Problem 3.25 Use the appropriate expression for the differential surface area ds to determine the area of each of the following surfaces:
(a) $r=3;\ 0\le\phi\le\pi/3;\ -2\le z\le 2$,
(b) $2\le r\le 5;\ \pi/2\le\phi\le\pi;\ z=0$,
(c) $2\le r\le 5;\ \phi=\pi/4;\ -2\le z\le 2$,
(d) $R=2;\ 0\le\theta\le\pi/3;\ 0\le\phi\le\pi$,
(e) $0\le R\le 5;\ \theta=\pi/3;\ 0\le\phi\le 2\pi$.
Also sketch the outlines of each of the surfaces.

Solution:

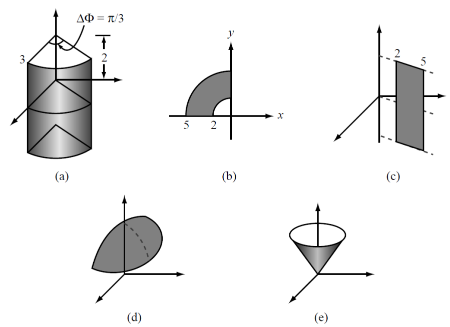


Figure P3.25: Surfaces described by Problem 3.25.

(a) Using Eq. (3.43a),

$$A=\int_{z=-2}^{2}\int_{\phi=0}^{\pi/3}(r)|_{r=3}\,d\phi\,dz=\left((3\phi z)|_{\phi=0}^{\pi/3}\right)\Big|_{z=-2}^{2}=4\pi.$$

(b) Using Eq. (3.43c),

$$A=\int_{r=2}^{5}\int_{\phi=\pi/2}^{\pi}(r)|_{z=0}\,d\phi\,dr=\left(\left(\tfrac{1}{2}r^2\phi\right)\Big|_{r=2}^{5}\right)\Big|_{\phi=\pi/2}^{\pi}=\frac{21\pi}{4}.$$

(c) Using Eq. (3.43b),

$$A=\int_{z=-2}^{2}\int_{r=2}^{5}(1)|_{\phi=\pi/4}\,dr\,dz=\left((rz)|_{z=-2}^{2}\right)\Big|_{r=2}^{5}=12.$$

(d) Using Eq. (3.50b),

$$A=\int_{\theta=0}^{\pi/3}\int_{\phi=0}^{\pi}(R^2\sin\theta)|_{R=2}\,d\phi\,d\theta=\left((-4\phi\cos\theta)|_{\theta=0}^{\pi/3}\right)\Big|_{\phi=0}^{\pi}=2\pi.$$

(e) Using Eq. (3.50c),

$$A=\int_{R=0}^{5}\int_{\phi=0}^{2\pi}(R\sin\theta)|_{\theta=\pi/3}\,d\phi\,dR=\left(\left(\tfrac{1}{2}R^2\phi\sin\frac{\pi}{3}\right)\Big|_{\phi=0}^{2\pi}\right)\Big|_{R=0}^{5}=\frac{25\sqrt{3}\pi}{2}.$$

Problem 3.26 Find the volumes described by
(a) $2\le r\le 5;\ \pi/2\le\phi\le\pi;\ 0\le z\le 2$,
(b) $0\le R\le 5;\ 0\le\theta\le\pi/3;\ 0\le\phi\le 2\pi$.
Also sketch the outline of each volume.

Solution:

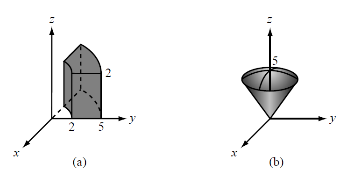


Figure P3.26: Volumes described by Problem 3.26 .

(a) From Eq. (3.44),

$$v=\int_{z=0}^{2}\int_{\phi=\pi/2}^{\pi}\int_{r=2}^{5}r\,dr\,d\phi\,dz=\left(\left(\left(\tfrac{1}{2}r^2\phi z\right)\Big|_{r=2}^{5}\right)\Big|_{\phi=\pi/2}^{\pi}\right)\Big|_{z=0}^{2}=\frac{21\pi}{2}.$$

(b) From Eq. (3.50e),

$$v=\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/3}\int_{R=0}^{5}R^2\sin\theta\,dR\,d\theta\,d\phi$$
$$=\left(\left(\left(-\cos\theta\frac{R^3}{3}\phi\right)\Big|_{R=0}^{5}\right)\Big|_{\theta=0}^{\pi/3}\right)\Big|_{\phi=0}^{2\pi}=\frac{125\pi}{3}.$$

Problem 3.40 For the scalar function $V=xy^2-z^2$, determine its directional derivative along the direction of vector $\mathbf{A}=(\hat{\mathbf{x}}-\hat{\mathbf{y}}z)$ and then evaluate it at $P=(1,-1,4)$.

