Homework 1

Lewis Collum

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1.4

$$2\sin\left(2\pi\left(2t+5x\right)\right)$$

1. Along the negative x direction.
2. $\boxed{\phi = 0}$
3. $f = \boxed{2\text{Hz}}$
4. $\lambda = \boxed{\frac{1}{5}\text{cm}}$
5. $u = \lambda f = \boxed{\frac{2}{5}\text{cm}\,\text{s}^{-1}}$

1.8

(a)

y_1 is headed right. y_2 is headed left.

1.14

$Ae^{-\alpha x}$

$98.02 = Ae^{-\alpha \cdot 10}$

$A_1 = \frac{98.02}{e^{-10\alpha}}$

$A_2 = \frac{81.87}{e^{-1000\alpha}}$

$A_1 < A_2$

$\frac{98.02}{e^{-100\alpha}} < \frac{81,87}{e^{-1000\alpha}}$

$\frac{98.02}{81.87} < \frac{e^{-10\alpha}}{e^{-100\alpha}}$

$1.20 = e^{-10\alpha + 100\alpha}$

$\ln(1.20) = 90\alpha \ln(e)^1$

$\alpha = \frac{\ln(1.20)}{90}$

$\boxed{\alpha = 0.0020}$

1.16

1.16.a $z_1 = 8e^{j\pi/3}$

$z_1 = 8\left(\cos\left(\frac{\pi}{3}\right) + j\sin\left(\frac{\pi}{3}\right)\right)$

$\boxed{z_1 = 4 + j4\sqrt{3}}$

1.16.b $z_2 = \sqrt{3}e^{j3\pi/4}$

$z_2 = \sqrt{3}\left(\cos\left(\frac{3\pi}{4}\right) + j\sin\left(\frac{3\pi}{4}\right)\right)$

$\boxed{z_2 = \sqrt{3}\left(-\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right)}$

1.16.c $z_3 = 2e^{-j\pi/2}$

$z_3 = 2\left(\cos\left(-\frac{\pi}{2}\right) + j\sin\left(-\frac{\pi}{2}\right)\right)$

$\boxed{z_3 = -j2}$

1.16.d $z_4 = j^3 = \boxed{-j}$

1.16.e $z_5 = j^{-4} = \boxed{1}$

1.16.f $z_6 = (1-j)^3$

$= (1-j)(1-j)(1-j)$

$= (1 - 2j - 1)(1-j)$

$= -2j + 2j^2$

$\boxed{= -2 - 2j}$

1.16.g $z_7 = (1-j)^{1/2}$

$\boxed{= \sqrt{1-j}}$?

1.17

(a) $\boxed{\begin{aligned} z_1 &= 3.6\angle -33.7^\circ \\ z_2 &= 5\angle -36.9^\circ \end{aligned}}$

(b) $\boxed{|z_1| = 3.6}$

(c) $z_1 z_2 = 3.6e^{-j0.59} \cdot 5e^{-j0.64}$

$= 18e^{-j1.23}$

$\boxed{= 18\angle 109^\circ}$

(d) $z_1 / z_2 = \frac{3.6}{5} e^{-j0.59 + j0.64}$

$= 0.72e^{j0.05}$

$\boxed{= 0.72\angle 3.18^\circ}$

(e) $z_1^3 = 3.6^3 e^{-j0.59 \times 3}$

$= 3.6^3 e^{-j1.77}$

$\boxed{= 46.66\angle -101.4^\circ}$

1.25

$v_s(t) = R i_s(t) + v_c(t)$

$\tilde{V}_s = R\tilde{I}_s + \tilde{V}_c$

$\tilde{V}_c = \tilde{V}_s - R\tilde{I}_s$

$\tilde{I}_s = \frac{\tilde{V}_s}{R + 1/j\omega C}$

$= \frac{25\angle -30^\circ}{1.28\times 10^6 \angle -38.5^\circ}$

$= 1.95\times 10^{-5}\angle 8.5^\circ$

$= 19.5\angle 8.5^\circ$ mA

$= 25\angle -30^\circ - 19.5\angle 8.5^\circ$

$= 15.6\angle -81.3^\circ$ V

$\boxed{v_c(t) = 15.6\cos(2\pi\times 10^3 t - 81.3^\circ)\ [V]}$

1.26

(a) $\tilde{V} = 9e^{-j\frac{\pi}{3}}$

(b) $\tilde{V} = 12e^{\frac{\pi}{2} - j\frac{\pi}{4}}$

(c) $\tilde{I} = 5e^{\frac{\pi}{2} - 3x + j\frac{\pi}{6}}$

(d) $\tilde{I} = -2e^{j3\pi/4}$

(e) $\tilde{I} = 4e^{\frac{\pi}{2} + j\frac{\pi}{3}} + 3e^{-j\frac{\pi}{6}}$

1.29

$\tilde{I}_L = \frac{\tilde{V}_A}{j\omega L}$

$\frac{\tilde{V}_s - \tilde{V}_A}{R_1} = \frac{\tilde{V}_A}{R_2} + \frac{\tilde{V}_A}{j\omega L}$

$\tilde{V}_A\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{j\omega L}\right) R_1 = \tilde{V}_s$

$V_A = \tilde{V}_s / \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{j\omega L}\right) R_1$

$\tilde{V}_s = 25\angle -45^\circ$

$= 12\angle -8.1^\circ$ * V_A

$= \frac{12\angle -8.1^\circ}{j4\times 10^4 \cdot 0.4\text{mH}}$

$= 0.75\angle -98^\circ$ A * $\tilde{I}_L$

$\boxed{i_L(t) = 0.75\cos(4\times 10^4 t - 98^\circ)\ A}$