EE381 HW 10

Lewis Collum

Updated: April 28, 2020

1

```
from numpy import pi, sin, e

f = 400E6
c = 3E8
wavelength = c/f

I0 = 3
r = 60
l = 0.02*wavelength

eta0 = 120*pi
theta = pi/2
k = 2*pi/wavelength
E = 1j*I0*l*k*eta0/4/pi * (e**(-1j*k*r)/r) * sin(theta)
print(f'\[|E| = {abs(E):.2f}\]')

H = E/eta0
print(f'\[|H| = {abs(H)*10000:.0f}\cdot 10^{{-4}}\]')

RRad = 80*pi**2*(l/wavelength)**2
print(f'\[R_{{rad}} = {RRad:.2f}\Omega\]')

PRad = 1/2 * I0**2 * RRad
print(f'\[P_{{rad}} = {PRad:.2f}W\]')
```

$$|E| = 0.19$$

$$|H| = 5 \cdot 10^{-4}$$

$$R_{rad} = 0.32\Omega$$

$$P_{rad} = 1.42W$$

2

```
from numpy import pi, sin, e, sqrt

PRad = 4
I0 = sqrt(PRad/36.6)
print(f'\[I_0 = {I0:.3f}A\]')
```

$$I_0 = 0.331A$$

3

(a)

$$\vec{H} = \frac{\cos 2\theta}{r\eta} e^{-j\beta r} \hat{\phi}$$

(b)

$$P_{rad} = R^2 \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} S(R,\theta,\phi) \sin\theta \, d\theta \, d\phi$$

$$= R^2 \int_{\theta=0}^{\pi} S(R,\theta) \sin\theta \, 2\pi \, d\theta$$

$$= \frac{2\pi R^2}{2\eta_0} \int_{\theta=0}^{\pi} \left| \frac{\cos 2\theta}{R} e^{-j\beta r} \hat{\theta} \right|^2 \sin\theta \, d\theta$$

$$= \frac{\pi R^2}{\eta_0} \int_{\theta=0}^{\pi} \left(\frac{\cos 2\theta}{R} \right)^2 \sin\theta \, d\theta$$

$$= \frac{1}{120} \int_0^{\pi} \cos^2 2\theta \sin\theta \, d\theta$$

$$P_{rad} = \frac{7}{900} = 7.78 \text{ mW}$$

(c)

$$P_{span} = \frac{\pi R^2}{\eta_0} \int_{\theta=60^\circ}^{120^\circ} \left(\frac{\cos 2\theta}{R} \right)^2 \sin\theta \, d\theta$$

$$= \frac{1}{120} \int_{60^\circ}^{120^\circ} \cos^2 2\theta \sin\theta \, d\theta$$

$$P_{span} = \frac{43}{7200} = 5.97 \text{ mW}$$

$$\frac{P_{span}}{P_{rad}} = \frac{5.97}{7.78} = 0.767$$

4

(a)

```
from numpy import pi, sin, e, sqrt, deg2rad, log10

B = deg2rad(4)
D = 4*pi / B**2
G = 10*log10(D)
print(f'Gain = {round(G, 2)}db')
```

```
Gain = 34.11db
```

(b)

```
from numpy import pi, sin, e, sqrt, deg2rad, log10, rad2deg

Bold = deg2rad(4)
Dold = 4*pi / Bold**2
Dnew = Dold * (1+0.5)
Bnew = sqrt(4*pi/Dnew)
Gnew = 10*log10(Dnew)
print(f'\[G = {round(Gnew, 2)}db\]')
print(f'\[\\beta = {rad2deg(Bnew):.2f}^{{\circ}}\]')
```

$$G = 35.87db$$

$$\beta = 3.27^\circ$$

(c)

```
from numpy import pi, sin, e, sqrt, deg2rad, log10, rad2deg

B0 = deg2rad(4)
D0 = 4*pi / B0**2
f0 = 15E9
c = 3E8
wavelength0 = c/f0
Ap = D0 * wavelength0**2 / 4/pi

f1 = 30E9
wavelength1 = c/f1
D1 = 4*pi*Ap/wavelength1**2
G1 = 10*log10(D1)
B1 = sqrt(4*pi/D1)

print(f'\[G = {round(G1, 2)}db\]')
print(f'\[\\beta = {rad2deg(B1):.2f}^{{\circ}}\]')
```

$$G = 40.13db$$

$$\beta = 2.00^\circ$$

5

(a)

$$F_A(\theta) = \left| \sum_{i=0}^{1} a_i e^{j\psi_i'} e^{jikd\cos\theta} \right|^2$$

$$= \left| 1 + e^{jikd\cos\theta} \right|^2$$

$$= \left| 1 + e^{j(2\pi/\lambda)(\lambda)\cos\theta} \right|^2$$

$$= \left| 1 + e^{j2\pi\cos\theta} \right|^2$$

$$= 4\cos^2\left(\frac{2\pi\cos\theta}{2}\right)$$

$$= 4\cos^2(\pi\cos\theta)$$

(b)

$$F_A(\theta) = 0 = 4\cos^2(\pi\cos\theta)$$

$$\pi n - \frac{\pi}{2} = \pi\cos\theta$$

$$n - \frac{1}{2} = \cos\theta$$

$$\cos^{-1}\left(n - \frac{1}{2}\right) = \theta$$

$$\theta = \left\{2\pi n \pm \frac{\pi}{3},\ 2\pi n \pm \frac{2\pi}{3}\right\}$$

$$= \frac{\pi}{3},\ \frac{2\pi}{3},\ -\frac{\pi}{3},\ -\frac{2\pi}{3} \quad \text{FOR } n=1$$

(c)

$$\frac{dF_A}{d\theta} = -4\pi\sin(2\pi\cos(\theta))\sin(\theta)$$

$$0 = 4\pi\sin(2\pi\cos(\theta))\sin(\theta)$$

$$\theta = 0,\ \frac{\pi}{2},\ \frac{3\pi}{2},\ \pi$$

(d)