Solution: The directional derivative is given by Eq. (3.75) as $dV/dl=\nabla V\cdot\hat{\mathbf{a}}_l$, where the unit vector in the direction of $\mathbf{A}$ is given by Eq. (3.2):

$$\hat{\mathbf{a}}_l=\frac{\hat{\mathbf{x}}-\hat{\mathbf{y}}z}{\sqrt{1+z^2}},$$

and the gradient of V in Cartesian coordinates is given by Eq. (3.72):

$$\nabla V=\hat{\mathbf{x}}y^2+\hat{\mathbf{y}}2xy-\hat{\mathbf{z}}2z.$$

Therefore, by Eq. (3.75),

$$\frac{dV}{dl}=\frac{y^2-2xyz}{\sqrt{1+z^2}}.$$

At $P=(1,-1,4)$,

$$\left(\frac{dV}{dl}\right)\Big|_{(1,-1,4)}=\frac{9}{\sqrt{17}}=2.18.$$

Problem 3.5 Given vectors $\mathbf{A}=\hat{\mathbf{x}}+\hat{\mathbf{y}}2-\hat{\mathbf{z}}3$, $\mathbf{B}=\hat{\mathbf{x}}2-\hat{\mathbf{y}}4$, and $\mathbf{C}=\hat{\mathbf{y}}2-\hat{\mathbf{z}}4$, find
(a) A and $\hat{\mathbf{a}}$,
(b) the component of $\mathbf{B}$ along $\mathbf{C}$,
(c) θ_{AC},
(d) $\mathbf{A}\times\mathbf{C}$,
(e) $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})$,
(f) $\mathbf{A}\times(\mathbf{B}\times\mathbf{C})$,
(g) $\hat{\mathbf{x}}\times\mathbf{B}$, and
(h) $(\mathbf{A}\times\hat{\mathbf{y}})\cdot\hat{\mathbf{z}}$.

Solution:
(a) From Eq. (3.4),

$$A=\sqrt{1^2+2^2+(-3)^2}=\sqrt{14},$$

and, from Eq. (3.5),

$$\hat{\mathbf{a}}_A=\frac{\hat{\mathbf{x}}+\hat{\mathbf{y}}2-\hat{\mathbf{z}}3}{\sqrt{14}}.$$

(b) The component of $\mathbf{B}$ along $\mathbf{C}$ (see Section 3-1.4) is given by

Problem 3.14 Show that, given two vectors $\mathbf{A}$ and $\mathbf{B}$,
(a) the vector $\mathbf{C}$ defined as the vector component of $\mathbf{B}$ in the direction of $\mathbf{A}$ is given by

$$\mathbf{C}=\hat{\mathbf{a}}(\mathbf{B}\cdot\hat{\mathbf{a}})=\frac{\mathbf{A}(\mathbf{B}\cdot\mathbf{A})}{|\mathbf{A}|^2},$$

where $\hat{\mathbf{a}}$ is the unit vector of $\mathbf{A}$, and
(b) the vector $\mathbf{D}$ defined as the vector component of $\mathbf{B}$ perpendicular to $\mathbf{A}$ is given by

$$\mathbf{D}=\mathbf{B}-\frac{\mathbf{A}(\mathbf{B}\cdot\mathbf{A})}{|\mathbf{A}|^2}.$$

Solution:
(a) By definition, $\mathbf{B}\cdot\hat{\mathbf{a}}$ is the component of $\mathbf{B}$ along $\hat{\mathbf{a}}$. The vector component of $(\mathbf{B}\cdot\hat{\mathbf{a}})$ along $\mathbf{A}$ is

$$\mathbf{C}=\hat{\mathbf{a}}(\mathbf{B}\cdot\hat{\mathbf{a}})=\frac{\mathbf{A}}{|\mathbf{A}|}\left(\mathbf{B}\cdot\frac{\mathbf{A}}{|\mathbf{A}|}\right)=\frac{\mathbf{A}(\mathbf{B}\cdot\mathbf{A})}{|\mathbf{A}|^2}.$$

