Chapter 5 Synchronous Machine

Instructor: Dr. Tuyen Vu

Assistant Professor

ECE Department, Clarkson University

tvu@Clarkson.edu

Agenda

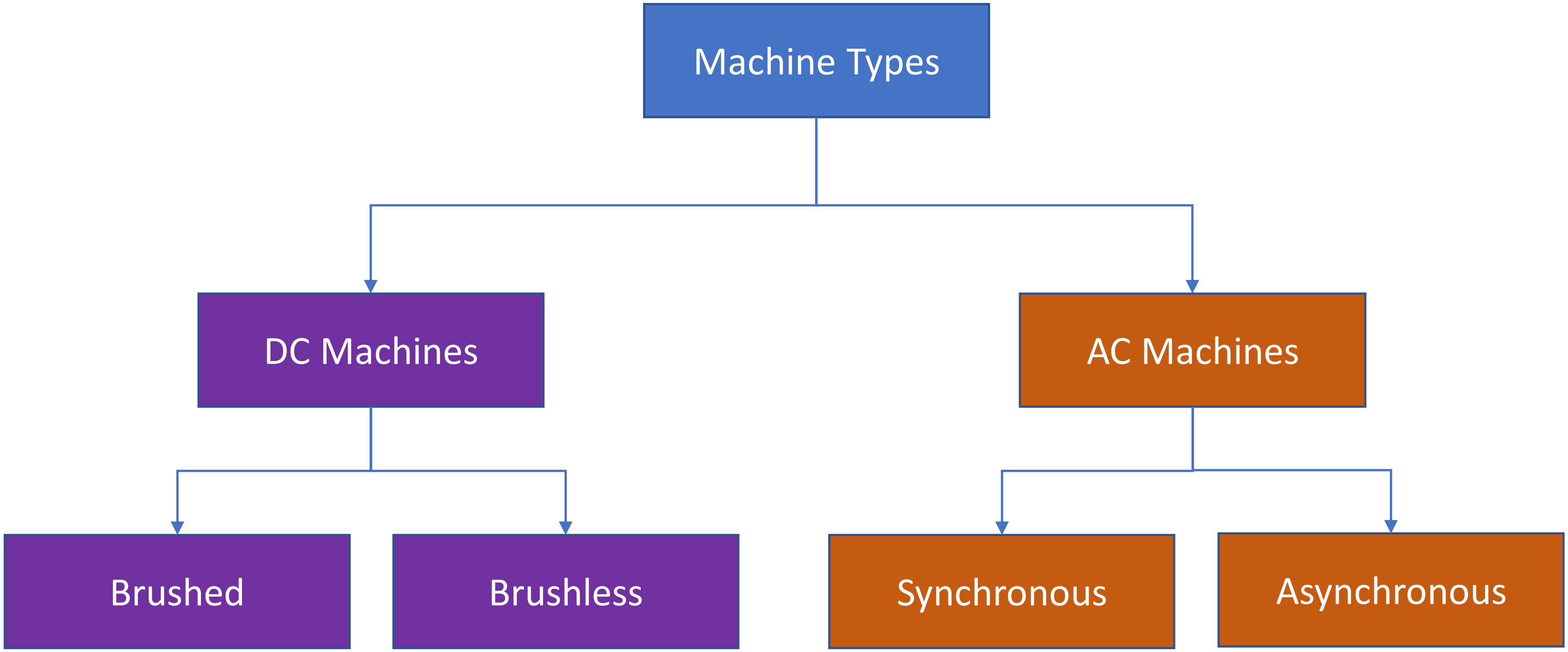
- **Overview**
- **Synchronous generator**
  - Construction
  - Working principle
    - Two-pole single-phase machine
    - Two-pole three-phase machine
    - Multi-pole three-phase machine
  - Equivalent circuit
  - Power and torque
- **Synchronous motor**
  - Equivalent circuit
  - Power and torque

Overview

- **Machine applications?**
  - Fans
  - Pumps
  - Conveyers
  - Elevators, escalators
  - Manufacturing machines
  - Vehicles

Overview

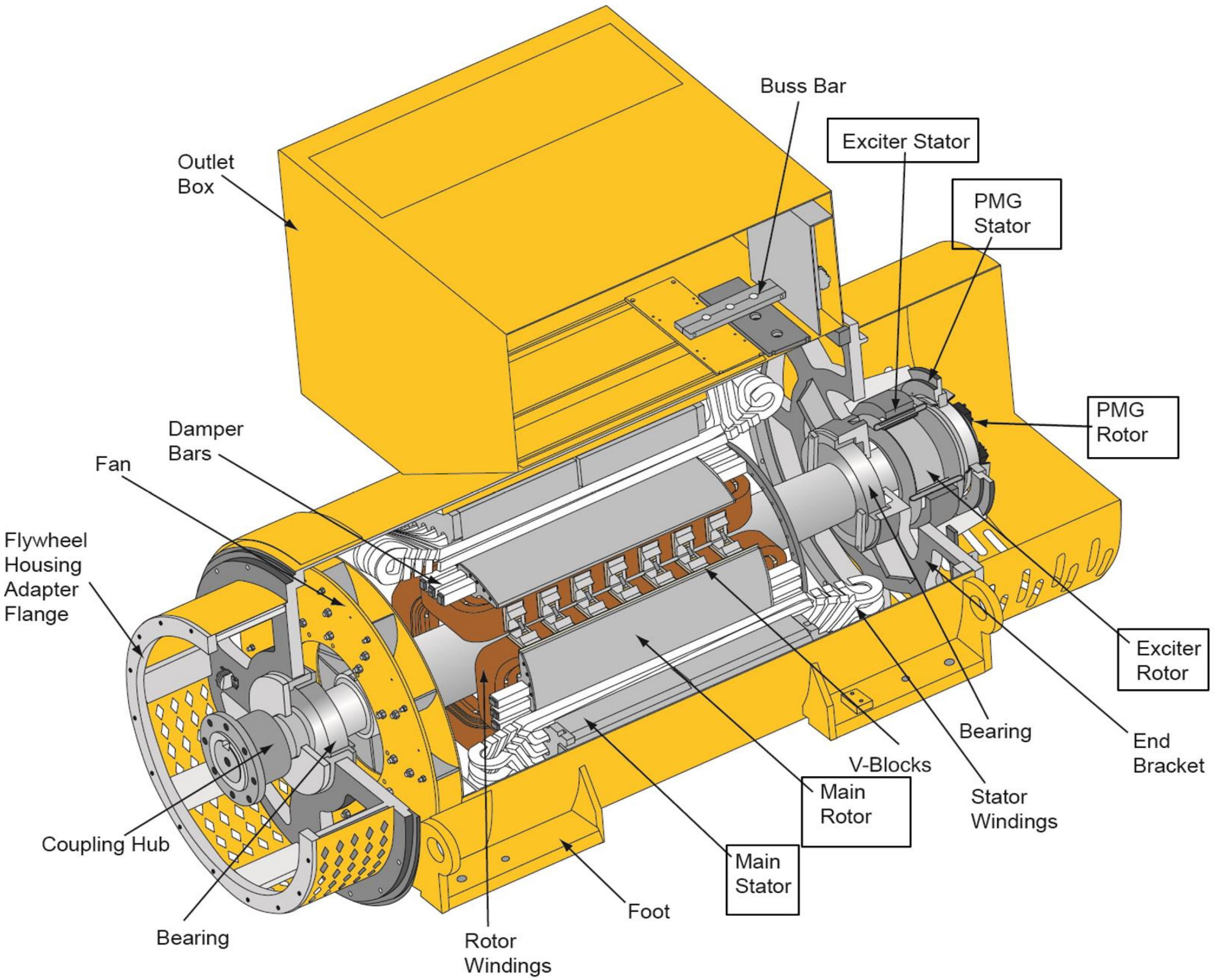
- **Motor types**



Synchronous Generators - Construction

Essential components?

- Rotor
  - Core (Laminated steel)
  - Winding: Energized by DC voltage source
  - Permanent magnet can be used instead of winding
  - May include slip rings to provide DC voltage for rotor winding
- Stator
  - Core (Laminated steel)
  - Stator winding to extract electrical energy
- Others
  - Bearing
  - Motor housing
  - Fan
  - Bus bar

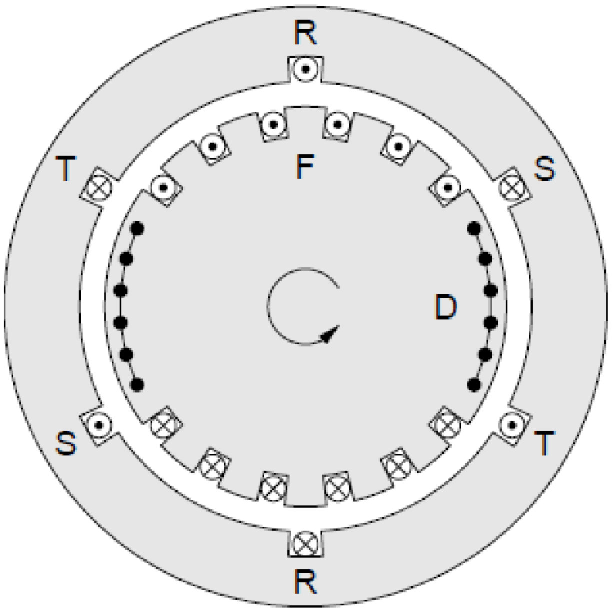


sites.ieee.org/houston/files/2016/10/2016-09-27-2-Generator-Basics-1.pdf

Construction

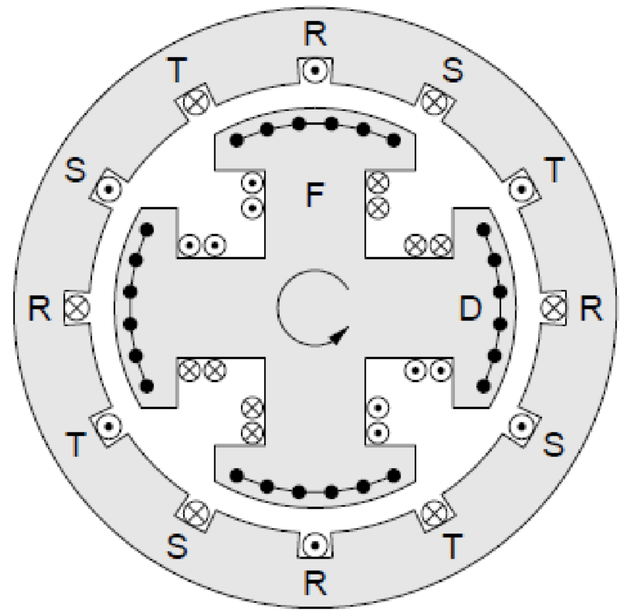
- **Rotor types**

2-4 poles for nuclear, gas, and thermal power plants??



Round rotor
$p = 2,\ n = 3000\,\mathrm{min}^{-1}$ for $f = 50\,\mathrm{Hz}$

4-60 poles for hydro power plants and wind power??



