Non-Local Image Denoising
&
Integral Images

Slides by: Stephan Garbin

Outline

1. **Non-Local Denoising**
   - Problem
   - Noise Model
   - Motivation for Non-Local Filters
   - Non-Local Means
   - Weighting for Non-Local Means

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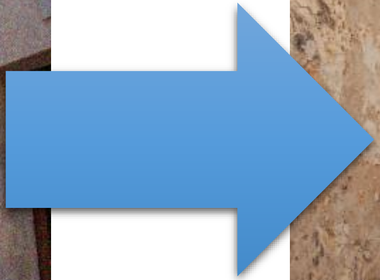
1. **Non-Local Denoising**
   - Problem
   - Noise Model
   - Motivation for Non-Local Filters
   - Non-Local Means
   - Weighting for Non-Local Means
2. **Efficient Template Matching**
   - Problem
   - What is an Integral Image?
   - How do we calculate it?
   - How do we use it?

Non-Local Denoising

Our Task

Noisy Image

Denoised Image



Noise Model

- Assumption: **Additive, Zero-Mean, ‘White’ Gaussian Noise**

Additive White Gaussian Noise

Noisy Image

$$\tilde{u} = u + n$$

Ideal (noiseless) Image

Noise Model

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Ideal (noiseless) Image

$$\mathbb{P}(\tilde{u} \mid u) = \frac{1}{(2\pi\sigma^2)^{\frac{N}{2}}} e^{-\frac{\|u - \tilde{u}\|_2^2}{2\sigma^2}}$$

Pixels in Image

Noise Model **Conditional Distribution**

Noise Model

$$\tilde{u} = u + n$$

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We want to recover u from $\tilde{u}$

How?

- (Gaussian) Low-Pass?

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- Bilateral Filter?

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- Bilateral Filter?

- **Too much detail lost**
- **Blurry results**

Exploiting Self-Similarity

- *Idea:* Image shows **self-similarity** ➔ many similar PATCHES

Exploiting Self-Similarity

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Non-Local Means: Concept

“Restore current pixel as a weighted average of all pixels in the image”

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<u>In Practise:</u>
Local neighbourhood
around pixel (e.g. 30x30)

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In Practise:
Local neighbourhood
around pixel (e.g. 30x30)

"Restore current pixel as a *weighted* average of *all* pixels in the image"

Similarity determined by
patches around pixels

- Similar pixels count more
- Dissimilar pixels should not count

Non-Local Filters – General Structure

1. **Block-Matching**
    - *Procedure:* For every reference patch, centred at every pixel, find k most similar patches
        - For NL-Means: *All* patches in local neighbourhood

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  - For NL-Means: *All* patches in local neighbourhood
- *Result:* 3d block of 2d patches for each reference patch

Note:
Does not need to be stored explicitly for NL-Means

Non-Local Filters – General Structure

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    - Use 3d blocks to derive denoised estimate

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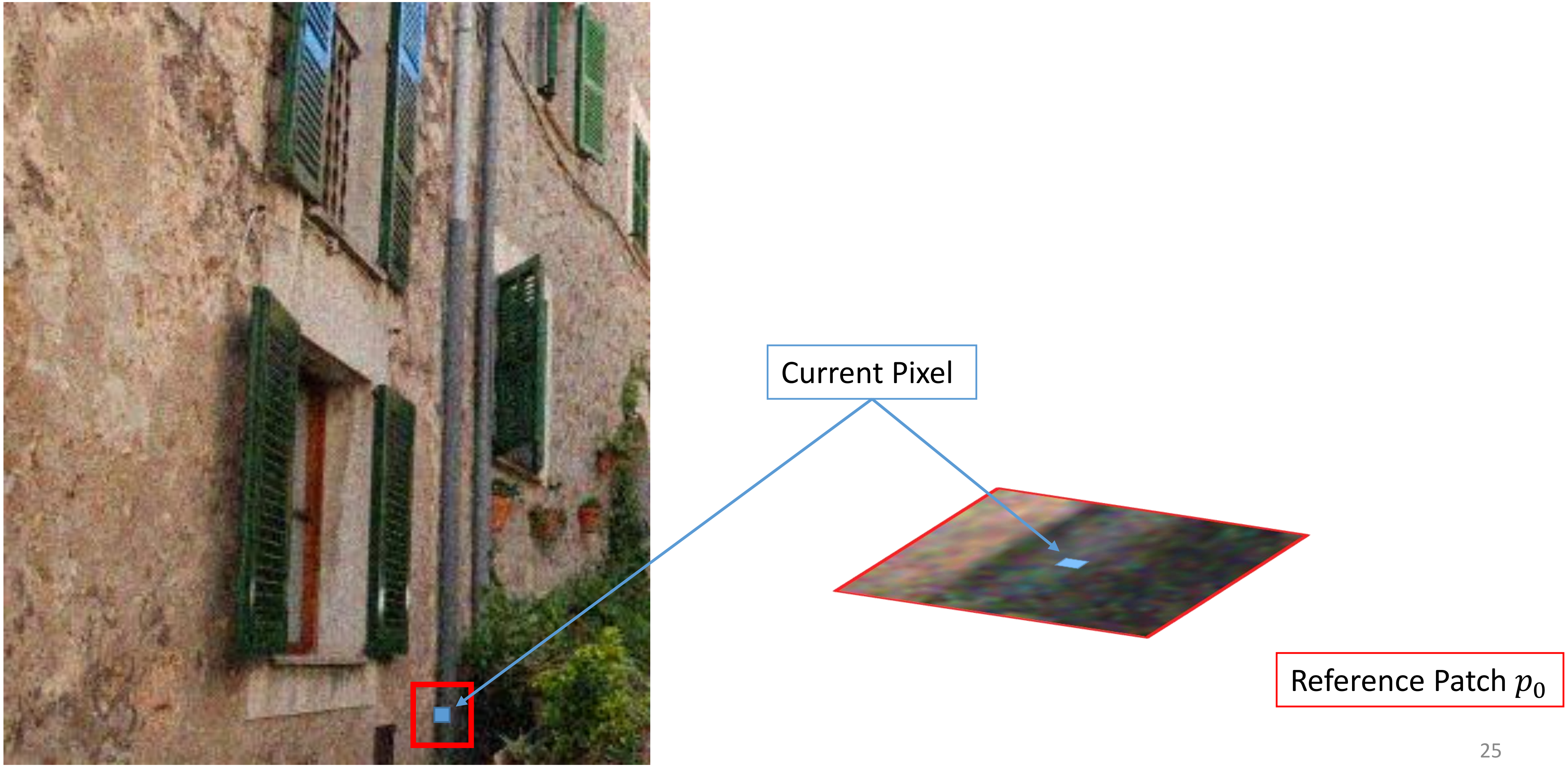
3. **(Aggregation)**
   - Combine (overlapping) patches to form final image

