Answer 2.1

Based on the Law of Mass Action,

E is consumed by forward reaction, and produced by 2 reverse reactions, hence,

$$\frac{d[E]}{dt} = k_2[ES] + k_3[ES] \; - \; k_1[E][S] = (k_2 + k_3)[ES] - k_1[E][S]$$

Similarly,

$$\frac{d[S]}{dt} = k_2[ES] - k_1[E][S]$$

$$\frac{d[ES]}{dt} = k_1[E][S] - k_2[ES] - k_3[ES] = k_1[E][S] - (k_2 + k_3)[ES]$$

$$\frac{d[P]}{dt} = k_3[ES]$$

Answer 2.2

For easy writing, let e=[E], s=[S], c=[ES], p=[P], so the 4 equations transform to:

$$\frac{de}{dt} = (k_2 + k_3)c - k_1 es$$

$$\frac{ds}{dt} = k_2 c - k_1 es$$

$$\frac{dc}{dt} = k_1 es - (k_2 + k_3)c$$

$$\frac{dp}{dt} = k_3 c$$

The formula of the fourth-order Runge-Kutta method is

$y_{i+1} \; = \; y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$,

$t_{i+1} = t_i + \Delta t$

where

$k_1 = \Delta t \cdot f(y_i, t_i)$

$k_2 = \Delta t \cdot f(y_i + \frac{k_1}{2}, t_i + \frac{\Delta t}{2})$

$k_3 = \Delta t \cdot f(y_i + \frac{k_2}{2}, t_i + \frac{\Delta t}{2})$

$k_4 = \Delta t \cdot f(y_i + k_3, t_i + \Delta t)$

For the first equation $\frac{de}{dt} = (k_2 + k_3)c - k_1 es$

$$e_{i+1} = e_i + \frac{1}{6}(k_{1e} + 2k_{2e} + 2k_{3e} + k_{4e})$$

$$k_{1e} = \Delta t \cdot ((k_2 + k_3)c - k_1 es)$$

$$k_{2e} = \Delta t \cdot f(y_i + \frac{k_{1e}}{2}, t_i + \frac{\Delta t}{2})$$

$$k_{3e} = \Delta t \cdot f(y_i + \frac{k_{2e}}{2}, t_i + \frac{\Delta t}{2})$$

$$k_{4e} = \Delta t \cdot f(y_i + k_{3e}, t_i + \Delta t)$$

Similarly, we can work out for s, c and p

Answer 2.3

The Michaelis-Menten equation is

$$\mathrm{v} = \mathrm{k}_3[\mathrm{E}]_0 \frac{[\mathrm{S}]}{\mathrm{K_m} + [\mathrm{S}]}$$

where,

v the rate at which the product P is formed

k_3 the rate constant for dissociation of the enzyme-product complex

$[E]_0$ the enzyme concentration

[S] the substrate concentration

K_m the Michaelis constant which measures the affinity of the substrate for the enzyme.

Here, $\mathrm{k}_3 = 150/\mathrm{min}, [\mathrm{E}]_0 = 1\mu\mathrm{M}, [\mathrm{S}] = 10\mu\mathrm{M}$,so

$\mathrm{v} = 150 * 1 * \frac{10}{\mathrm{K_m}+10} = \frac{1500}{\mathrm{K_m}+10}$,hence

$$V_m = 150$$