Poisson Image Editing

Goal

Direct cloning

Why does it look wrong?

- Land and McCann, 1971

- For the human eye
  - Low gradients
    - Hard to notice difference between pixels
  - Second order variations (Laplace)
    - We notice them a lot

Seamless cloning

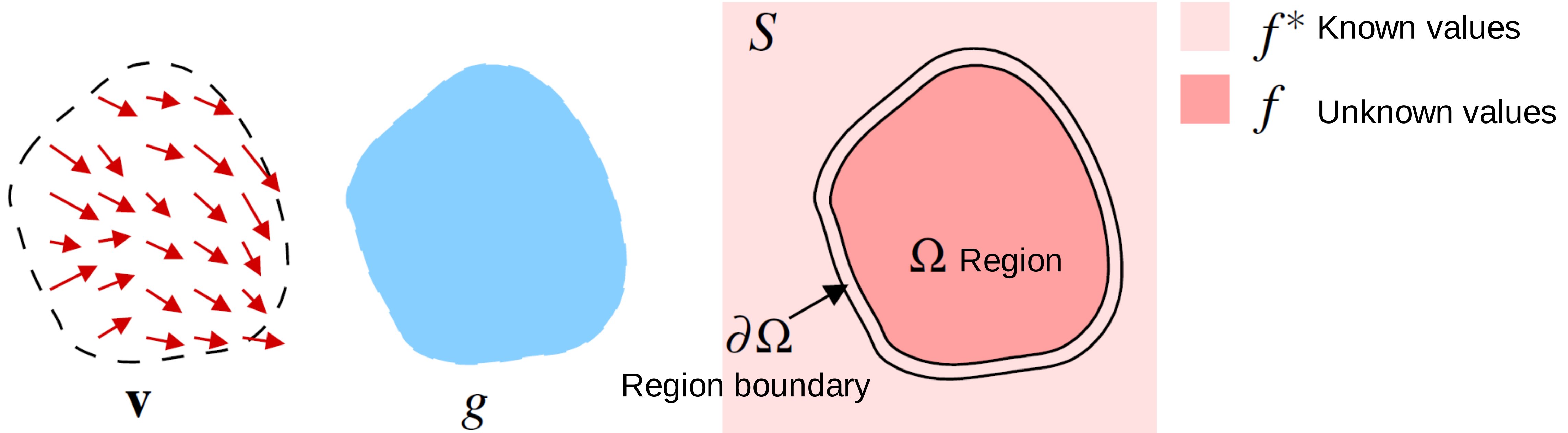
Reminder

- For the discrete case of an image $f\colon \mathbb{R}^2 \to \mathbb{R}$

$$\nabla f(\mathrm{x},\mathrm{y}) = \left(\nabla_x f(\mathrm{x},\mathrm{y}), \nabla_y f(\mathrm{x},\mathrm{y})\right) = \left(f(\mathrm{x}+1,\mathrm{y}) - f(\mathrm{x},\mathrm{y}), f(\mathrm{x},\mathrm{y}+1) - f(\mathrm{x},\mathrm{y})\right)$$

$$\begin{aligned}
\mathrm{div}\left(\nabla f(\mathrm{x},\mathrm{y})\right) &= \mathrm{div}\left(\nabla_x f(\mathrm{x},\mathrm{y}), \nabla_y f(\mathrm{x},\mathrm{y})\right) = \nabla_x^2 f(\mathrm{x},\mathrm{y}) + \nabla_y^2 f(\mathrm{x},\mathrm{y}) = \Delta f(\mathrm{x},\mathrm{y}) \\
&= 4f(\mathrm{x},\mathrm{y}) - f(\mathrm{x}+1,\mathrm{y}) - f(\mathrm{x}-1,\mathrm{y}) - f(\mathrm{x},\mathrm{y}+1) - f(\mathrm{x},\mathrm{y}-1) \\
&= 4f(\mathrm{x},\mathrm{y}) - \sum_{f(i,j)\in\mathcal{N}(x,y)} f(i,j)
\end{aligned}$$