(b) The figure shows vectors $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$, where $\mathbf{C}$ is the projection of $\mathbf{B}$ along $\mathbf{A}$. It is clear from the triangle that

$$\mathbf{B}=\mathbf{C}+\mathbf{D},$$

or

$$\mathbf{D}=\mathbf{B}-\mathbf{C}=\mathbf{B}-\frac{\mathbf{A}(\mathbf{B}\cdot\mathbf{A})}{|\mathbf{A}|^2}.$$

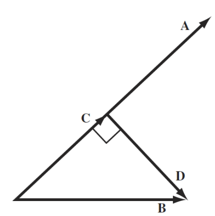


Figure P3.14: Relationships between vectors A, B, C, and D

Problem 2.2 A two-wire copper transmission line is embedded in a dielectric material with $\varepsilon_r=2.6$ and $\sigma=2\times10^{-6}$ S/m. Its wires are separated by 3 cm and their radii are 1 mm each.
(a) Calculate the line parameters R', L', G', and C' at 2 GHz.
(b) Compare your results with those based on CD Module 2.1. Include a printout of the screen display.

Solution:
(a) Given:

$$f=2\times10^9\text{ Hz},$$
$$d=2\times10^{-3}\text{ m},$$
$$D=3\times10^{-2}\text{ m},$$
$$\sigma_c=5.8\times10^7\text{ S/m (copper)},$$
$$\varepsilon_r=2.6,$$
$$\sigma=2\times10^{-6}\text{ S/m},$$
$$\mu=\mu_c=\mu_0.$$

From Table 2-1:

$$R_s=\sqrt{\pi f\mu_c/\sigma_c}$$
$$=[\pi\times2\times10^9\times4\pi\times10^{-7}/5.8\times10^7]^{1/2}$$
$$=1.17\times10^{-2}\ \Omega,$$
$$R'=\frac{2R_s}{\pi d}=\frac{2\times1.17\times10^{-2}}{2\pi\times10^{-3}}=3.71\ \Omega/\text{m},$$
$$L'=\frac{\mu}{\pi}\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]$$
$$=1.36\times10^{-6}\text{ H/m},$$
$$G'=\frac{\pi\sigma}{\ln[(D/d)+\sqrt{(D/d)^2-1}]}$$
$$=1.85\times10^{-6}\text{ S/m},$$
$$C'=\frac{G'\varepsilon}{\sigma}$$
$$=\frac{1.85\times10^{-6}\times8.85\times10^{-12}\times2.6}{2\times10^{-6}}$$
$$=2.13\times10^{-11}\text{ F/m}.$$

(b) Solution via Module 2.1:

2.75

$$T=\frac{1\text{m}}{2c/3}=5\text{ns}$$

$$\Gamma_L=\frac{R_L-Z_0}{R_L+Z_0}=\frac{25-50}{25+50}=-\frac{1}{3}$$

$$\Gamma_g=\frac{R_g-Z_0}{R_g+Z_0}=\frac{100-50}{100+50}=\frac{1}{3}$$

$$V_1^+=V_g\cdot\frac{Z_0}{Z_0+R_g}=60\ \frac{50}{50+100}=20\text{V}$$

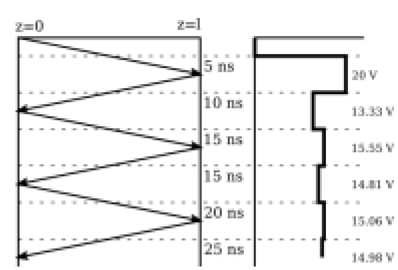
$$V_1^-=V_1^+\cdot\Gamma_L=20\cdot\frac{1}{3}=-6.67\text{V}$$

$$V_2^+=V_1^-\cdot\Gamma_g=6.67\ \frac{1}{3}=2.22\text{V}$$

$$V_2^-=-0.74\text{V}$$

$$V_3^+=0.25\text{V}$$

$$V_3^-=-0.083\text{V}$$



$$Z_{\text{in}} = \frac{Z_0^2}{Z_{\text{L}}}, \qquad \text{for } l = \lambda/4 + n\lambda/2.$$

