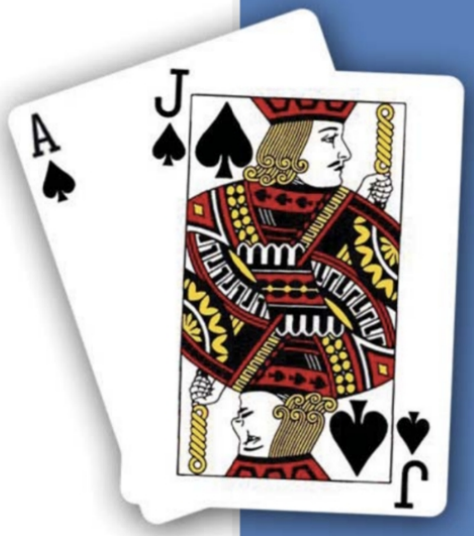
MATH 3808

Professor
Dr. Jason Gao

Blackjack Game Project Report



Group Members

Eren Sulutas
101101873

Nabeel Warsalee
101103167

Cailyn Edwards
100956026

Jingyi Wang
101082676

Date

April.23.2021

Project Name	Write a program to find the house edge for Blackjack using the basic strategy and single deck.
Descriptions for Methods	Our method for computing the House Edge will be to create a Monte-Carlo experiment in which we run multiple trials of a Blackjack Game having the player utilize the basic strategy. From these multiple trials, we extract the win/lose probabilities for both Dealer and Player and from that are able to compute the House Edge.
Instructions for running the program	1. Download the latest version of python (if not installed): https://www.python.org/downloads/release 2. Install the dependencies by entering the following into the terminal: a. pip install pydealer b. pip install yaspin 3. Start the simulation by entering the following into the terminal: py simulator.py
Repository	https://github.com/Math3808-Project/Blackjack-Simulator

Blackjack Rules Assumed

- Single deck
- Dealer stands on any 17 (including soft 17)
- No hole card: dealer does not draw nor consult their second card until after the player's final decision
- Split up to four hands
- Double down on any two initial cards, except for split Aces
- Split Aces may not hit (stand after drawing second card)
- No Blackjack after receiving a 10-value card after splitting Aces
- Player can only double/split on the first move, or the first move of a hand created by a split
- Surrender not permitted

Goals

Output the house edge with respect to Basic Strategy we use based on great quantities of (Monte-Carlo) simulations.

Basic Strategy

Each blackjack game has a *basic strategy*, which prescribes the optimal method of playing any hand against any dealer up card so that the long-term house advantage (the expected loss of the player) is minimized. A standard basic strategy chart can only provide with the strategy for one set of rules, such as dealer hits on Soft 17.

Composition-dependent:

Optimal strategy based on the composition of cards in a player's hand and the dealer's up card.

Basic strategy is based upon a player's point total and the dealer's visible card. Players may be able to improve on this decision by considering the precise composition of their hand, not just the point total. For example, players should ordinarily stand when holding 12 against a dealer 4. However, in a single deck game, players should hit if their 12 consists of a 10 and a 2. The presence of a 10 in the player's hand has two consequences:

- It makes the player's 12 a worse hand to stand on (since the only way to avoid losing is for the dealer to go bust, which is less likely if there are fewer 10s left in the shoe).
- It makes hitting safer, since the only way of going bust is to draw a 10, and this is less likely with a 10 already in the hand.

However, even when basic and composition-dependent strategies lead to different actions, the difference in expected reward is small, and it becomes even smaller with more decks. Using a composition-dependent strategy rather than basic strategy in a single deck game reduces the house edge by 4 in 10,000, which falls to 3 in 100,000 for a six-deck game.

Total-dependent: *(our approach)*

Optimal strategy based on the total (hard or soft) in a player's hand and the dealer's up card. Hence the player does not distinguish among (10,6), (9,7), and (2,4,5,5).

Most blackjack basic strategy charts are said to be "total dependent." That means the total of the player's cards is considered, but not the specific composition. Total dependent basic strategy also considers whether the hand is soft or hard, and whether doubling, splitting, or surrender is possible.

Basic Strategy Specifications (total-dependent)

There are two slightly different strategies depending on whether Dealer stands or hits on Soft 17 based on total-dependent basic strategy.

In our case, we assume Dealer stands on Soft 17, chart shows below:

Single-Deck, Dealer Stands on Soft 17

Player	Dealer's card									
Hard	2	3	4	5	6	7	8	9	10	A
5-7	H	H	H	H	H	H	H	H	H	H
8	H	H	H	Dh	Dh	H	H	H	H	H
9	Dh	Dh	Dh	Dh	Dh	H	H	H	H	H
10	Dh	Dh	Dh	Dh	Dh	Dh	Dh	Dh	H	H
11	Dh	Dh	Dh	Dh	Dh	Dh	Dh	Dh	Dh	Dh
12	H	H	S	S	S	H	H	H	H	H
13	S	S	S	S	S	H	H	H	H	H
14	S	S	S	S	S	H	H	H	H	H
15	S	S	S	S	S	H	H	H	H	H
16	S	S	S	S	S	H	H	H	Rh	Rh
17+	S	S	S	S	S	S	S	S	S	S
Soft	2	3	4	5	6	7	8	9	10	A
13	H	H	Dh	Dh	Dh	H	H	H	H	H
14	H	H	Dh	Dh	Dh	H	H	H	H	H
15	H	H	Dh	Dh	Dh	H	H	H	H	H
16	H	H	Dh	Dh	Dh	H	H	H	H	H
17	Dh	Dh	Dh	Dh	Dh	H	H	H	H	H
18	S	Ds	Ds	Ds	Ds	S	S	H	H	S
19	S	S	S	S	Ds	S	S	S	S	S
20+	S	S	S	S	S	S	S	S	S	S
Splits	2	3	4	5	6	7	8	9	10	A
2,2	Ph	P	P	P	P	P	H	H	H	H
3,3	Ph	Ph	P	P	P	P	Ph	H	H	H
4,4	H	H	Ph	Pd	Pd	H	H	H	H	H
6,6	P	P	P	P	P	Ph	H	H	H	H
7,7	P	P	P	P	P	P	Ph	H	Rs	H
8,8	P	P	P	P	P	P	P	P	P	P
9,9	P	P	P	P	P	S	P	P	S	S
A,A	P	P	P	P	P	P	P	P	P	P

www.wizardofodds.com

H	Hit
S	Stand
Dh	Double if allowed, otherwise hit
Ds	Double if allowed, otherwise stand
P	Split
Ph	Split if double after split is allowed, otherwise hit
Pd	Split if double after split is allowed, otherwise double
Rh	Surrender if allowed, otherwise hit
Rs	Surrender if allowed, otherwise stand

Also, the other case is Dealer hits on Soft 17:

Single-Deck, Dealer Hits on Soft 17

Your hand	Dealer's card 2	3	4	5	6	7	8	9	10	A
4-7	H	H	H	H	H	H	H	H	H	H
8	H	H	H	Dh	Dh	H	H	H	H	H
9	Dh	Dh	Dh	Dh	Dh	H	H	H	H	H
10	Dh	Dh	Dh	Dh	Dh	Dh	Dh	Dh	H	H
11	Dh	Dh	Dh	Dh	Dh	Dh	Dh	Dh	Dh	Dh
12	H	H	S	S	S	H	H	H	H	H
13	S	S	S	S	S	H	H	H	H	H
14	S	S	S	S	S	H	H	H	H	H
15	S	S	S	S	S	H	H	H	H	Rh
16	S	S	S	S	S	H	H	H	Rh	Rh
17	S	S	S	S	S	S	S	S	S	Rs
18+	S	S	S	S	S	S	S	S	S	S
Soft	2	3	4	5	6	7	8	9	10	A
13	H	H	Dh	Dh	Dh	H	H	H	H	H
14	H	H	Dh	Dh	Dh	H	H	H	H	H
15	H	H	Dh	Dh	Dh	H	H	H	H	H
16	H	H	Dh	Dh	Dh	H	H	H	H	H
17	Dh	Dh	Dh	Dh	Dh	H	H	H	H	H
18	S	Ds	Ds	Ds	Ds	S	S	H	H	H
19	S	S	S	S	Ds	S	S	S	S	S
20+	S	S	S	S	S	S	S	S	S	S
Splits	2	3	4	5	6	7	8	9	10	A
2,2	Ph	P	P	P	P	P	H	H	H	H
3,3	Ph	Ph	P	P	P	P	Ph	H	H	H
4,4	H	H	Ph	Pd	Pd	H	H	H	H	H
6,6	P	P	P	P	P	Ph	H	H	H	H
7,7	P	P	P	P	P	P	Ph	H	Rs	Rh
8,8	P	P	P	P	P	P	P	P	P	P
9,9	P	P	P	P	P	S	P	P	S	Ps
A,A	P	P	P	P	P	P	P	P	P	P

www.wizardofodds.com

H	Hit
S	Stand
Dh	Double if allowed, otherwise hit
Ds	Double if allowed, otherwise stand
P	Split
Ph	Split if double after split is allowed, otherwise hit
Pd	Split if double after split is allowed, otherwise double
Ps	Split if double after split is allowed, otherwise stand
Rh	Surrender if allowed, otherwise hit
Rs	Surrender if allowed, otherwise stand

Other basic strategy rules:

- Never take insurance or "even money." The house edge on insurance is 5.9%, based on one deck.

- If there is no row for splitting (fives and tens), then look up your hand as a hard total (10 or 20).

- If you cannot split because of a limit on re-splitting, then look up your hand as a hard total, except aces. In the extremely unlikely event, you have a pair of aces you cannot re-split and drawing to split aces is allowed, then double against a 5 or 6, otherwise hit.

Computing the House Edge

As mentioned earlier, we attempted to determine the house edge of Blackjack with one deck where the player utilizes the basic strategy by performing a Monte-Carlo simulation. We achieved this by running multiple games of Blackjack with the player using the basic strategy and the dealer using the default strategy of standing on soft 17. The simulator supported a wide range of available Blackjack options, such as splits and doubles.