Problem Formulation



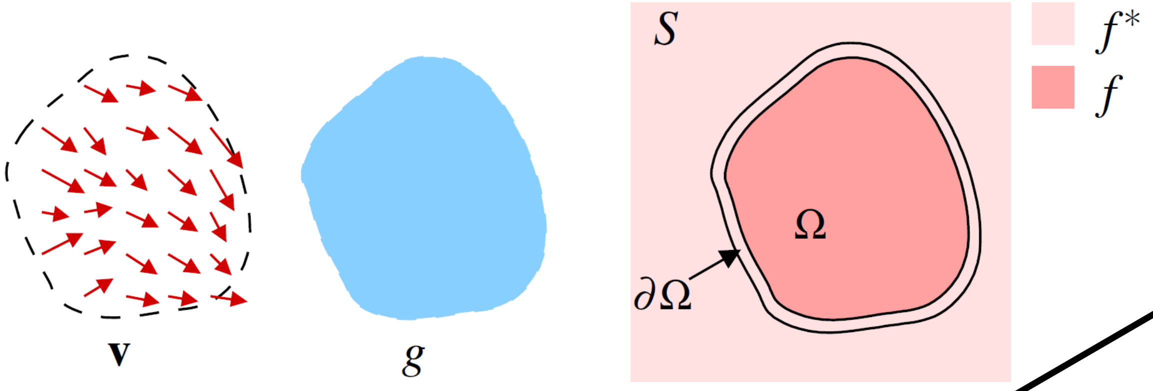
Guidance vector field

Image source region

Destination image

$$\Delta f = \operatorname{div}\mathbf{v} \; with \; f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega}$$

Poisson Equation



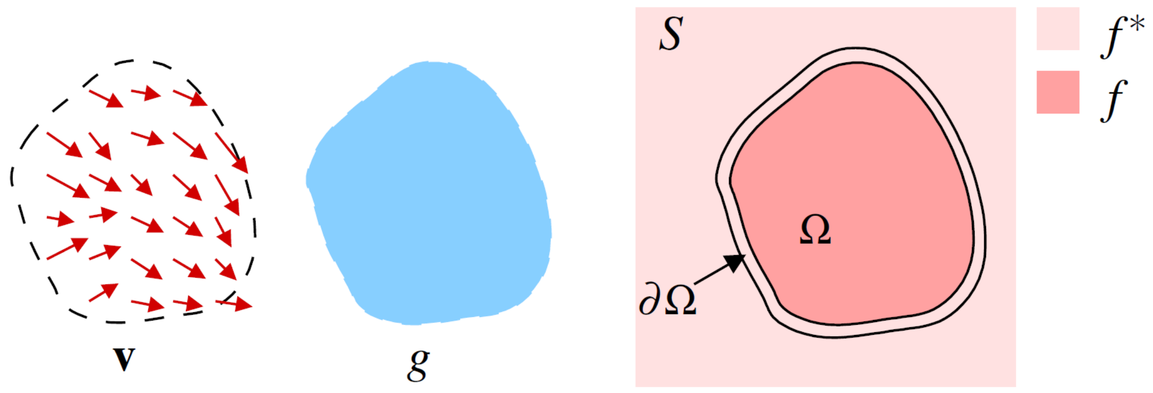
$$\Delta f = \mathrm{div}\,\mathbf{v} \; with \; f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega}$$

$$\Delta f_p = |\mathcal{N}_p| f_p - \sum_{q \in \mathcal{N}_p \cap \Omega} f_q - \sum_{q \in \mathcal{N}_p \cap \partial\Omega} f_q^*$$

$$\mathrm{div}\,\boldsymbol{v} = \sum_{q \in \mathcal{N}_p} v_{pq} \qquad (v_{pq} = g_p - g_q)$$

$$\boxed{|\mathcal{N}_p| f_p - \sum_{q \in \mathcal{N}_p \cap \Omega} f_q = \sum_{q \in \mathcal{N}_p \cap \partial\Omega} f_q^* + \sum_{q \in \mathcal{N}_p} v_{pq}}$$

