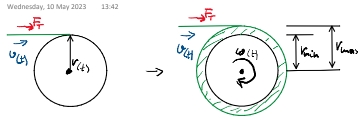
Antriebsberechnung Hauptachse

Wednesday, 10 May 2023 13:42



$F_T(t) \approx$ const. … Drahtspannung
$\omega(t)$ … Frequenz Hauptachse
r_{min}/r_{max} … Wickelradien
$v(t) \approx$ const. … Drahtgeschwindigkeit
$r(t)$ … Wickelradius

$$M_{ges} = M_T(r) + M_{I_0}(\omega(t)) + M_{I_r}(r, v(t))$$

↳ Drehmoment durch veränderliches Spulenträgheitsmoment
↳ Beschleunigungsdrehmoment der leeren Hauptachse + Spulenkörper
↳ Drehmoment durch Drahtspannung

$$M_T(r(t)) = F_T \cdot r(t)$$

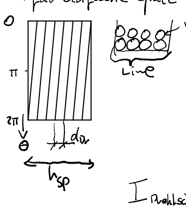
$$\frac{d}{dt}\left(I(t) \cdot \omega(t)\right)$$

$$M_{I_0}(\omega(t)) = I_0 \cdot \dot{\omega}(t) = M(\dot{\omega}(t))$$

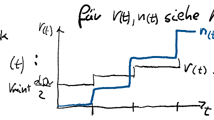
$$M_{I_r}(r, \omega(t), v(t)) = \frac{d L_{I_r}(t)}{dt} = \frac{d}{dt}\left(I_r(r(t)) \cdot \omega(t)\right) = I_r(r(t)) \cdot \dot{\omega}(t) + \dot{I}_r(r(t)) \cdot \omega(t) = M(\omega(t), \dot{\omega}(t))$$

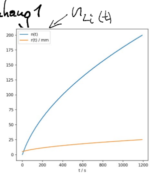
$I_r(r(t))$ … Trägheitsmoment der Spulenwicklung

↳ für einfache Spule:



für $r(t), n(t)$ siehe Anhang 1





$$I_{Drahtschleife}(t) = m_{U_{Wk}} \cdot \left(\frac{3}{2} \cdot \left(\frac{d_{Dr}}{2}\right)^2 + r^2(t)\right) \cdot \underbrace{\left(\frac{\theta}{2\pi}\right)}_{\text{für Teilschleife}}$$

$$m_{U_{Wk}} = \sigma \cdot 2\pi r(t) \qquad [\sigma] = \frac{kg}{m}$$

$$I_{Li}(t) = N_{Wk,L} \cdot \sigma 2\pi r(t) \left(6 d_{Dr}^2 + r^2(t)\right)$$

$$I_{Li,Wk}(t) = n_{Li,Wk}(t) \cdot \sigma 2\pi\, r(t) \cdot \left(6 d_{Dr}^2 + r^2(t)\right)$$ geht mit $I_i(n) = I_i(r(n))$

$$I_{Li,Wk,\theta}(t) = \sigma 2\pi\, r(t) \cdot \left(6 d_{Dr}^2 + r^2(t)\right) \cdot \frac{\theta(t)}{2\pi}$$

$$I_{sp}(t) = \sum_{n_{Li}^i = 0}^{N_{Li}(t)} I_{Li}(n_{Li}^i) + \sum_{n_{Li,wk}^i = 0}^{N_{Li,Wk}(t)} I_{Li,Wk}(n_{Li,wk}^i) \left(+ I_{Li,Wk,\theta}(t)\right)$$

$$I_{Wk}(r(t)) = I_{Wk}(r(n)) = \sigma 2\pi\, r(n) \left(6 d_{Dr}^2 + r^2(n)\right)$$

$r(\theta) = k \sin(2\theta) + d$
$r_{max} = k + d = r_g$
$r_{min} = d = r$

grobe Abschätzung Pickup Coil winding

$$M_T(r(t)) = F_T \cdot r(t) = F_T \cdot \left(\sqrt{r_g^2 \cos^2(\theta(t)) + r_k^2 \sin^2(\theta(t))} + r^+(n_{Line})\right)$$

$\cos(\theta) \to -\sin(\theta) \cdot \dot{\theta}(t)$

$$v = \int_0^{2\pi} |\dot{\vec{r}}| \cdot d\theta = \int_0^{2\pi} \sqrt{r_g^2 \sin^2(\theta) + r_k^2 \cos^2 \theta}\, d\theta$$

M (') – T

o $\omega(t)$ – –o · $\omega(t)$

$$v_{Dr}^2 = \dot{\vec{r}}^2(t) = r_g^2 \cdot \sin^2(\theta(t)) \cdot \dot{\theta}^2(t) + r_k^2 \cdot \cos^2(\theta(t)) \cdot \dot{\theta}^2(t)$$

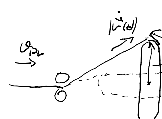
i hab ma gedacht i approximier den zerst mal mit ana ellipse dann krieg i für v_Dr die folgende Wurscht ausi bei am algemmein zeitabhängigen Hauptachsenwinkel theta(t) und Hauptachsen Winkelgschwindigkeit omega(t)