Note:
Only necessary for patch-wise implementation

Building Blocks of 2d Patches



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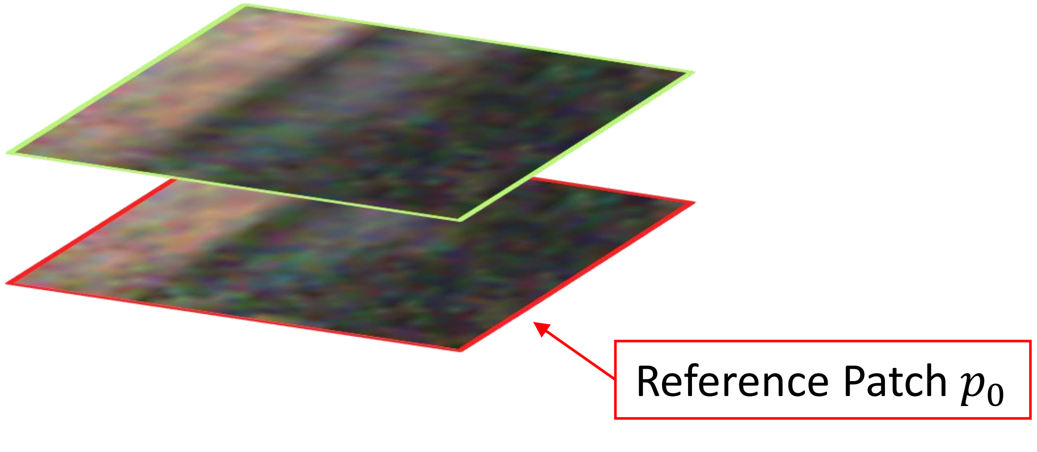

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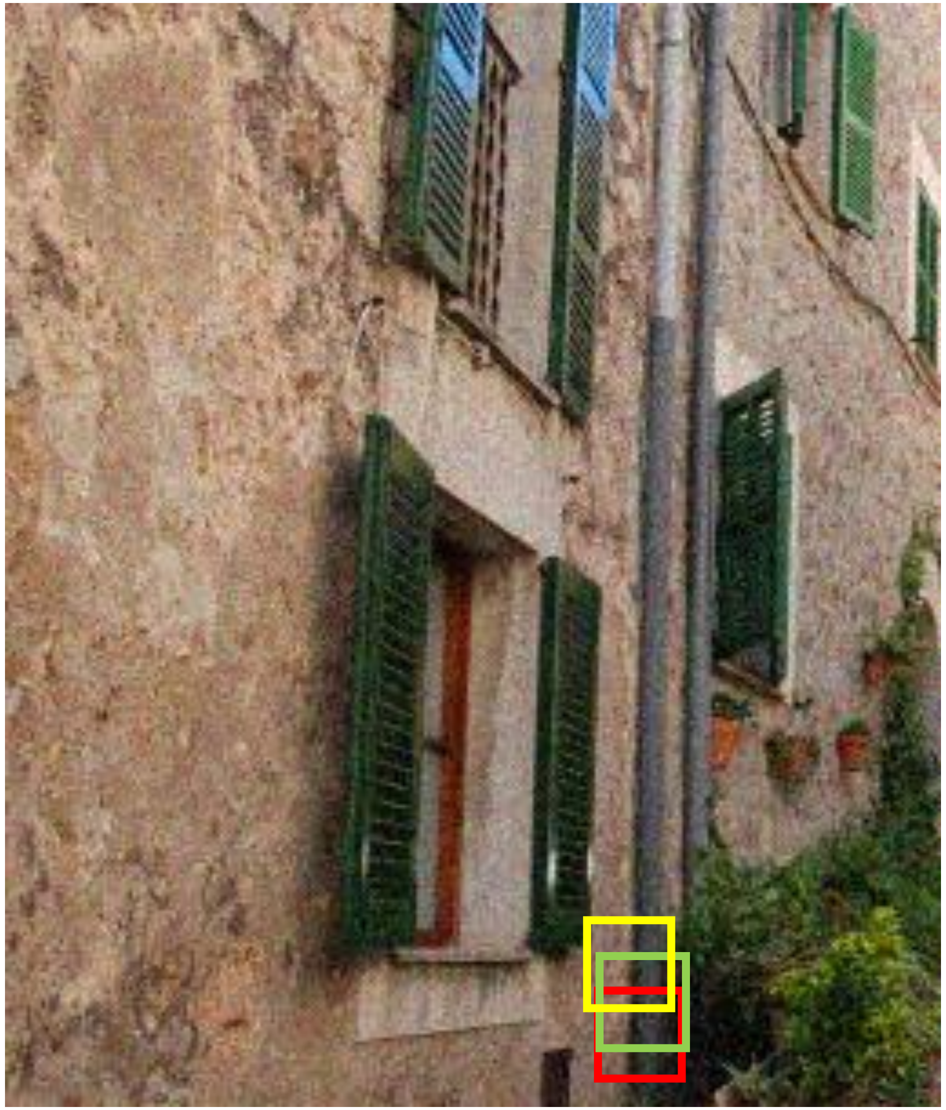
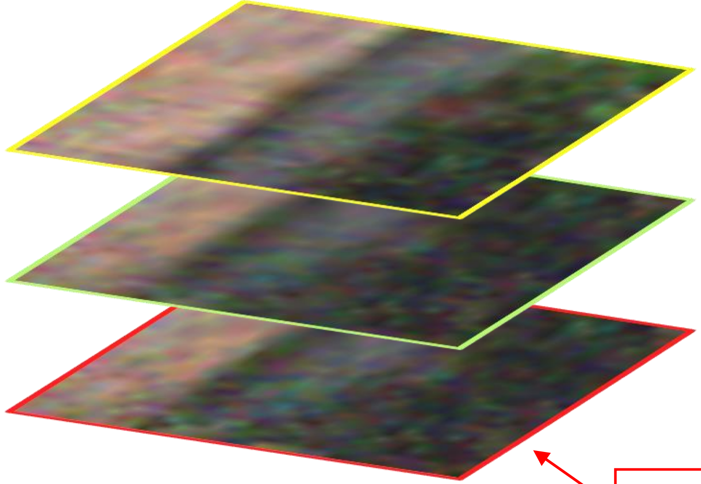
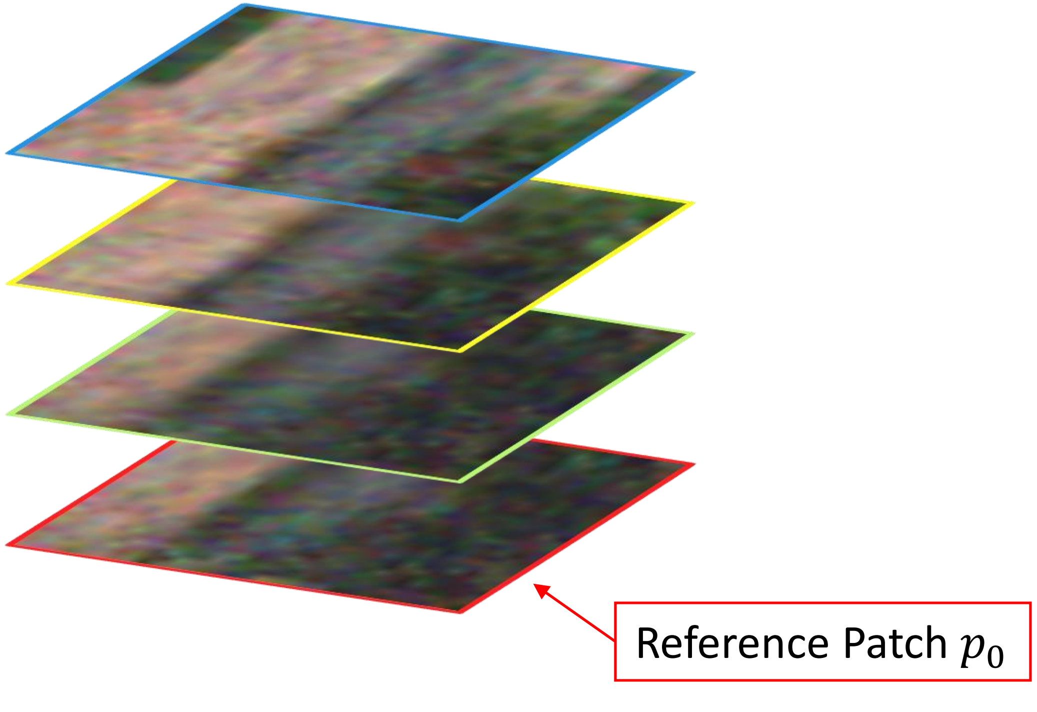




