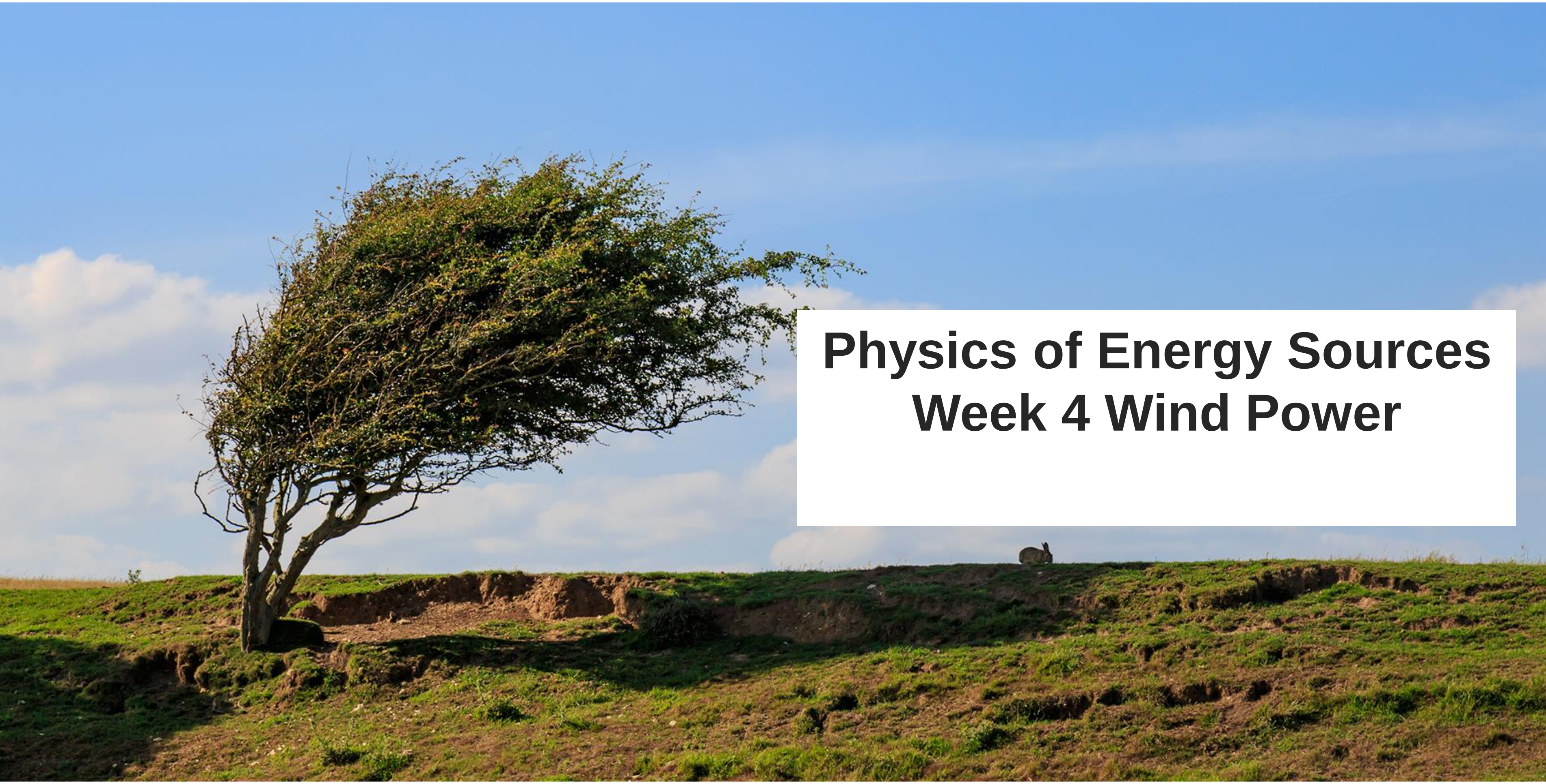
MANCHESTER
1824
The University of Manchester



Physics of Energy Sources
Week 4 Wind Power

Philippa Browning

Department of Physics and Astronomy

p.browning@manchester.ac.uk

Wind energy ultimately comes from solar energy

- Wind power has been in use for many centuries
- Modern rejuvenation in electrical generation during the 1973 oil crisis
- Recent technological progress accelerated wind power construction

(a)

(b)

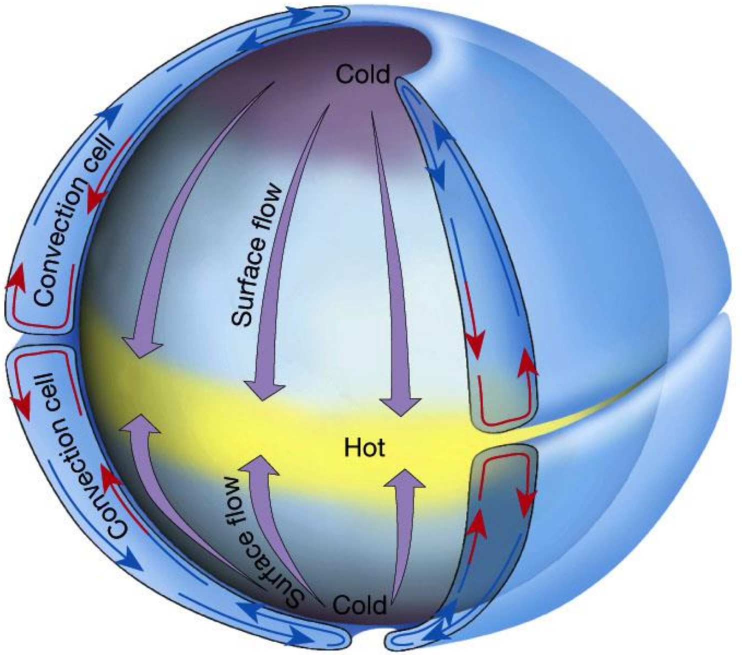
- Why do winds happen, and how do they vary with location and time?
- How much power is in the wind?
- How can we extract this power?
(How do turbines work?)
- How do we build a wind farm?
- What are the pros and cons of wind power?

The origin and nature of wind (1)

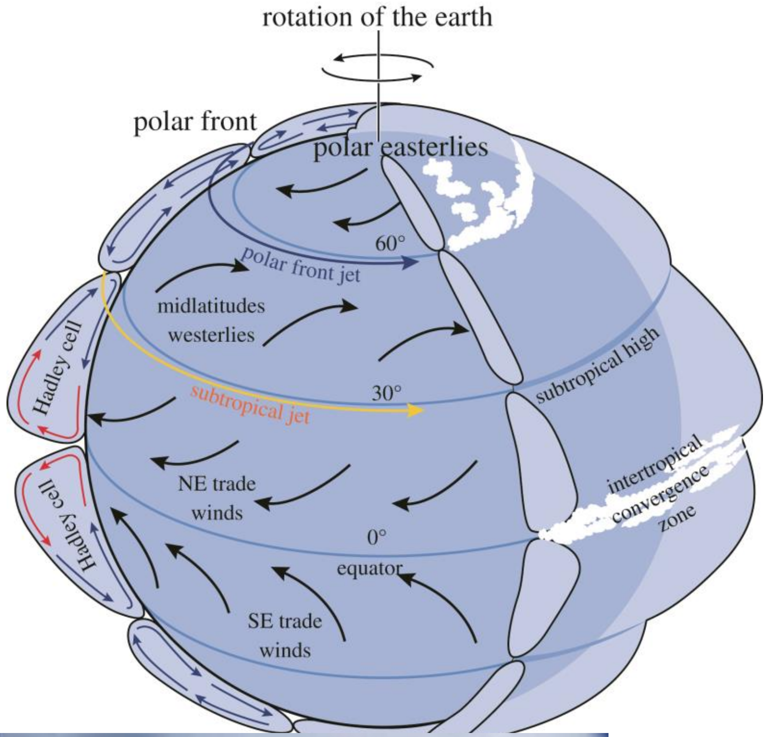
- 0.5%-3% of solar energy on Earth is fed into wind
- If all this power were dissipated, could generate 1000 TW
- But wind is highly variable in time and distributed unevenly in location

- **Large-scale** ~10,000 km average motion driven by solar forcing
  - Hot air rises at equator, convects to poles, cool air sinks

Without rotation



With rotation



- Strongly affected by rotation – Coriolis forces

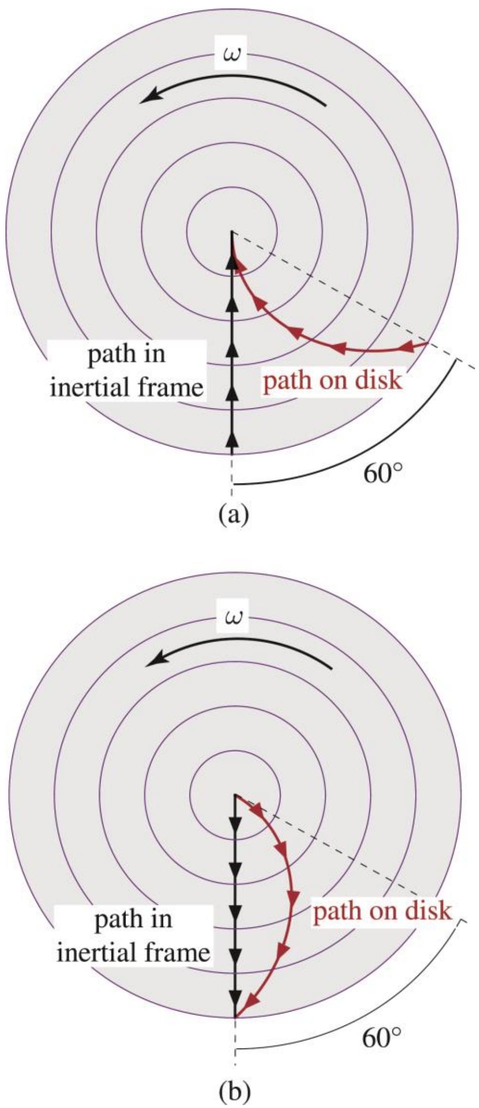
- Creates large-scale **primary circulation** - **Hadley cells** - also Ferrel, Polar cells
  - Wind strength stronger at some latitudes than others, prevailing winds e.g. Trade winds

