ASP 4200 A1 Lewis Picker

Q1 i) convert L.E.E into a system of 1st order linear differential equations

begin with L.E.E.

$$\frac{d}{d\xi}\left(\xi^2 \frac{d\theta}{d\xi}\right) = -\xi^2 \theta^n$$

make substitution $U = \frac{d\theta}{d\xi}$

such that,

$$\frac{d}{d\xi}(\xi^2 U) = -\xi^2\theta^n = 2\xi U + \xi^2 U'$$

So $$U' = -\frac{2}{\xi} U - \theta^n$$

ii) find inner boundary condition needed for integration. i.e

$$\lim_{\xi\to 0} \frac{d^2\theta(\xi)}{d\xi^2} = \ldots \quad \text{well, } \frac{d^2\theta(\xi)}{d\xi^2} \Rightarrow -\frac{2}{\xi}U - \theta^n$$

Use l'hopital's rule where

$$\lim_{x\to a} \frac{f(x)}{h(x)} = \lim_{x\to a} \frac{f'(x)}{h'(x)} \quad \text{So,}$$

$$\lim_{\xi\to 0}\left(-\frac{2}{\xi}U - \theta^n\right) = \lim_{\xi\to 0}\left(\frac{-2U - \xi\theta^n}{\xi}\right) = \lim_{\xi\to 0} U'$$

where $f(\xi) = -2U - \xi\theta^n$, $g(\xi) = \xi$,

so

$$f'(\xi) = -2U' - \left(\theta^n + \xi\frac{d\theta^n}{d\xi}\right) \quad g'(\xi) = 1$$

$\therefore$

$$\lim_{\xi\to 0} \upsilon' = \lim_{\xi\to 0}\left(-2\upsilon' - \theta^n - \xi \frac{d\theta^n}{d\xi}\right)$$

$$= -2\lim_{\xi\to 0}\upsilon' - \lim_{\xi\to 0}\theta^n - \lim_{\xi\to 0} \underbrace{\xi \frac{d\theta^n}{d\xi}}_{=0}$$

by deffinition $\theta^n = 1$ when $\xi = 0$, so we get

$$3\lim_{\xi\to 0}\upsilon' = -1$$

$$\therefore \lim_{\xi\to 0}\upsilon' = -\frac{1}{3}.$$

iii) will atatch code to submission.

iv) this is in the code, since it breaks when theta < 0, the last values will be at ξ_1, so ξ_1 = Zeta_Sol[-1], $\left.\frac{d\theta}{d\xi}\right|_{\xi_1}$ = U_Sol[-1].

QI). from my model $\xi_1 = 3.65$, $\left.\frac{d\theta}{d\xi}\right|_{\xi_1} = -0.204$.

we calculate α via,

$M = -4\pi\alpha^3 \rho_c \xi_1^2 \left.\frac{d\theta}{d\xi}\right|_{\xi_1}$ such that $\alpha = 7.14\times10^9$ and we can ge K from

$\alpha^2 = (n+1)K/(4\pi G \rho_c^{\frac{n-1}{n}}) \Rightarrow K = 9.3\times10^{13}$

So that $P_c = K\rho_c^{\frac{n+1}{n}} = 4.4\times10^{17}$ Barye

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Q7) now that we have the central ~~temperature~~, pressure and density we can get the central temperature from the Ideal gas law,

$$P_c = \frac{k_B T_c \rho_c}{N m_u} \Rightarrow T_c = 2.01 \times 10^7 \, K.$$

The radius of the star,

$$R = \alpha \xi_1 = 2.61 \times 10^{10} \, cm.$$

Q 7 b) The actual values of the sun is,

$$R_\odot = 7 \times 10^{10} \, cm.$$

$$T_c = 1.5 \times 10^7 \, K.$$

$$P_c = 2.65 \times 10^{17} \, barye,$$

We are within the order of magnitude to the actual results which is good, reasons for some of these discrepancy might be the simplistic nature of the model, i.e we dont take into consideration the radiation pressure in the EOS, we also do not take into account energy generation that contributes to holding up the star, which would increase (or should increase) the size of our model. Finally the last but not the least possible reason would be that we are using a simple polytropic model with constant n, real stars are not polytropes and perhaps the model that yielded the solar quantities takes ~~into account~~ this into account. i.e it uses a more complex model with varying n values ect, I wouldn't really know.

Q2c) Lets first calculate the reduced mass.

$X_H = 0.715$, $X_{He} = 0.271$, $X_n = 1 - X_H - X_{He} = 0.014$.

