Assignemnet 2 Comp Astro

Lewis Picker

March 31, 2022

1 Question 1

1.1 Plots

1.1.1 Leapfrog

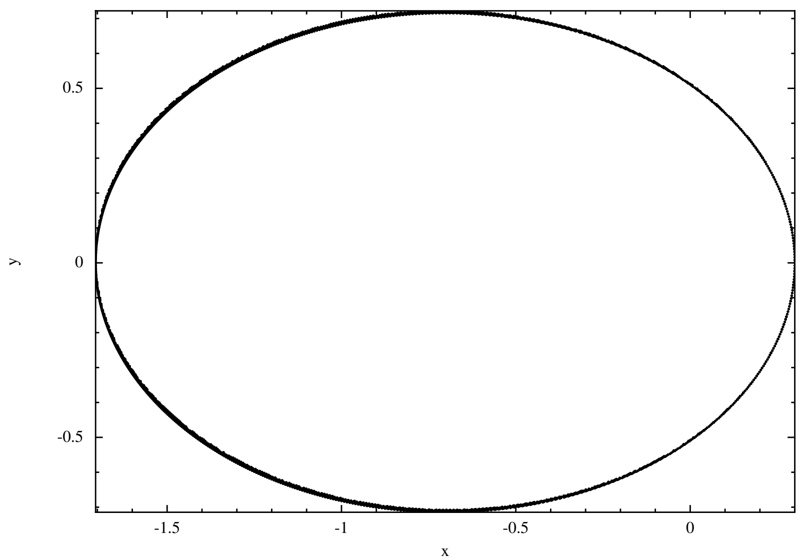

Figure 1: Trajectory of a test particle in orbit around a central point mass. With eccentricity 0.7 with time steps of 0.01 and 5000 iterations of the leapfrog method

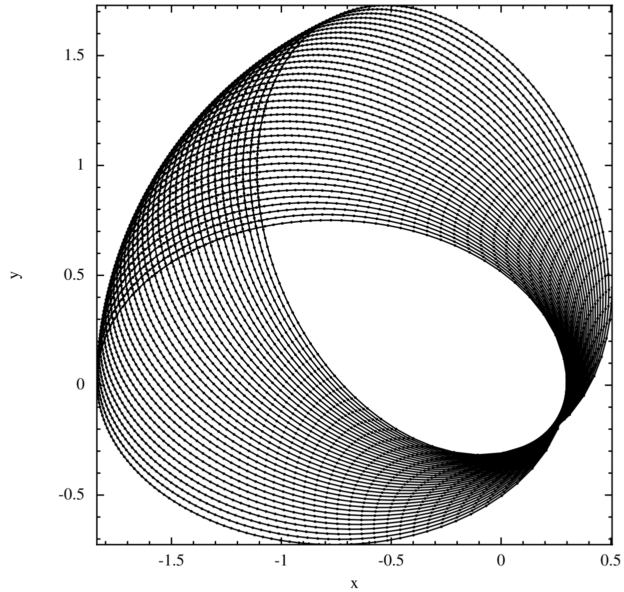

Figure 2: Trajectory of a test particle in orbit around a central point mass. With eccentricity 0.7 with time steps of 0.05 and 5000 iterations of the leapfrog method