Voltage maximum	$\lvert\widetilde{V}\rvert_{\max} = \lvert V_0^+\rvert[1+\lvert\Gamma\rvert]$
Voltage minimum	$\lvert\widetilde{V}\rvert_{\min} = \lvert V_0^+\rvert[1-\lvert\Gamma\rvert]$
Positions of voltage maxima (also positions of current minima)	$d_{\max} = \frac{\theta_r\lambda}{4\pi} + \frac{n\lambda}{2}, \quad n = 0, 1, 2, \ldots$
Position of first maximum (also position of first current minimum)	$d_{\max} = \begin{cases} \frac{\theta_r\lambda}{4\pi}, & \text{if } 0 \le \theta_r \le \pi \\ \frac{\theta_r\lambda}{4\pi} + \frac{\lambda}{2}, & \text{if } -\pi \le \theta_r \le 0 \end{cases}$
Positions of voltage minima (also positions of current maxima)	$d_{\min} = \frac{\theta_r\lambda}{4\pi} + \frac{(2n+1)\lambda}{4}, \quad n = 0, 1, 2, \ldots$
Position of first minimum (also position of first current maximum)	$d_{\min} = \frac{\lambda}{4}\left(1 + \frac{\theta_r}{\pi}\right)$
Input impedance	$Z_{\text{in}} = Z_0\left(\frac{z_{\text{L}} + j\tan\beta l}{1 + jz_{\text{L}}\tan\beta l}\right) = Z_0\left(\frac{1+\Gamma_l}{1-\Gamma_l}\right)$
Positions at which Z_{in} is real	at voltage maxima and minima
Z_{in} at voltage maxima	$Z_{\text{in}} = Z_0\left(\frac{1+\lvert\Gamma\rvert}{1-\lvert\Gamma\rvert}\right)$
Z_{in} at voltage minima	$Z_{\text{in}} = Z_0\left(\frac{1-\lvert\Gamma\rvert}{1+\lvert\Gamma\rvert}\right)$
Z_{in} of short-circuited line	$Z_{\text{in}}^{\text{sc}} = jZ_0\tan\beta l$
Z_{in} of open-circuited line	$Z_{\text{in}}^{\text{oc}} = -jZ_0\cot\beta l$
Z_{in} of line of length $l = n\lambda/2$	$Z_{\text{in}} = Z_{\text{L}}, \quad n = 0, 1, 2, \ldots$
Z_{in} of line of length $l = \lambda/4 + n\lambda/2$	$Z_{\text{in}} = Z_0^2/Z_{\text{L}}, \quad n = 0, 1, 2, \ldots$
Z_{in} of matched line	$Z_{\text{in}} = Z_0$

$\lvert V_0^+\rvert$ = amplitude of incident wave; $\Gamma = \lvert\Gamma\rvert e^{j\theta_r}$ with $-\pi < \theta_r < \pi$; θ_r in radians; $\Gamma_l = \Gamma e^{-j2\beta l}$.

Table 3-1 Summary of vector relations.

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation $\mathbf{A} =$	$\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{r}}A_r + \hat{\boldsymbol{\phi}}A_\phi + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{R}}A_R + \hat{\boldsymbol{\theta}}A_\theta + \hat{\boldsymbol{\phi}}A_\phi$
Magnitude of $\mathbf{A}$ $\lvert\mathbf{A}\rvert =$	$\sqrt[+]{A_x^2 + A_y^2 + A_z^2}$	$\sqrt[+]{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\overrightarrow{OP_1} =$	$\hat{\mathbf{x}}x_1 + \hat{\mathbf{y}}y_1 + \hat{\mathbf{z}}z_1$, for $P(x_1, y_1, z_1)$	$\hat{\mathbf{r}}r_1 + \hat{\mathbf{z}}z_1$, for $P(r_1, \phi_1, z_1)$	$\hat{\mathbf{R}}R_1$, for $P(R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{\mathbf{x}}\cdot\hat{\mathbf{x}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}}\cdot\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}}\cdot\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = 1$ $\hat{\mathbf{R}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}}\times\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$ $\hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\mathbf{A}\cdot\mathbf{B} =$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\mathbf{A}\times\mathbf{B} =$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length $d\mathbf{l} =$	$\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}r\,d\phi + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{R}}\,dR + \hat{\boldsymbol{\theta}}R\,d\theta + \hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$
Differential surface areas	$d\mathbf{s}_x = \hat{\mathbf{x}}\,dy\,dz$ $d\mathbf{s}_y = \hat{\mathbf{y}}\,dx\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}\,dx\,dy$	$d\mathbf{s}_r = \hat{\mathbf{r}}r\,d\phi\,dz$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}r\,dr\,d\phi$	$d\mathbf{s}_R = \hat{\mathbf{R}}R^2\sin\theta\,d\theta\,d\phi$ $d\mathbf{s}_\theta = \hat{\boldsymbol{\theta}}R\sin\theta\,dR\,d\phi$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}R\,dR\,d\theta$
Differential volume $d\mathcal{V} =$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

Table 3-2 Coordinate transformation relations.

