Student: Lewis C **Date:** 05/11/20	**Instructor:** Scott Fulton **Course:** MA339 Applied Linear Algebra	**Assignment:** Section 3.1 Homework

1. Compute the determinant using a cofactor expansion across the first row. Also compute the determinant by a cofactor expansion down the second column.

$$\begin{vmatrix} 2 & 0 & 3 \\ 2 & 3 & 3 \\ 0 & 5 & -1 \end{vmatrix}$$

Compute the determinant using a cofactor expansion across the first row. Select the correct choice below and fill in the answer box to complete your choice.

(Simplify your answer.)

- **A.** Using this expansion, the determinant is $-(0)(-2)+(3)(-2)-(5)(0)=$ ______.
- **B.** Using this expansion, the determinant is $(0)(-2)-(3)(-2)+(5)(0)=$ ______.
- **C.** Using this expansion, the determinant is $-(2)(-18)+(0)(-2)-(3)(10)=$ ______.
- **D.** Using this expansion, the determinant is $(2)(-18)-(0)(-2)+(3)(10)=$ -6.

Compute the determinant using a cofactor expansion down the second column. Select the correct choice below and fill in the answer box to complete your choice.

(Simplify your answer.)

- **A.** Using this expansion, the determinant is $(2)(-18)-(0)(-2)+(3)(10)=$ ______.
- **B.** Using this expansion, the determinant is $-(2)(-18)+(0)(-2)-(3)(10)=$ ______.
- **C.** Using this expansion, the determinant is $(0)(-2)-(3)(-2)+(5)(0)=$ ______.
- **D.** Using this expansion, the determinant is $-(0)(-2)+(3)(-2)-(5)(0)=$ -6.

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2. Compute the determinant using a cofactor expansion across the first row. Also compute the determinant by a cofactor expansion down the second column.

$$\begin{vmatrix} 8 & -8 & 9 \\ 9 & 1 & 8 \\ 1 & 9 & -1 \end{vmatrix}$$

Write the expression for the determinant using a cofactor expansion across the first row. Choose the correct answer below.

- **A.** Using this expansion, the determinant is $(8)(71) - (-8)(89) + (9)(82)$.
- **B.** Using this expansion, the determinant is $(8)(71) + (-8)(89) + (9)(82)$.
- **C.** Using this expansion, the determinant is $(8)(-73) + (-8)(-17) + (9)(80)$.
- **D.** Using this expansion, the determinant is $(8)(-73) - (-8)(-17) + (9)(80)$.

Write the expression for the determinant using a cofactor expansion down the second column. Choose the correct answer below.

- **A.** Using this expansion, the determinant is $(-8)(-1) + (1)(1) + (9)(145)$.
- **B.** Using this expansion, the determinant is $(-8)(-17) + (1)(-17) + (9)(-17)$.
- **C.** Using this expansion, the determinant is $-(-8)(-1) + (1)(1) - (9)(145)$.
- **D.** Using this expansion, the determinant is $-(-8)(-17) + (1)(-17) - (9)(-17)$.

The determinant is 0 .
(Simplify your answer.)

3. Compute the determinant using a cofactor expansion down the first column.

$$A = \begin{Vmatrix} 7 & -5 & 2 \\ 1 & 1 & 3 \\ 0 & 4 & -2 \end{Vmatrix}$$

Determine the value of the first term in the cofactor expansion. Substitute the value for a_{11} and complete the matrix for C_{11} below.

$$a_{11}C_{11} = (\ 7\ )\det\begin{bmatrix} 1 & 3 \\ 4 & -2 \end{bmatrix}$$

Determine the value of the second term in the cofactor expansion. Substitute the value for a_{21} and complete the matrix for C_{21} below.

$$a_{21}C_{21} = -(\ 1\ )\det\begin{bmatrix} -5 & 2 \\ 4 & -2 \end{bmatrix}$$

Determine the value of the third term in the cofactor expansion. Substitute the value for a_{31} and complete the matrix for C_{31} below.

$$a_{31}C_{31} = (\ 0\ )\det\begin{bmatrix} -5 & 2 \\ 1 & 3 \end{bmatrix}$$

Complete the cofactor expansion to compute the determinant.

