EE381 HW 7

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1 - Biot-Savart Law

A - Compared to Coloumb's Law

Similarities:

- Proportional to $1/R^2$
- Work on the principle of superposition

Differences:

- Coulomb's
  - Point charge produces electric field
  - Direction of E is radial to point charge
- Biot-Savart's
  - Current element produces magnetic field
  - Direction of B is perpependicular to $\hat{r}$

B - Magnetic Field Intensity (H) from a line at a point

$$\vec{H} = \frac{I}{4\pi}\int_\ell \frac{d\vec{\ell}\times\hat{R}}{r^2}$$

$$d\vec{\ell}\times\hat{R} = dz(\hat{z}\times\hat{R}) = dz\cdot\hat{\phi}\sin\theta$$

$$= \frac{I\hat{\phi}}{4\pi}\int_a^b \frac{\sin\theta\, dz}{r^2}$$

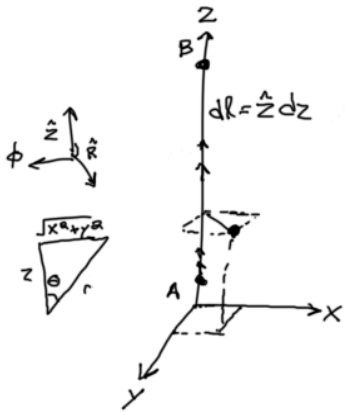
$$\sin\theta = \frac{z}{r}$$

$$r = x^2+y^2+z^2$$

$$= \frac{I\hat{\phi}}{4\pi}\int_a^b \frac{\sqrt{x^2+y^2}\, dz}{(x^2+y^2+z^2)^{3/2}}$$

$$\text{IDENTITY}: \int_a^b \frac{dz}{(x^2+y^2+z^2)^{3/2}} = \frac{1}{(\sqrt{x^2+y^2})^2}\left[\frac{z}{(x^2+y^2+z^2)^{1/2}}\right]_a^b$$

$$= \frac{I\hat{\phi}}{4\pi\sqrt{x^2+y^2}}\cdot\left[\frac{b}{\sqrt{x^2+y^2+b^2}} - \frac{a}{\sqrt{x^2+y^2+a^2}}\right] \checkmark$$



2 - Solenoid

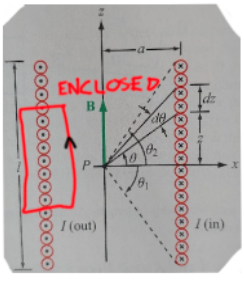
$$\oint \vec{B}\,d\vec{s} = \mu_0 I_{enc}$$

$$\int_{TOP}\vec{B}d\vec{s} + \int_{BOTTOM}\vec{B}d\vec{s} + \int_{LEFT}\vec{B}d\vec{s} + \int_{RIGHT}\vec{B}d\vec{s} = \mu_0 I_{enc}$$

$$0 + 0 + 0 + \vec{B}R = \mu_0 I_{enc}$$

$$\vec{B} = \frac{\mu_0 I_{enc}}{\ell}$$

$$\vec{B} = \frac{\mu_0 I N}{\ell}\hat{z} \checkmark$$



3 - Tokamak

A - without E

$$F = F_e + F_m$$

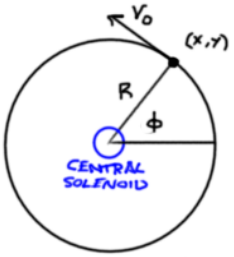
$$= e\vec{E} + e\vec{v}_0\times\vec{B}$$

$$= ev_0B\sin 90$$

$$= ev_0B$$

$$F_c = \frac{mv_0^2}{R} = ev_0B \quad \text{CENTRIPETAL FORCE}$$

$$\rightarrow R = \frac{mv_0}{eB} \checkmark$$



$$x(t) = R\cos\left(\frac{v_0 t}{R}\right)\hat{x} = \frac{mv_0}{eB}\cos\left(\frac{eBt}{m}\right)$$

$$y(t) = R\sin\left(\frac{v_0 t}{R}\right)\hat{y} = \frac{mv_0}{eB}\sin\left(\frac{eBt}{m}\right)$$

$$v_x(t) = -v_0\sin\left(\frac{v_0 t}{R}\right)\hat{x} = -v_0\sin\left(\frac{eBt}{m}\right)$$

$$v_y(t) = v_0\cos\left(\frac{v_0 t}{R}\right)\hat{y} = v_0\cos\left(\frac{eBt}{m}\right)$$

$$\phi = \frac{v_0 t}{R} = \frac{v_0 t}{\frac{mv_0}{eB}} = \frac{eBt}{m}$$

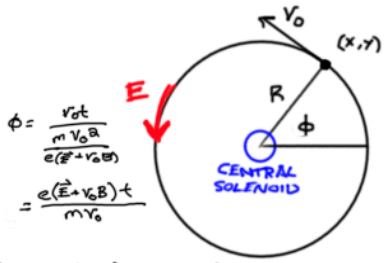
B - with E

$$F = F_e + F_m$$

$$= e\vec{E} + e\vec{v}_0\times B$$

$$= e\vec{E} + ev_0B = \frac{mv_0^2}{R}$$

$$\rightarrow R = \frac{mv_0^2}{e(\vec{E}+v_0B)}$$



$$\phi = \frac{v_0 t}{\frac{mv_0^2}{e(\vec{E}+v_0B)}} = \frac{e(\vec{E}+v_0B)t}{mv_0}$$

$$x(t) = R\cos\left(\frac{v_0 t}{R}\right)\hat{x} = \frac{mv_0^2}{e(\vec{E}+v_0B)}\cos\left(\frac{e(\vec{E}+v_0B)t}{mv_0}\right)\hat{x}$$

$$y(t) = R\sin\left(\frac{v_0 t}{R}\right)\hat{y} = \frac{mv_0^2}{e(\vec{E}+v_0B)}\sin\left(\frac{e(\vec{E}+v_0B)t}{mv_0}\right)\hat{y}$$

$$v_x(t) = -v_0\sin\left(\frac{v_0 t}{R}\right)\hat{x} = -v_0\sin\left(\frac{e(\vec{E}+v_0B)t}{mv_0}\right)\hat{x}$$

$$v_y(t) = v_0\cos\left(\frac{v_0 t}{R}\right)\hat{y} = v_0\cos\left(\frac{e(\vec{E}+v_0B)t}{mv_0}\right)\hat{y}$$

4 - Sliding Bar