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt[+]{x^2+y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{\mathbf{x}} = \hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{r}}\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt[+]{x^2+y^2+z^2}$ $\theta = \tan^{-1}[\sqrt[+]{x^2+y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{\mathbf{R}} = \hat{\mathbf{x}}\sin\theta\cos\phi + \hat{\mathbf{y}}\sin\theta\sin\phi + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{x}}\cos\theta\cos\phi + \hat{\mathbf{y}}\cos\theta\sin\phi - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$	$A_R = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$ $A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{\mathbf{x}} = \hat{\mathbf{R}}\sin\theta\cos\phi + \hat{\boldsymbol{\theta}}\cos\theta\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{R}}\sin\theta\sin\phi + \hat{\boldsymbol{\theta}}\cos\theta\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_x = A_R\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$ $A_y = A_R\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$
Cylindrical to spherical	$R = \sqrt[+]{r^2+z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{\mathbf{R}} = \hat{\mathbf{r}}\sin\theta + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{r}}\cos\theta - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{\mathbf{r}} = \hat{\mathbf{R}}\sin\theta + \hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$

Problem 2.13 In addition to not dissipating power, a lossless line has two important features: (1) it is dispertionless (μ_p is independent of frequency) and (2) its characteristic impedance Z_0 is purely real. Sometimes, it is not possible to design a transmission line such that $R' \ll \omega L'$ and $G' \ll \omega C'$, but it is possible to choose the dimensions of the line and its material properties so as to satisfy the condition

$$R'C' = L'G' \quad \text{(distortionless line)}.$$

Such a line is called a *distortionless* line because despite the fact that it is not lossless, it does nonetheless possess the previously mentioned features of the loss line. Show that for a distortionless line,

$$\alpha = R'\sqrt{\frac{C'}{L'}} = \sqrt{R'G'}, \qquad \beta = \omega\sqrt{L'C'}, \qquad Z_0 = \sqrt{\frac{L'}{C'}}.$$

Solution: Using the distortionless condition in Eq. (2.22) gives

$$\begin{aligned}
\gamma = \alpha + j\beta &= \sqrt{(R' + j\omega L')(G' + j\omega C')} \\
&= \sqrt{L'C'}\sqrt{\left(\frac{R'}{L'} + j\omega\right)\left(\frac{G'}{C'} + j\omega\right)} \\
&= \sqrt{L'C'}\sqrt{\left(\frac{R'}{L'} + j\omega\right)\left(\frac{R'}{L'} + j\omega\right)} \\
&= \sqrt{L'C'}\left(\frac{R'}{L'} + j\omega\right) = R'\sqrt{\frac{C'}{L'}} + j\omega\sqrt{L'C'}.
\end{aligned}$$

Hence,

$$\alpha = \Re\mathfrak{e}(\gamma) = R'\sqrt{\frac{C'}{L'}}, \qquad \beta = \Im\mathfrak{m}(\gamma) = \omega\sqrt{L'C'}, \qquad u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{L'C'}}.$$

Similarly, using the distortionless condition in Eq. (2.29) gives

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} = \sqrt{\frac{L'}{C'}}\sqrt{\frac{R'/L' + j\omega}{G'/C' + j\omega}} = \sqrt{\frac{L'}{C'}}.$$

Problem 2.20 A 300-Ω lossless air transmission line is connected to a complex load composed of a resistor in series with an inductor, as shown in Fig. P2.20. At 5 MHz, determine: (a) Γ, (b) S, (c) location of voltage maximum nearest to the load, and (d) location of current maximum nearest to the load.

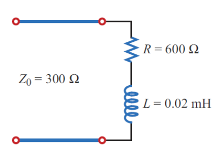


Figure P2.20: Circuit for Problem 2.20.

Solution:

(a)

$$\begin{aligned}
Z_{\text{L}} &= R + j\omega L \\
&= 600 + j2\pi \times 5 \times 10^6 \times 2 \times 10^{-5} = (600 + j628)\ \Omega.
\end{aligned}$$

$$\begin{aligned}
\Gamma &= \frac{Z_{\text{L}} - Z_0}{Z_{\text{L}} + Z_0} \\
&= \frac{600 + j628 - 300}{600 + j628 + 300} \\
&= \frac{300 + j628}{900 + j628} = 0.63e^{j29.6^\circ}.
\end{aligned}$$

(b)

$$S = \frac{1 + \lvert\Gamma\rvert}{1 - \lvert\Gamma\rvert} = \frac{1 + 0.63}{1 - 0.63} = 1.67.$$