Poisson Equation



$$\Delta f = \mathrm{div}\mathbf{v} \; with \; f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega}$$

$$\boxed{|\mathcal{N}_p| f_p - \sum_{q\in\mathcal{N}_p\cap\Omega} f_q = \sum_{q\in\mathcal{N}_p\cap\partial\Omega} f_q^* + \sum_{q\in\mathcal{N}_p} v_{pq}}$$

$$\mathbf{A}\mathrm{x} = \mathrm{b} \quad \rightarrow \begin{pmatrix} |\mathcal{N}_{p_1}| & a_{p_1p_2} & a_{p_1p_3} & \cdots & a_{p_1p_{|\Omega|}} \\ a_{p_2p_1} & |\mathcal{N}_{p_2}| & a_{p_2p_3} & \cdots & a_{p_2p_{|\Omega|}} \\ a_{p_3p_1} & a_{p_3p_2} & |\mathcal{N}_{p_3}| & \cdots & a_{p_3p_{|\Omega|}} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{p_{|\Omega|}p_1} & a_{p_{|\Omega|}p_2} & a_{p_{|\Omega|}p_3} & \cdots & |\mathcal{N}_{p_{|\Omega|}}| \end{pmatrix} \begin{pmatrix} f_{p_1} \\ f_{p_2} \\ f_{p_3} \\ \vdots \\ f_{p_{|\Omega|}} \end{pmatrix} = \begin{pmatrix} b_{p_1} \\ b_{p_2} \\ b_{p_3} \\ \vdots \\ b_{p_{|\Omega|}} \end{pmatrix}$$

$$a_{p_ip_j} = \begin{cases} -1, & p_j \in \mathcal{N}_{p_i} \cap \Omega \\ 0, & otherwise \end{cases}$$

$$b_{p_i} = \sum\nolimits_{q\in\mathcal{N}_{p_i}\cap\partial\Omega} f_q^* + \sum\nolimits_{q\in\mathcal{N}_{p_i}} v_{p_iq}$$

Model and solve SLE in Matlab

- Matrix $\mathbf{A}$ is a really sparse matrix
  - At most, five non-zero elements per row
  - Define using the function `sparse`
- Solve SLE
  - Use backslash operator: `x=A\b`
  - Returns the least square solution

Guidance vector field

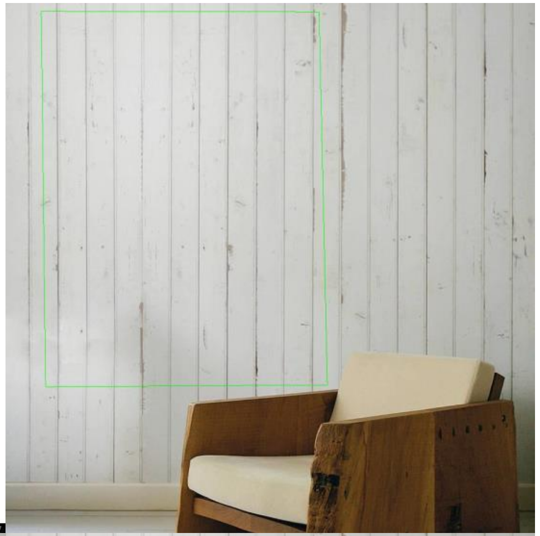
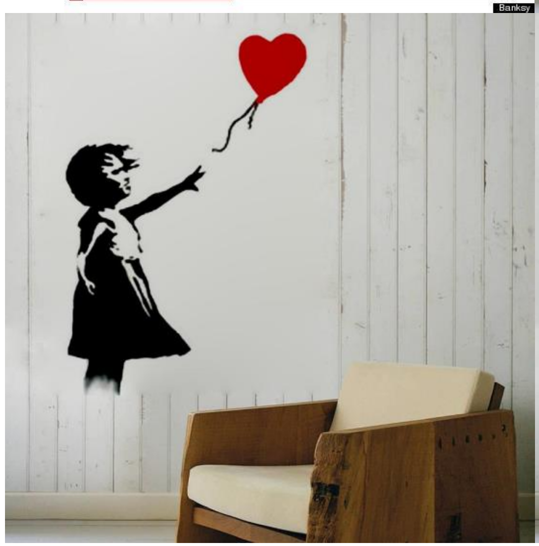
Different effects depending on chosen v_{pq}:

- $v_{pq} = 0$
- $v_{pq} = g_p - g_q$
- $v_{pq} = \begin{cases} f_p^* - f_q^*, & if \left|f_p^* - f_q^*\right| > \left|g_p - g_q\right| \\ g_p - g_q, & otherwise \end{cases}$
- …

Have fun!






References

[1] P. Perez, M. Gangnet, and A. Blake. 2003. *Poisson Image Editing.* SIGGRAPH ’03, pp. 313–318.