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Direct Proof & Counterexample: Rational Numbers

Notes

- A real number r is **rational** if, and only if, it can be expressed as a quotient of two integers with a nonzero denominator. A real number that is not rational is **irrational**.

$$\forall\ r \in \mathbb{R}, \exists \text{ integers } a \text{ and } b \text{ such that } r = \frac{a}{b} \text{ and } b \neq 0$$

- **Zero Product Property:** If neither of two real numbers is zero, then their product is also non zero.

- A **corollary** is a statement whose truth can be immediately deduced from a theorem that has already been proved. E.g.

  The double of a rational number is rational.

  Proof. Suppose r is any rational number.
  Then $2r = r + r$ is a sum of two rational numbers.
  So, by Theorem 4.2.2, $2r$ is rational.

  □

Test Yourself

1. the quotient of two integers.
2. real number; not the quotient of two integers.
3. it is the quotient of two integers, 0 and 1.