Student: Lewis C	**Instructor:** Scott Fulton	**Assignment:** Section 2.5 Homework
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1. Solve the equation A**x** = **b** by using the LU factorization given for A. Also solve A**x** = **b** by ordinary row reduction.

$$A = \begin{bmatrix} 3 & -6 & -4 \\ -3 & 2 & 3 \\ 6 & 0 & -6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} 3 & -6 & -4 \\ 0 & -4 & -1 \\ 0 & 0 & -1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ -14 \\ 48 \end{bmatrix}$$

Let L**y** = **b** and U**x** = **y**. Solve for **x** and **y**.

$$\mathbf{y} = \begin{bmatrix} 0 \\ -14 \\ 6 \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} 2 \\ 5 \\ -6 \end{bmatrix}$$

Row reduce the augmented matrix [A **b**] and use it to find **x**.

The reduced row echelon form of [A **b**] is $\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -6 \end{bmatrix}$, yielding $\mathbf{x} = \begin{bmatrix} 2 \\ 5 \\ -6 \end{bmatrix}$.

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2. Solve the equation A**x** = **b** by using the LU factorization given for A.

$$A = \begin{bmatrix} 2 & -4 & 2 \\ -6 & 9 & -1 \\ 4 & -5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -4 & 2 \\ 0 & -3 & 5 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 4 \\ 0 \\ -4 \end{bmatrix}$$

Let L**y** = **b** and U**x** = **y**. Solve for **x** and **y**.

$$\mathbf{y} = \begin{bmatrix} 4 \\ 12 \\ 0 \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} -6 \\ -4 \\ 0 \end{bmatrix}$$

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3. Solve the equation A**x** = **b** by using the LU factorization given for A.

$$A = \begin{bmatrix} 2 & -4 & 4 \\ 1 & -4 & 11 \\ 3 & -16 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{3}{2} & 5 & 1 \end{bmatrix} \begin{bmatrix} 2 & -4 & 4 \\ 0 & -2 & 9 \\ 0 & 0 & -45 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 0 \\ -1 \\ -50 \end{bmatrix}$$

Let L**y** = **b**. Solve for **y**.

$$\mathbf{y} = \begin{bmatrix} 0 \\ -1 \\ -45 \end{bmatrix}$$

Let U**x** = **y**. Solve for **x**.

$$\mathbf{x} = \begin{bmatrix} 8 \\ 5 \\ 1 \end{bmatrix}$$

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4. Find an LU factorization of the matrix A (with L unit lower triangular).

$$A = \begin{bmatrix} 5 & 3 \\ -4 & -3 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 \\ -\frac{4}{5} & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 5 & 3 \\ 0 & -\frac{3}{5} \end{bmatrix}$$

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5. Find an LU factorization of the matrix A (with L unit lower triangular).

$$A = \begin{bmatrix} 9 & 12 \\ 27 & 33 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 9 & 12 \\ 0 & -3 \end{bmatrix}$$

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6. Find an LU factorization of the matrix A (with L unit lower triangular).

$$A = \begin{bmatrix} -2 & 0 & 4 \\ 6 & 3 & -8 \\ 6 & 12 & 13 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -3 & 4 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} -2 & 0 & 4 \\ 0 & 3 & 4 \\ 0 & 0 & 9 \end{bmatrix}$$

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7. Find an LU factorization of the matrix A (with L unit lower triangular).

$$A = \begin{bmatrix} 4 & 4 & -2 & 3 \\ 4 & 1 & 1 & 4 \\ 16 & 4 & 6 & 17 \\ 4 & -8 & 12 & 9 \end{bmatrix}$$

$$U = \begin{bmatrix} 4 & 4 & -2 & 3 \\ 0 & -3 & 3 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(Simplify your answer.)

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 4 & 4 & 1 & 0 \\ 1 & 4 & 1 & 1 \end{bmatrix}$$

(Simplify your answer.)