An aside – Coriolis force

- Coriolis force is a fictitious force in rotating reference frame (c.f. centrifugal force) which affects bodies MOVING in rotating frame
- Consider body moving with constant velocity (in inertial frame) viewed from rotating disk – see right
  - Path on disk is curved
  - Thus has acceleration requiring fictitious force
- Acts at right-angles to the (apparent) velocity

$$\mathbf{F}_{cor} = -2m\boldsymbol{\omega}\times\mathbf{v}$$

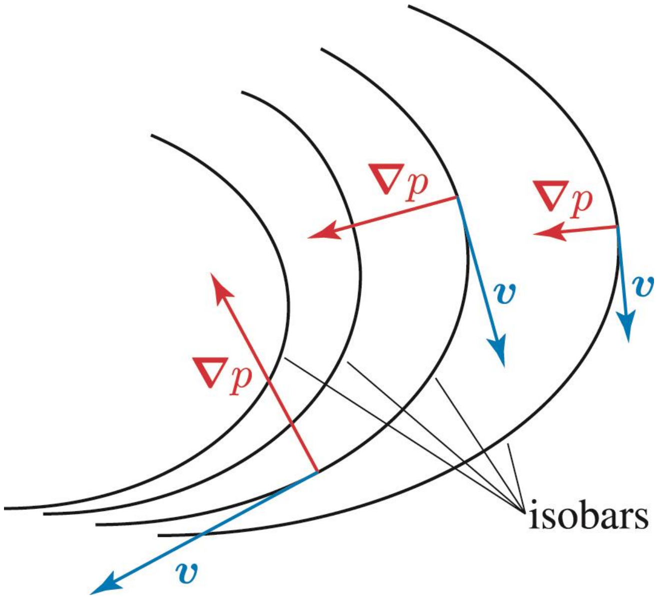
  - Deflects winds to the right in the Northern hemisphere
  - Deflects winds to the left in the Southern hemisphere



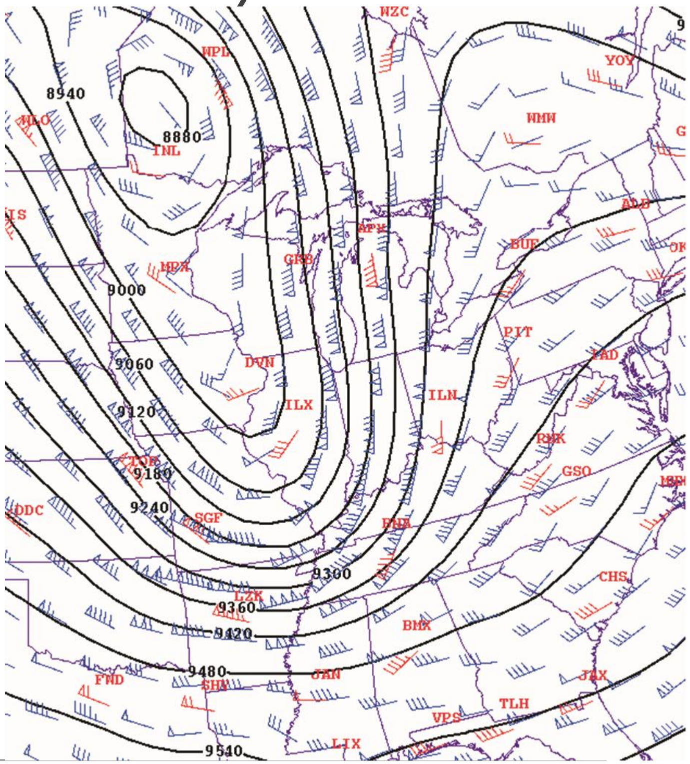
Jaffe and Taylor

The origin and nature of wind (2)

- **Secondary circulation ~ 1000 km**
  - Cyclones and anticyclones – regions of high and low pressure – drive weather patterns
  - Also monsoons etc
- Because of Coriolis forces, wind flows approximately *along* isobars (contours of constant pressure)

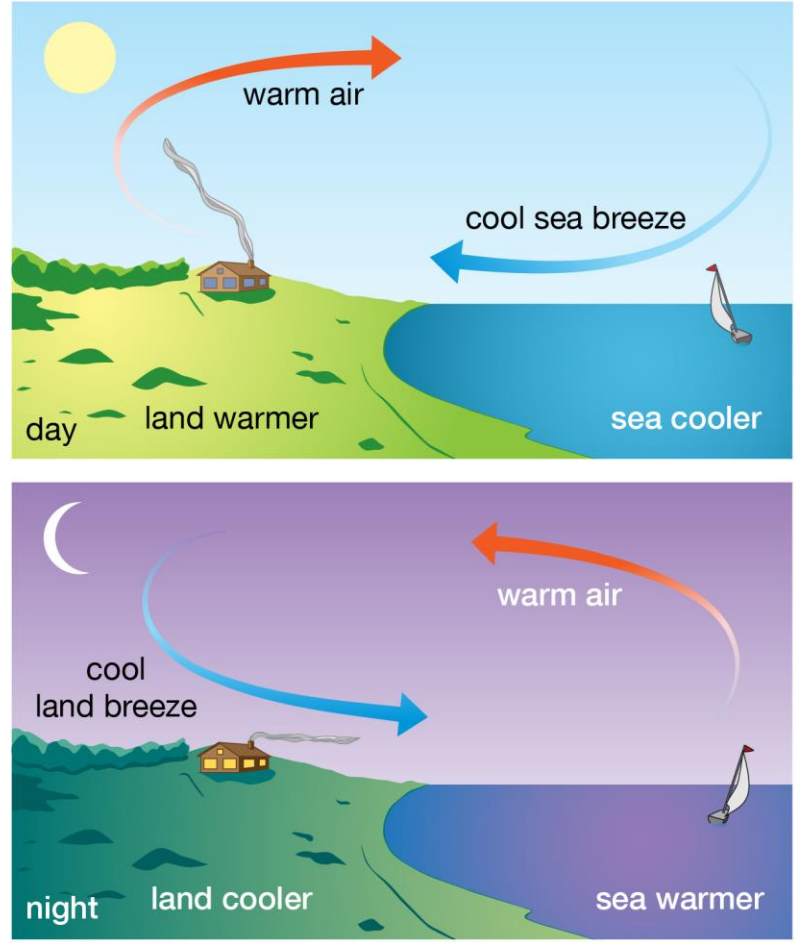


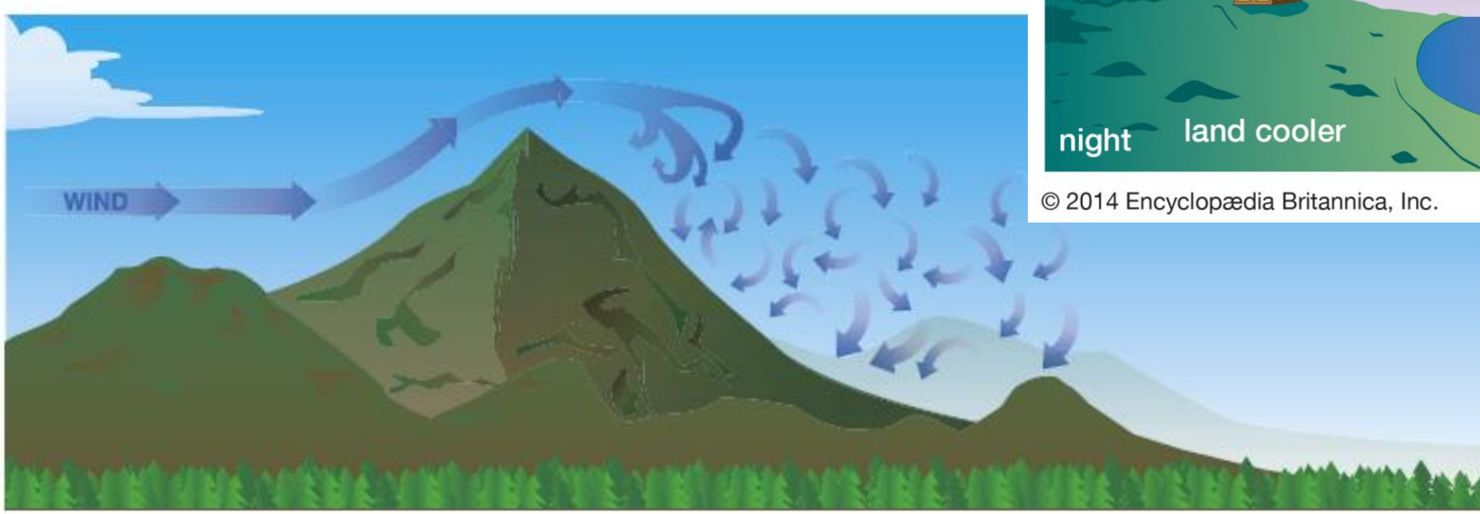
Wind (observed red, blue predicted) and isobars over USA 1996
From Jaffe and Taylor