Salient pole rotor
$p = 4,\ n = 1500\,\mathrm{min}^{-1}$ for $f = 50\,\mathrm{Hz}$

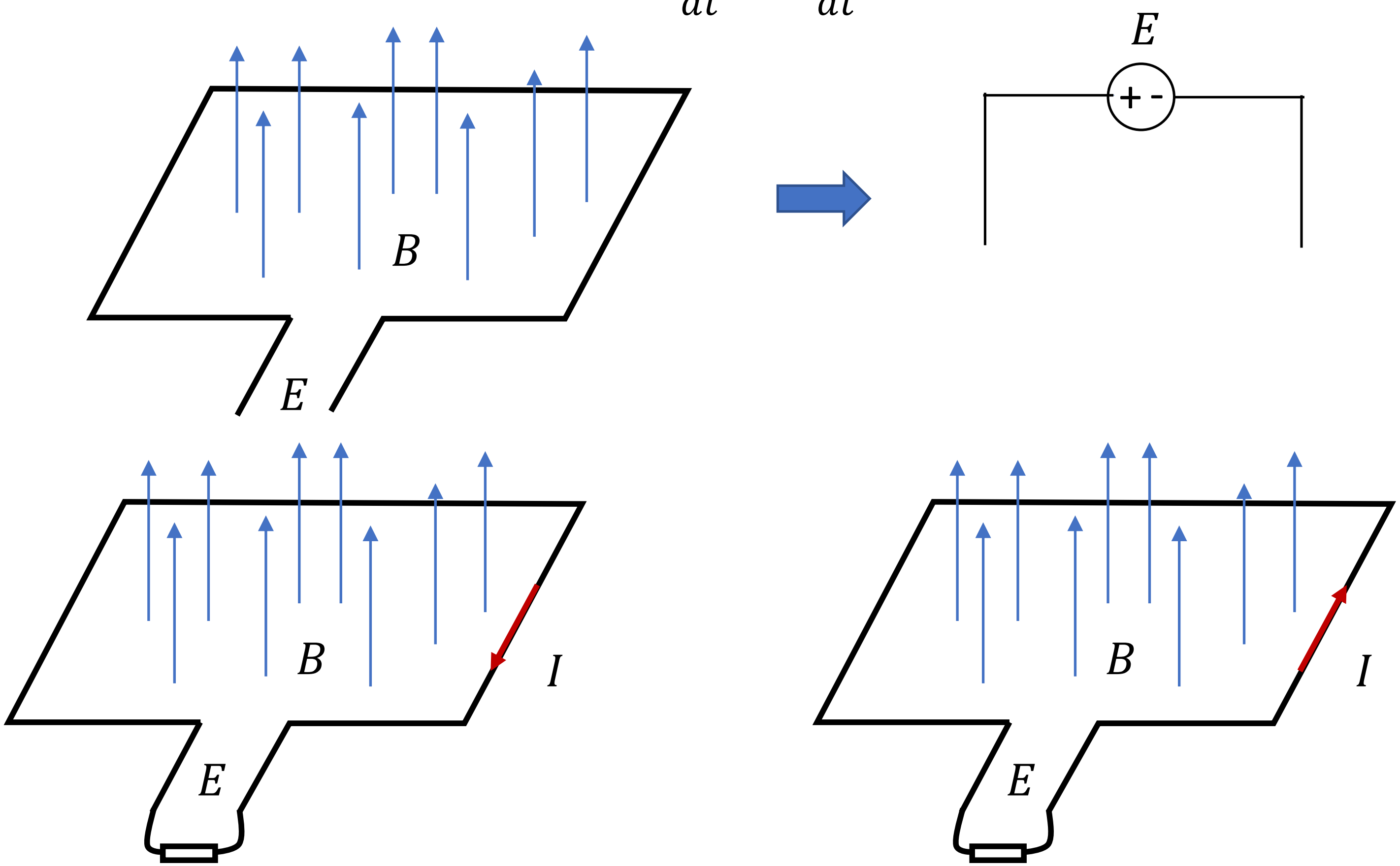
sites.ieee.org/houston/files/2016/10/2016-09-27-2-Generator-Basics-1.pdf

Working Principle

- **Faraday-Lenz's Law**

Precisely indicates the direction of EMF, which is in the opposite of the rate of change in the magnetic flux

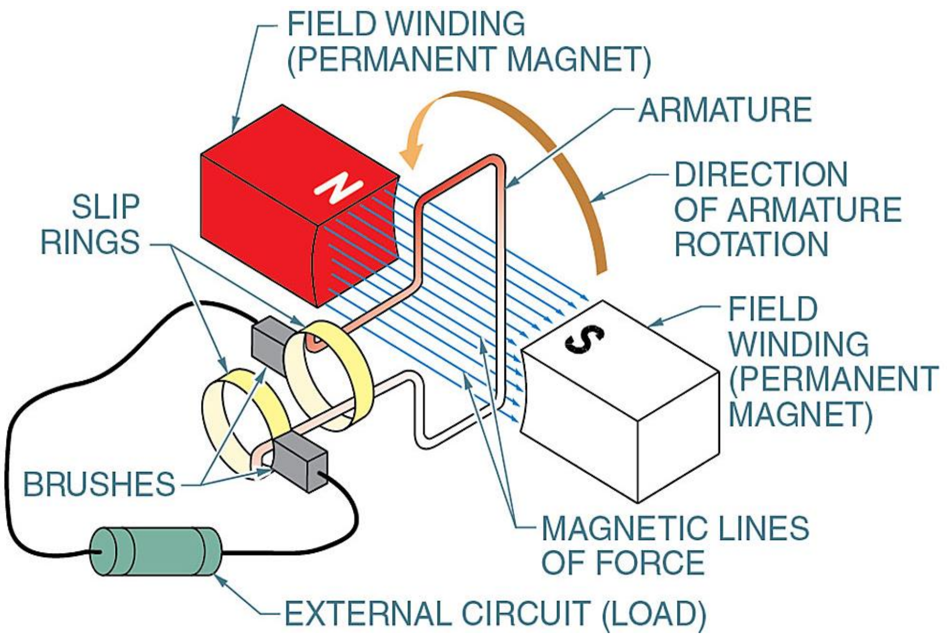
$$E = -N\frac{d\Phi}{dt} = -\frac{d\lambda}{dt}$$



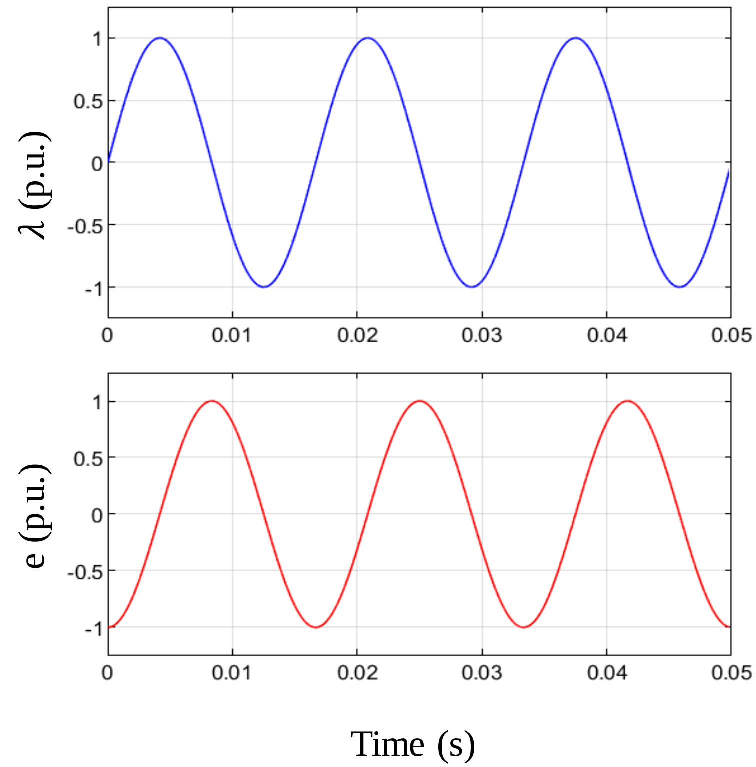
Flux increases

Flux decreases

Working Principle



Illustrative example [1]



Magnetic flux → Mechanical power → Rotate field winding → Varying magnetic flux → Electric voltage

$$\lambda \rightarrow P_m \rightarrow \omega_r \rightarrow \lambda_{\max}\cos(p_p\omega_r t) \rightarrow e = -\frac{\mathrm{d}\lambda}{dt}$$

[1] http://electricala2z.com/ac-machines/ac-generator-parts-functions/

Working Principle

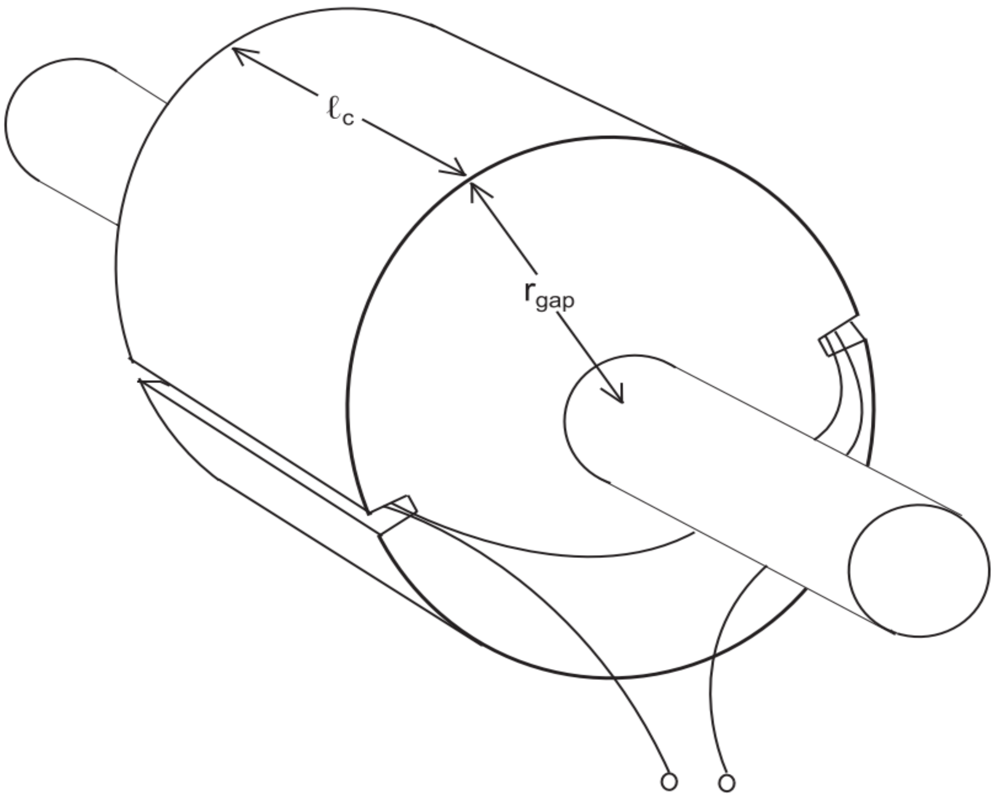
Two-pole Single-Phase Machine

Rotor parameters

- Active length l_c
- Radius r_{gap}

Rotor coil (F)