So $\mu_I^{-1} = \sum \frac{X_i}{A_i m_u N_a} = \frac{1}{m_u N_a}\left(\left(\frac{0.715}{1}\right) + \frac{0.271}{4} + \frac{0.014}{14}\right)$

$\mu_E^{-1} = \frac{1}{m_u N_a}\left(0.715 + \frac{1}{2}0.271 + \frac{1}{2}0.014\right) = \cancel{0.738} = 1.165$

$\Rightarrow \mu_I = 1.28$. so $\mu = (\mu_I^{-1} + \mu_E^{-1})^{-1} = 0.61$.

from before we get $\xi_1 = 3.65$, $\left.\frac{d\theta}{d\xi}\right|_{\xi_1} = -0.204$.

Such that $\alpha = 6.24 \times 10^{10}$, $K = 2.37 \times 10^{15}$

and,

$R = \alpha \xi_1 = 2.28 \times 10^{11}$ cm. $P_c = 4.7 \times 10^{16}$ ~~g/cm~~ barye

In order to get the central temperature of the star one needs to consider the EoS

$$P = P_{gas} + P_{rad} = \frac{k_B T \rho}{\mu m_u} + \frac{a}{3} T^4$$

so

$$P_c = \frac{k_B T_c \rho}{\mu m_u} + \frac{a}{3} T_c^4 \Rightarrow 4.7 \times 10^{16} = 3.9 \times 10^8 T + 2.5 \times 10^{-15} T_c^4$$

Using a solver I get. ~~(from wolfram mathematica)~~
I used $\mu = \mu_I = 1.28$, corrected to $\mu = 0.61$

$T_c = \cancel{5.62 \times 10^7}$ K $= 4.49 \times 10^7$ K.

now $P_g = \frac{k_B T \rho}{\mu m_u} = \cancel{2.2 \times 10^{16}}$ ~~barye~~. 3.67×10^{16}.

So $\beta = \frac{P_{gas}}{P} = \frac{2.2 \times 10^{16}}{4.7 \times 10^{16}} = \cancel{0.47} = 0.78$.

Corrections are due to me forgetting it was a fully ionized gas.

Q2 a) as before,

$\mu_I = 1.28$ $\mu_E = 1.165$ so $\mu = (1.28^{-1} + 1.165^{-1})^{-1}$
$= 0.61$

from Kipp we have,

$$\varepsilon_{pp} = 2.57 \times 10^4 \psi f_{11} g_{11} \rho X_1^2 T_9^{-2/3} e^{-3.381/T_9^{1/3}}$$

and

$$g_{11} = (1 + 3.82 T_9 + 1.51 T_9^2 + 0.144 T_9^3 - 0.0114 T_9^4)$$

where $T_9 = T \times 10^{-9} K^{-1}$ and $X_1 = X_H = 0.715$.

and for simplicity. $\psi = 1$, $f_{11} = 2$.
and hydro equilibrium such that $E_{nuc} \equiv L_{star}$.

So $\varepsilon_{pp} = 5.14 \times 10^4 g_{11} \rho X_1^2 T_9^{-2/3} e^{-3.381/T_9^{1/3}}$

This time we need to solve for ρ_c
to achieve this we calculate every thing in ρ_c

So, $\rho(\rho_c)$, $P(\rho_c)$ and $T(\rho_c)$,
we obtain T from the EOS with radiation.

Once we have those quantities it is possible to calculate the total luminosity in ρ_c via,

$$L = \int_0^M \varepsilon_{pp} dm \equiv \sum_i^N \varepsilon_{pp}(\rho_c) \Delta m.$$

given that the star is in hydro static equilibrium.
therefore by setting $L \equiv L_\odot$ the suitable value for ρ_c was found to be

$\rho_c = 50 \, g/cm^3$

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The Radius of this star is

$R = 0.85 R_\odot$

The main reason for this discrepancy might be the polytropic index used for this model. ($n = 1.5$) where as Main sequence stars are usually modelled with $n = 3$. This has a significant effect on the ξ_1 value that determines the Radius of the Star. Other potential reasons might be that we have neglected other forms/modes of energy generation that typically Potts on the star.

2 b). Simmilar to part a) I computed the model for $n=3$, $M=10^5 M_\odot$ and use formula 18.65 in Kipp's book, to calculate the energy production from the CNO cycle.

This time I solve for ρ_c from the EOS,

where $$P(\rho_c) = \frac{R \rho_c T_c}{\mu} + \frac{a}{3} T_c^4$$

to get,

$\rho_c = [0.0033, 0.0065, 0.011, 0.018]$ g/cm^3

now that ρ_c is defined for each central temperature the modes are fully defined and, I calculate the Radius and Luminosity to be,

$R = [1317, 1053, 877, 752]$ Rsol

$L = [22, 2251, 79200, 1391000]$ Lsol.

