Lewis Collum
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Set Theory: Disproofs and Algebraic Proofs

Notes

- Counterexample for all sets A, B, and C, $(A - B) \cup (B - C) = A - C$.
  Counterexample: Let $A = \emptyset$, $B = \{3\}$, $C = \emptyset$. Then

  $$A - B = \emptyset,\ B - C = \{3\},\ A - C = \emptyset.$$

  Hence $(A - B) \cup (B - C) = \emptyset \cup \{3\} = \{3\}$, whereas $A - C = \emptyset$.
  Since $\{3\} \neq \emptyset$, $(A - B) \cup (B - C) \neq A - C$.

- Algebraic proof that for all sets A and B, $(A \cup B) - C = (A - C) \cup (B - C)$.

$$\begin{aligned}
(A \cup B) - C &= (A \cup B) \cap C^n && \text{(set difference law)}\\
&= C^n \cap (A \cup B) && \text{(commutative law)}\\
&= (C^n \cap A) \cup (C^n \cap B) && \text{(distributive law)}\\
&= (A \cap C^n) \cup (B \cap C^n) && \text{(commutative law)}\\
&= (A - C) \cup (B - C) && \text{(set difference law)}
\end{aligned}$$

Test Yourself

1. make the sides of the equation unequal.
2. cite the algebraic laws used
3. exactly