- Number of turns N_f
- Current i_f
- Current i_f $\rightarrow$ Magnetic field



Machine rotor with a single coil

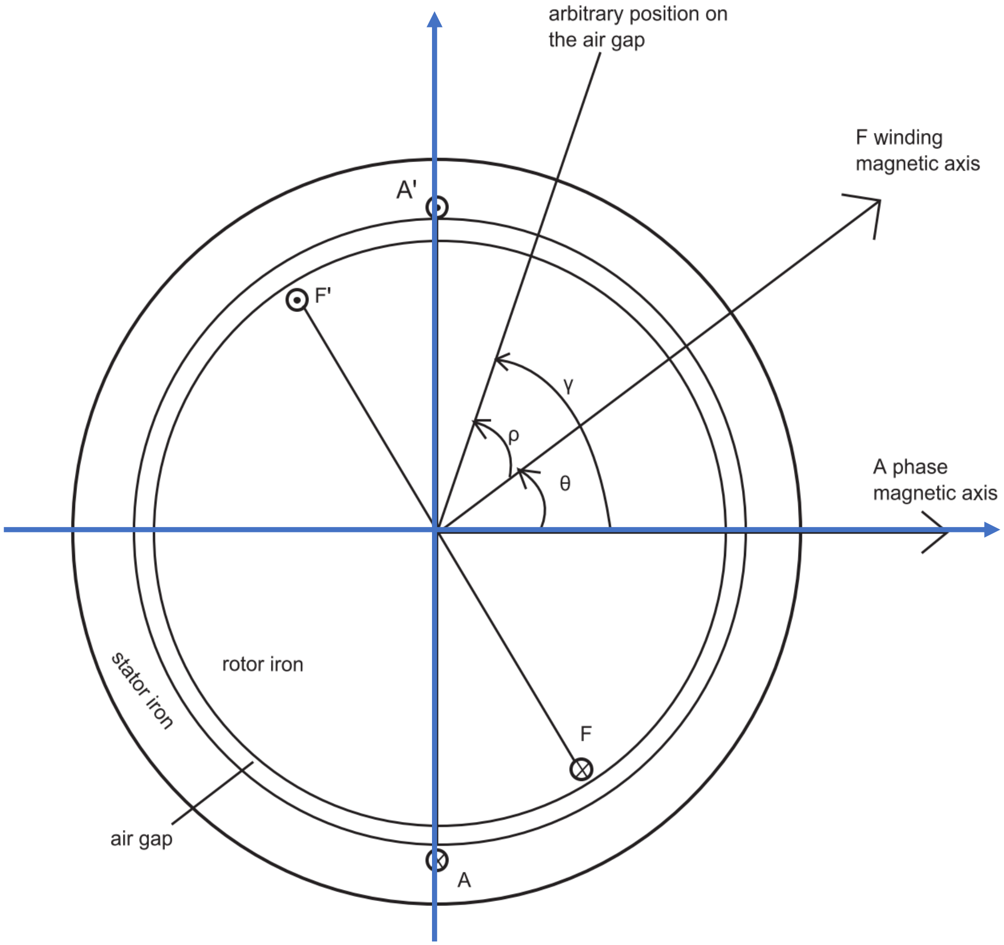
Working Principle

Two-pole Single-Phase Machine

Stator coil (A)

Other parameters

- Reference axis: A-phase stator magnetic axis
- Rotor position θ
- Absolute arbitrary position of magnetic flux γ for a random flux vector
- Relative arbitrary position $\rho = \gamma - \theta$ compared to F-axis for the random flux vector



Cross section of a motor
stator coil A-phase and rotor winding F

Working Principle

Two-pole Single-Phase Machine

Airgap flux density

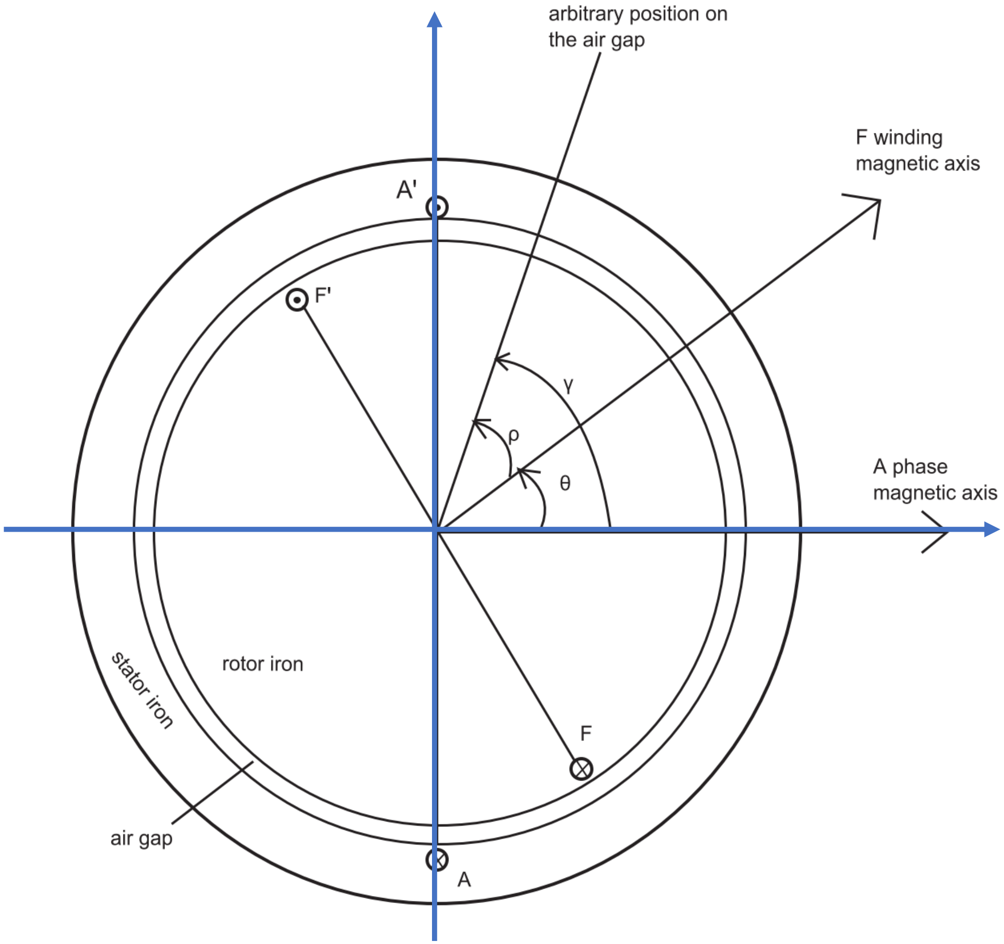
- Airgap flux density (distributed throughout the airgap)

$$B_F(\rho) = B_{Fmax}\cos(\rho)$$

$$B_F(\gamma) = B_{Fmax}\cos(\gamma - \theta\ )$$

where

- Arbitrary position of magnetic flux γ
- Rotor position $\theta = \omega t$
- Relative arbitrary position compared to F axis ρ (ρ)



Cross section of a motor
stator coil A-phase and rotor winding F

Working Principle

Two-pole Single-Phase Machine

- A current flows in the field winding so that $B_{Fmax} = 1$ T. The rotor speed is 1000 rpm, with a rotor position $\theta = 0^{\circ}$at $t = 0\ s$. What is the machine flux density created by this field current, with reference to the stator magnetic axis?

$$B_F(\gamma) = B_{Fmax} \cos(\gamma - \omega t)$$

$$B_F(\gamma) = 1 \cos(\gamma - 104.7t)\ (\mathrm{T})$$

Working Principle

Two-pole Single-Phase Machine

Flux linkage: Created by rotor winding magnetic field through coil A

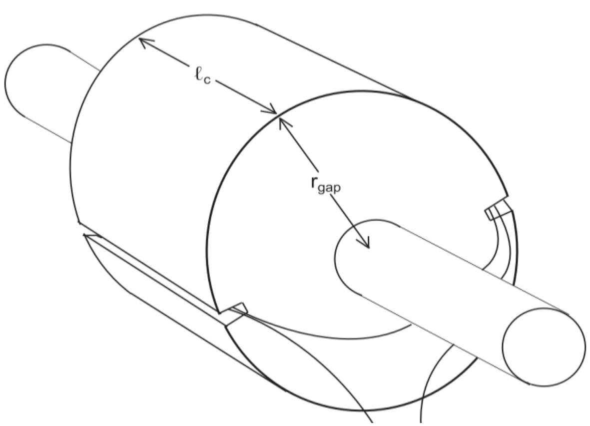
Surface area = $l_c r_{gap}$

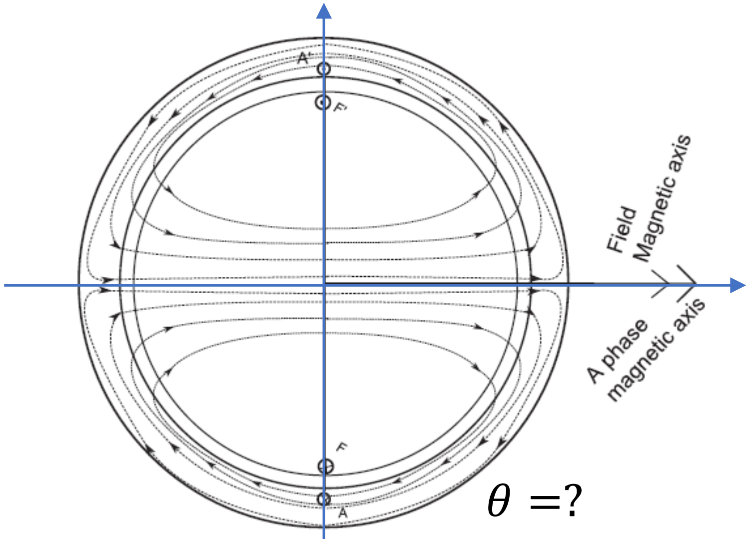
$$\phi_{Af}(\theta) = l_c r_{gap} \int_{-\pi/2}^{\pi/2} B(\gamma,\theta) d\gamma$$

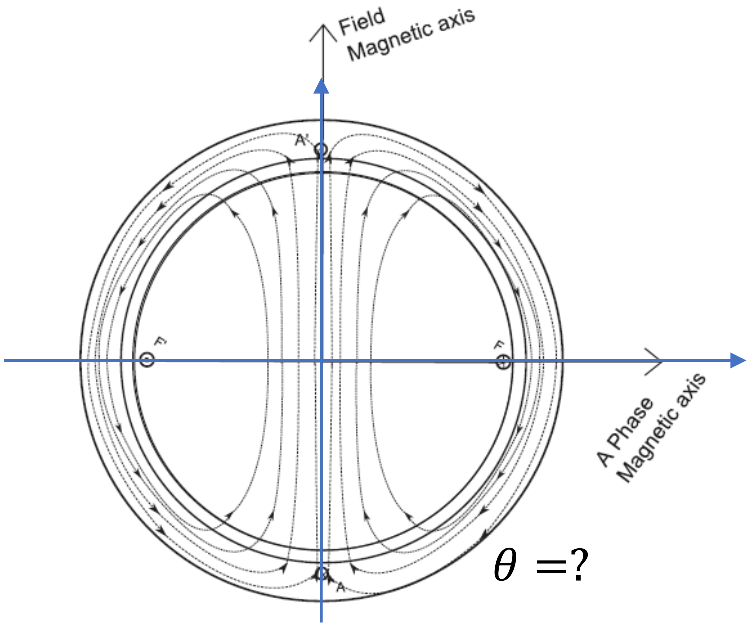
$$\phi_{Af}(\theta) = 2 l_c r_{gap} B_{\max} \cos\theta$$

