Symbols

c	Speed of light (3×10^8 m/s).
ω	Angular velocity (m/s).
u	Phase velocity (m/s).
γ	(Complex) propogati on constant.
β	Phase constant, also called "wavenumber" (rad/m).
α	Attenuation constant.
ϕ	Phase.
ϕ_0	Phase offset. If positive, wave leads (left-shifted). If negative, wave lags (right-shifted).
$e^{-\alpha x}$	Attenuation factor $+x$ direction (in a lossy medium).
Z_0	Characteristic impedance (Ω).
Z_{L}	Load impedance.
z_{L}	Normalized load impedance [eq (10)].
Γ	Voltage reflection coefficient. The amplitudes of the reflected and incident voltage waves at the load.
L'	The combined inductance of both conductors per unit length, in H/m.
G'	The conductance of the insulation medium between the two conductors per unit length, in S/m.
C'	The capacitance of the two conductors per unit length in, F/m.
R'	The combined resistance of both conductors per unit length, in Ω/m.
ϵ	Permittivity (dielectric insulator).
ϵ_0	Permittivity of free space (8.854×10^{-12} F/m).
ϵ_{r}	Relative permittivity.
ϵ_{eff}	Effective relative permittivity.
μ	Permeability (dielectric insulator).
μ_0	Permeability of free space ($4\pi \times 10^{-7}$ H/m).
μ_c	Permeability of conducting strip.
σ	Conductivity (dielectric insulator).
σ_c	Conductivity of conducting strip.

Equations

$$\omega = 2\pi f = \frac{2\pi}{T} \tag{1}$$

$$\beta = \frac{2\pi}{\lambda} \tag{2}$$

$$u_p = \frac{\lambda}{T} = f\lambda = \frac{\omega}{\beta} \tag{3}$$

$$\phi(x,t) = \omega t - \beta x + \phi_0 \tag{4}$$

$$y(x,t) = A\cos(\phi(x,t)) \tag{5}$$

$$e^{j\theta} = \cos(\theta) + j\sin(\theta) \qquad \text{Euler's Identity}$$

$$\frac{d\tilde{V}(z)}{dz} = -(R' + j\omega L')\tilde{I}(z)$$
$$\frac{d\tilde{I}(z)}{dz} = -(G' + j\omega C')\tilde{V}(z) \qquad \text{Telegrapher's equations}$$

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')} = \alpha + j\beta \tag{6}$$

$$\alpha = \Re(\gamma);\ \beta = \Im(\gamma) \tag{7}$$

$$\frac{d^2\tilde{I}(z)}{dz^2} - \gamma^2\tilde{I}(z) = 0$$
$$\frac{d^2\tilde{V}(z)}{dz^2} - \gamma^2\tilde{V}(z) = 0 \qquad \text{Wave equations}$$

$$Z_0 = \frac{R' + j\omega L'}{\gamma} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} = \frac{V_0^+}{I_0^+} = \frac{-V_0^-}{I_0^-} \tag{8}$$

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_{\mathrm{L}} - Z_0}{Z_{\mathrm{L}} + Z_0} = \frac{Z_{\mathrm{L}}/Z_0 - 1}{Z_{\mathrm{L}}/Z_0 + 1} = \frac{z_{\mathrm{L}} - 1}{z_{\mathrm{L}} + 1} \tag{9}$$

$$z_{\mathrm{L}} = \frac{Z_{\mathrm{L}}}{Z_0} \tag{10}$$

$$V_0^+ = |V_0^+|e^{j\phi^+};\ V_0^- = |V_0^-|e^{j\phi^-} \tag{11}$$

$$\begin{aligned} v(x,t) &= \Re(\tilde{V}(x)e^{j\omega t}) \\ &= |V_0^+|e^{-\alpha x}\cos(\omega t - \beta x + \phi^+) \\ &\quad + |V_0^-|e^{\alpha x}\cos(\omega t + \beta x + \phi^-) \end{aligned} \tag{12}$$

$$\epsilon_{\mathrm{r}} = \frac{\epsilon}{\epsilon_0} \tag{13}$$

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}} \tag{14}$$

For coaxial, two-wire, parallel-plate:

$$u_p = \frac{c}{\sqrt{\epsilon_{\mathrm{r}}}} \tag{15}$$

For microstrip:

$$u_p = \frac{c}{\sqrt{\epsilon_{\mathrm{eff}}}} \tag{16}$$

$$R' = 0\ (\because \sigma_c = \infty); \tag{17}$$

$$G' = 0\ (\because \sigma = 0); \tag{18}$$

$$C' = \frac{\sqrt{\epsilon_{\mathrm{eff}}}}{Z_0 c}; \tag{19}$$

$$L' = Z_0^2 C'; \tag{20}$$

$$\alpha = 0\ (\because R' = G' = 0); \tag{21}$$

$$\beta = \frac{\omega}{c}\sqrt{\epsilon_{\mathrm{eff}}}. \tag{22}$$

For all TEM lines:

$$L'C' = \mu\epsilon \tag{23}$$

$$\frac{G'}{C'} = \frac{\sigma}{\epsilon} \tag{24}$$

Lossless Line

$$R' \ll \omega L';\ G' \ll \omega C' \to R' = G' \approx 0 \to \tag{25}$$

$$\gamma = \alpha + j\beta = j\omega\sqrt{L'C'} \tag{26}$$

From equation (8),

$$Z_0 = \sqrt{\frac{L'}{C'}} \tag{27}$$

From equations (2)/(3) and (26)

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{L'C'}} \tag{28}$$

$$u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{L'C'}} \tag{29}$$

From equation (23), (26) and (29)

$$\beta = \omega\sqrt{\mu\epsilon} \tag{30}$$

$$u_p = \frac{1}{\sqrt{\mu\epsilon}} \tag{31}$$

Notes

Fundamental Properties of EM waves:

- A *monochromatic* (single frequency) EM wave consists of electric and magnetic fields that oscillate at the same frequency f.
- The phase velocity of an EM wave in a vacuum is the speed of light, c.
- In vacuum, the wavelength of an EM wave is related to its oscillation frequency f by $\lambda = \frac{c}{f}$.

- For passive transmission lines, α is either zero or positive. The gain region of a laser is an example of an active transmission line with a negative α.

- Microstrip line is considered a *quasi-TEM* because **E** and **F** are not everywhere perfectly orthogonal.

- From equation (12), or a similar equation, the term $...e^{-j\beta x}$ is the **incident wave** (travelling from source to load, or in the positive x direction). The term $...e^{j\beta x}$ is the **reflected wave** (travelling from load to source, or in the negative x direction).