$\det A = -100$

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4. Compute the determinant by cofactor expansion. At each step, choose a row or column that involves the least amount of computation.

$$\begin{vmatrix} 5 & 0 & 0 & 5 \\ 3 & 7 & 3 & -2 \\ 3 & 0 & 0 & 0 \\ 7 & 3 & 1 & 9 \end{vmatrix}$$

$$\begin{vmatrix} 5 & 0 & 0 & 5 \\ 3 & 7 & 3 & -2 \\ 3 & 0 & 0 & 0 \\ 7 & 3 & 1 & 9 \end{vmatrix} = -30$$ (Simplify your answer.)

5. Compute the determinant by cofactor expansion. At each step, choose a row or column that involves the least amount of computation.

$$\begin{vmatrix} 2 & 6 & -1 & 4 \\ 0 & -3 & 9 & -6 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 5 \end{vmatrix}$$

$$\begin{vmatrix} 2 & 6 & -1 & 4 \\ 0 & -3 & 9 & -6 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 5 \end{vmatrix} = -30$$ (Simplify your answer.)

6. Compute the determinant by cofactor expansion. At each step, choose a row or column that involves the least amount of computation.

$$\begin{vmatrix} 9 & 3 & 3 & 4 & 0 \\ 6 & 0 & -3 & 1 & 0 \\ 2 & -8 & 3 & 8 & 1 \\ 3 & 0 & 0 & 0 & 0 \\ 7 & 2 & 4 & 2 & 0 \end{vmatrix}$$

$$\begin{vmatrix} 9 & 3 & 3 & 4 & 0 \\ 6 & 0 & -3 & 1 & 0 \\ 2 & -8 & 3 & 8 & 1 \\ 3 & 0 & 0 & 0 & 0 \\ 7 & 2 & 4 & 2 & 0 \end{vmatrix} = 0$$ (Simplify your answer.)

7. Explore the effects of an elementary row operation on the determinant of a matrix. State the row operation and describe how it affects the determinant.

$$\left\|\begin{matrix} 5 & 4 \\ 6 & 6 \end{matrix}\right\|, \left\|\begin{matrix} 5 & 4 \\ 6+5k & 6+4k \end{matrix}\right\|$$

What is the elementary row operation?

- **A.** Replace row 2 with k times row 1 plus row 2.
- **B.** Replace row 2 with k times row 2.
- **C.** Replace row 2 with k times row 1.
- **D.** Replace row 2 with row 1 plus k times row 2.

How does the row operation affect the determinant?

- **A.** The determinant is increased by 20k.
- **B.** The determinant is increased by 40k.
- **C.** The determinant is decreased by 20k.
- **D.** The determinant does not change.

8. Explore the effect of an elementary row operation on the determinant of a matrix. State the row operation and describe how it affects the determinant.

$$\begin{bmatrix} 9 & 2 & 7 \\ 1 & 5 & 8 \\ a & b & c \end{bmatrix}, \begin{bmatrix} 9 & 2 & 7 \\ a & b & c \\ 1 & 5 & 8 \end{bmatrix}$$

What is the elementary row operation?

- **A.** Rows 1 and 3 are interchanged.
- **B.** Rows 1 and 2 are interchanged.
- **C.** Row 3 is replaced with the sum of rows 2 and 3.
- **D.** Row 2 is replaced with the sum of rows 1 and 2.
- **E.** Row 3 is replaced with the sum of rows 1 and 3.
- **F.** Row 2 is replaced with the sum of rows 2 and 3.
- **G.** Rows 2 and 3 are interchanged.

How does the row operation affect the determinant?

- **A.** It increases the determinant by 1.
- **B.** It changes the sign of the determinant.
- **C.** It multiplies the determinant by 2.
- **D.** It does not change the determinant.

9. Explore the effects of an elementary row operation on the determinant of a matrix. State the row operation and describe how it affects the determinant.

$$\begin{Vmatrix} 1 & 1 & 1 \\ -2 & 9 & -4 \\ 4 & -2 & 3 \end{Vmatrix}, \begin{Vmatrix} k & k & k \\ -2 & 9 & -4 \\ 4 & -2 & 3 \end{Vmatrix}$$

What is the elementary row operation?

- **A.** Replace row 1 with row 1 minus k.
- **B.** Replace row 1 with row 1 divided by k.
- **C.** Replace row 1 with k times row 1.
- **D.** Replace row 1 with k plus row 1.

