

$$= -B_0 \ell \int_0^{x_0} x\, dx$$

$$= -\frac{B_0 \ell x_0^2}{2}$$

$$= -\frac{B_0 \ell (vt)^2}{2}$$

$$= -\frac{B_0 \cdot 1m \cdot (1\,m/s \cdot t)^2}{2}$$

$$= -\frac{B_0 t^2}{2}$$

$$V_{emf} = -\frac{d\Phi}{dt}$$

$$= -\frac{d}{dt}\left[-\frac{B_0 t^2}{2}\right]$$

$$\boxed{= B_0 t}$$

$$I_R = \frac{V_{emf}}{R}$$

$$\boxed{= \frac{1}{10} B_0 t}$$

CURRENT FLOWING FROM BOTTOM OF RESISTOR TO TOP.

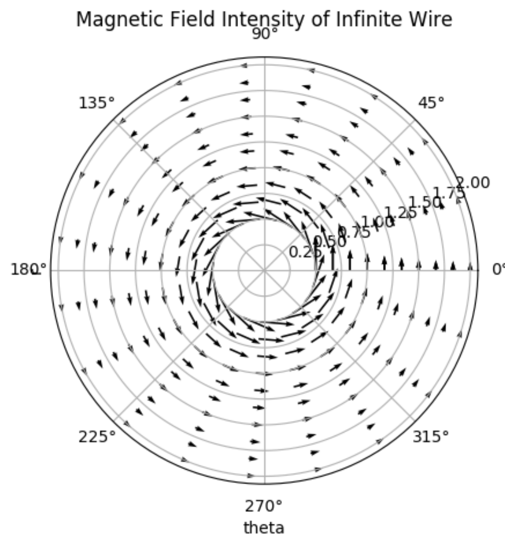
5 - Magnetic Vector Potential

Magnetic Field Intensity

From Ampere's law applied to an infinite wire:

$$\vec{B} = \hat{\phi}\frac{I}{2\pi r}$$

```python
import matplotlib.pyplot as pyplot
import numpy

import my #some helper functions I made

phi, r = my.makePolarGrid(start=0.5, stop=2)

I = 1
H = I/2/numpy.pi/r

my.plotPolarField(phi, r, H, 0)
pyplot.xlabel('theta')
pyplot.ylabel('r')
pyplot.title('Magnetic Field Intensity of Infinite Wire')
#pyplot.show()

pyplot.savefig('./5-a.png')
```



Magnetic Vector Potential

$$\vec{A} = \hat{\phi}\frac{I}{2\pi} \cdot \ln(r)$$

```python
import matplotlib.pyplot as pyplot
import numpy

import my

phi, r = my.makePolarGrid(start=0.5, stop=2.5)

I = 1
A = I/2/numpy.pi * numpy.log(r)

my.plotPolarField(phi, r, A, 0)
pyplot.xlabel('theta')
pyplot.ylabel('r')
pyplot.title('Magnetic Vector Potential of Infinite Wire')
#pyplot.show()

pyplot.savefig('5-b.png')
```