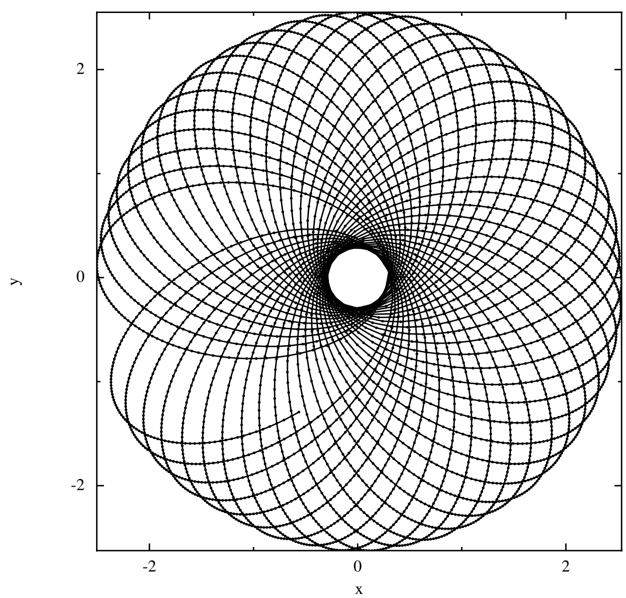

Figure 3: Trajectory of a test particle in orbit around a central point mass. With eccentricity 0.7 with time steps of 0.1 and 5000 iterations of the leapfrog method

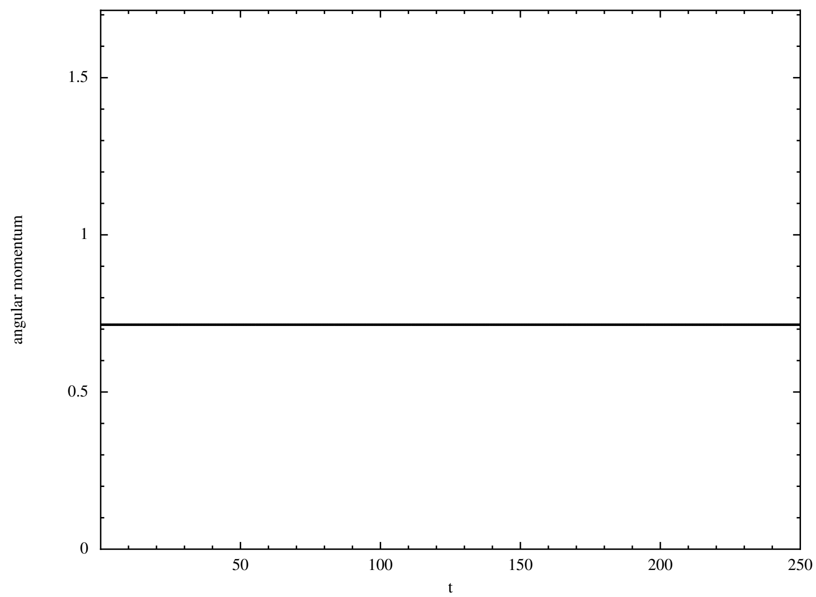

Figure 4: Angular momentum against time for the particle in orbit with timestep 0.05 for the leapfrog method

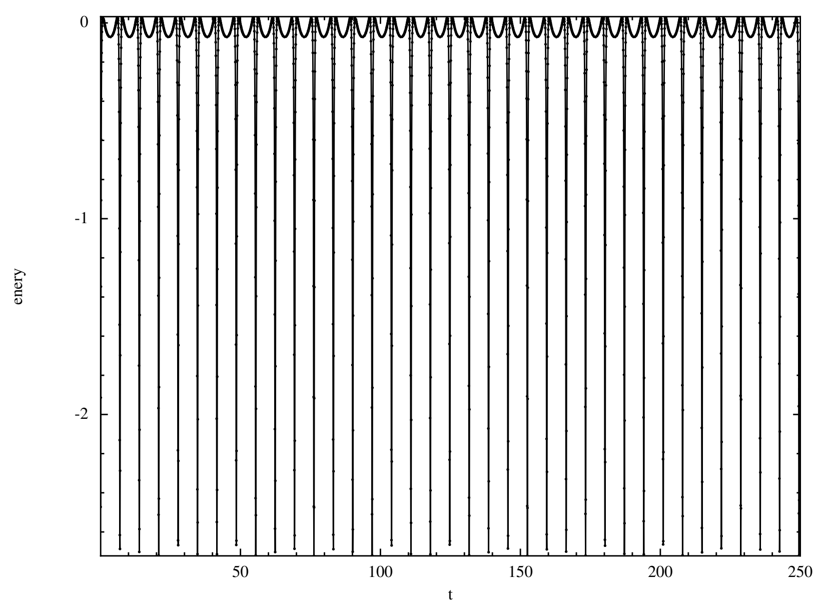

Figure 5: Total energy against time for the particle in orbit with timestep 0.05 for the leapfrog method