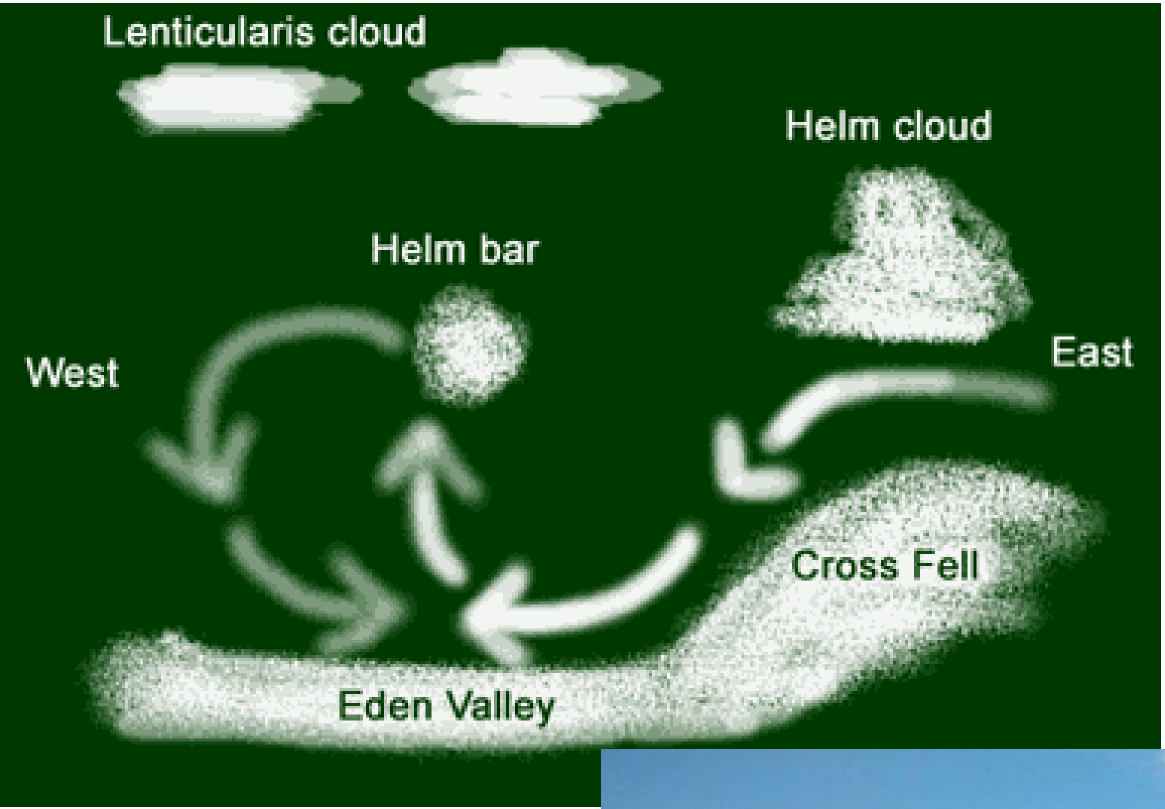


✧ Tertiary scale – 10 – 100 km

- Local scales affected by topography etc
- e.g. Land/sea breezes, mountains and valleys



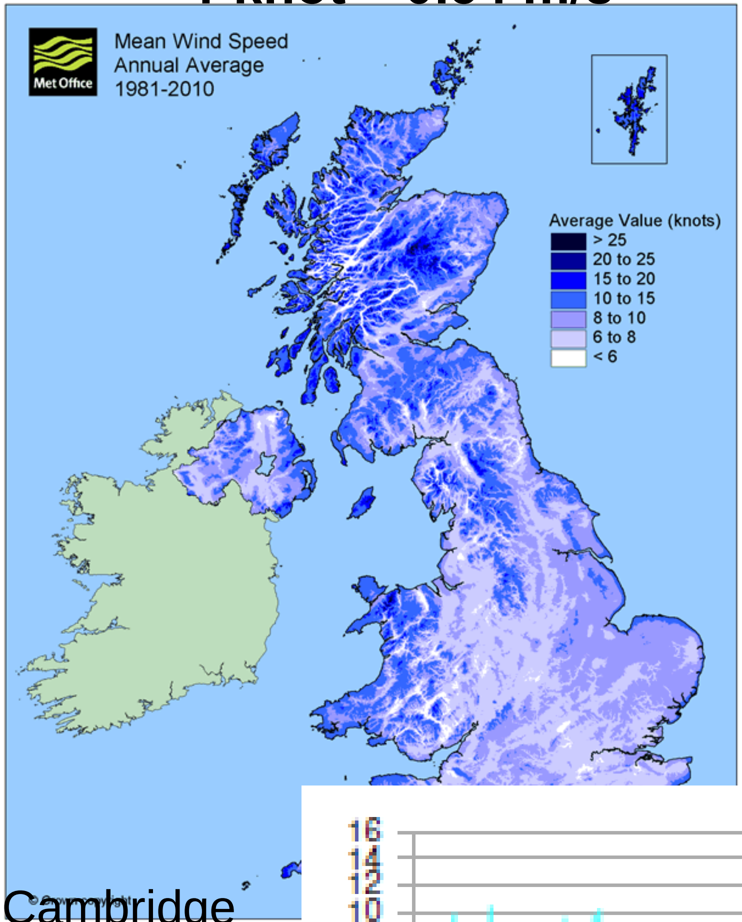



Lenticularis cloud
Helm cloud
Helm bar
West
East
Cross Fell
Eden Valley


MOUDY55/WEATHER WATCHERS

Wind in the UK

1 knot ≈ 0.54 m/s



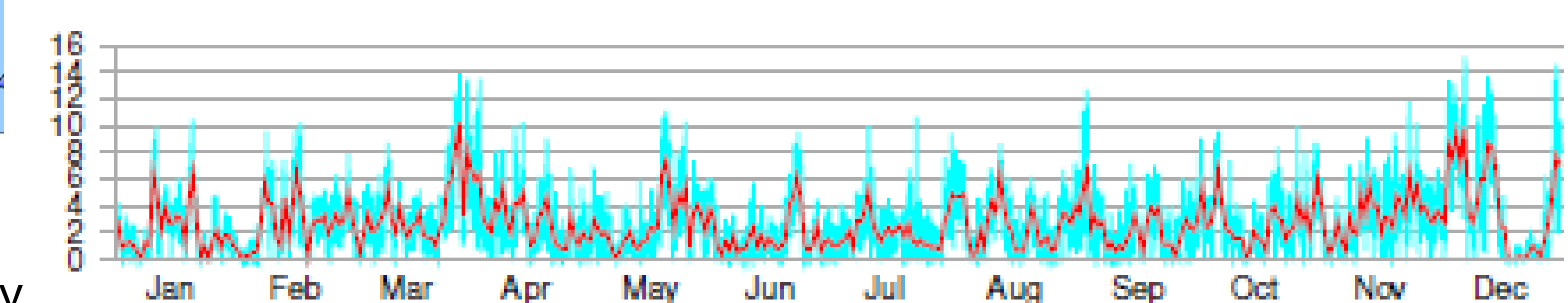
- Strongest recorded wind 150.3 knots (193 mph) – Cairngorm summit 1986

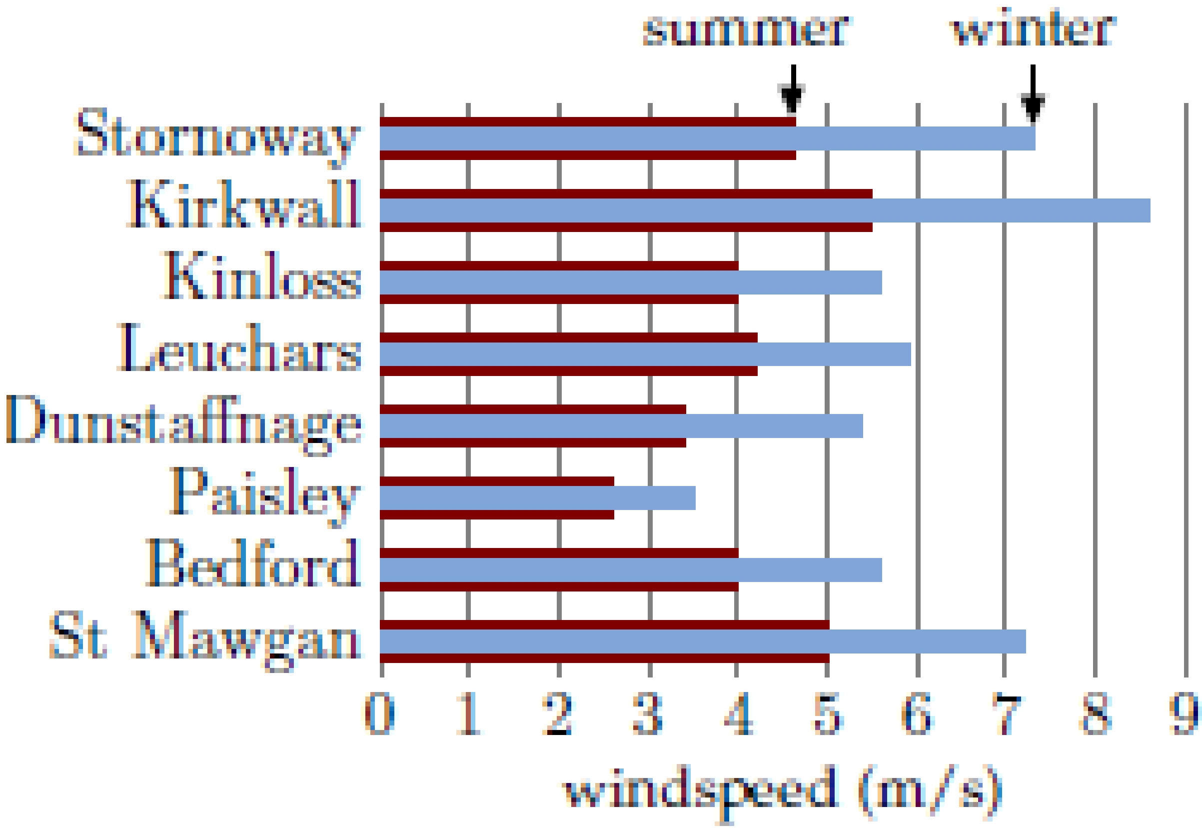
- Windiest region – Shetland
- Average wind speed 14.7 knots