Reference Patch p_0

Building Blocks of 2d Patches



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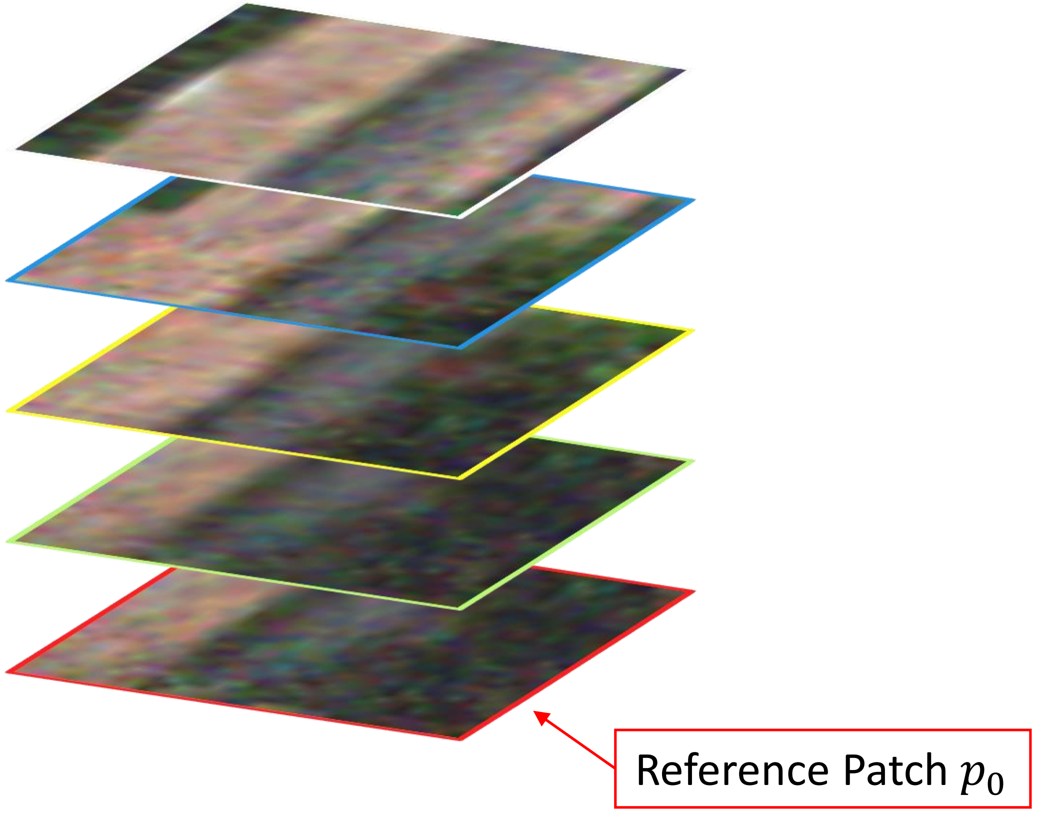
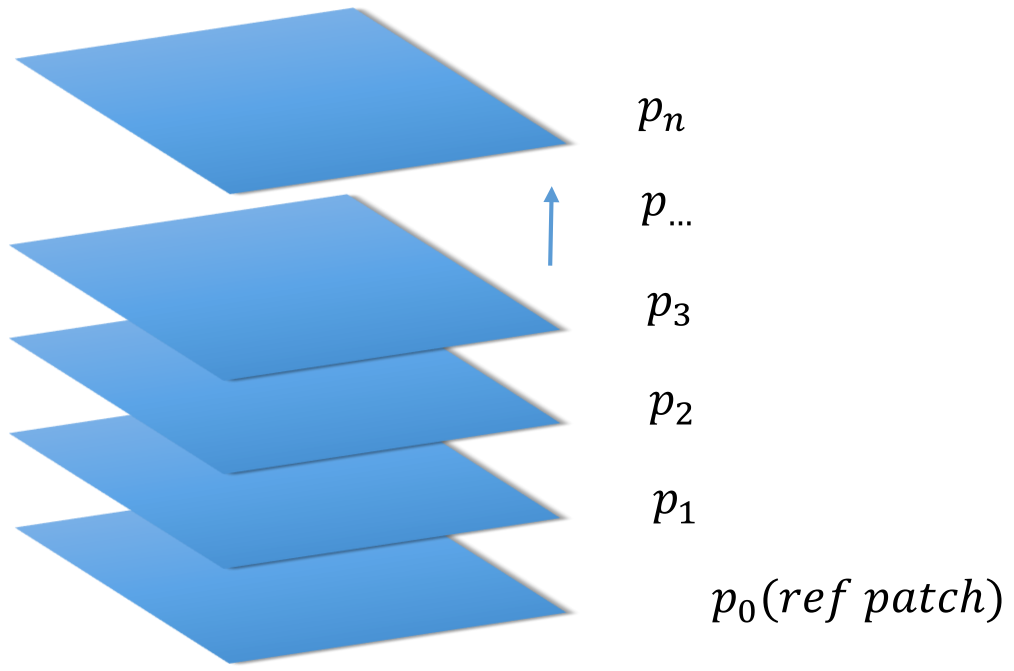


Building Blocks of 2d Patches

3D Block of Similar Patches



Two variants of NL-Means

Pixel-Wise

- Use *only pixels at the* ***centre*** *of each patch* for final estimate

Denoised pixel at location p

Noisy pixel at location q

$$\hat{u}(p) = \frac{1}{C(p)} \sum_{q \in B(p,r)} \tilde{u}(q)\, w(p,q)$$

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Weight between patches centred at p (reference patch) and q

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<u>Note:</u>

- We now get multiple estimates per pixel ➔ take average
- *Reason:* Patches centred at every pixel overlap

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- Average pixels across all patches in a block?