1.1.2 RK4

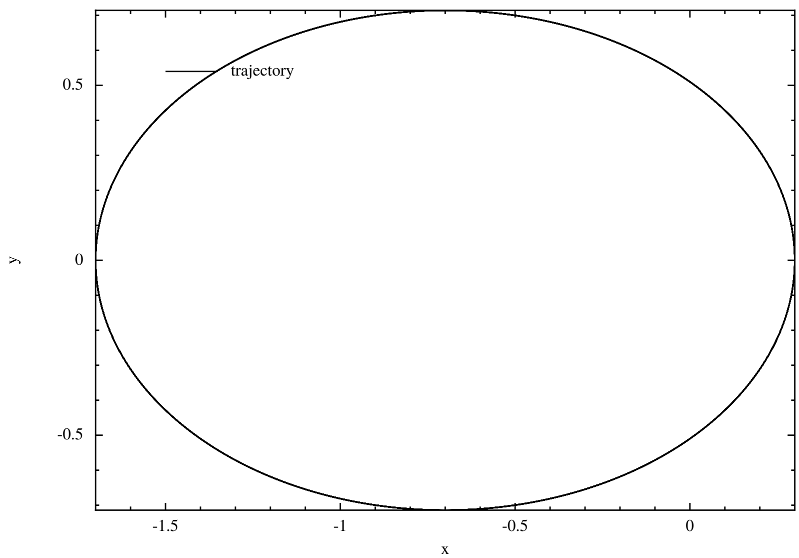

Figure 6: Trajectory of a test particle in orbit around a central point mass. With eccentricity 0.7 with time steps of 0.01 and 5000 iterations of the RK4 method

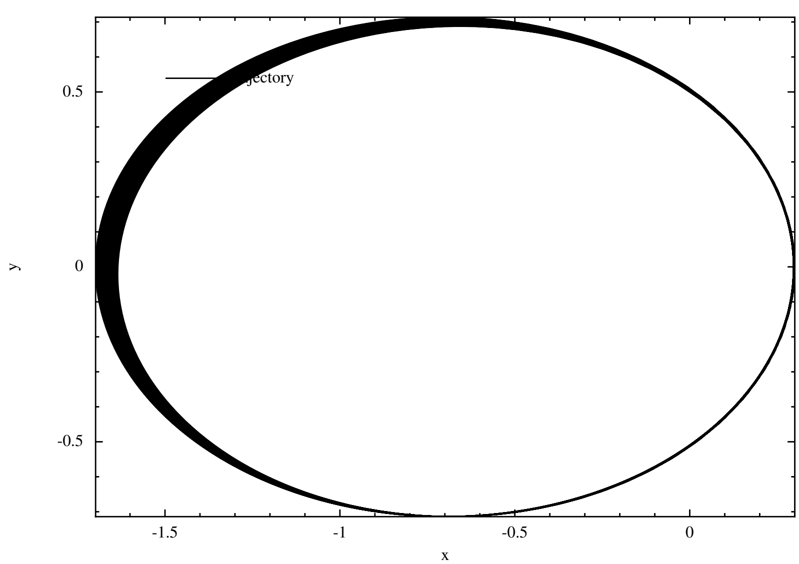

Figure 7: Trajectory of a test particle in orbit around a central point mass. With eccentricity 0.7 with time steps of 0.05 and 5000 iterations of the RK4 method