(c)

$$\begin{aligned}
l_{\max} &= \frac{\theta_r\lambda}{4\pi} \quad \text{for } \theta_r > 0. \\
&= \left(\frac{29.6^\circ\pi}{180^\circ}\right)\frac{60}{4\pi}, \qquad \left(\lambda = \frac{3 \times 10^8}{5 \times 10^6} = 60\ \text{m}\right) \\
&= 2.46\ \text{m}
\end{aligned}$$

(d) The locations of current maxima correspond to voltage minima and vice versa. Hence, the location of current maximum nearest the load is the same as location of voltage minimum nearest the load. Thus

$$\begin{aligned}
l_{\min} &= l_{\max} + \frac{\lambda}{4}, \qquad \left(l_{\max} < \frac{\lambda}{4} = 15\ \text{m}\right) \\
&= 2.46 + 15 = 17.46\ \text{m}.
\end{aligned}$$

Problem 2.42 A generator with $\widetilde{V}_{\text{g}} = 300$ V and $Z_{\text{g}} = 50\ \Omega$ is connected to a load $Z_{\text{L}} = 75\ \Omega$ through a 50-Ω lossless line of length $l = 0.15\lambda$.

(a) Compute Z_{in}, the input impedance of the line at the generator end.

(b) Compute $\widetilde{I}_{\text{i}}$ and $\widetilde{V}_{\text{i}}$.

(c) Compute the time-average power delivered to the line, $P_{\text{in}} = \frac{1}{2}\Re\mathfrak{e}[\widetilde{V}_{\text{i}}\widetilde{I}_{\text{i}}^*]$.

(d) Compute $\widetilde{V}_{\text{L}}$, $\widetilde{I}_{\text{L}}$, and the time-average power delivered to the load, $P_{\text{L}} = \frac{1}{2}\Re\mathfrak{e}[\widetilde{V}_{\text{L}}\widetilde{I}_{\text{L}}^*]$. How does P_{in} compare to P_{L}? Explain.

(e) Compute the time-average power delivered by the generator, P_{g}, and the time-average power dissipated in Z_{g}. Is conservation of power satisfied?

Solution:

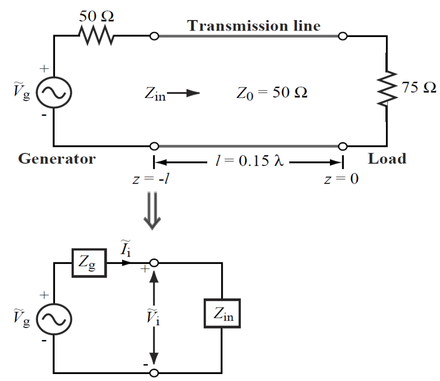


Figure P2.42: Circuit for Problem 2.42.

(a)

$$\beta l = \frac{2\pi}{\lambda} \times 0.15\lambda = 54^\circ,$$

$$Z_{\text{in}} = Z_0\left[\frac{Z_{\text{L}} + jZ_0\tan\beta l}{Z_0 + jZ_{\text{L}}\tan\beta l}\right] = 50\left[\frac{75 + j50\tan 54^\circ}{50 + j75\tan 54^\circ}\right] = (41.25 - j16.35)\ \Omega.$$

(b)

$$\begin{aligned}
\widetilde{I}_{\text{i}} &= \frac{\widetilde{V}_{\text{g}}}{Z_{\text{g}} + Z_{\text{in}}} = \frac{300}{50 + (41.25 - j16.35)} = 3.24\,e^{j10.16^\circ} \quad \text{(A)}, \\
\widetilde{V}_{\text{i}} &= \widetilde{I}_{\text{i}}Z_{\text{in}} = 3.24\,e^{j10.16^\circ}(41.25 - j16.35) = 143.6\,e^{-j11.46^\circ} \quad \text{(V)}.
\end{aligned}$$

(c)

$$\begin{aligned}
P_{\text{in}} &= \frac{1}{2}\Re\mathfrak{e}[\widetilde{V}_{\text{i}}\widetilde{I}_{\text{i}}^*] = \frac{1}{2}\Re\mathfrak{e}[143.6\,e^{-j11.46^\circ} \times 3.24\,e^{-j10.16^\circ}] \\
&= \frac{143.6 \times 3.24}{2}\cos(21.62^\circ) = 216 \quad \text{(W)}.
\end{aligned}$$