(Met office website)

Cambridge mean wind speed daily/halfhourly

from Mackay





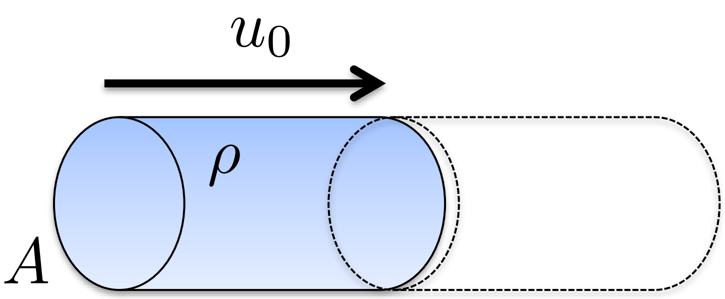
UK windspeed at 10 m in m/s 1971 – 2000

From Mackay

Typical UK mean wind speed ≈ 4 m/s

Power in moving air

- Consider a fluid that is moving smoothly (uniform motion)
  - Velocity u_0
  - Fluid mass density ρ
- **Kinetic Power**: Energy crossing area A per unit time



$$P = \frac{1}{2}\dot{m}u_0^2 = \frac{1}{2}\rho A u_0^3$$

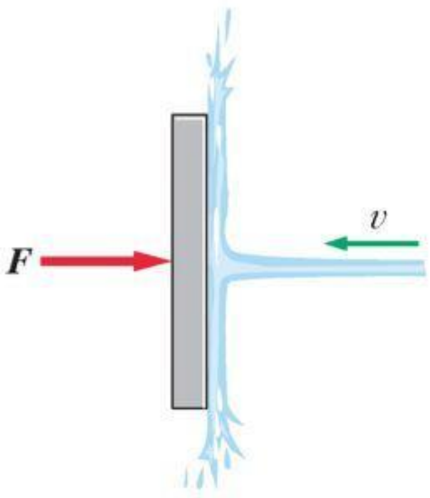
See board/visualiser

Power is strongly dependent on fluid velocity

Example: typical values: $u_0 \sim 10\,\mathrm{m\ s}^{-1}, \rho \sim 1.2 kgm^{-3}$

Extracting power from moving fluid

- Consider jet of water hitting a wall.
- Water is stopped – exerting a force on the wall
- Power delivered to the wall is

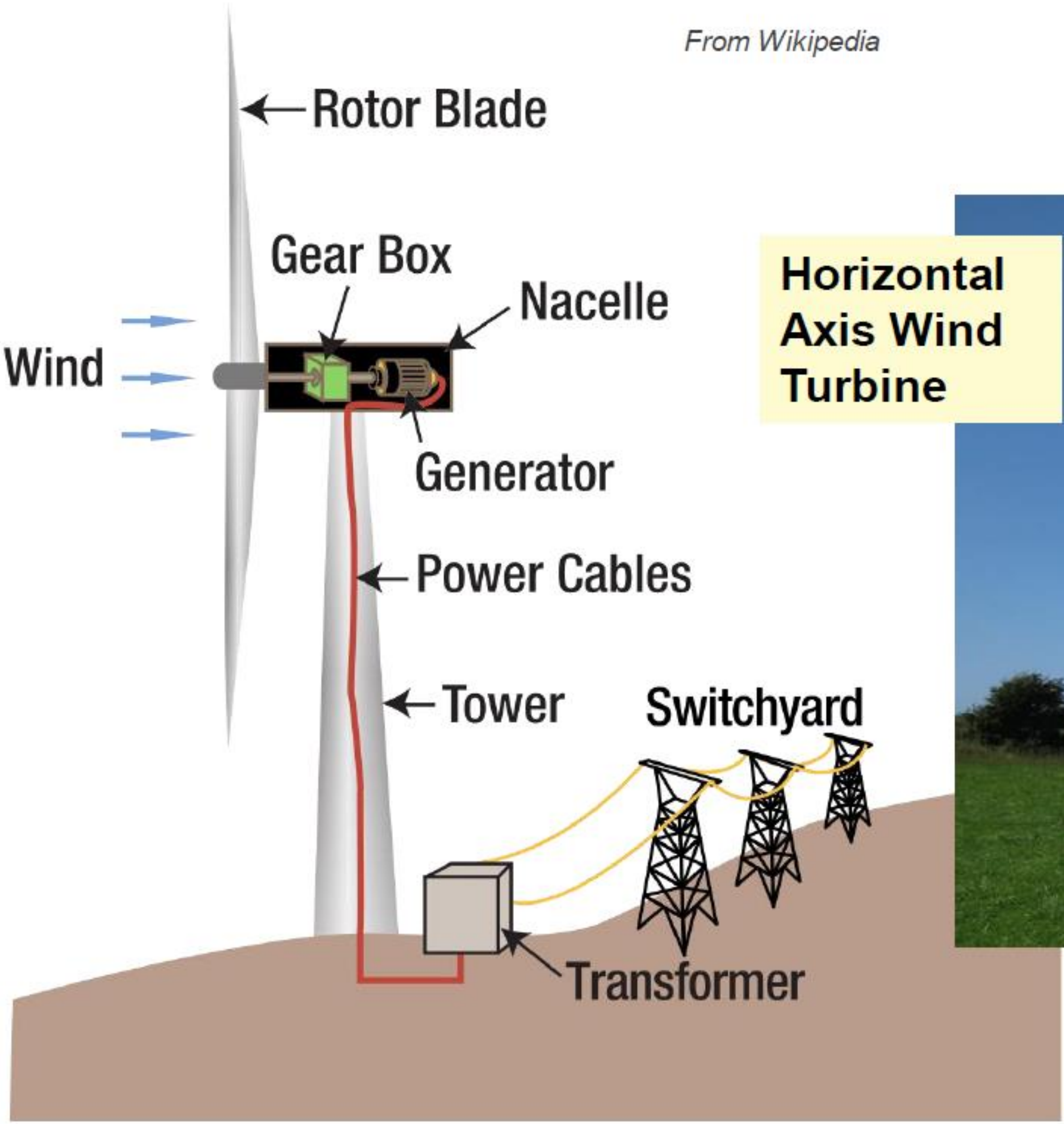


$$P = \frac{1}{2}\rho A u_0^3$$

- But a wind turbine is a bit different – why?

Wind turbines

From Wikipedia

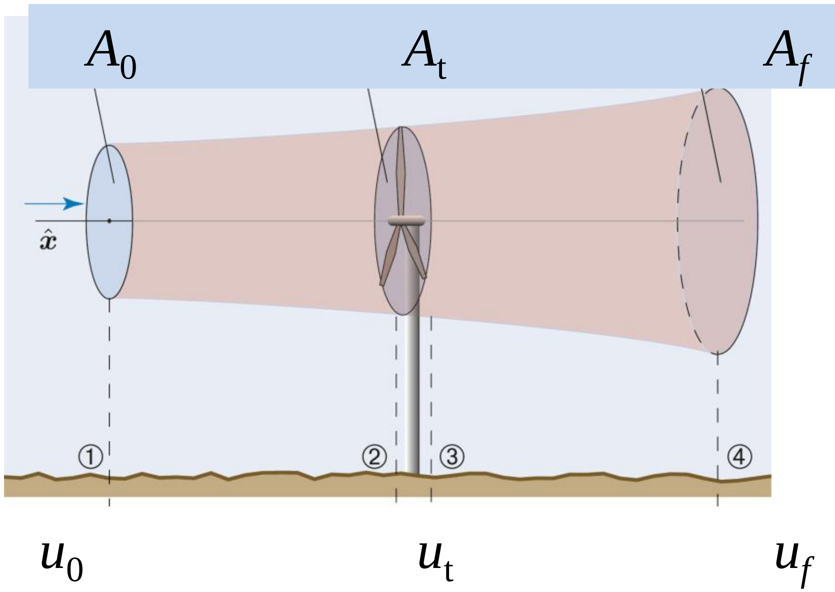


Horizontal Axis Wind Turbine

- Suppose that a constant velocity air stream goes through a turbine in ***laminar*** (non-turbulent), ***incompressible*** (no change in density) flow
- Wind flow cannot be stopped as air must flow through the turbine to turn the blades – hence not all kinetic power can be extracted

Energy Extraction from wind

- Consider column of air flowing through turbine
- Mass must be conserved – hence constant flux of air



$$\dot{m} \equiv \rho u A = \text{constant}$$

Air:

- Comes in with speed u_0
- Speed u_t at turbine
- Flows out with (lower) final speed u_f

A_t: Rotor sweep area
A_0 and A_f: areas of air flowing into/out of turbine

Energy extraction and power coefficient

- Define **fractional wind speed decrease**:

$$a = \frac{u_0 - u_t}{u_0}$$

- **Power extracted** by turbine = Power lost by wind:

$$P_t = \frac{1}{2}\dot{m}\left(u_0^2 - u_f^2\right)$$

- **Wind power (undisturbed)**: $P_0 = \frac{1}{2}\rho A u_0^3$

- Define **power coefficient** $C_p = \frac{P_t}{P_0}$ = fraction of wind power extracted by turbine

- Apply **conservation of mass**

$$\dot{m} \equiv \rho A_0 u_0 = \rho A_t u_t = \rho A_f u_f$$

and equate power drop across the turbine

$$P_t = \frac{1}{2}\dot{m}\left(u_0^2 - u_f^2\right)$$

to force x velocity, where force = rate of change of momentum

  - Find wind speed at turbine is mean of initial and final speeds $u_t = \frac{1}{2}\left(u_0 + u_f\right)$