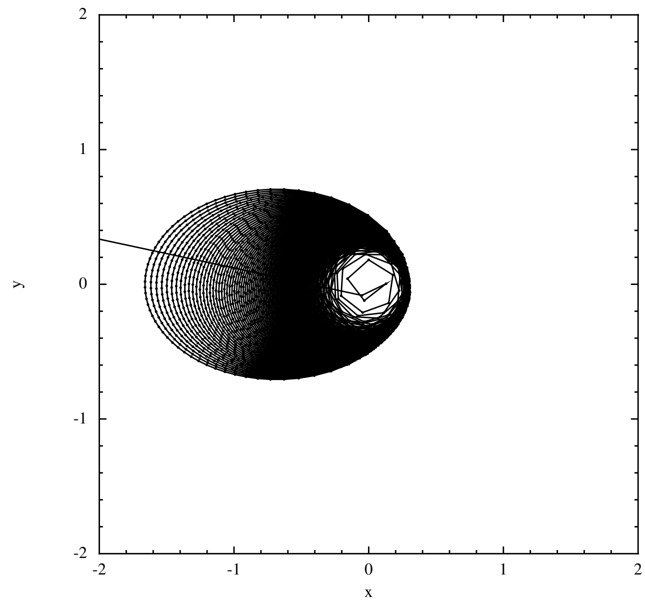

Figure 8: Trajectory of a test particle in orbit around a central point mass. With eccentricity 0.7 with time steps of 0.1 and 5000 iterations of the RK4 method

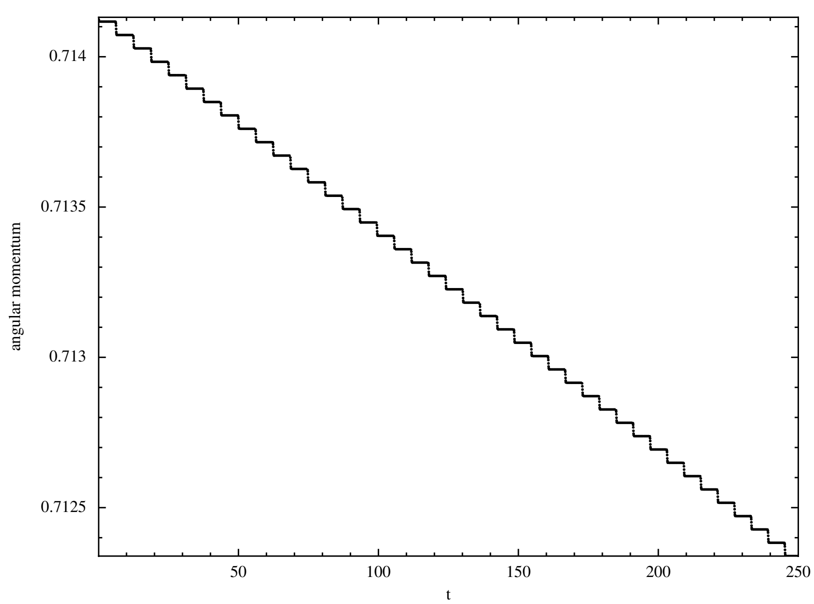

Figure 9: Angular momentum against time for the particle in orbit with timestep 0.05 for the RK4 method