Question:

$$\theta = 0^{\mathrm{o}} \rightarrow \phi_{Af} = ?, \theta = 90^{\mathrm{o}} \rightarrow \phi_{Af} = ?$$







Working Principle

Two-pole Single-Phase Machine

Induced voltage in coil A

$$e_{Af} = -\frac{d\lambda_{Af}}{dt} = -N_s \frac{d\phi_{Af}}{dt}$$

$$e_{Af} = -N_s 2 l_c r_{gap} B_{\max} \frac{d\cos\theta}{dt}$$

$$e_{Af} = 2N_s l_c r_{gap} B_{\max} \frac{d\theta}{dt} \sin\theta$$

$$e_{Af} = 2N_s l_c r_{gap} B_{\max} \omega_r \sin(\omega_r t)$$

$$\boldsymbol{e_{Af} = \sqrt{2} E_f \, sin(\omega_r t)}$$

Where $E_f = \sqrt{2} N_s l_c r_{gap} B_{\max} \omega_r$

Working Principle

Two-pole Three-phase Machine

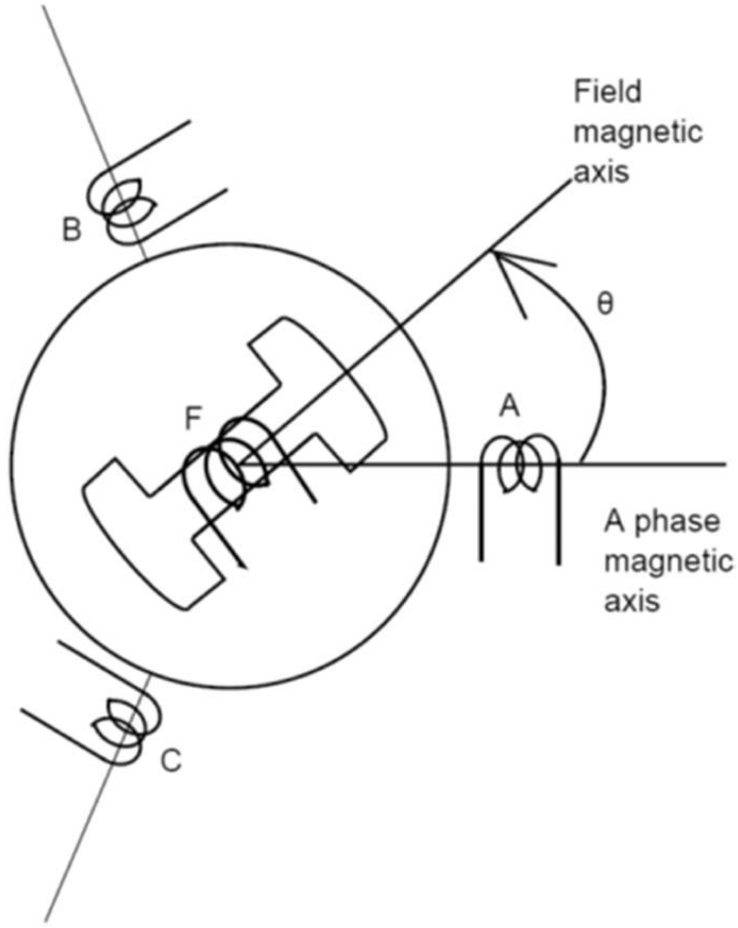
Flux linkage

$$\lambda_{Af} = N_s \phi_{Af\max} cos\theta = \lambda_{\max} cos\theta$$

$$\lambda_{Bf} = N_s \phi_{Bf\max} \cos(\theta - \frac{2\pi}{3}) = \lambda_{\max} \cos(\theta - \frac{2\pi}{3})$$

$$\lambda_{Cf} = N_s \phi_{Cf\max} \cos(\theta + \frac{2\pi}{3}) = \lambda_{\max} \cos(\theta + \frac{2\pi}{3})$$

Where $\lambda_{\max} = 2N_s l_c r_{gap} B_{\max}$



Two-pole, three-phase machine

Two-pole Three-phase Machine

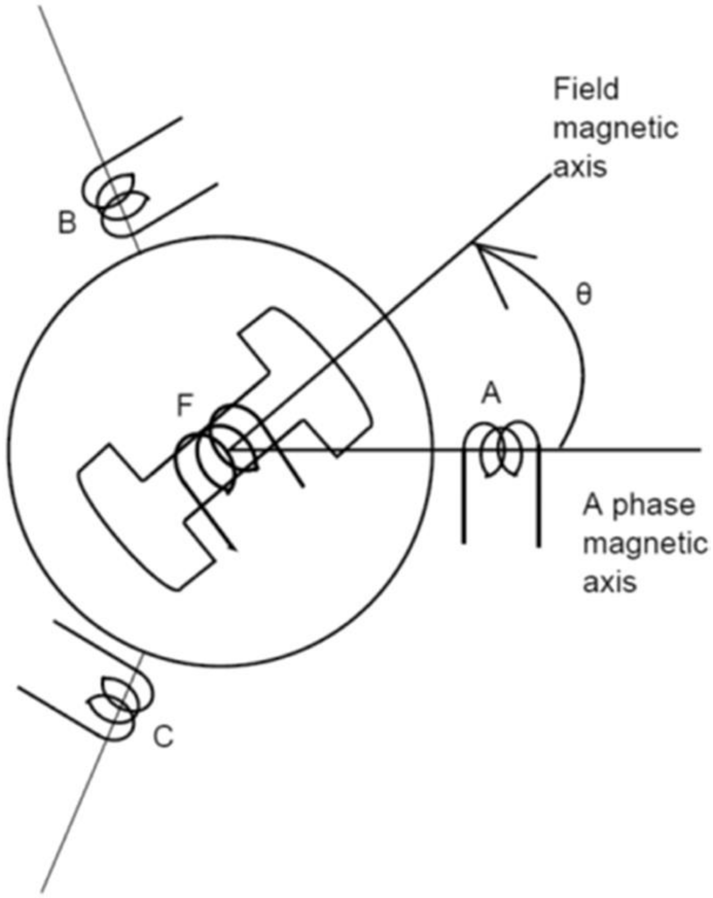
Induced voltage

$$e_{Af} = -\frac{d\lambda_{Af}}{dt} = \sqrt{2}E_f \sin(\omega_r t)$$

$$e_{Bf} = -\frac{d\lambda_{Bf}}{dt} = \sqrt{2}E_f \sin(\omega_r t - \frac{2\pi}{3})$$

$$e_{Cf} = -\frac{d\lambda_{Cf}}{dt} = \sqrt{2}E_f \sin(\omega_r t + \frac{2\pi}{3})$$

where $E_f = \sqrt{2} N_s l_c r_{gap} B_{\max} \omega_r$



Two-pole, three-phase machine

Working Principle

Multi-pole Three-phase Machine

- **Four-pole, three-phase machine**

  Flux linkage (A-phase)

  $$\lambda_{Af} = \lambda_{\max} cos\mathbf{2}\theta$$

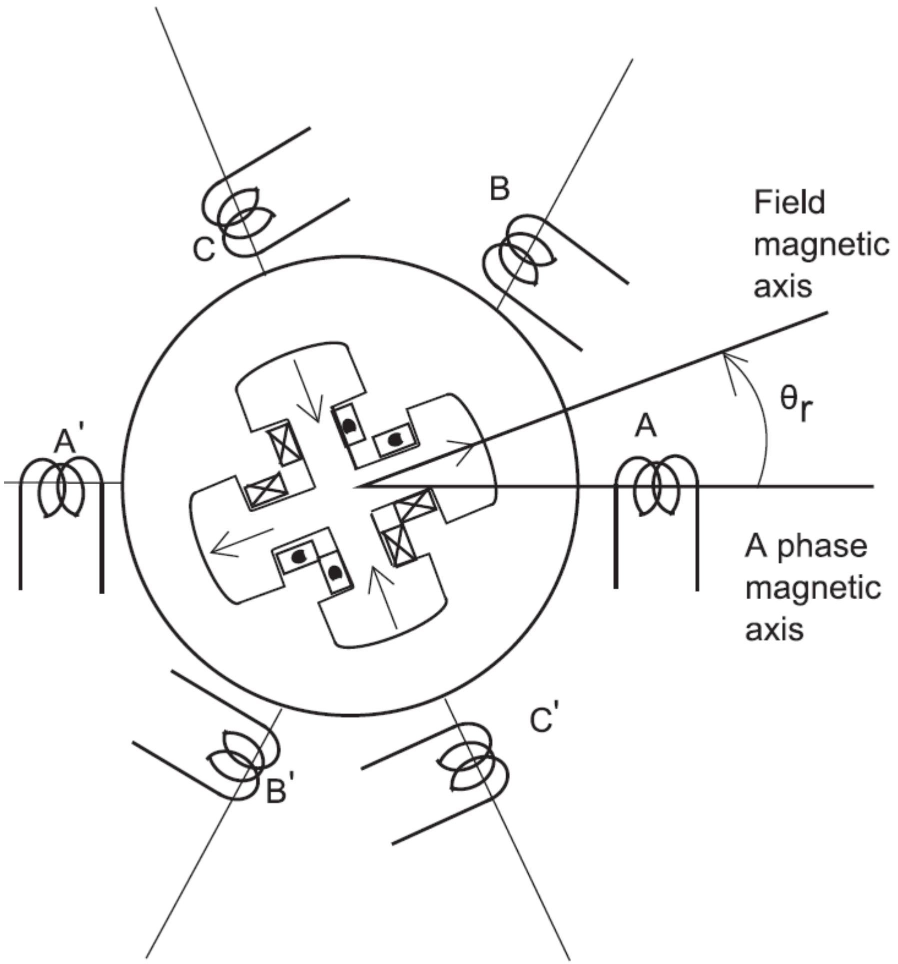
  Why 2θ?

- **Multi-pole, three-phase machine**

  Flux linkage (A-phase)

  $$\lambda_{Af} = \lambda_{\max} cos\boldsymbol{p_p}\theta$$

  p_p: number of pole pairs ($p_p = p/2$)



Four-pole, three-phase machine

Working Principle

Multi-pole Three-phase Machine

Relationship between electrical frequency and mechanical speed

$$f_e = p_p f_r = p_p \frac{N_r(rpm)}{60}$$

Or

$$\omega_e = p_p \omega_r$$

Eg. A synchronous machine operates at 900 RPM power via 60 Hz power source. How many poles does the machine rotor have?