- Substituting for u_f and $\dot{m}$, find

$$P_t = \left[4a(1-a)^2\right]\left(\frac{1}{2}\rho A_t u_0^3\right)$$

$$= C_p P_0$$

Power coefficient X power in unobstructed air column at turbine

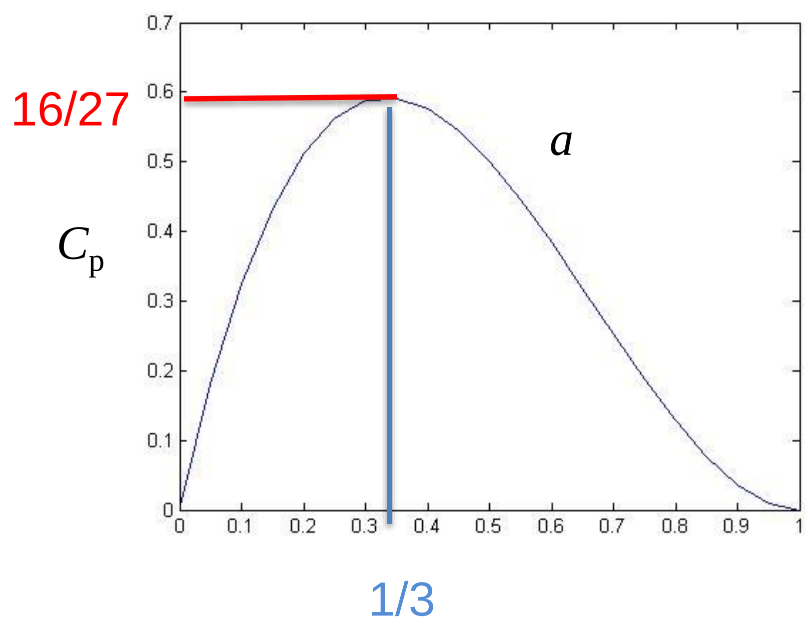
See Problem sheet

Energy Extraction: Betz Criterion

$$C_p \equiv \frac{P_t}{P_0} = 4a(1-a)^2$$

$$P_t = C_p\left(\frac{1}{2}\rho A_t u_0^3\right)$$

Can show that C_p has maximum where a = 1/3, with value:
Maximum C_p = 16/27 = 0.59



$$a = \frac{1}{3} \Rightarrow u_t = \frac{2}{3}u_0$$

$$\text{Since } u_t = \frac{1}{2}\left(u_0 + u_f\right),\ u_f = \frac{1}{3}u_0$$

More about design of turbines

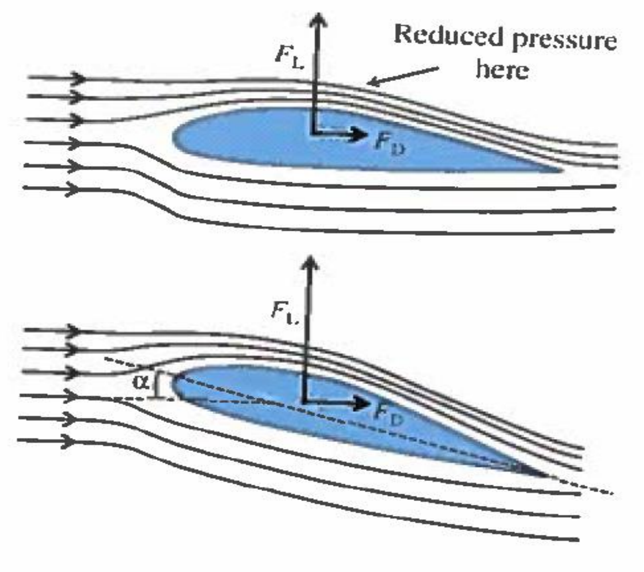
- How does wind make a turbine rotate?
- Optimising turbine operation

Worlds largest wind turbine blade 107 m
https://www.ge.com/news/reports/extreme-measures-107-meters-worlds-largest-wind-turbine-blade-longer-football-field-heres-looks-like

Turbine design and relative wind

- The **turbine** converts **linear wind momentum** into **rotational motion** (Ω) of rotor blades

- Mainly based on principle of "**lift**" of aerofoil, similar to aeroplane wing
- Lift is perpendicular to relative velocity of air to wing (turbine blade)
  - Caused by lower pressure on upper side of aerofoil



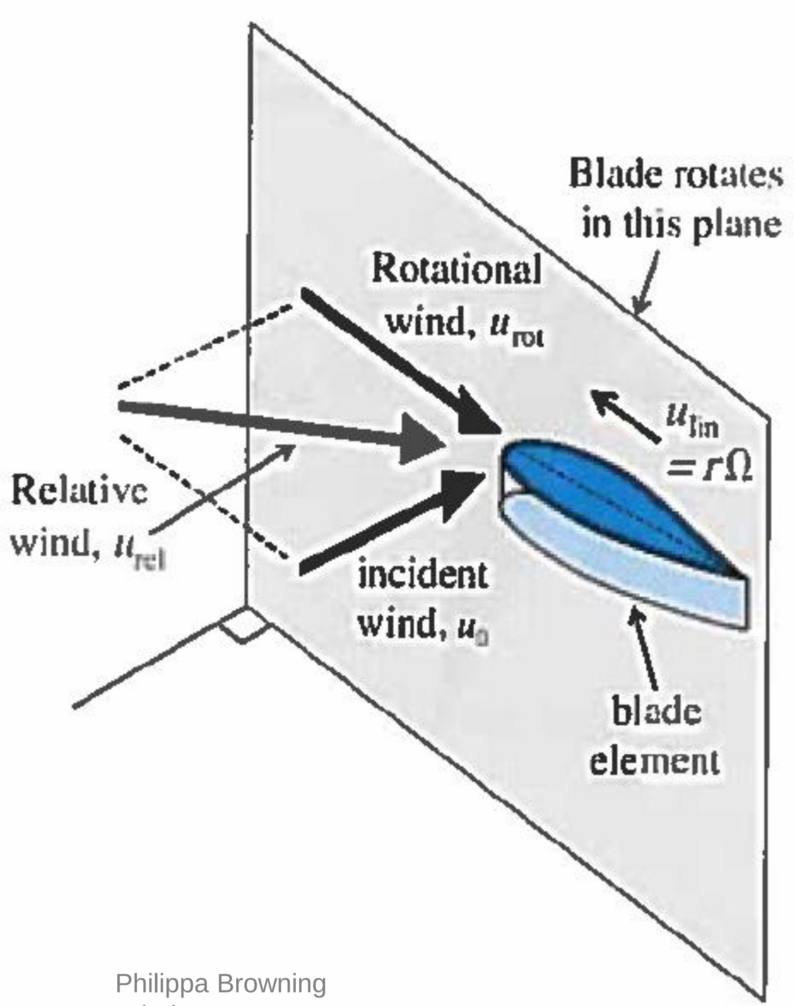
Lift force F_L increases with angle of attack α – up to a maximum value