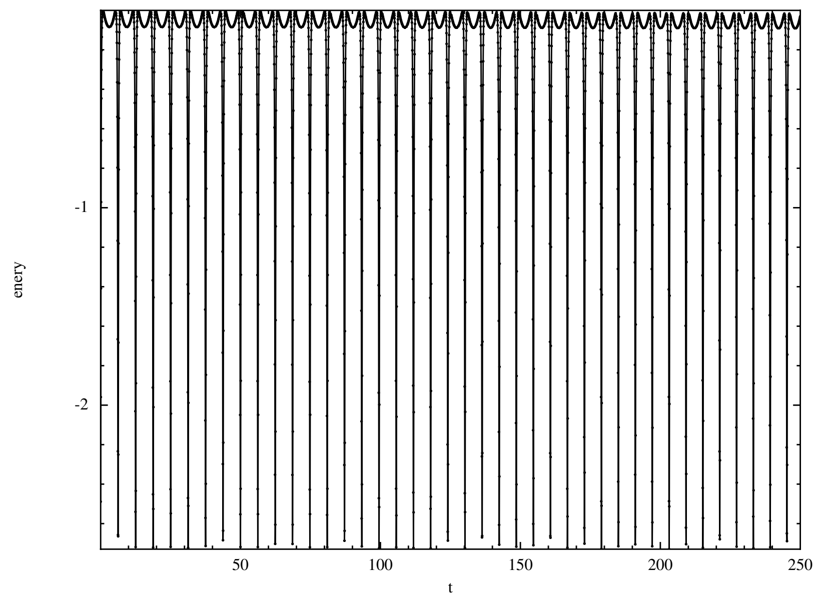

Figure 10: Total energy against time for the particle in orbit with timestep 0.05 for the RK4 method. I know it looks identical to the leapfrog method. In my code the energy is calculated simultaneously with the angular momentum with the same values I have no Idea how this happened since I cleaned all the leapfrog results. I will provide my code.

1.2 Explanations

Looking at 4 and 5 it is at least clear that angular momentum is being conserved, where as the total energy tends to dip dramatically. These dips are cause by errors picked up from the time steps. However it is important to notice that these dips are cyclic with a period of about t = 7. Otherwise the period of the particle in the orbit and hence when the completes its orbit it has the same energy. This implies that energy is indeed conserved and it is due to the time reversal properties of the leapfrog integrator. 9 it is clear that angular momentum is not conserved and nor is the energy (from my knowledge). This is because RK4 is simply designed for accuracy and not for conservation and therefore drops every orbit. However comparing 1 with 6 and 2 with 7 it should be clear that RK4 is depicting a steadier elliptical orbit. This is because on shorter timescales the energy loss from RK4 is negligible and hence more accurate. Comparing 3 to 8 the timescale is large enough for the orbit to decrease and eventually break down. Since Leapfrog is conserving energy the solution is somewhat plausible as if the orbit was processing.