Multi-pole Three-phase Machine

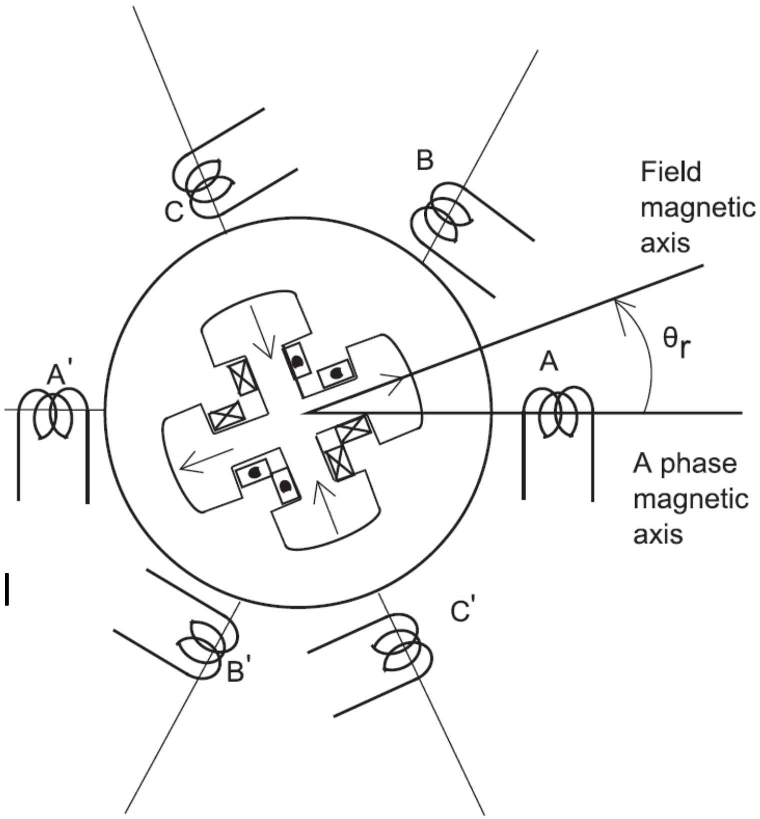
Induced voltage

$$e_{Af} = -\frac{d\lambda_{Af}}{dt} = \frac{d(\lambda_{\max}\cos(p_p\theta))}{dt} = \sqrt{2}E_f\sin(\omega_e t)$$

$$e_{Bf} = -\frac{d\lambda_{Bf}}{dt} = \sqrt{2}E_f\sin(\omega_e t - \frac{2\pi}{3})$$

$$e_{Cf} = -\frac{d\lambda_{Cf}}{dt} = \sqrt{2}E_s\sin(\omega_e t + \frac{2\pi}{3})$$

Where $E_f = \sqrt{2}N_s l_c r_{gap} B_{\max}\omega_r$ and θ_e is the electrical angle



Four-pole, three-phase machine

What is the relationship between θ_e and θ?

$$\theta_e = \omega_e t = p_p(\omega_r t) = p_p\theta$$

Working Principle

Summary

- Magnetic flux → Mechanical power → Rotate field winding → Varying magnetic flux → Electric voltage

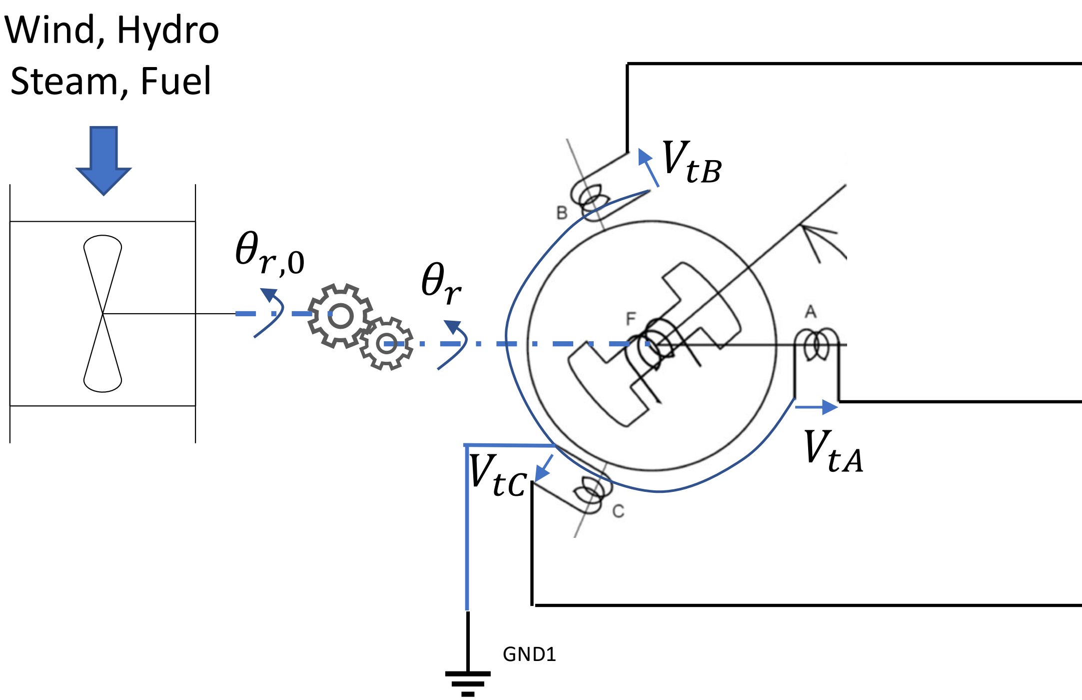
$$\lambda \rightarrow P_m \rightarrow \omega_r \rightarrow \lambda_{\max}\cos(p_p\omega_r t) \rightarrow e = -\frac{\mathrm{d}\lambda}{dt}$$

https://www.youtube.com/watch?v=OOeFhL92vC8

Conversion Stages

- Magnetic flux → Mechanical power → Rotate field winding → Varying magnetic flux → Electric voltage

$$\lambda \rightarrow P_m \rightarrow \omega_r \rightarrow \lambda_{\max} \cos(p_p \omega_r t) \rightarrow e = -\frac{\mathrm{d}\lambda}{dt}$$



Transformer
- Step-up for transmission
- Step-down for consumption

Three-phase power systems
- Supply power to power grids

Load
- Deliver power to load

Analysis

Equivalent electrical circuit

Equivalent stator circuit

- Impedance X_s caused by flux linkage and phase leakage inductance

$$X_s = \omega_e L_s$$

- Resistance R_s represents stator coil resistance and stray loss

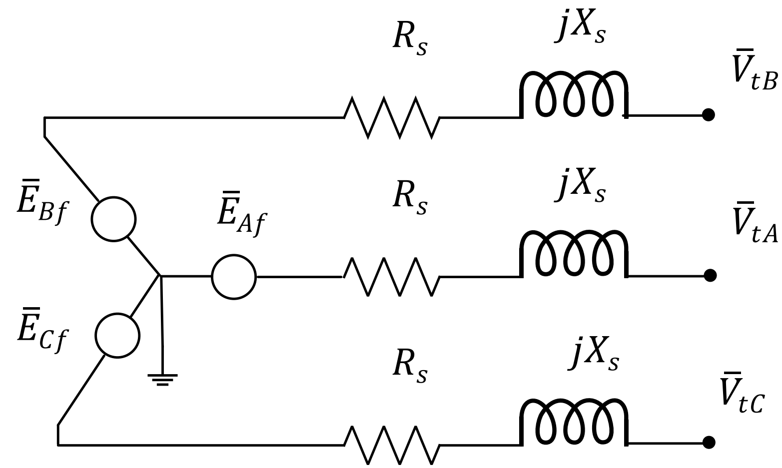
- Induced internal stator voltage E_f

$$E_f = K_e \omega_e I_f$$

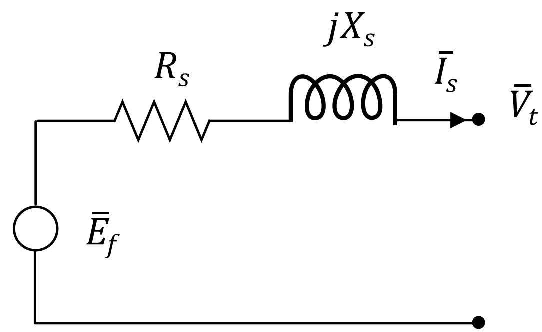
ω_e: Electrical frequency $(\mathrm{rad/s})$

K_e: EMF constant for electrical induced current (V-s/rad-A)