In the design of our simulator, each game returns a report in the form of a Python dictionary that contains important metadata about the game that was just played. The report includes information such as the overall result of the game (i.e., the player's net win/loss), the sum of the player's hand of cards, the sum of the dealer's hand of cards and the specific scenario that the game saw. Examples of scenarios include if there was a blackjack, if the game saw a double down initiated by the player and if there was a split initiated by the player. Each instance of a Blackjack game assumes that the Player's bet was $1 and performs the necessary checks and calculations to account for doubling of the bet to ensure the proper net result is returned.

After each instance of a Blackjack game has concluded, the overall result of the game is extracted from the report and added onto a total that represents

the player's net winnings for the course of the simulation, in other words their net earnings over the number of trials/games that were run. The net win total for a given simulation run along with the number of trials run for that simulation are added onto a global total for games run and net win. This global total is stored in the form of a CSV file and has its values adjusted after every simulation run to account for the new results for that simulation. It is from this CSV file that that player's expected value is computed for all the collected runs by simply dividing the player's net earnings by the total number of games that have been recorded. This value is then multiplied by negative one to reflect the House's expected value.

Trials Result Conclusion

The number of trials ran to produce an expected value result that was accurate to a hundredth was a million. A million trials take roughly four minutes, and still produce a variance in the result. An increased number of trials, say 10 million or 20 million for instance, will yield closer results to the expected value, however, at these ranges the program will be expected to run for larger periods of time.

In attempt to counteract the resulted variance levels, our methodology employed the implementation of an external file to maintain data from previous runs, namely the number of trials, the betting amount, and the overall result. Through this, we were able to average out the results from various smaller trials to achieve results closer to the true house edge in the game of Blackjack with the rules defined on page 1 of this document.

In attempt to counteract the resulted variance levels, our methodology employed the implementation of an external file to maintain data from previous runs, namely the number of trials, the betting amount, and the overall result.

Through this, we were able to average out the results from various smaller trials to achieve results closer to the true house edge in the game of Blackjack with the rules defined on page 1 of this document.

At the time of writing, there has been a recording of 40,000,000 total games run in the simulation. The total net sum of player results in the CSV file database is $\$ - 4307.0$ and the player's expected value is:

$$\$ - \frac{-4307.0}{40{,}000{,}000} = -0.000107675.$$

Resultantly, the observed house edge is 0.0107%. This value signifies the average or expected house win per $1 player bet. Thus, for a game of Blackjack with a single deck of cards and the rules outlined on page 1, the house edge when the player utilizes the basic strategy will be approximately 0.0107%.

Another set of values provided by the simulator is the final hand distribution for the dealer. The dealer's final hand is defined as a hand that was essential in determining the outcome of the game, meaning that the player did not bust or win with a Blackjack or natural starting hand. According to the dealer rule, a dealer must hit on 16 and stand on 17, soft hands included. This denotes that the dealer's final hand sum could be in the range of 17 to 21 and include cases such as a natural (a hand with only an ace and a 10-value card) and busts (hands that exceed a sum of 21).

In his lecture on March 21, 2021, Professor Jason Gao provided the probabilities of the dealer's final hand distribution given a single deck. Comparing these values with a sample run of a million trials from the simulation will provide further insight as to where a sample size of 1 million

trials fails to reach a precise result:

Dealer's Final Hand:	17	18	19	20	21	BJ	Bust
Target Distribution	.1458	.1381	.1348	.1758	.0736	.0483	.2836
Simulation Distribution (With 1 Million Trials)	.1444	.1355	.1326	.1672	.0787	.0441	.2975

Upon initial inspection, the dealer's final hand distribution from a simulation of 1 million games of Blackjack appear to be mostly accurate, within a degree of two decimal places. The distribution for the dealer's final hand becoming 17, 18, 19, 21, and a Blackjack are consistently near their expected values when running the simulation. However, the distribution for the dealer final hands of 20 and Bust are off, and by more than a percent in average cases. With higher trials, it is expected that these values become more accurate as well, though it is intriguing how, for the most part, these two hands that strike the greatest difference between the expected and observed results. A possible theory is that the 10-value cards that could be used to make up a 20-sum hand are most often than not the cards being drawn that result in a bust for the dealer. Since the dealer must hit when their sum is below 17, a dealer with a sum between 12 and 16 risks busting their hand with the chance that they draw a 10-value card.

Annotated Bibliography

Single-Deck Blackjack Strategy - Wizard of Odds,August.13.2019

https://wizardofodds.com/games/blackjack/strategy/1-deck/

Total Dependent vs. Composition Dependent Basic

https://wizardofodds.com/games/blackjack/composition-dependent-benefit/

Blackjack - Wikipedia

https://en.wikipedia.org/wiki/Blackjack

Blackjack Basic Strategy Engine - BlackjackInfo.com

https://www.blackjackinfo.com/blackjack-basic-strategy-engine/?numdecks=1&soft17=s17&dbl=all&das=yes&surr=ns&peek=yes