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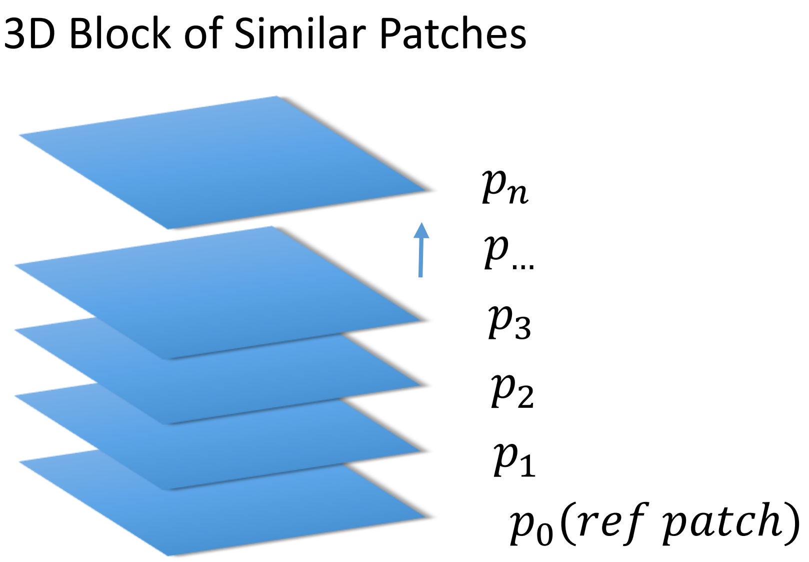

Filtering – Choosing the weights

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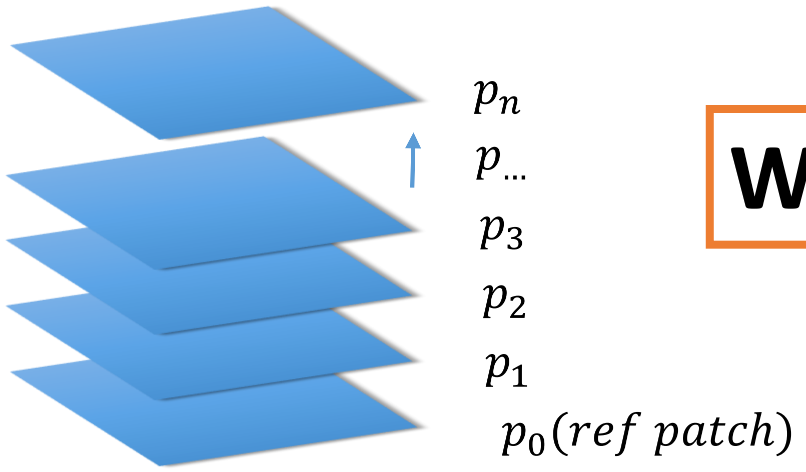
3D Block of Similar Patches



We need a weight for each patch

Filtering – Weighting Function

- We want
  - Similar patches to count more
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  - (Impose *additional* constraint on which patches to keep)

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- So far, we have assumed using *all* patches in a local neighbourhood
- However, we could exclude dissimilar patches from the start
  ➔ We keep only a *subset* of patches in $B(p, r)$
- *Note: Weighting function already does this to some degree (can be 0)*

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Squared Patch Distance (SSD)

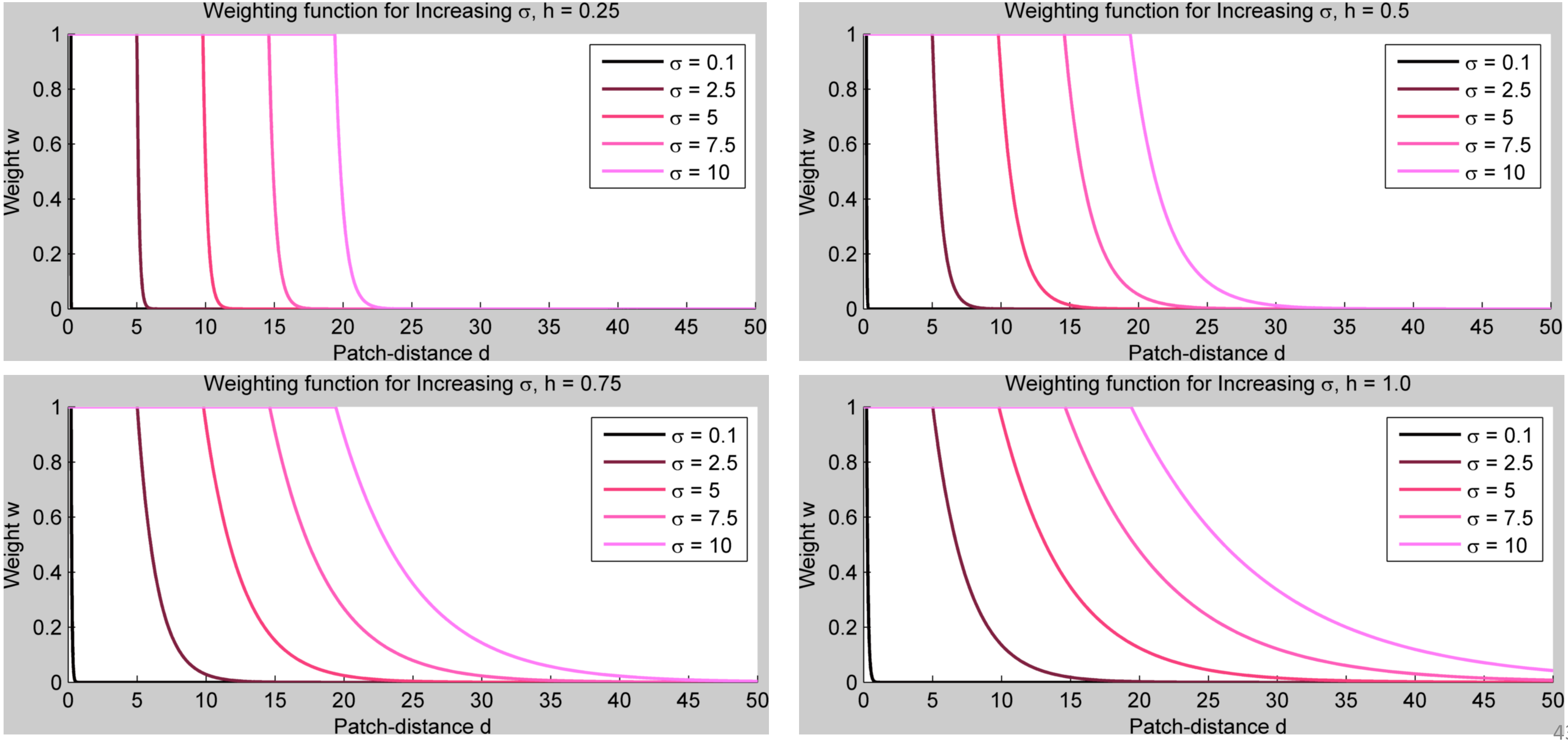
Noise Standard Deviation

$$\boxed{w(p,q) = e^{-\frac{\max(d^2 - 2\sigma^2, 0)}{h^2}}}$$

Weight between patches centred at p and q

Decay Parameter

Filtering – Weighting Function



Problems with Non-Local Methods

- Complexity (Can be quadratic or worse w.r.t. image size)
  - ➔ Takes a long time to run
  - ➔ In the real world, we deal with 2k, 4k, 6k Images

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- Not all images are highly self-similar everywhere

- Search **Database** of similar images
- Search **other frames** (if video)

Efficient Template Matching

The Problem

- Calculating SSD for every patch centred at each pixel with every other patch in local neighbourhood is *VERY* expensive

$$\boldsymbol{O(MNn^2)}$$

M = # offsets
N = # pixels in the image
n = patch size

Grow **quadratically** for square images / search regions as well!

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Grow **quadratically** for square images / search regions as well!

Integral Images can reduce number of arithmetic operations required to:

$$\boldsymbol{O}(\boldsymbol{MN})$$

What is an Integral Image?