2 Question 2

2.1 1.75D MHD shock tube

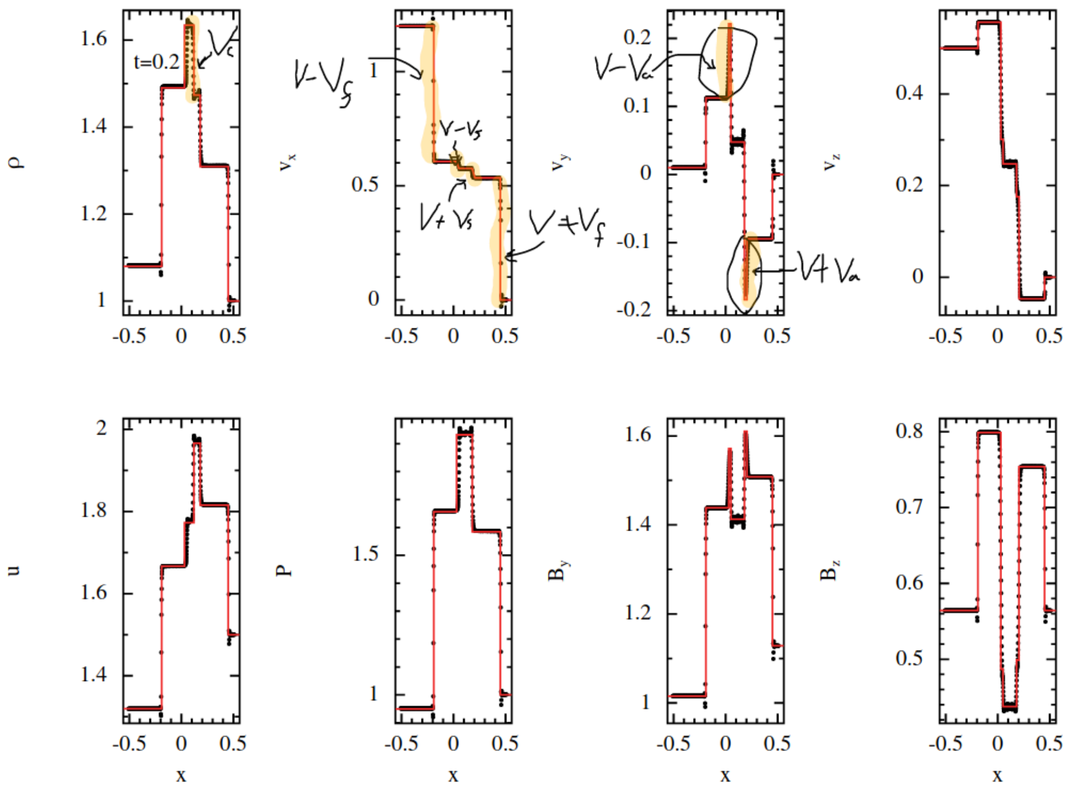

Figure 11: Magnetised shock tube results at 0.15t displaying various parameters against x

From the theory we expect 7 kinds of wave speeds 2 traverse waves deriving from the Aflven velocity ($V - V_a, V + V_a$) and 4 longitudinal waves ($V \pm V_f, V \pm V_s$) and finally the discontinuity wave travelling at constant speed. From 11 we can see clearly see 4 discontinuities in the V_x vs x coming from the slow and fast waves. By looking at the V_y vs x we can observe the transverse Alfven velocities. Finally we can observe the discontinuity travelling at a constant speed in the ρ vs x plot.