(d)

$$\begin{aligned}
\Gamma &= \frac{Z_{\text{L}} - Z_0}{Z_{\text{L}} + Z_0} = \frac{75 - 50}{75 + 50} = 0.2, \\
V_0^+ &= \widetilde{V}_{\text{i}}\left(\frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}}\right) = \frac{143.6\,e^{-j11.46^\circ}}{e^{j54^\circ} + 0.2\,e^{-j54^\circ}} = 150e^{-j54^\circ} \quad \text{(V)}, \\
\widetilde{V}_{\text{L}} &= V_0^+(1 + \Gamma) = 150e^{-j54^\circ}(1 + 0.2) = 180e^{-j54^\circ} \quad \text{(V)}, \\
\widetilde{I}_{\text{L}} &= \frac{V_0^+}{Z_0}(1 - \Gamma) = \frac{150e^{-j54^\circ}}{50}(1 - 0.2) = 2.4\,e^{-j54^\circ} \quad \text{(A)}, \\
P_{\text{L}} &= \frac{1}{2}\Re\mathfrak{e}[\widetilde{V}_{\text{L}}\widetilde{I}_{\text{L}}^*] = \frac{1}{2}\Re\mathfrak{e}[180e^{-j54^\circ} \times 2.4\,e^{j54^\circ}] = 216 \quad \text{(W)}.
\end{aligned}$$

$P_{\text{L}} = P_{\text{in}}$, which is as expected because the line is lossless; power input to the line ends up in the load.

(e)

Power delivered by generator:

$$P_{\text{g}} = \frac{1}{2}\Re\mathfrak{e}[\widetilde{V}_{\text{g}}\widetilde{I}_{\text{i}}] = \frac{1}{2}\Re\mathfrak{e}[300 \times 3.24\,e^{j10.16^\circ}] = 486\cos(10.16^\circ) = 478.4 \quad \text{(W)}.$$

Power dissipated in Z_{g}:

$$P_{Z_{\text{g}}} = \frac{1}{2}\Re\mathfrak{e}[\widetilde{I}_{\text{i}}\widetilde{V}_{Z_{\text{g}}}] = \frac{1}{2}\Re\mathfrak{e}[\widetilde{I}_{\text{i}}\widetilde{I}_{\text{i}}^*Z_{\text{g}}] = \frac{1}{2}\lvert\widetilde{I}_{\text{i}}\rvert^2 Z_{\text{g}} = \frac{1}{2}(3.24)^2 \times 50 = 262.4 \quad \text{(W)}.$$

Note 1: $P_{\text{g}} = P_{Z_{\text{g}}} + P_{\text{in}} = 478.4$ W.

Problem 3.48 A vector field $\mathbf{D} = \hat{\mathbf{r}}r^3$ exists in the region between two concentric cylindrical surfaces defined by $r = 1$ and $r = 2$, with both cylinders extending between $z = 0$ and $z = 5$. Verify the divergence theorem by evaluating:

(a) $\oint_S \mathbf{D}\cdot d\mathbf{s}$,

(b) $\int_{\mathcal{V}} \nabla\cdot\mathbf{D}\,d\mathcal{V}$.

Solution:

(a)

$$\begin{aligned}
\iint \mathbf{D}\cdot d\mathbf{s} &= F_{\text{inner}} + F_{\text{outer}} + F_{\text{bottom}} + F_{\text{top}}, \\
F_{\text{inner}} &= \int_{\phi=0}^{2\pi}\int_{z=0}^{5}\left((\hat{\mathbf{r}}r^3)\cdot(-\hat{\mathbf{r}}r\,dz\,d\phi)\right)\Big|_{r=1} \\
&= \int_{\phi=0}^{2\pi}\int_{z=0}^{5}\left(-r^4\,dz\,d\phi\right)\Big|_{r=1} = -10\pi, \\
F_{\text{outer}} &= \int_{\phi=0}^{2\pi}\int_{z=0}^{5}\left((\hat{\mathbf{r}}r^3)\cdot(\hat{\mathbf{r}}r\,dz\,d\phi)\right)\Big|_{r=2} \\
&= \int_{\phi=0}^{2\pi}\int_{z=0}^{5}\left(r^4\,dz\,d\phi\right)\Big|_{r=2} = 160\pi, \\
F_{\text{bottom}} &= \int_{r=1}^{2}\int_{\phi=0}^{2\pi}\left((\hat{\mathbf{r}}r^3)\cdot(-\hat{\mathbf{z}}r\,d\phi\,dr)\right)\Big|_{z=0} = 0, \\
F_{\text{top}} &= \int_{r=1}^{2}\int_{\phi=0}^{2\pi}\left((\hat{\mathbf{r}}r^3)\cdot(\hat{\mathbf{z}}r\,d\phi\,dr)\right)\Big|_{z=5} = 0.
\end{aligned}$$

Therefore, $\iint \mathbf{D}\cdot d\mathbf{s} = 150\pi$.