- The **Integral Image** $\boldsymbol{I_{int}}$ *for an Image* I stores at every pixel location,
  - The sum of all pixels in I to the left *of the current location*
  - And all pixels in I above *of the current location*
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Integral Image location: x, y

$$I_{int}(x, y) = \sum_{p \leq x, q \leq y} I(p, q)$$

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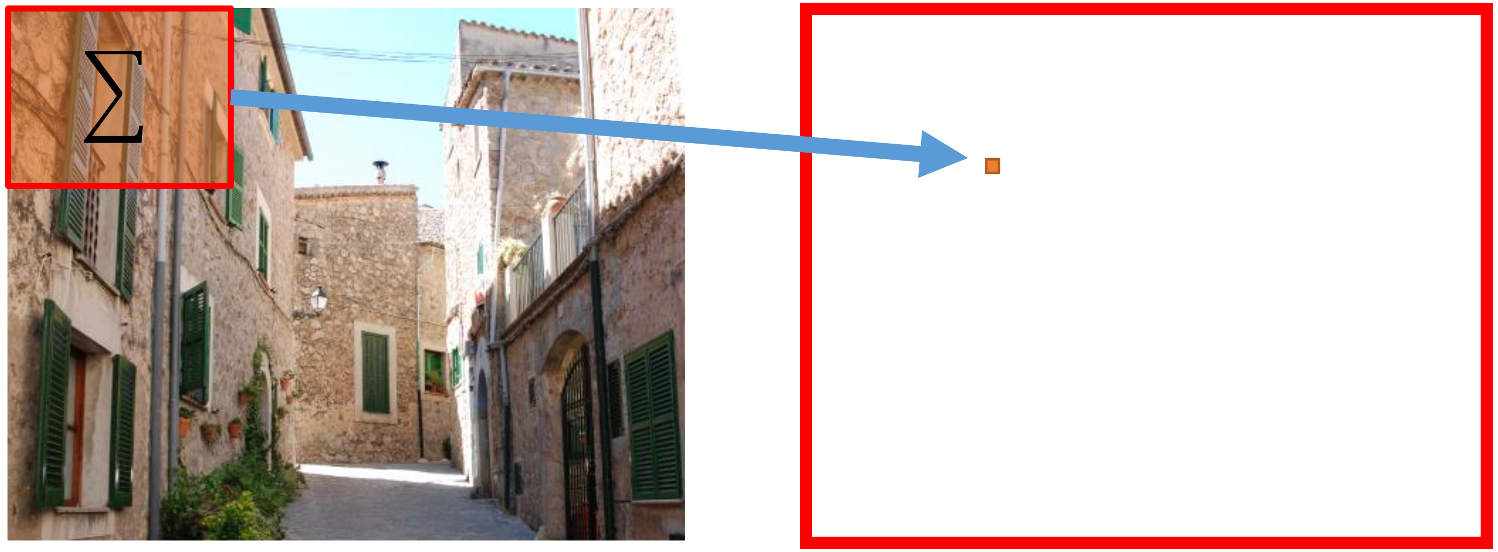

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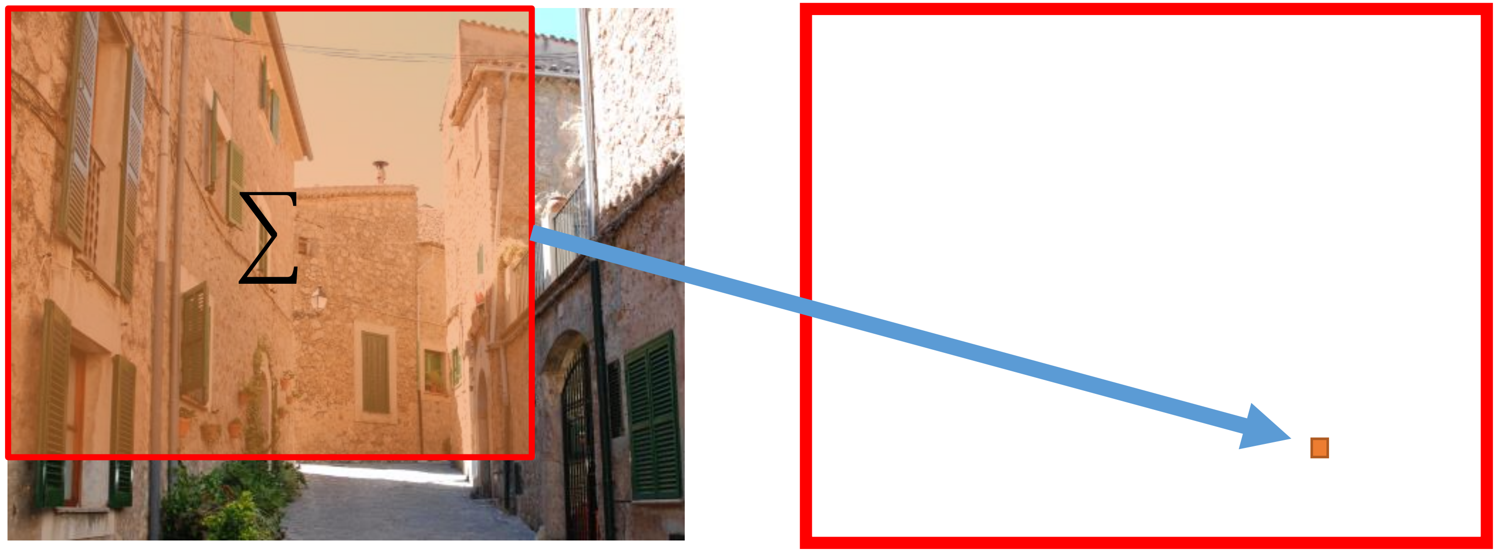

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Computing it efficiently (serially)

- Remember:

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- Remember:

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- Use recurrence relation:

Cumulative sum (column-wise)

$$s(x, y) = s(x, y - 1) + I(x, y)$$

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Note Starting Point:

$$s(x, -1) = I_{int}(-1, y) = 0$$

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- We can calculate the sum of all pixels in a patch in **constant time, regardless of patch size**!