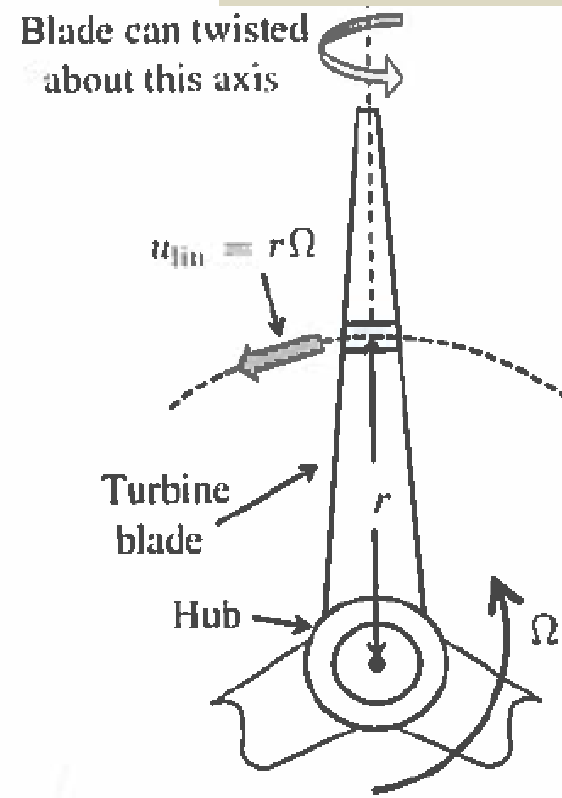
Also drag force F_D

Pictures from King

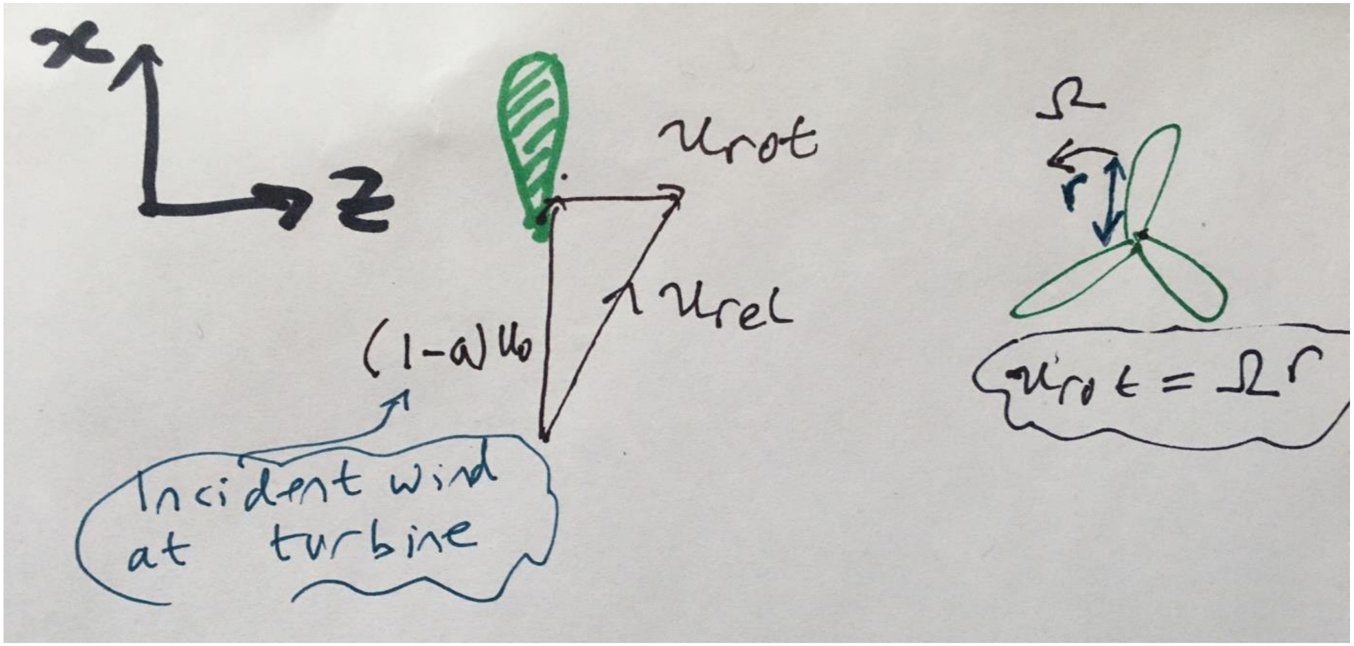
- Air velocity relative to blade is "**relative wind**" – vector sum of incident wind at wing $(1-a)u_0$ and linear speed $u_{\mathrm{lin}} = r\Omega$ of rotating blade at r

$$a = \frac{u_0 - u_t}{u_0}$$





Relative wind example



Ex $r = 25$ m $\quad u_0 = 12$ m s^{-1}

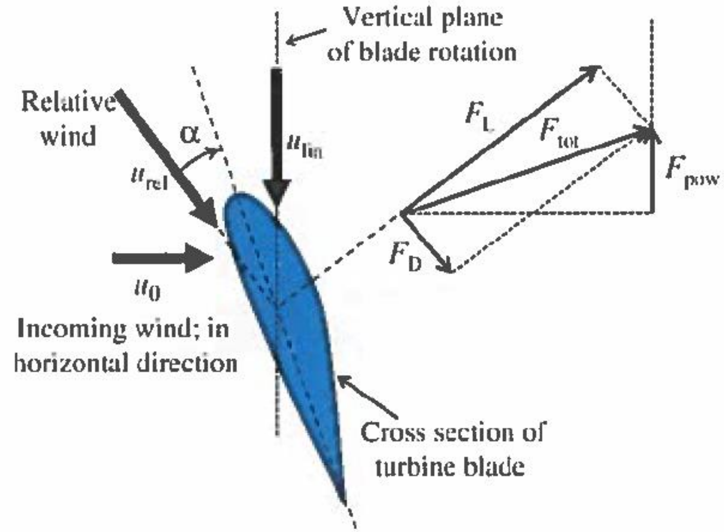
$a = 1/3$ (optimal for Betz)

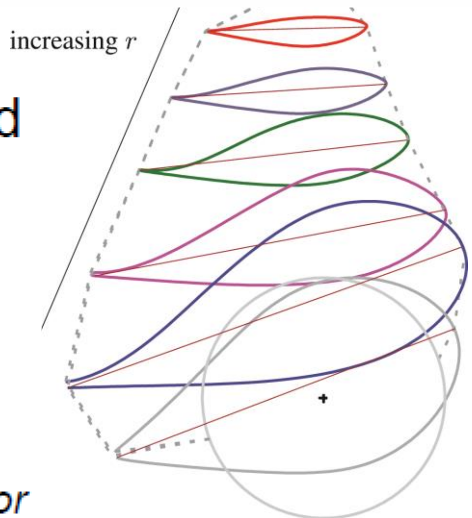
Rotation 15 rpm (revolutions per minute)

Turbine design: blade shape

- ❖ Angle of attack of “aerofoil” with respect to relative wind direction should be optimised
- ❖ But relative wind depends on distance from hub (r)

- ❖ Hence blades are twisted – profile and orientation are varied along the length

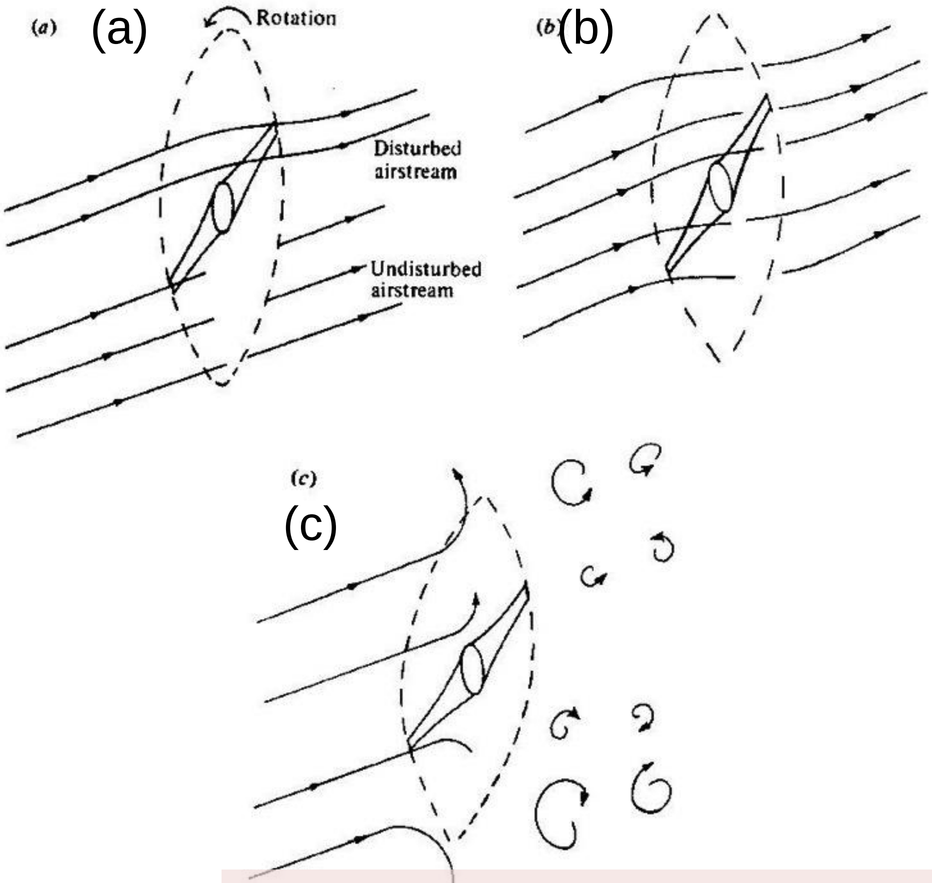




Variation of wind turbine blade profile as function of distance from hub (r)

From Jaffe and Taylor

Turbine Design: rotation speed

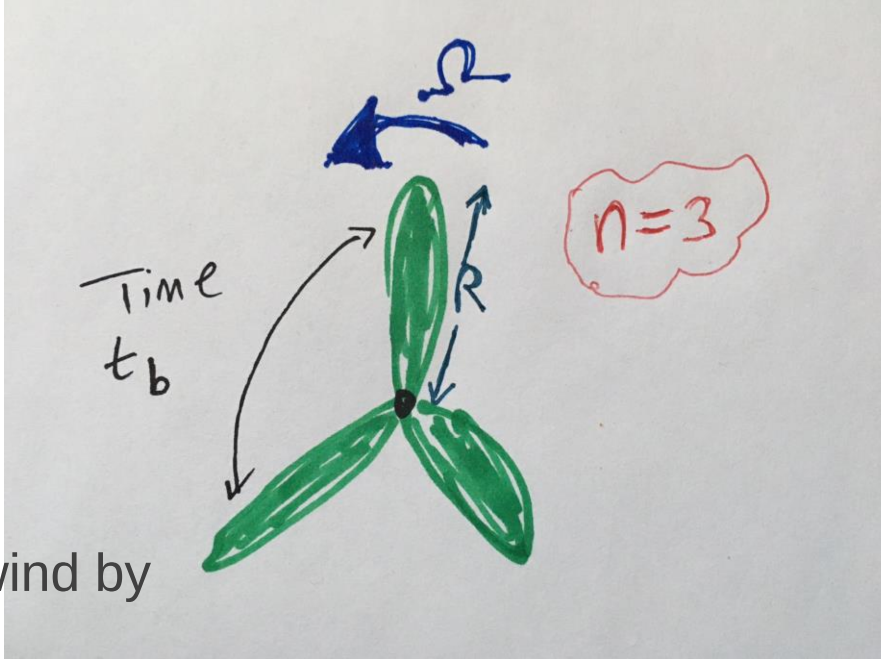


- **Rotation speed** and **number of blades** is very important to optimise the power of the turbine
  a) Too slow or blades too far apart: *some wind goes through without interaction with turbine*
  b) Optimum: whole stream interacts without turbulence
  c) Too fast or blades too close together: *creation of turbulence decreasing efficiency*

We need to match the rotational speed to a range of wind speeds

Turbine Design: tip speed ratio

t_b: Time for one blade to move into position previously occupied by another
n: number of blades
Ω: angular velocity of turbine
R: radius of blade



t_w: Characteristic lifetime of disturbance created by one blade
u_0: Incoming wind velocity
d: characteristic perturbation length of wind by a single rotating blade

$$t_w = d/u_0$$

Tip speed ratio:

$$\lambda \equiv \frac{\text{Speed of blade tip}}{\text{Speed of oncoming wind}} = \frac{R\Omega}{u_0}$$