How does the row operation affect the determinant?

- **A.** The determinant is decreased by 3k.
- **B.** The determinant is increased by 3k.
- **C.** The determinant is multiplied by k.
- **D.** The determinant does not change.

10. Let $A = \begin{bmatrix} 2 & 8 \\ 9 & 3 \end{bmatrix}$. Write 4A. Is det(4A) equal to 4det(A)?

$4A = \begin{bmatrix} 8 & 32 \\ 36 & 12 \end{bmatrix}$

(Type an integer or decimal for each matrix element.)

Select the correct choice below and fill in the answer box(es) to complete your choice.

- **A.** Yes, det(4A) is equal to 4det(A). The value of both expressions is ______.
- **B.** No, det(4A) is not equal to 4det(A). The value of det(4A) is -1056, whereas the value of 4det(A) is -264.

11. Let A be an $n \times n$ matrix. Mark each statement as true or false. Justify each answer.
a. An $n \times n$ determinant is defined by determinants of $(n-1) \times (n-1)$ submatrices.
b. The (i,j)-cofactor of a matrix A is the matrix A_{ij} obtained by deleting from A its ith row and jth column.

a. Choose the correct answer below.

- **A.** The statement is false. Although determinants of $(n-1) \times (n-1)$ submatrices can be used to find $n \times n$ determinants, they are not involved in the definition of $n \times n$ determinants.
- **B.** The statement is false. An $n \times n$ determinant is defined by determinants of $(n-1) \times (n-1)$ submatrices only when $n > 3$. Determinants of 1×1, 2×2, and 3×3 matrices are defined separately.
- **C.** The statement is true. The determinant of an $n \times n$ matrix A can be computed by a cofactor expansion across any row or down any column. Each term in any such expansion includes a cofactor that involves the determinant of a submatrix of size $(n-1) \times (n-1)$.
- **D.** The statement is true. The determinant of an $n \times n$ matrix A can be computed by a cofactor expansion along either diagonal. Each term in any such expansion includes a cofactor that involves the determinant of a submatrix of size $(n-1) \times (n-1)$.

b. Choose the correct answer below.

- **A.** The statement is false. The (i,j)-cofactor of A is the number $C_{ij} = \det(A_{ij})$, where A_{ij} is the submatrix obtained by deleting from A its ith row and jth column.
- **B.** The statement is false. The (i,j)-cofactor of a matrix A is the matrix A_{ij} obtained by deleting from A its jth row and ith column.
- **C.** The statement is true. It is the definition of the (i,j)-cofactor of a matrix A.
- **D.** The statement is false. The (i,j)-cofactor of A is the number $C_{ij} = (-1)^{i+j}\det(A_{ij})$, where A_{ij} is the submatrix obtained by deleting from A its ith row and jth column.

12. Let A be an $n \times n$ matrix. Mark each statement True or False. Justify each answer.

a. The cofactor expansion of det A down a column is the negative of the cofactor expansion along a row.
b. The determinant of a triangular matrix is the sum of the entries on the main diagonal.

a. Choose the correct answer below.

- **A.** False, because the determinant of A can be computed by cofactor expansion across any row or down any column. Since the determinant of A is well defined, both of these cofactor

- **B.** True, because the plus or minus sign of the (i,j)-cofactor depends on the position of a_{ij} in matrix A. Cofactor expansion down a column switches the order of i and j, thereby switching the sign of the cofactor expansion across a row.
- **C.** True, because cofactor expansion across a row adds each of the cofactors together. Cofactor expansion down a column subtracts each cofactor from one another. This causes the two cofactor expansions to have opposite signs.
- **D.** False, because the determinant of A can only be calculated by cofactor expansion across a row. Cofactor expansion down a column has no relation to the determinant.

b. Choose the correct answer below.

- **A.** False, because the determinant of a matrix is the arithmetic mean of the entries along the main diagonal.
- **B.** True, because the determinant of A is the following finite series.

$$\det A = \sum_{j=1}^{n} (-1)^{1+j} a_{1j} \det A_{1j}$$

  In a triangular matrix, this series simplifies to the sum of the entries along the main diagonal.
- **C.** True, because cofactor expansion along the row (or column) with the most zeros of a triangular matrix produces a determinant equal to the sum of the entries along the main diagonal.
- **D.** False, because the determinant of a triangular matrix is the product of the entries along the main diagonal.

