

DIRECT PROOF & COUNTEREXAMPLE: DIVISIBILITY

Notes

- **Divisibility.** ($d|n$ reads “ d divides n ”)

$$d|n \leftrightarrow \exists \text{ an integer } k \text{ such that } n = dk.$$

- Division is transitive.
- **Unique Factorization of Integers Theorem.** Given any integer $n > 1$, there exists a positive integer k , distinct prime numbers p_1, p_2, \dots, p_k , and positive integers e_1, e_2, \dots, e_k such that

$$n = p_1^{e_1} p_2^{e_2} p_3^{e_3} \dots p_k^{e_k}$$

Test Yourself

1. there exists k such that $n = kd$.
2. n ; d .
3. a ; b .
4. $\frac{n}{d}$ is not an integer.
5. a divides b ; a divided by b .
6. $a|b$ and $b|c$; $a|c$.
7. divisible by at least one prime.
8. prime; a product of primes; order.