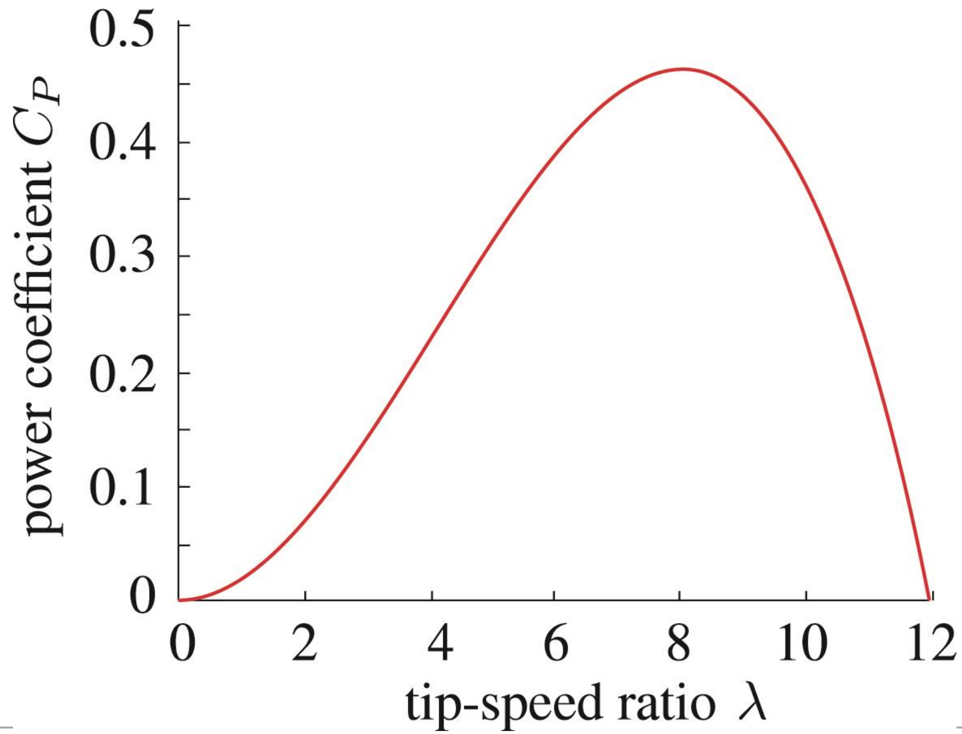
Optimised tip speed ratio:

$$\lambda_0 = \frac{2\pi R}{nd}$$

See Board/visualiser

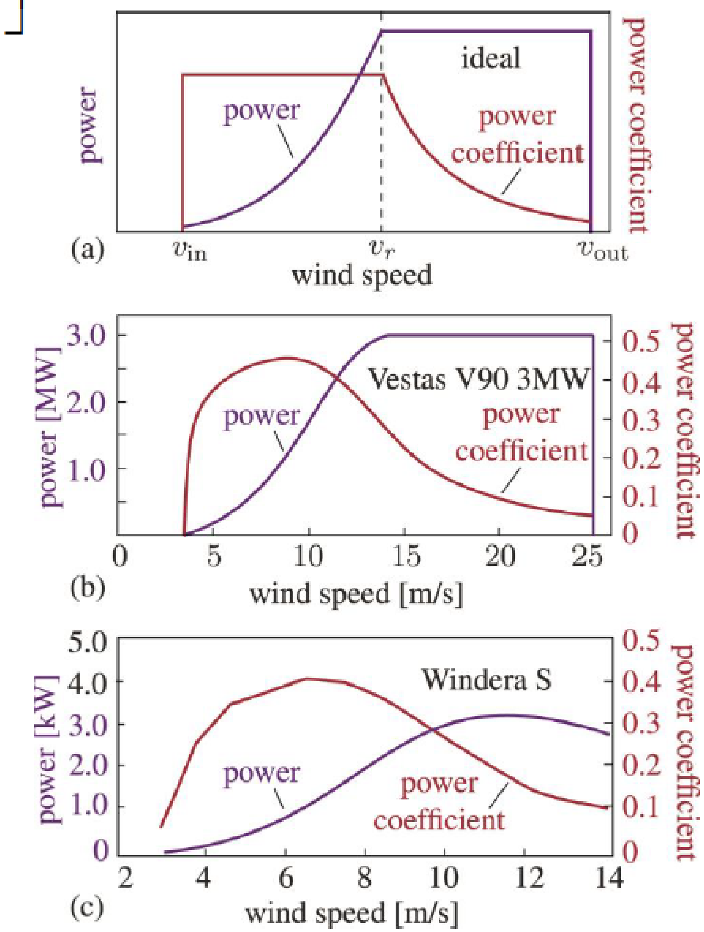
Since empirically $d \approx R/2$:

$$\lambda_0 \approx \frac{4\pi}{n}$$



Wind turbine power outputs

- Recall Betz: $$P_{t,\max} = 0.59\left[\frac{1}{2}\rho A_0 u_0^3\right]$$
- Optimal power extraction relies also on optimising tip speed ratio which depends both on turbine rotation velocity and wind speed (variable!)
  - **Fixed-speed turbines** reach design power coefficient only for one wind speed
  - **Variable-speed turbines** can adjust to wind speed – within limits
- For small wind speeds, below **cut-in speed** (typically 3 – 5 m/s), blades are fixed and no power generated
- Above this, power increases with cube of wind speed up to **rated wind speed** (typically 12 -15 m/s)
- Above rated wind speed, power absorbed is moderated to match electrical/mechanical limits
- Above **cut-out speed**, braking system stops turbine to avoid damage



P_t and C_p vs wind speed – ideal case and two real turbines

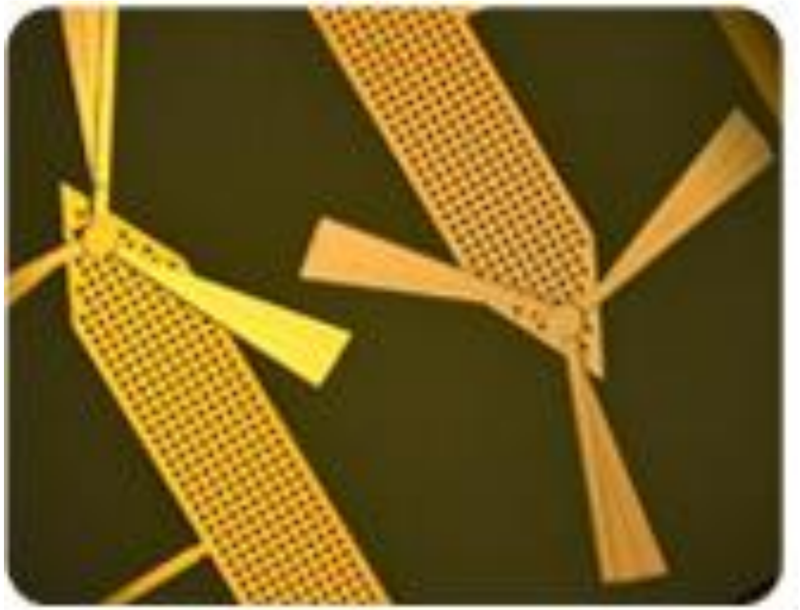
From Jaffe and Taylor

A big turbine!

- ***Vestas (2014 - today)***: 9 MW Turbine: 220 m high, 80 m blades
  - Cross-section Area ~ 3 football pitches
- Big power, but problems in high, variable winds

Extreme end of the scale: Micro turbines!

- 1.8 mm diameter – highly replicable manufacturing
  - MEMs: **Microelectromechanical systems**
- 200 – 500 mW (10 m/s wind) ? (claimed)
  - → Enough to recharge a smart phone?
- http://www.uta.edu/news/releases/2014/01/microwindmill-rao-chiao.php



A few turbine examples:

Micro – Rotor diameter ~ 2 mm – Power ~ 0.5 mW

Small – Rotor diameter ~ 10m – Power ~ 15 kW

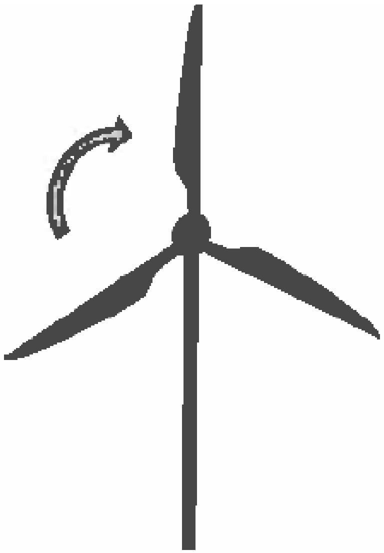
Medium - Rotor diameter ~ 25m – Power ~ 90 kW

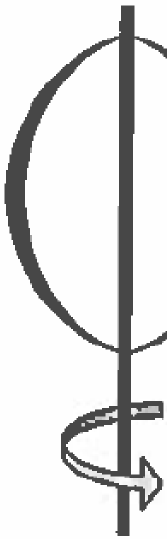
Largest– Rotor diameter ~ 160m – Power ~ 3600 kW

Assumes:
C_p=30% (average)
Wind speed 10 m/s
ρ = 1.2 kg.m^{-3}
λ = 6

Types of turbine

Horizontal axis | **Vertical axis (Darrieus)**





Darrieus

giromill

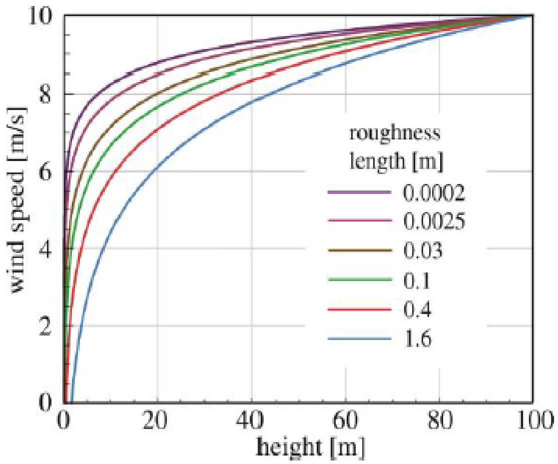
Many odd designs!