2.2 2D MHD rotor test

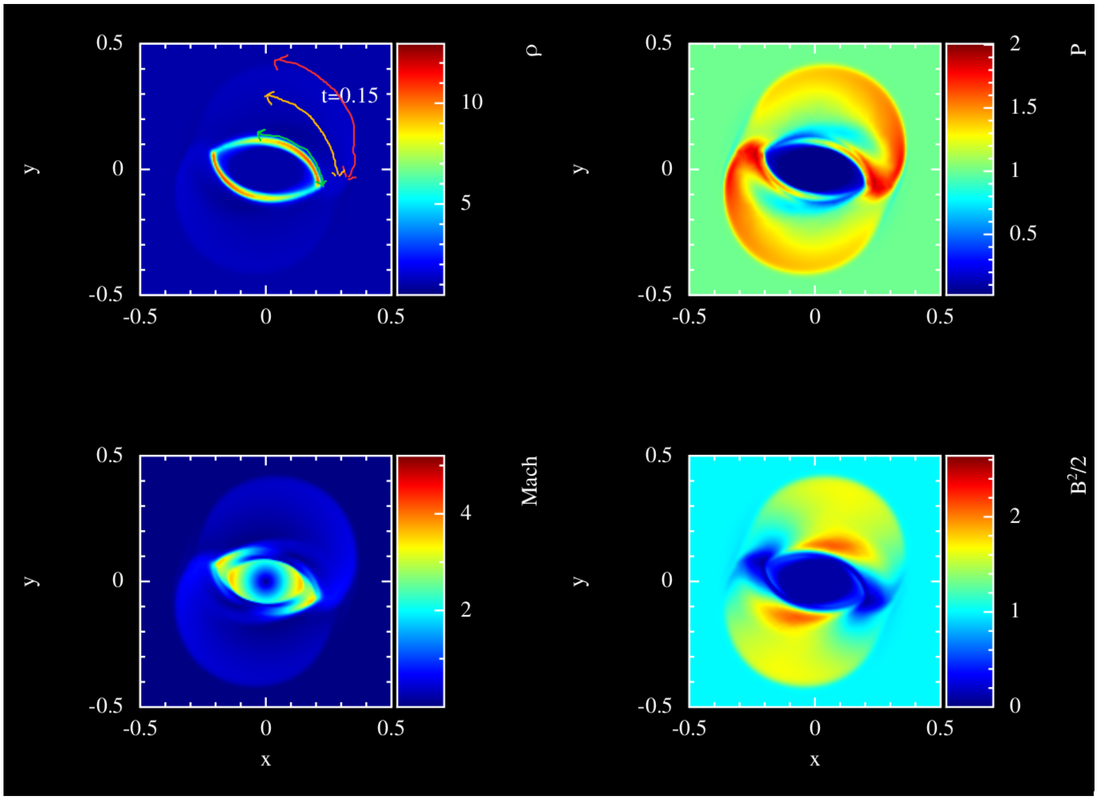

Figure 12: A plot rendering the density, pressure magnetic energy and Mach number at t = 0.15 for a rotating high density disk embedded in a lower density. Where the red line shows the fast wave front, the green line the slow wave-front and the orange line the Alfven wave-front

To calculate the Alfven velocity I used the following expression:

$$v_a^2 = \frac{B_0^2}{\mu_0 \rho_0}$$

Where in our case $\mu_0 = \rho_0 = 1$ and $\vec{B} = [\frac{5}{\sqrt{4\pi}}, 0, 0]$, hence $v_a \approx 1.41$. Therefore after 0.15 seconds the wave should move about 0.21 units, however the initial discontinuity was at 0.1 so we expect it at 0.31. Our velocities of our waves are calculated by

$$\frac{\omega^2}{k^2} = V^2 = \frac{1}{2}(c_s^2 + v_a^2 \pm \sqrt{(c_s^2 + v_a^2)^2 - 4c_s^2 v_a^2 cos(\theta)})$$

Where θ is the angle between the magnetic field and the wave direction. Since the magnetic filed is entirely in the x direction for simplicity we can calculate the velocity along the y-axis hence $\theta = 90°$ and since $c_s = 1$ hence we can calculate the wave speeds to be. $V_f = 1.73$ and $V_s = 0$ and hence after $t = 0.15s$ we expect the wave-front to have traveled a distance of 0.36 and 0.1 for the fast and slow waves respectively. Solving for the velocities parallel to the field would mean that $\theta = 0°$ and hence the velocities would be $v_f = 1.41$ and $v_s = 0.986$ traveling 0.31 and 0.25 respectively.

3 Question 3

3.1 plots

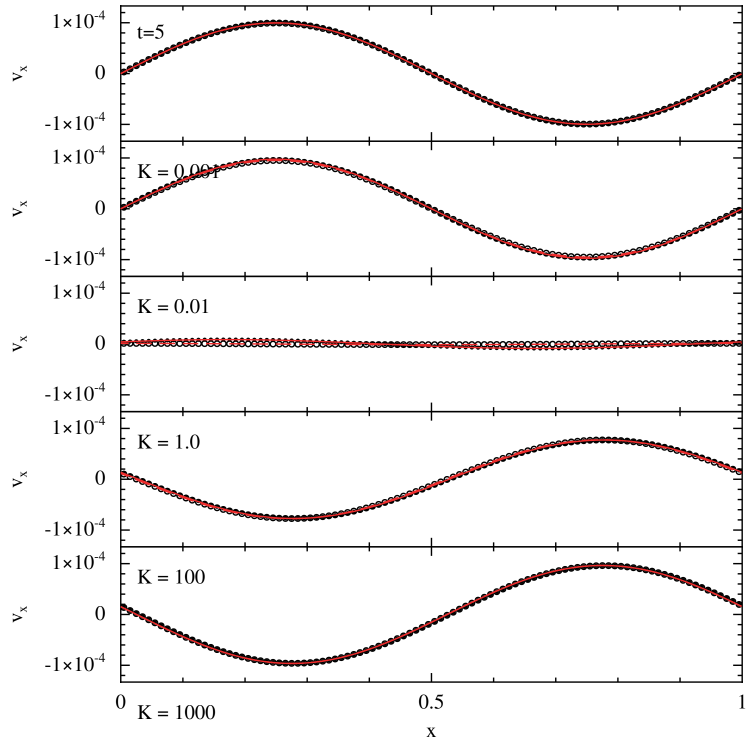

Figure 13: A plot at t = 5 (at 5 wave periods) for K values of 0.001, 0.01, 1.0, 100 and 1000. Using the one fluid equation