I_f: Field winding current



Generator equivalent circuit



Single-phase equivalent circuit

Equivalent electrical circuit

Equivalent rotor circuit

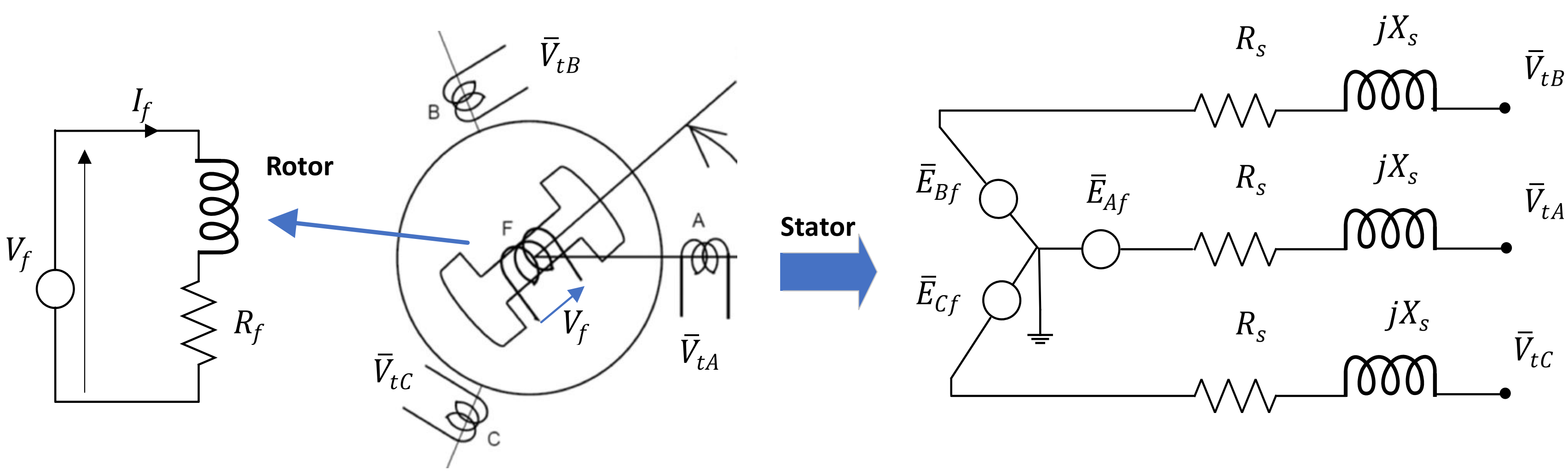
- Field current

$$I_f = \frac{V_f}{R_f}$$

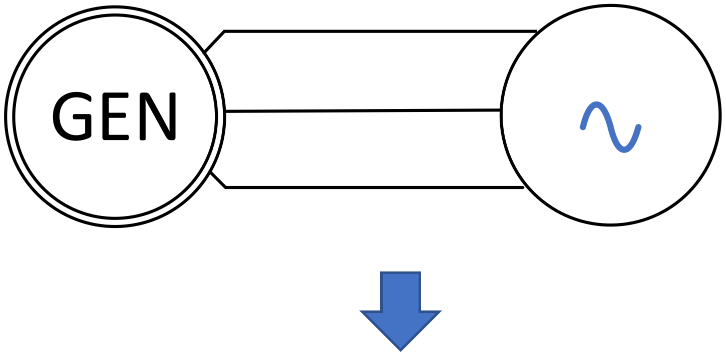
V_f: DC field voltage

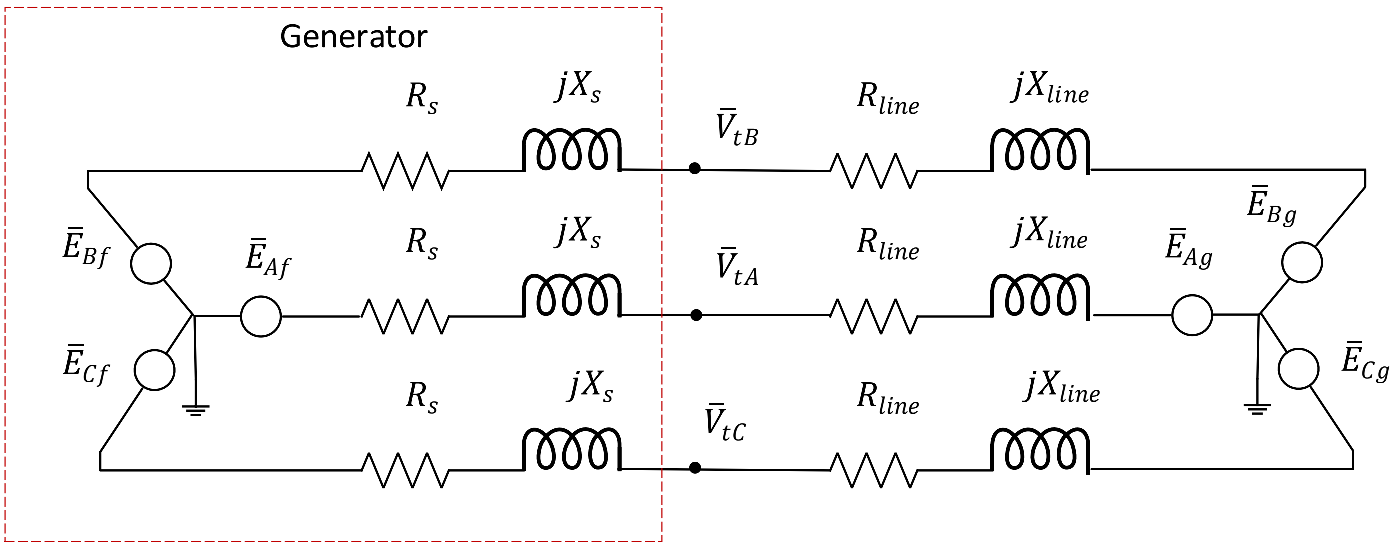
R_f: Field winding resistance

Generator Model



Equivalent electrical circuit connecting to external power systems





System equivalent circuit

Analysis

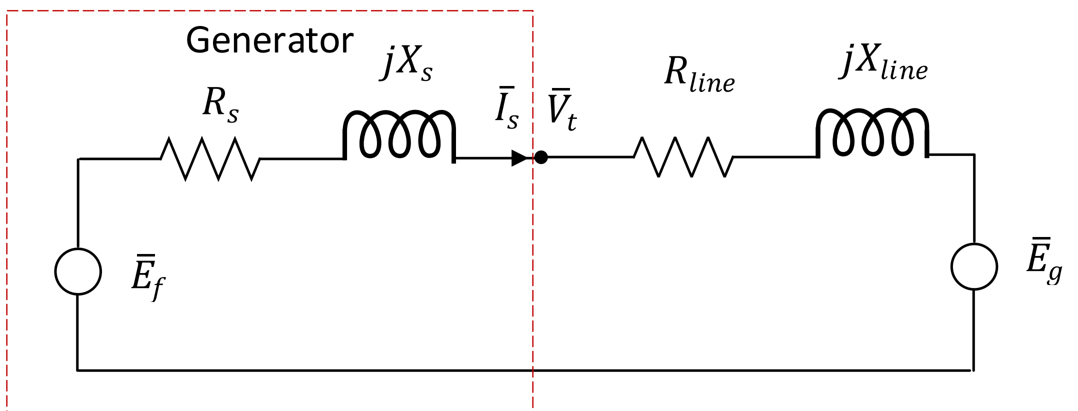
Single phase representation

- Power at terminal $\bar{S}_t$?

$$\bar{S}_t = 3\bar{V}_t \bar{I}_s^* = P_s + jQ_s$$

- Power converted to electrical energy P_e?

$$\bar{S}_e = 3\bar{E}_f \bar{I}_s^* = P_e + jQ_e$$



System equivalent circuit

Example 1

A three phase four pole synchronous generator has ratings 300 MVA, 60 Hz, 13.2 kV/7.62 kV wye. When the generator is operating at rated speed and frequency, a field current of 150 A is required to develop rated open circuit voltage. The generator synchronous reactance X_s = 1.5 ohms, the resistance is ignored. The resistance of the generator phases can be neglected.

Q1. Find the rated speed (rpm) of the generator

Q2. The generator is connected to an infinite bus which has a voltage of 13.2 kV/7.62 kV wye. Draw the per phase equivalent circuit of the generator.

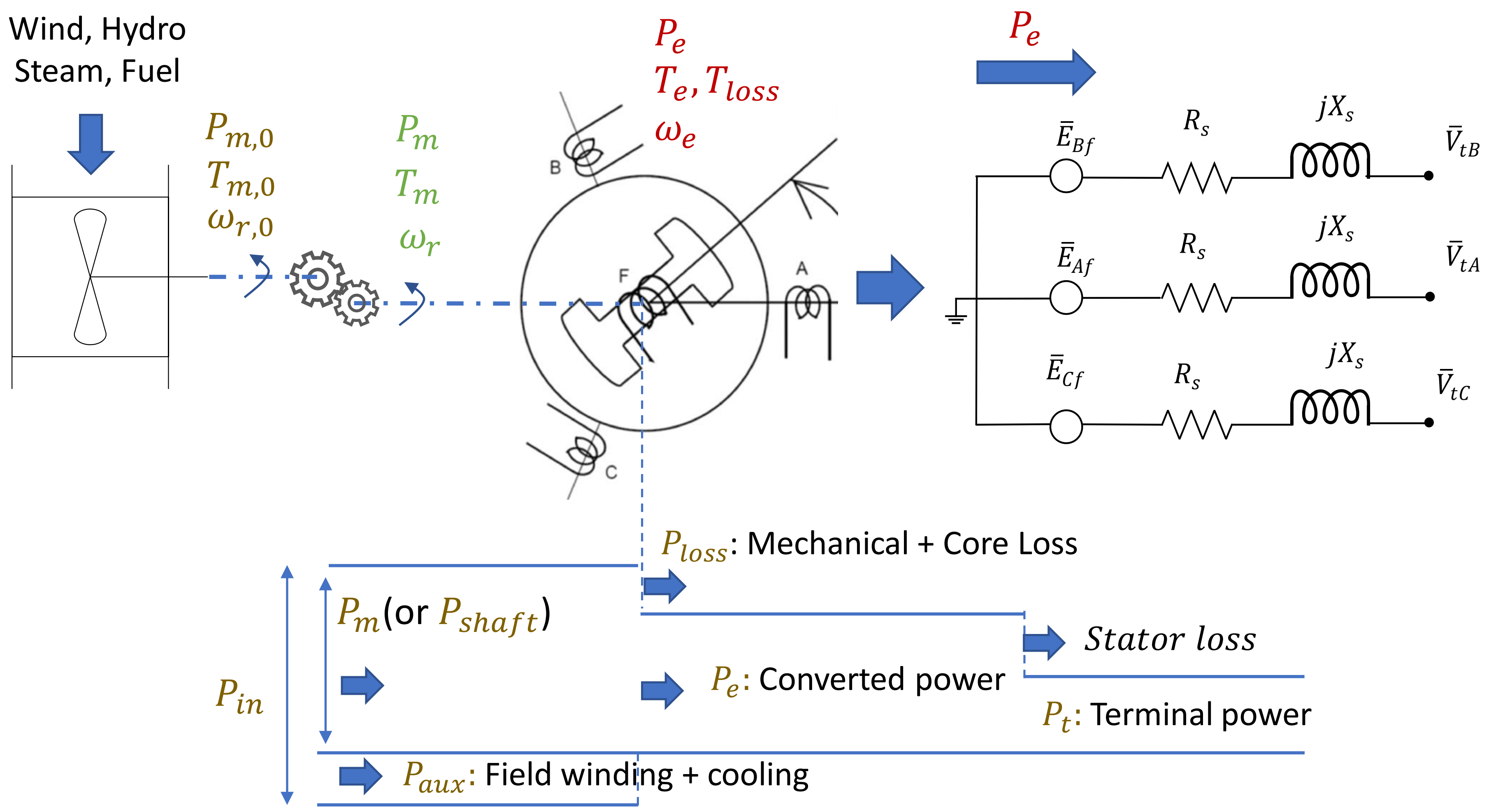
Q3. The generator is supplying 80% of rated load at unity power factor. Calculate the magnitude and angle of the generator phase current.

Q4. Find the internal line-neutral voltage

Q5. Find generator field current

Power and Torque

Energy conversion



Energy conversion

Mechanical power (Shaft power)

$$P_m = T_m \omega_r$$

T_m: Mechanical torque (Nm)

ω_r: Rotor speed (rad/s)