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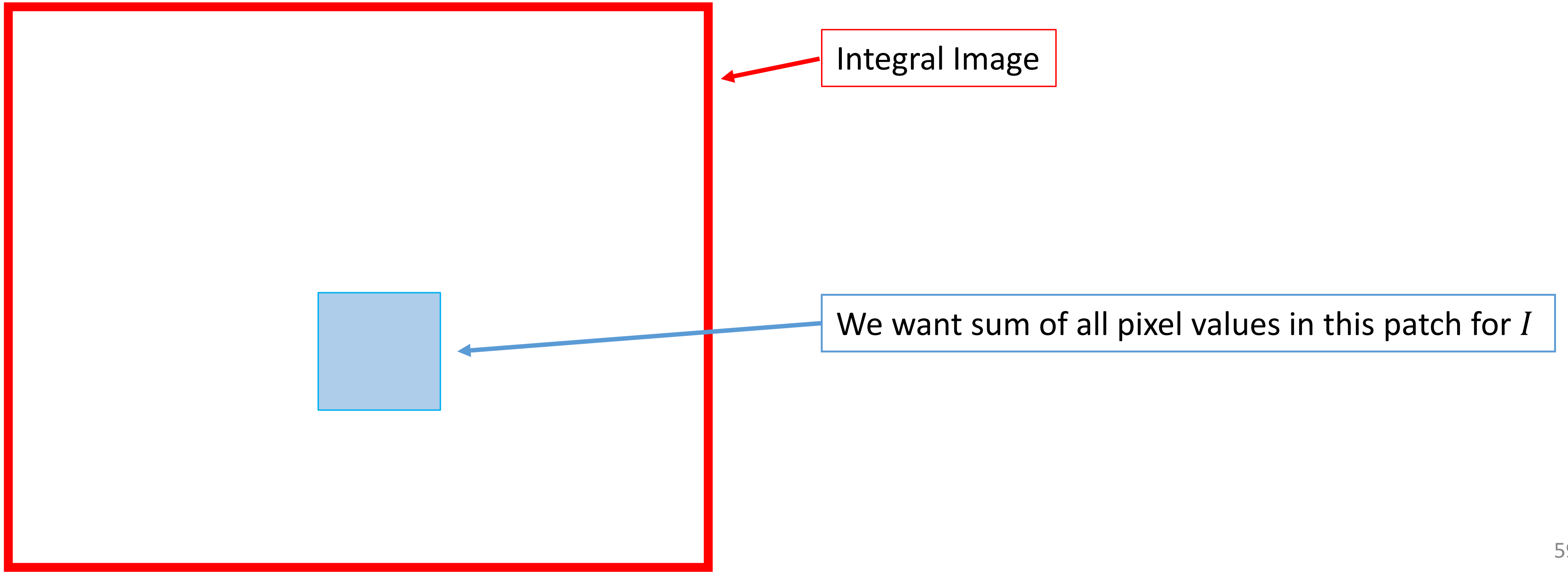

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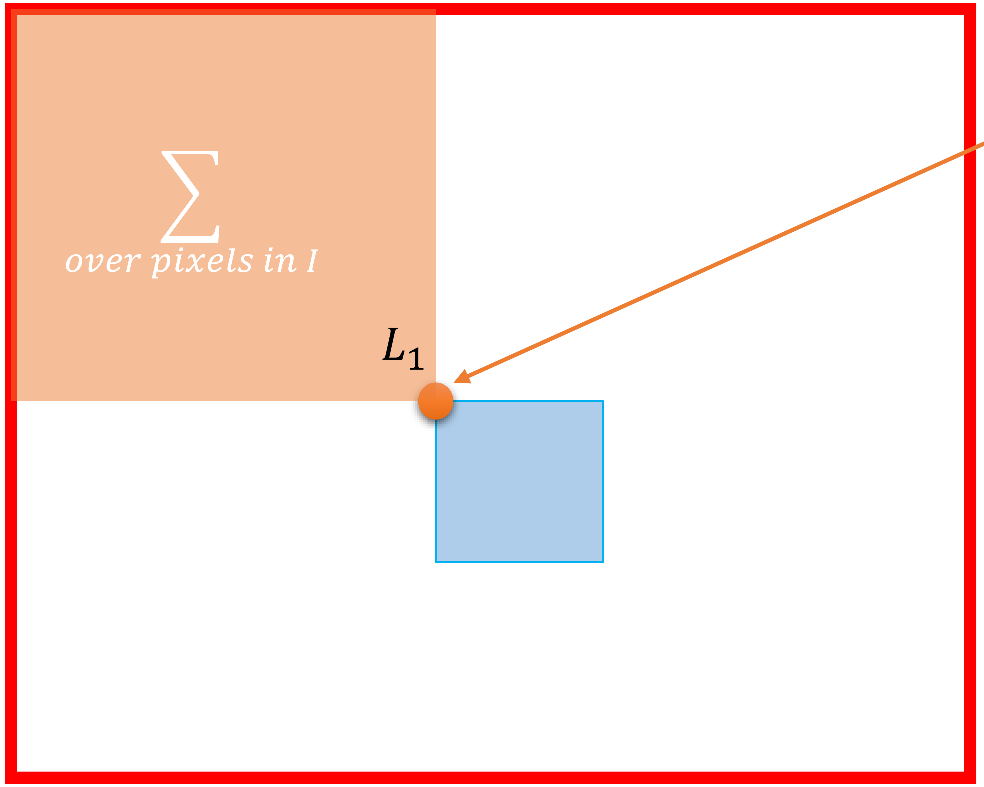

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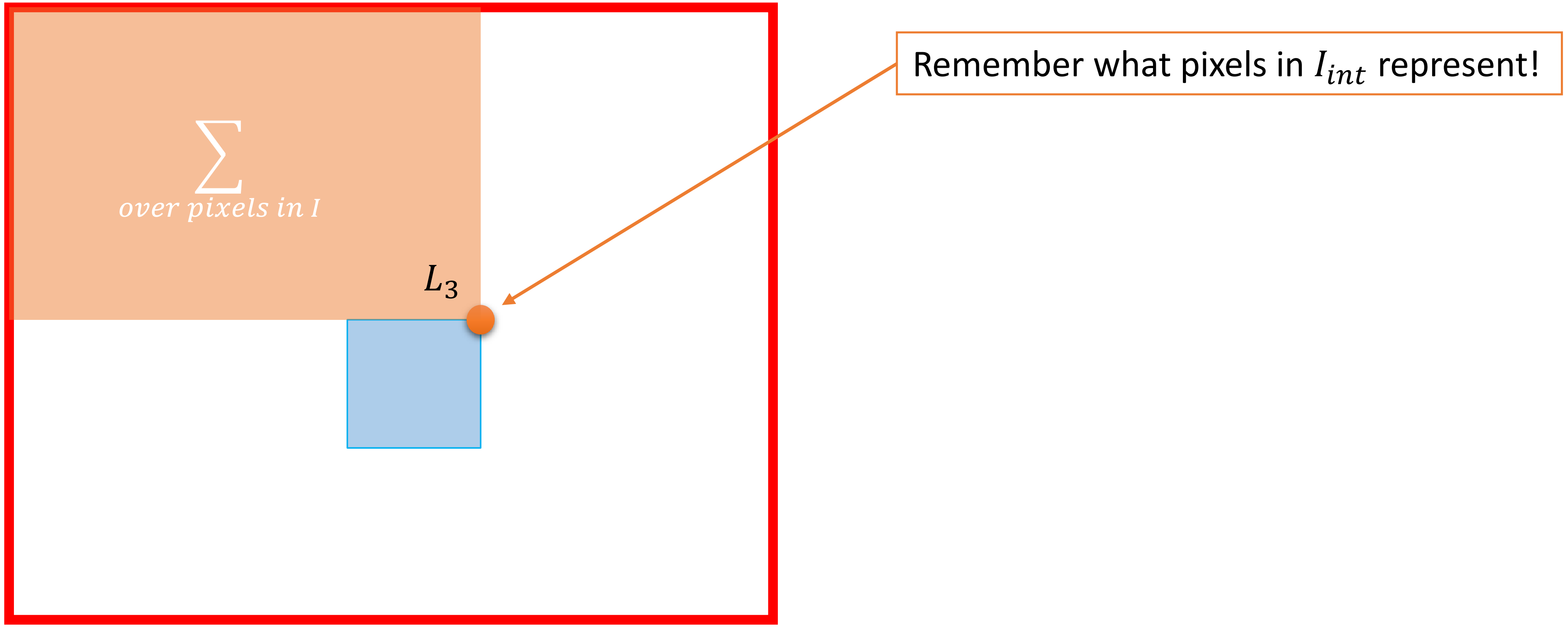