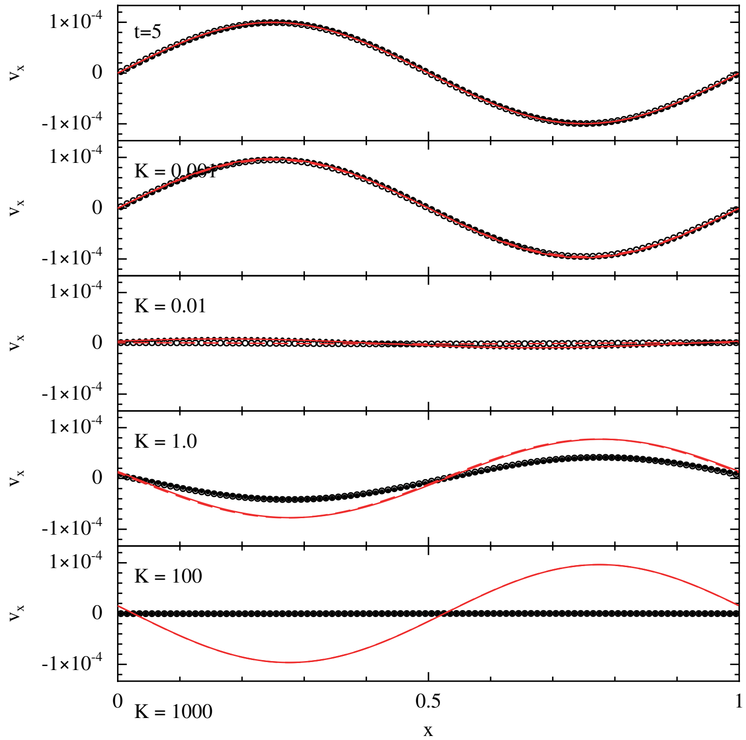

Figure 14: A plot at t = 5 (at 5 wave periods) for K values of 0.001, 0.01, 1.0, 100 and 1000. Using the two fluids equations

3.2 PSQ 8

Refer to the attached PDF for working out of question 8.

3.3 Solving K

Using the expression for stopping time

$$t_s = \rho_d \rho_g / k(\rho_d + \rho_g)$$

where the density of the gas and dust are both 1 respectively. We obtain the stopping time of 500, 50, 0.5, 0.005 and 0.0005, for the values of k = 0.001, 0.01, 1.0, 100 and 1000 respectively. Referring to the dispersion relation:

$$(\omega - k^2 c_s^2) + \frac{i}{\omega t_s}(\omega^2 - \tilde{c_s}^2 k^2)$$

We notice that for low values of t_S results in $\omega^2 \approx c_s^2 K^2$ which can be interpreted as undamped waves in a gas. Where as for larger values of t_s results in $\omega^2 \approx \tilde{c_s}^2 K^2$. undamped waves in a mixture. Which is what we more or less witness in 13. The sweet spot as my lecturer would call it is in the middle where the interesting physics happen. The term $\frac{i}{\omega t_s}$ provides a dampening on the waves and is largest when $\omega t_s \approx 1$ which is obvious occurring in 13 for k = 1.0.

3.4 Analysis of K

Referring to 14 the two fluid method gives accurate results w.r.t the analytical solution when $k \leq 1.0$. In the stokes regime this correlates to a larger stopping time which is in term proportional to the grain size (large and medium grains). And for the single fluid method 13 the method matches the analytical solution for all values of K corresponding to small and large dust grains and everything in between.