Electrical power converted

$$P_e = T_e \omega_r = T_e \frac{\omega_e}{p_p}$$

T_e: Electrical torque

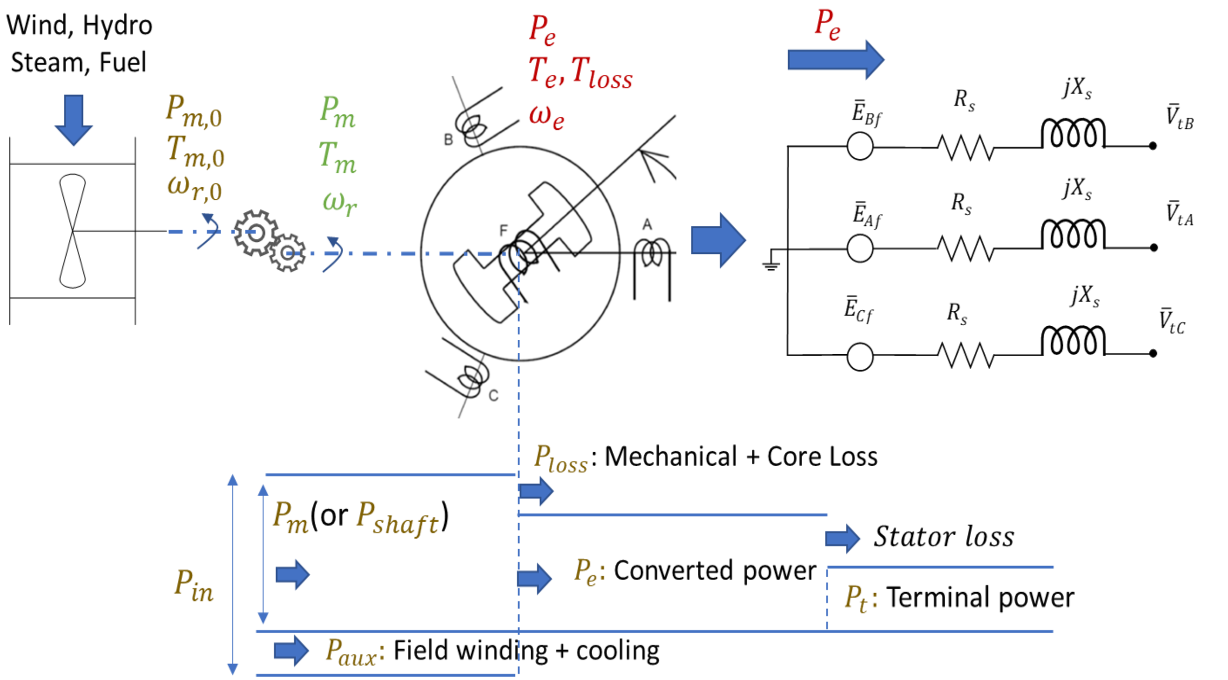
ω_e: Electrical speed (rad/s)

p_p: Number of pole pairs

Power loss

P_{loss}: Mechanical + Core loss

P_{aux}: Field winding + cooling



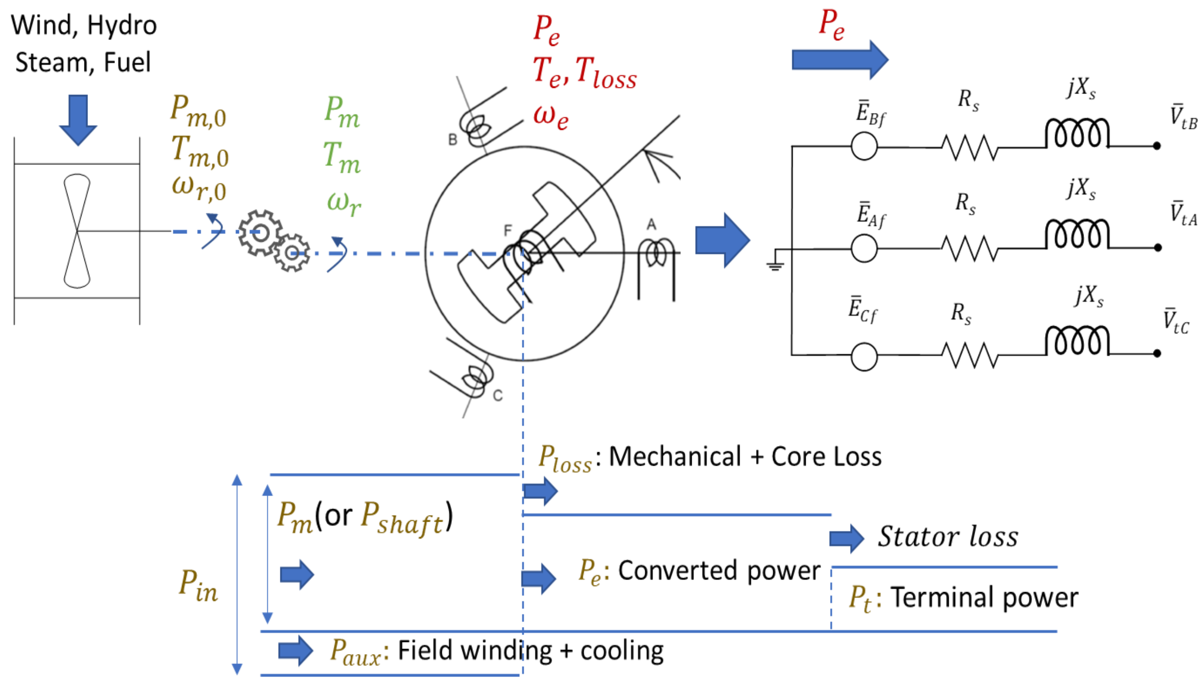
Energy conversion

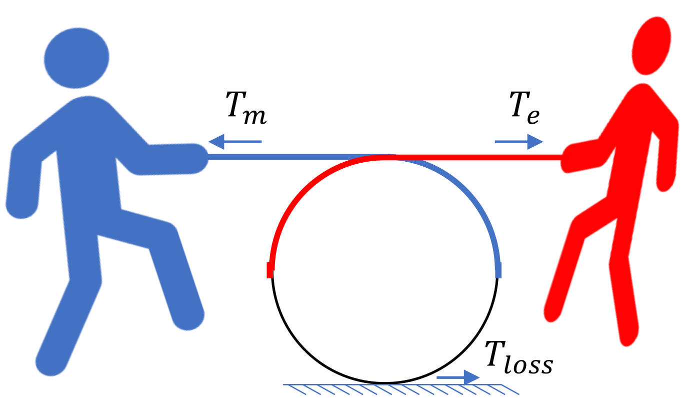
Dynamic condition

$$J\frac{d\omega_r}{dt} = T_m - T_e - T_{loss}$$

T_{loss}: Torque loss due to windage and friction

J: Reflected rotor inertia





Example 2

A three-phase eight-pole 60 Hz wye connected synchronous generator is rated at 1.0 MVA, 4.8/2.77 kV. The generator has a stator resistance of $R_s = 0.15\ \Omega$ and a synchronous reactance of $X_s = 4.0\ \Omega$. The generator operates at rated terminal voltage and delivers power to a 3-phase wye load, which has single phase impedance Z_L, $Z_L = 10 + j20\ \Omega$.

a. Find the internal voltage $\bar{E}_f$ of the generator.

b. Find the complex power generated at the terminal of the generator.

c. Find the real power converted from mechanical to electrical power.

d. Find the developed torque T_e.

Example 3

Example

An 8-pole 60 Hz synchronous generator has 100 MW flowing from its terminals. It has a stator copper loss of 1 MW, mechanical loss of 3 MW, core losses of 1.2 MW and field winding and cooling losses of 2 MW. The generator is operating with electrical frequency of 60 Hz.

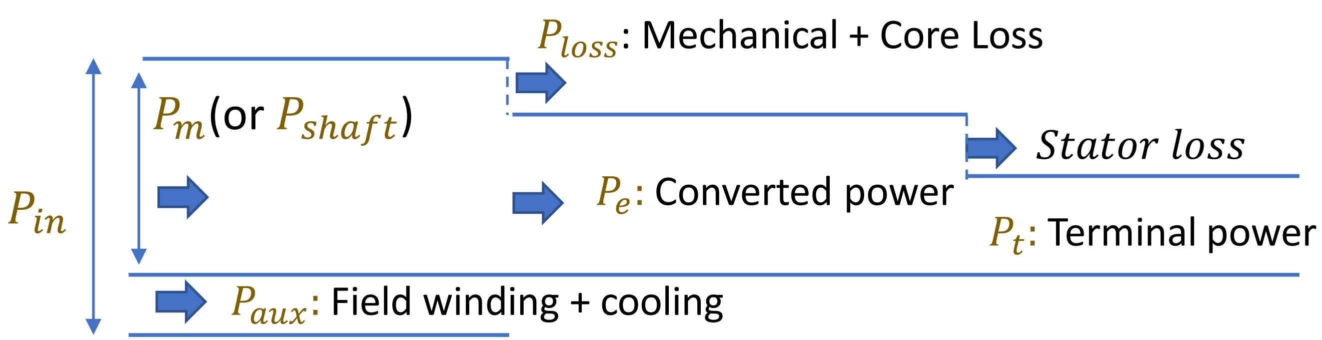
a. Determine the rotational speed of the generator in RPM and in rad/s.

b. Using the power flow diagram below, determine the converted power of the machine. Then find T_e.

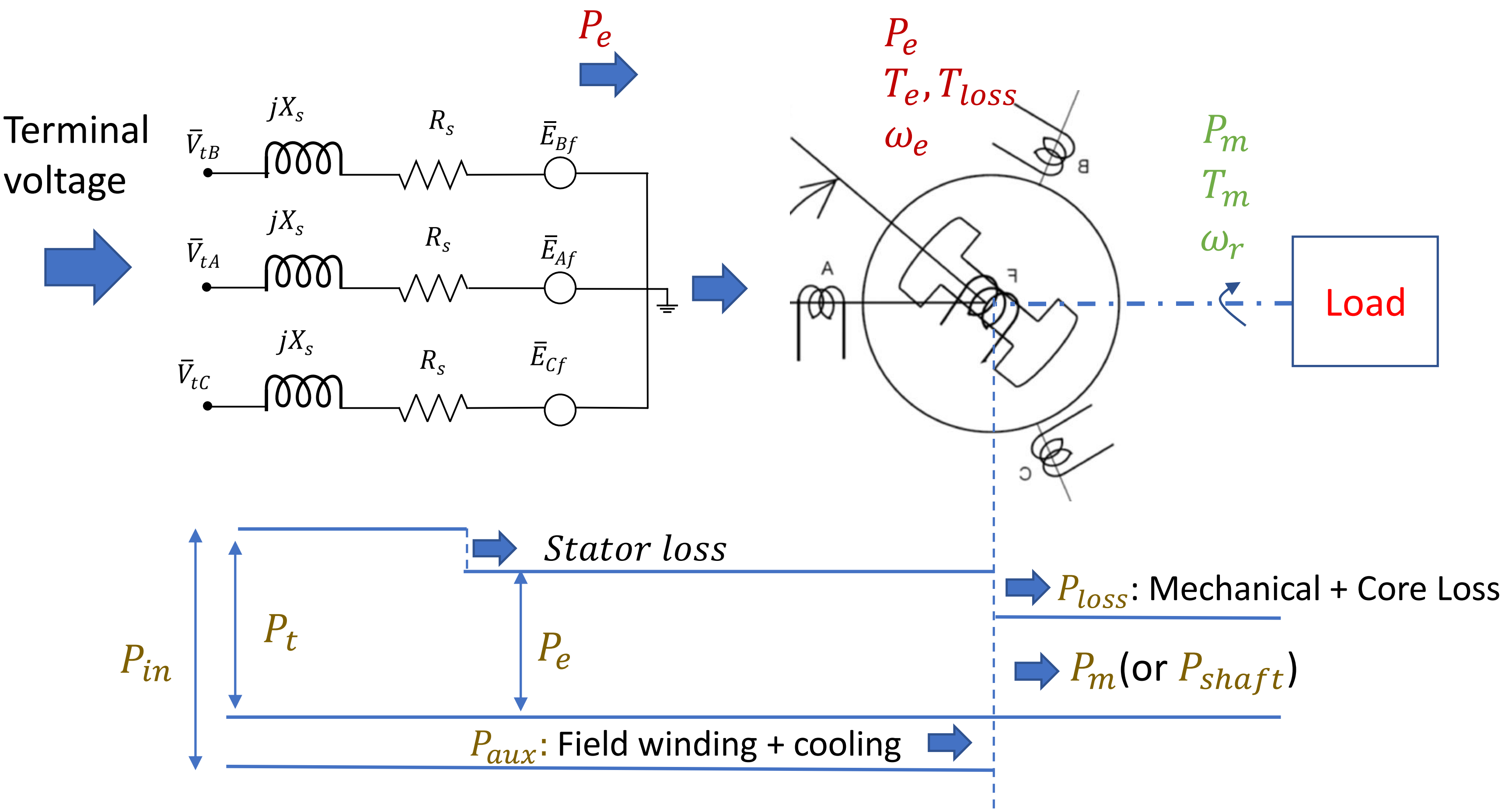
c. Find the shaft power P_{shaft} delivered by the turbine, and the corresponding torque T_m.

d. Compare the difference between T_e and T_m.

e. Determine the overall efficiency of the generator at this operating point.