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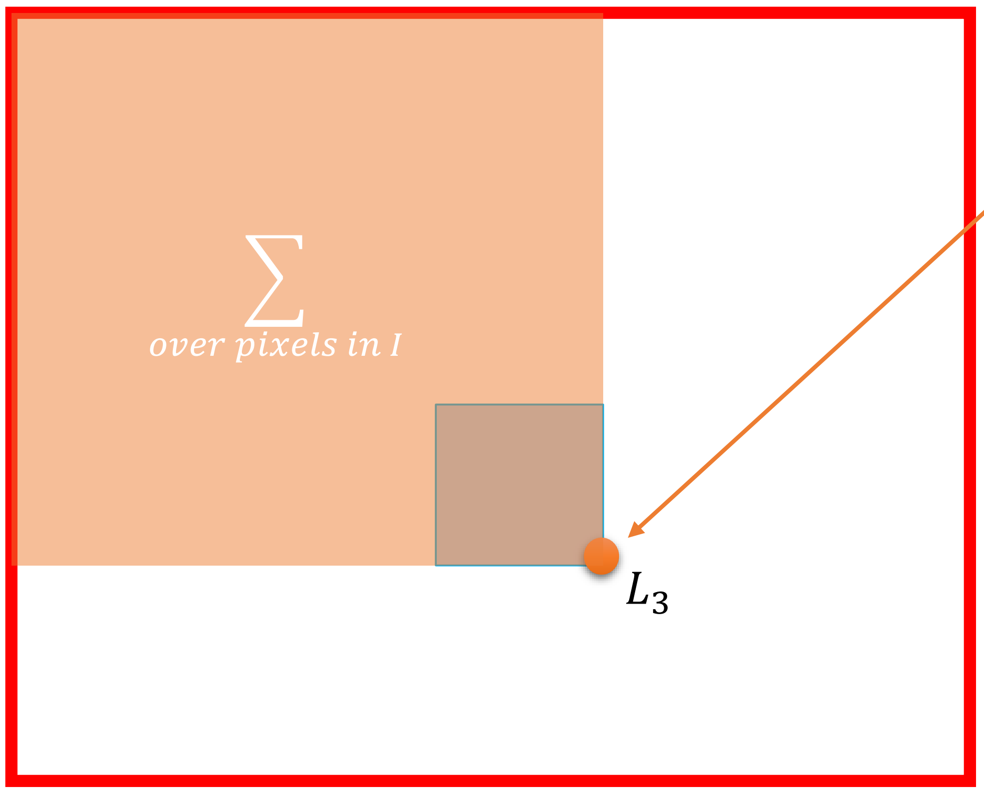

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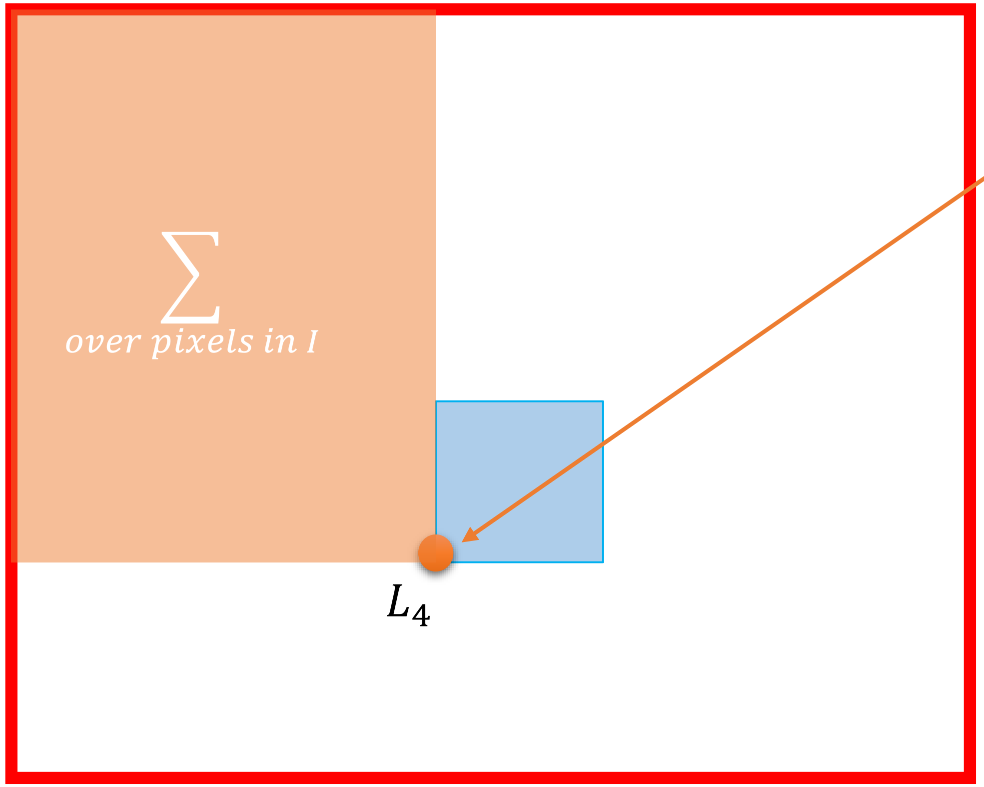

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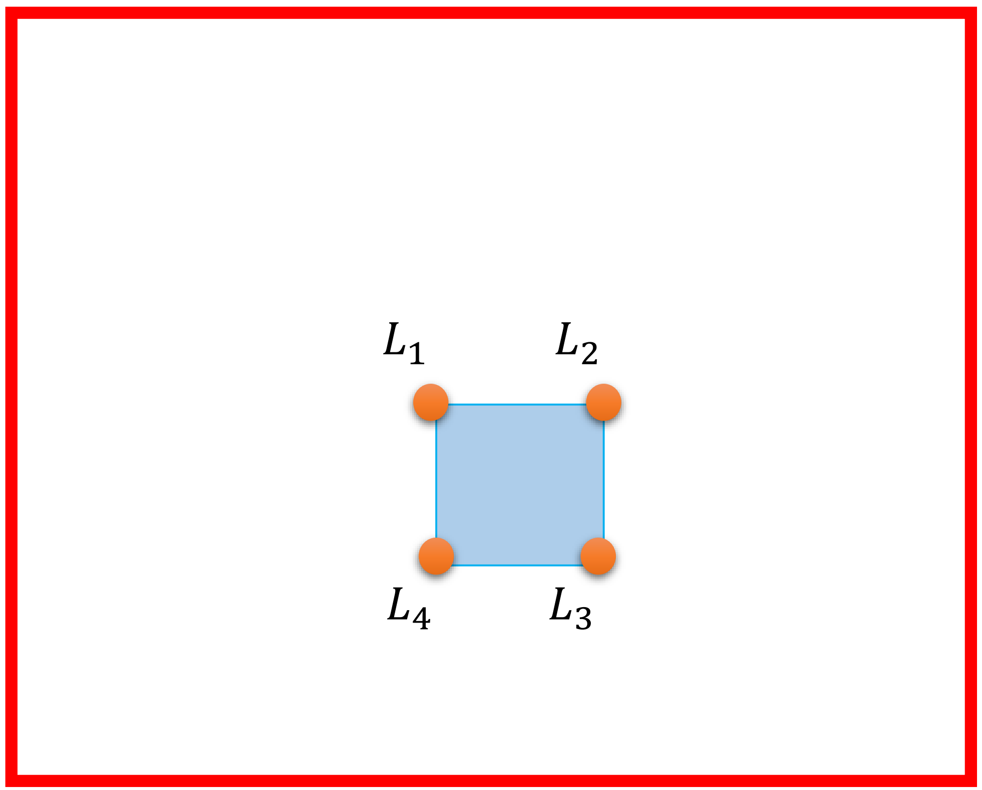

Sum of patch is:

$$\boldsymbol{L_3 - L_2 - L_4 + L_1}$$

Necessary as pixels used to calculate $L_1 \subseteq L_2$ AND $L_1 \subseteq L_4$

How does this help with template matching?