13. Let $\mathbf{u} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$. Compute the area of the parallelogram determined by **u**, **v**, **u** + **v**, and **0**, and compute the determinant of $\begin{bmatrix} \mathbf{u} & \mathbf{v} \end{bmatrix}$. How do they compare? Replace the first entry of **v** by an arbitrary number x, and repeat the problem. Draw a picture and explain what you find.

Select the correct choice below and fill in the answer box(es) to complete your choice.
(Simplify your answer.)

A. The area of the parallelogram and the determinant of $\begin{bmatrix} \mathbf{u} & \mathbf{v} \end{bmatrix}$ both equal 10.

B. The area of the parallelogram, ______, is less than the determinant of $\begin{bmatrix} \mathbf{u} & \mathbf{v} \end{bmatrix}$, ______.

C. The area of the parallelogram, ______, is greater than the determinant of $\begin{bmatrix} \mathbf{u} & \mathbf{v} \end{bmatrix}$, ______.

Replace the first entry of **v** by an arbitrary number x to make $\mathbf{w} = \begin{bmatrix} x \\ 2 \end{bmatrix}$. Select the correct choice below and fill in the answer box(es) to complete your choice.
(Simplify your answer.)

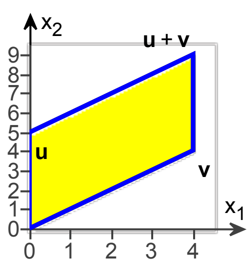
A. The area of the parallelogram, ______, is greater than the determinant of $\begin{bmatrix} \mathbf{u} & \mathbf{w} \end{bmatrix}$, ______.

B. The area of the parallelogram and the determinant of $\begin{bmatrix} \mathbf{u} & \mathbf{w} \end{bmatrix}$ both equal 10.

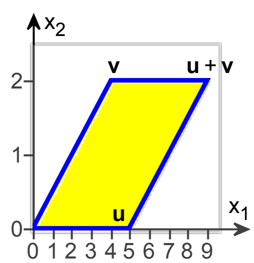
C. The area of the parallelogram, ______, is less than the determinant of $\begin{bmatrix} \mathbf{u} & \mathbf{w} \end{bmatrix}$, ______.

Which of the following shows the parallelogram determined by **u**, **v**, **u** + **v**, and **0**?

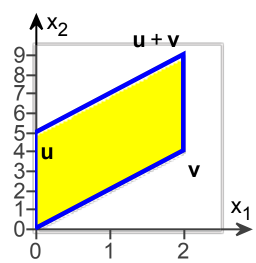
A.



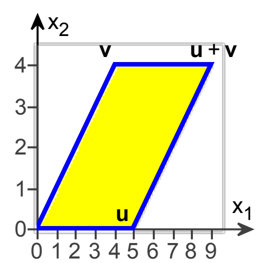
B.



C.

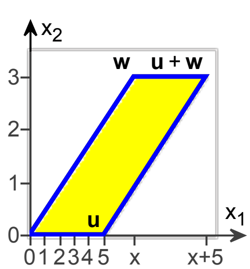


D.

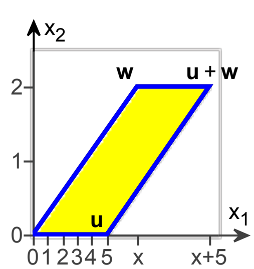


Which of the following shows the parallelogram determined by **u**, **w**, **u** + **w**, and **0**?

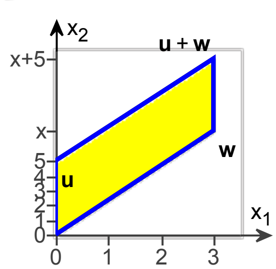
A.



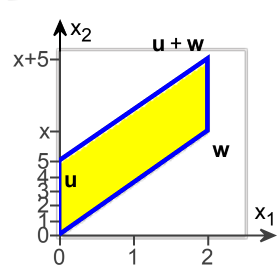
B.



C.



D.



Describe the results of the previous steps.

The absolute value of the determinant of the matrix whose columns are vectors which define the sides of a parallelogram adjacent to one another is equal to the area of the parallelogram.