Energy Conversion



Energy Conversion

Energy conversion

Mechanical power (Shaft power)

$$P_m = T_m \omega_r$$

T_m: Mechanical torque (Nm)

ω_r: Rotor speed (rad/s)

Electrical power converted

$$P_e = T_e \omega_r = T_e \frac{\omega_e}{p_p}$$

T_e: Electrical torque

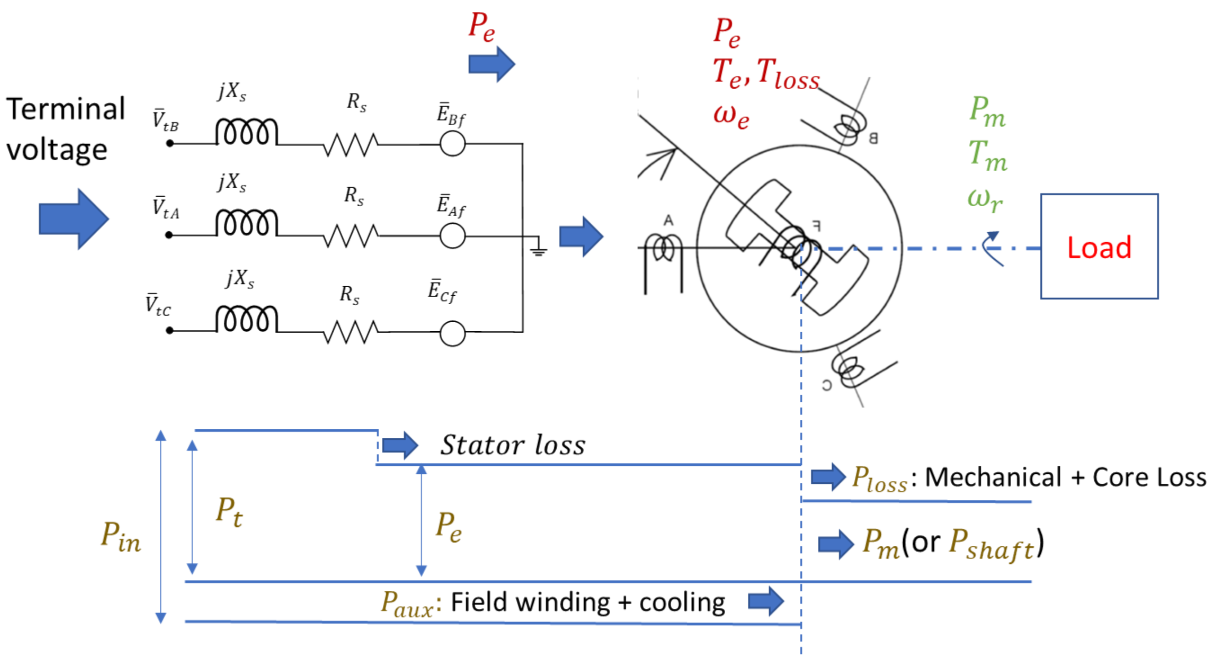
ω_e: Electrical speed (rad/s)

p_p: Number of pole pairs

Power loss

P_{loss}: Mechanical + Core loss

P_{aux}: Field winding + cooling



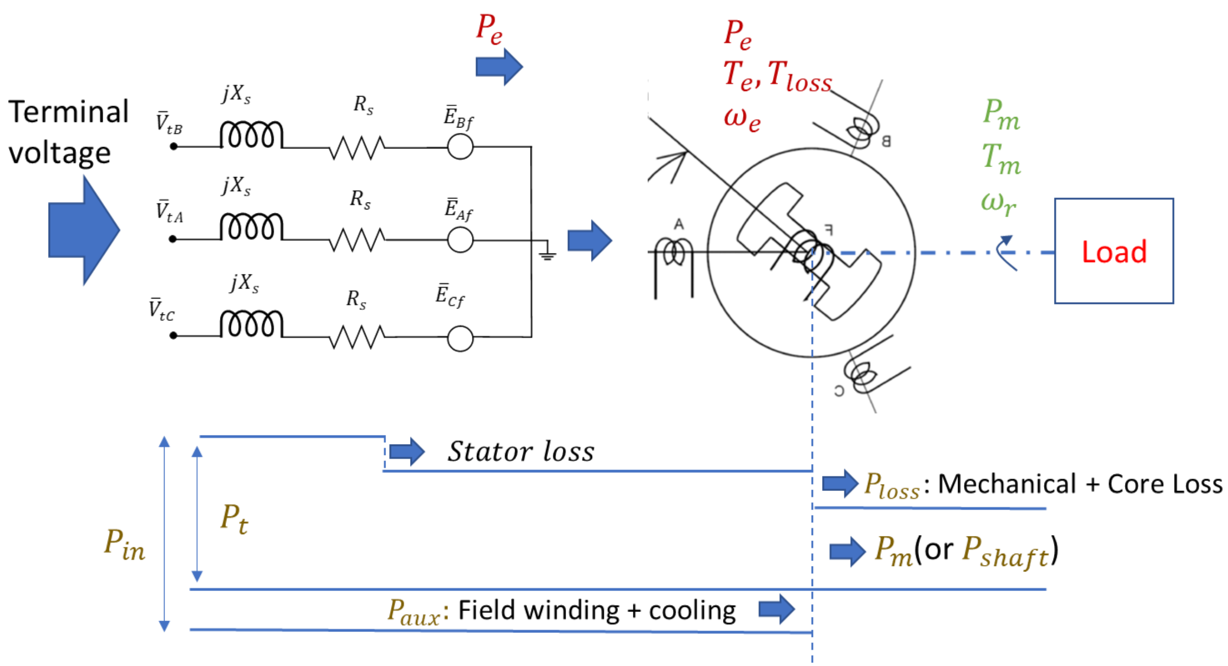
Energy conversion

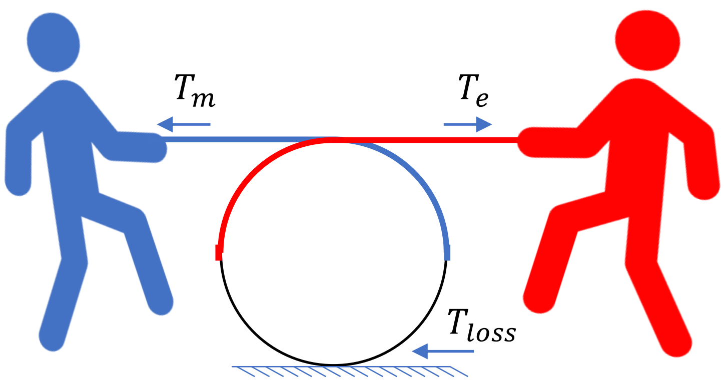
Dynamic condition

$$J\frac{d\omega_r}{dt} = T_e - T_m - T_{loss}$$

T_{loss}: Torque loss due to windage and friction

J: Reflected rotor inertia





Equivalent circuit

- Power at terminal $\bar{S}_t$?

$$\bar{S}_t = 3\bar{V}_t\bar{I}_s^* = P_s + jQ_s$$

- Power entering internal source $\bar{S}_e$?

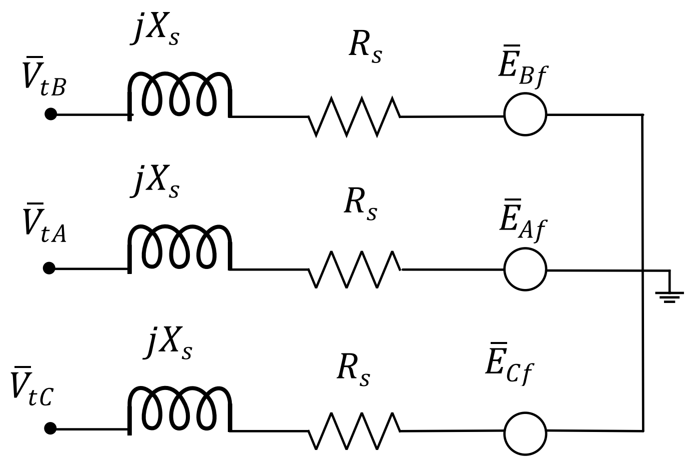
$$\bar{S}_e = 3\bar{E}_f\bar{I}_s^* = P_e + jQ_e$$

- Mechanical power (Shaft power)

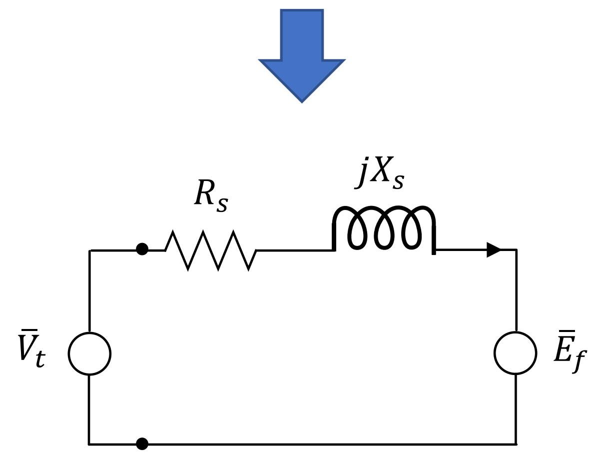
$$P_m = T_m\omega_r$$

- Electrical power converted

$$P_e = T_e\omega_r = T_e\frac{\omega_e}{p_p}$$



Motor equivalent circuit



Single-phase equivalent circuit

Synchronous Motor

Working principle: www.youtube.com/watch?v=Vk2jDXxZIhs

Example

A three phase 60 Hz synchronous motor has 8 poles. The per-phase diagram of the motor is shown in the figure. The motor rated voltage is 480V/277 V wye. The stator rated current is 150 A. The motor is operating with a terminal voltage magnitude of V_t = 270 V with a frequency of 60 Hz. The current magnitude $I_s = 150\,A$. $\bar{I}_s$ lags $\bar{V}_t$ by 20 degrees.

a. Find $\bar{E}_f$ in phasor domain.

b. The internal voltage magnitude $E_f = 0.05\omega_e I_f$, where ω_e is the electrical frequency (rad/s). Find the field voltage supply V_f for this operating point if field winding resistance is 2 Ω.

c. The converted power for the motor is the real power entering the internal voltage source E_f. Find the converted power for this motor.

d. From the converted power and the rotor speed ω_r, find the developed torque of the motor.