(b) From the back cover, $\nabla\cdot\mathbf{D} = (1/r)(\partial/\partial r)(rr^3) = 4r^2$. Therefore,

$$\iiint \nabla\cdot\mathbf{D}\,d\mathcal{V} = \int_{z=0}^{5}\int_{\phi=0}^{2\pi}\int_{r=1}^{2} 4r^2 r\,dr\,d\phi\,dz = \left(\left(\left(r^4\right)\Big|_{r=1}^{2}\right)\Big|_{\phi=0}^{2\pi}\right)\Big|_{z=0}^{5} = 150\pi.$$

Problem 3.50 For the vector field $\mathbf{E} = \hat{\mathbf{x}}xy - \hat{\mathbf{y}}(x^2 + 2y^2)$, calculate

(a) $\oint_C \mathbf{E}\cdot d\mathbf{l}$ around the triangular contour shown in Fig. P3.50(a), and

(b) $\int_S (\nabla\times\mathbf{E})\cdot d\mathbf{s}$ over the area of the triangle.

Solution: In addition to the independent condition that $z = 0$, the three lines of the triangle are represented by the equations $y = 0$, $x = 1$, and $y = x$, respectively.

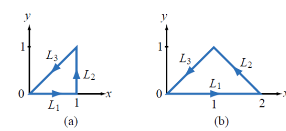


Figure P3.50: Contours for (a) Problem 3.50 and (b) Problem 3.51.

(a)

$$\begin{aligned}
\oint \mathbf{E}\cdot d\mathbf{l} &= L_1 + L_2 + L_3, \\
L_1 &= \int\left(\hat{\mathbf{x}}xy - \hat{\mathbf{y}}(x^2 + 2y^2)\right)\cdot\left(\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz\right) \\
&= \int_{x=0}^{1}(xy)\big|_{y=0,z=0}\,dx - \int_{y=0}^{0}(x^2 + 2y^2)\big|_{z=0}\,dy + \int_{z=0}^{0}(0)\big|_{y=0}\,dz = 0, \\
L_2 &= \int\left(\hat{\mathbf{x}}xy - \hat{\mathbf{y}}(x^2 + 2y^2)\right)\cdot\left(\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz\right) \\
&= \int_{x=1}^{1}(xy)\big|_{z=0}\,dx - \int_{y=0}^{1}(x^2 + 2y^2)\big|_{x=1,z=0}\,dy + \int_{z=0}^{0}(0)\big|_{x=1}\,dz \\
&= 0 - \left(y + \frac{2y^3}{3}\right)\Big|_{y=0}^{1} + 0 = \frac{-5}{3}, \\
L_3 &= \int\left(\hat{\mathbf{x}}xy - \hat{\mathbf{y}}(x^2 + 2y^2)\right)\cdot\left(\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz\right) \\
&= \int_{x=1}^{0}(xy)\big|_{y=x,\ z=0}\,dx - \int_{y=1}^{0}(x^2 + 2y^2)\big|_{x=y,\ z=0}\,dy + \int_{z=0}^{0}(0)\big|_{y=x}\,dz \\
&= \left(\frac{x^3}{3}\right)\Big|_{x=1}^{0} - \left(y^3\right)\Big|_{y=1}^{0} + 0 = \frac{2}{3}.
\end{aligned}$$

Therefore,

$$\oint \mathbf{E}\cdot d\mathbf{l} = 0 - \frac{5}{3} + \frac{2}{3} = -1.$$

(b) From Eq. (3.105), $\nabla\times\mathbf{E} = -\hat{\mathbf{z}}3x$, so that

$$\begin{aligned}
\iint \nabla\times\mathbf{E}\cdot d\mathbf{s} &= \int_{x=0}^{1}\int_{y=0}^{x}\left((-\hat{\mathbf{z}}3x)\cdot(\hat{\mathbf{z}}\,dy\,dx)\right)\big|_{z=0} \\
&= -\int_{x=0}^{1}\int_{y=0}^{x} 3x\,dy\,dx = -\int_{x=0}^{1} 3x(x - 0)\,dx = -\left(x^3\right)\Big|_0^1 = -1.
\end{aligned}$$