Since the stars are assumed to be in Hydrostatic equilibrium L, was calculated via,

$$L = \int_0^M \varepsilon_{cno}\, dm = \sum_0^N \varepsilon_{cno} \Delta m.$$

Plots for $T(m)$ $\rho(m)$ against ~~closed~~ enclosed mass are supplied in the note book.

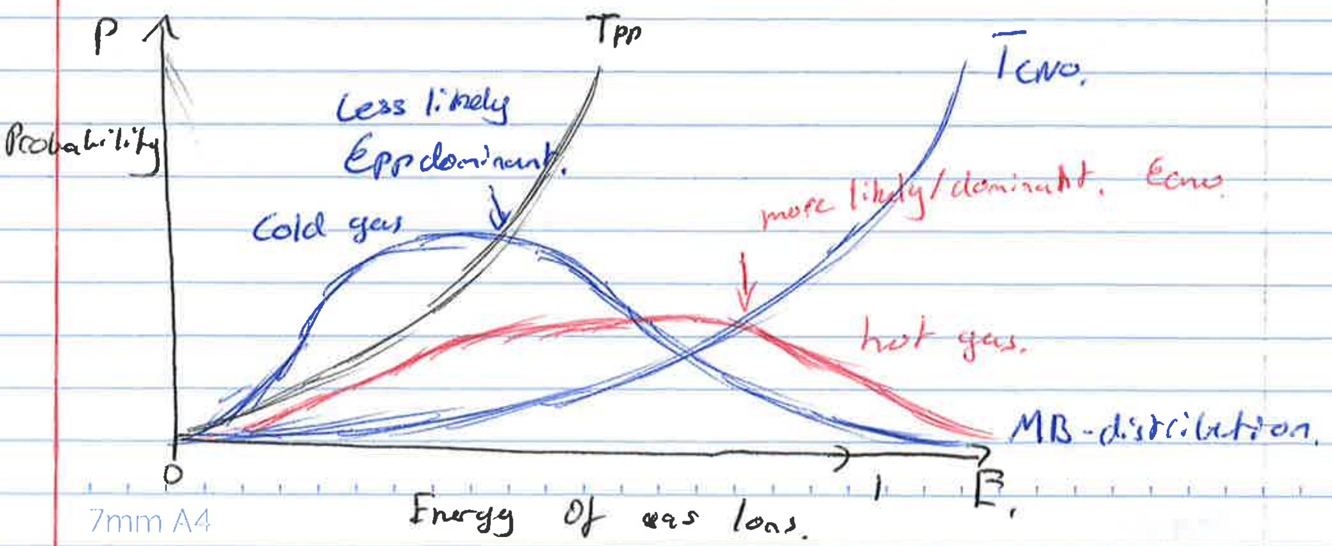
Q2 c)

~~In the note book I plotted the energy generation of both the PP chain and CNO cycle for central densities of $\rho_c = 50\ g\,cm^{-3}$ and $\rho_c =$~~
~~as was found in Part a) and b).~~
~~The temperature at which ~~the~~ ϵ_{pp} ~~dominates~~ $= \epsilon_{cno}$ is, $T_c =$ and $T_c =$ (from the plot).~~

~~Therefore the ~~energy~~ PP energy generation is ~~dominant~~ dominant for temperatures of~~

~~$T_c <$ and $T_c <$~~

The reason for this temperature sensitivity is due to the coloumb barrier of the atoms involved in the process, specifically.
The CNO cycle works via proton capture of ^{12}C, ^{13}C and ^{14}N for which the coloumb barrier is much higher than, ^{1}H, ^{2}H, ^{3}He, ^{7}Be, ^{7}Li, that are involved in the pp-chain. One can also think of this problem in terms of the Gamow window where Ions need more energy in order to overcome the coloumb barrier.



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~~Q2 c)~~

Q2 c) In order to find the necassary central temperature for which energy generation via the P-P chain dominates I calculated the total luminosity generated from the PP chain and the CNO cycle. Although in parts a, b we only considered energy generation via one or the other in reality it is both. I.e $L_{TOT} = L_{CNO} + L_{pp}$. (However due to temperature sensitivity one form will usually dominate)

I then plot the luminosities and it's clear that for $T_c < 2\times10^7$ K, pp energy is dominating.

Note! All plots are in the notebook. ~~with~~ along with the modules that was used in this assignment.

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