- We can build an integral image of a *difference image*

Example: *Calculate SSD for offset (-10, -15)*

I.e. for a patch shifted -10 pixels down and -15 pixels to the right

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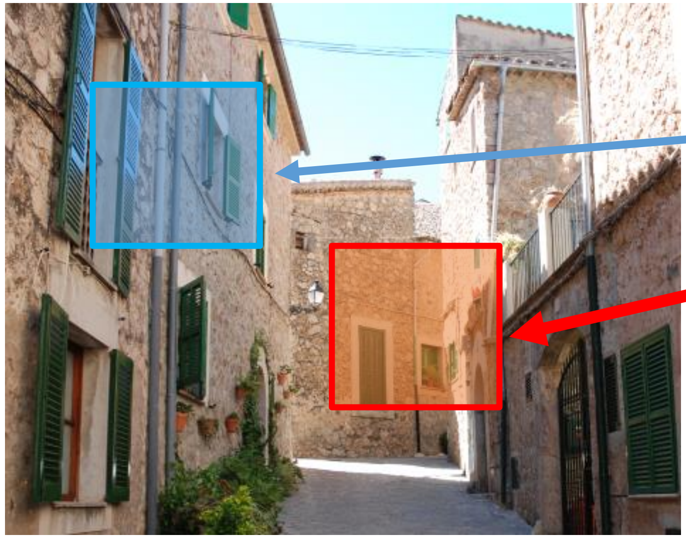

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➔ Difference Image

2. Calculate Integral Image of the difference image

➔ We can now read SSD *for all patches* ***for this offset*** in constant time!

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➔ Difference Image

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<u>Note:</u> There are as many offset as there are pixels in the search window!

Problems

- Memory Overhead?
- Parallel Computation of the Integral Image?
- No benefit for smaller patch-sizes
- Some Distance Metrics not supported

Results

Original

Results

Noisy

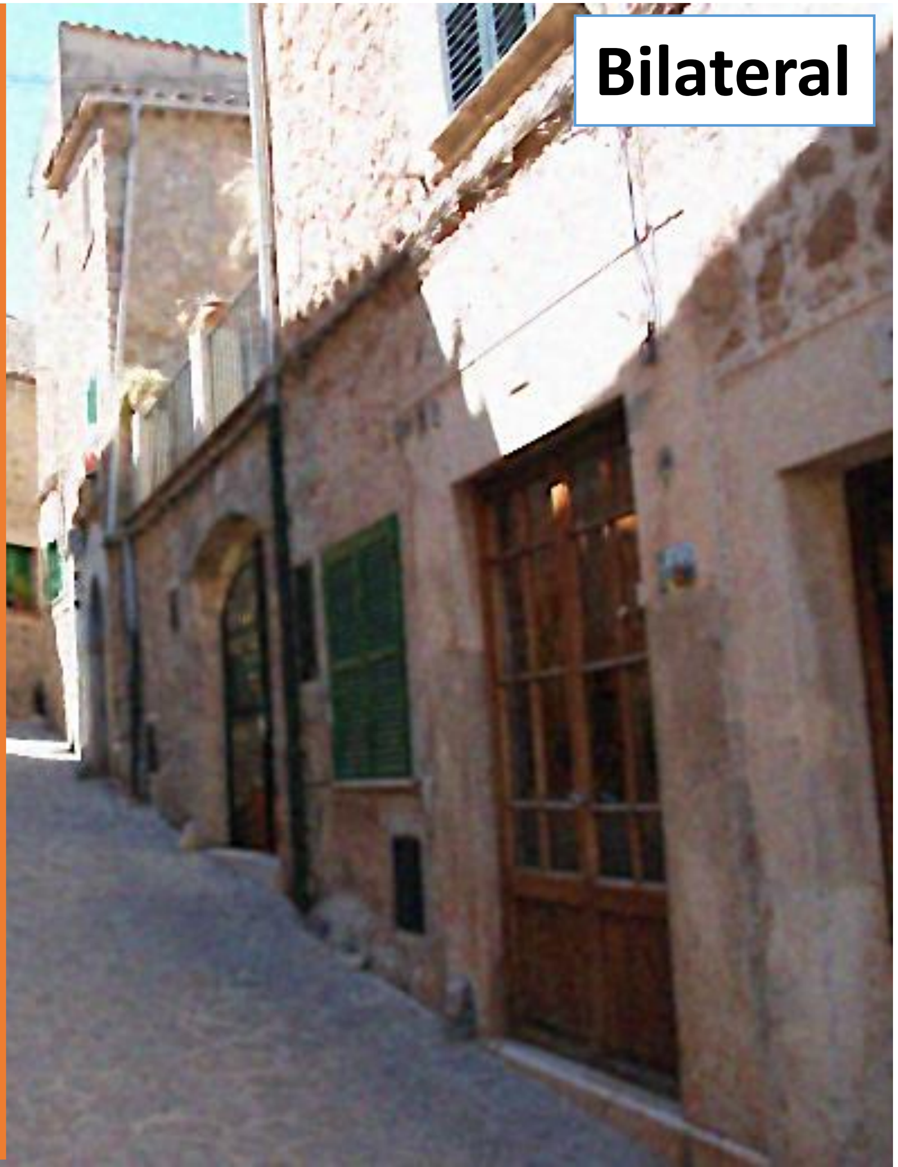
Results

Denoised

Results

Error

Results



Closing Remarks

- We have just implemented & understood the NL-Means filter:

$$\boldsymbol{NL}\big(\boldsymbol{u}(\boldsymbol{p})\big) = \frac{\boldsymbol{1}}{\boldsymbol{C}(\boldsymbol{p})} \int \boldsymbol{f}\Big(\boldsymbol{d}\big(\boldsymbol{B}(\boldsymbol{p}), \boldsymbol{B}(\boldsymbol{q})\big)\boldsymbol{u}(\boldsymbol{q})\Big)\, \boldsymbol{dq}$$

- Many other more complicated non-local methods:
  - BM3D/BM3D-SAPCA,
  - Non-Local Bayes,
  - PLOW, etc…

References

- Antoni Buades, Bartomeu Coll, and Jean-Michel Morel, *Non-Local Means Denoising*, Image Processing On Line, 1 (2011).http://dx.doi.org/10.5201/ipol.2011.bcm_nlm
- Gabriele Facciolo, Nicolas Limare, and Enric Meinhardt-Llopis, *Integral Images for Block Matching*, Image Processing On Line,4 (2014), pp. 344–369. http://dx.doi.org/10.5201/ipol.2014.57