Turbine positioning: height

- **Planetary boundary layer** near Earth's surface
  - Generally about 1 km high but can be 3 m – 3 km
  - Often lower over ocean – hence benefits for offshore wind farms
- Semi-empirical dependence of wind speed with height above ground, z

$$u_0 = U_0 \ln\left(z/z_0\right)$$

- The roughness length is z_0 - depends on nature of terrain



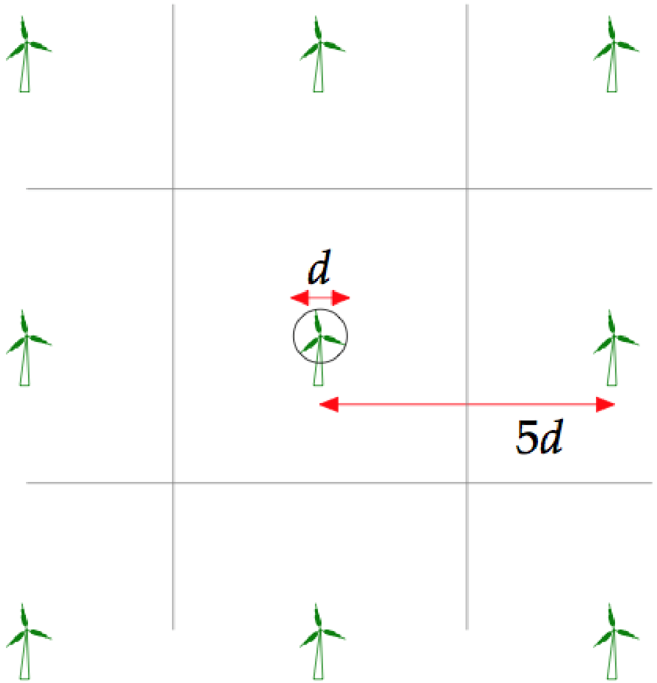
- Much variability superimposed on this mean profile (winds are gusty!)
- **Clearly, better to have tall turbines!**

Wind farm layout

- Turbines should not interfere with each other
  - Each turbine has a “wind shadow”
- **Empirically, if blade diameter is** d (twice radius), then **separate by ~ 5d**
- Turbine area (power) goes as d^2 and farm area goes as d^2

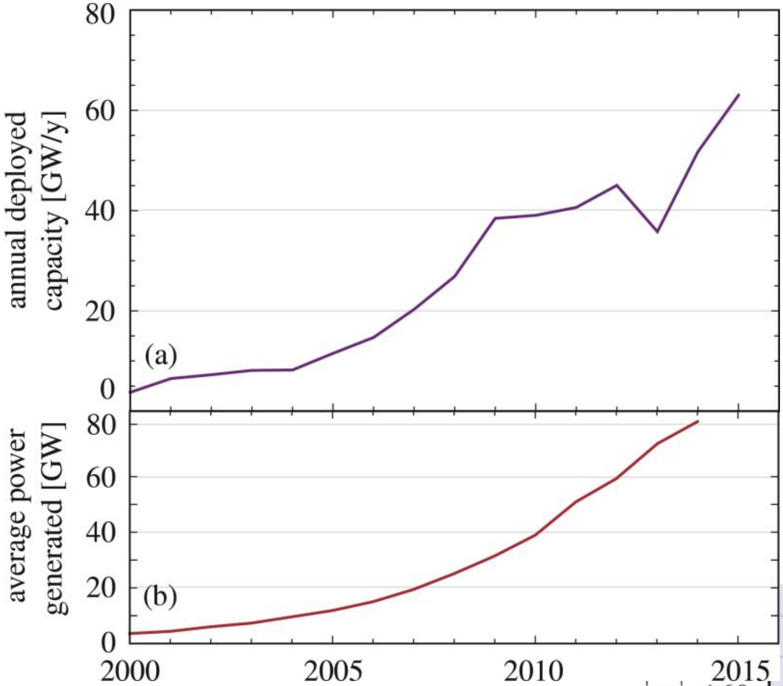
→ **Power goes as farm area**

- For average conditions in the UK: P/area ~ 2 - 3 W / m^2 (mean)
- **Need ~ 1000 km^2 for 2 – 3 GW**
  - ***Compare with biomass ~ 3000 km^2***



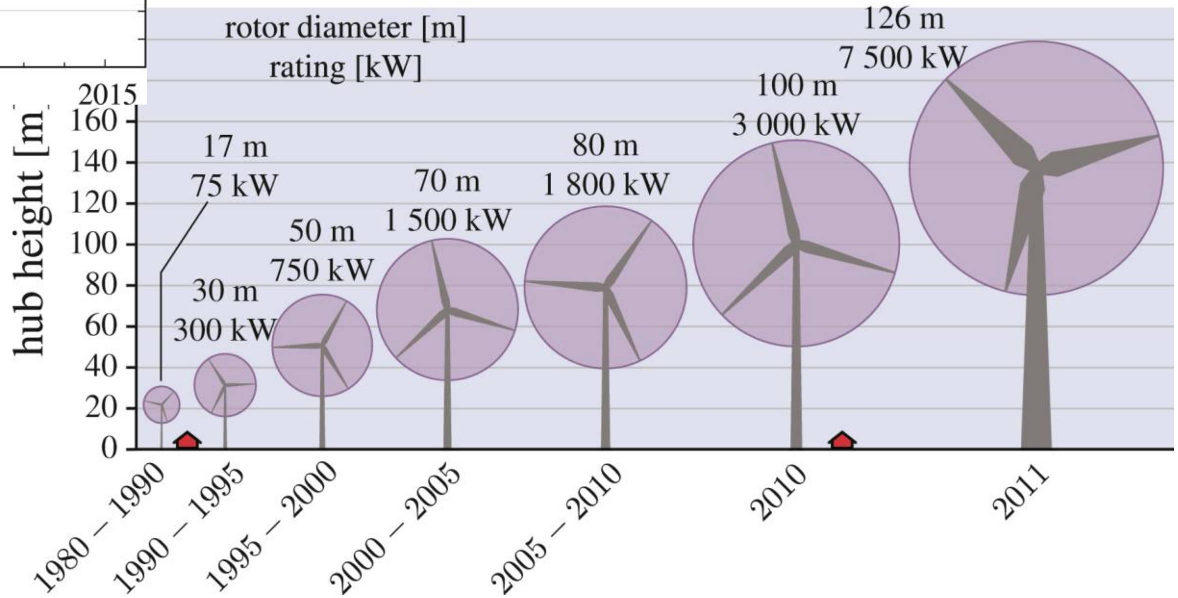
Ref. 5

Development of wind power



Jaffe and Taylor

Jaffe and Taylor



Wind power – pros and cons

- **Advantages**
  - No pollution
  - Renewable
  - Getting cheaper
  - Reliable
  - Can be done on small scale, good for small/remote communities
- **Disadvantages**
  - Preferably in a windy location
    - High wind variability in direction and speed
    - If too high → cannot operate/possible destruction
  - Needs to be orientated for maximum efficiency → moving parts
  - ~ 150 of max power turbines = nuclear power plant
  - Some of the best locations for wind (e.g. coastlines) might be problematic for other reasons (corrosion, etc.)
  - **Environment / visual and acoustic disturbances**
    - Need to cover large areas, often in areas of natural beauty e.g. coasts, moorlands
    - Migrating birds, problems with marine habitat

Example - Orkney wind power

- 500 domestic wind turbines – more than any other UK county
- Produces more than 100% of its energy needs from renewables – mostly wind

World's 2nd largest offshore wind farm

- Gwynt y Mor – off coast of North Wales
- 2 billion £
- 576 MW – enough to supply 33% of Welsh homes

"One of world's largest wind farms off coast of North Wales could be about to get BIGGER

The 160 turbine Gwynt y Mor, situated around 10 miles off the coast of Colwyn Bay, could get extended"

December 18th 2018 https://www.dailypost.co.uk/business/business-news/one-worlds-largest-wind-farms-15553952

13:59
◀ Facebook
5G

Wind power

UK windfarms generate record amount of electricity during Storm Malik

Wind speeds of up to 100 miles an hour recorded in Scotland helped send power generation soaring

Jillian Ambrose

Sun 30 Jan 2022 13.27 GMT

The UK's windfarms generated a record amount of renewable electricity over the weekend as Storm Malik battered parts of Scotland and northern England.

Wind speeds of up to 100 miles an hour recorded in Scotland helped wind power generation to rise to a provisional all-time high of more than 19,500 megawatts - or more than half the UK's electricity -

theguardian.com

Guardian 30/1/2022

Record breaking wind power in Storm Malik

https://www.theguardian.com/environment/2022/jan/30/uk-windfarms-generate-record-electricity-storm-malik?CMP=twt_a-environment_b-gdneco&fbclid=IwAR3kdN-VGpLhcmeZG_SzL-qIxNIGKVXWHgH5AgwsE3pMkze66PmtzjbWIug

Wider reading

- King *Physics of Energy Sources* Chapter 6
- MacKay. *Sustainable Energy Without the Hot Air* Chapters 4 and IIB
- Jaffe and Taylor *The physics of energy Chapters 27, 28, 30*