$\theta(t)$
$0° \to \dot{\theta}(t) = \frac{v_{Dr}}{r_k}$
$90° \to \dot{\theta}(t) = \frac{v_{Dr}}{r_g}$

$$v_{Dr}^2 = \dot{\theta}^2(t) \left(\frac{1}{2} r_g^2 + \frac{1}{2} r_k^2\right)$$

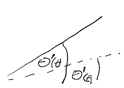
$$45° \to \dot{\theta}(t) = \sqrt{\frac{2}{r_g^2 + r_k^2}}\, v_{Dr}$$

$\theta(t) =$



$$r_y'(t) = \vec{r}(t) \cdot \hat{y}$$

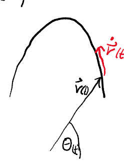
$$\theta'(t) = \operatorname{atan}\left(\frac{|\vec{r}(t)|}{d}\right)$$



↙ achtung bewegte Koordinatentransformation
↳ schnell überlegt: $|\dot{\vec{r}}(t)|$ = Draht aufwickelspeed
$= v_x'(t)$

$$v_y'(t) = \frac{\partial}{\partial t}\left(|\vec{r}(t)|\right)$$

$$v_{Dr} = |\vec{v}'(t)| = \left|\begin{pmatrix} \sqrt{(r_g \sin(\theta(t))\omega(t))^2 + (r_k \cos(\theta(t))\omega(t))^2} \\ \frac{\partial}{\partial t}\sqrt{(r_g \cos(\theta(t)))^2 + (r_k \sin(\theta(t)))^2} \end{pmatrix}\right|$$



$$v_y'(t) = \partial_t \sqrt{(r_g \cos(\theta(t)))^2 + (r_k \sin(\theta(t)))^2}$$

$$= \frac{1}{2} \cdot \left[(r_g \cos(\theta(t)))^2 + (r_k \sin(\theta(t)))^2\right]^{-\frac{1}{2}} \cdot \left[2 r_g^2 \cos(\theta(t)) \cdot (-\sin(\theta(t))) \cdot \omega(t) + 2 r_k^2 \sin(\theta(t)) \cdot \cos(\theta(t)) \cdot \omega(t)\right]$$

$$= \frac{(r_k^2 - r_g^2) \cdot \sin(2\theta(t))\,\omega(t)}{2\sqrt{r_g^2 \cos^2(\theta(t)) + r_k^2 \sin^2(\theta(t))}}$$

$$\to v_{Dr} = \sqrt{\left[r_g \sin(\theta)\omega\right]^2 + \left[r_k \cos(\theta)\omega\right]^2 + \frac{\left[(r_k^2 - r_g^2) \sin(2\theta(t))\omega\right]^2}{4\left[r_g^2 \cos^2\theta + r_k^2 \sin^2\theta\right]}}$$

$$v_{Dr}(0°) = \sqrt{(r_k \omega(0°))^2} = r_k\, \omega(0°)$$

$$(180°) = r_k\, \omega(180°)$$

$$y'(t) = \sqrt{\left(c^2 - \left[r_g \sin(\theta)\omega\right]^2 + \left[r_k \cos(\theta)\omega\right]^2\right) \cdot \frac{4\left[r_g^2 \cos^2\theta + r_k^2 \sin^2\theta\right]}{(r_k^2 - r_g^2)^2 \sin^2(2\theta(t))}}$$

$N_{Wk,L} = \frac{h_{sp}}{d_{Dr}}$ … Anzahl Wicklungen pro Spulenhöhe

$U_{Dr,Wk}(t) = 2\pi r(t)$ … Drahtumfang abh. von Wickelradius

$$U_{Dr,Line}(t) = N_{Wk,L} \cdot U_{Dr,Wk}(t) \qquad r$$

$$\int v(t) = l_{Wk}(t) \to l_{Wk}(t) \approx v(t) \cdot t$$

$$N_{Wk}(t) = \frac{l_{Wk}(t)}{U_{Dr,Wk}(t)} \to N_L(t) = \operatorname{round}_\downarrow\left(\frac{N_{Wk}(t)}{N_{Wk,L}}\right)$$

$$r(t) = r_{min} + \frac{d_{Dr}}{2} \cdot (N_L(t) \cdot 2 + 1) = \underbrace{r_{min} + \frac{d_{Dr}}{2}}_{r_0} + d_{Dr} \cdot \operatorname{round}_\downarrow\left(\frac{N_{Wk}(t)}{N_{Wk,L}}\right) = r_0 + d_{Dr} \cdot \operatorname{round}_\downarrow\left(\frac{\frac{l_{Wk}(t)}{U_{Dr,Wk}(t)}}{\frac{h_{sp}}{d_{Dr}}}\right)$$

$$\dots_{Wk}(t) = \frac{\dots}{U_{Dr,Wh}(t)} \Rightarrow N_L(t) = \text{round}_\downarrow\left(\frac{\dots_{Wk}(t)}{N_{Wk,L}}\right)$$

$$y'(t) = \sqrt{\left(c^2 - [v_g \sin(\theta)\omega]^2 + [v_h \cos(\theta)\omega]^2\right) \cdot \frac{4\left[v_g^2\cos^2\theta + v_h^2 \sin^2\theta\right]}{\left(v_h^2 - v_g^2\right)^2 \sin^2(2\theta(t))}}$$

$$r(t) = r_{min} + \frac{d_{Dr}}{2}\cdot\left(N_L(t)\cdot 2 + 1\right) = \underbrace{r_{min} + \frac{d_{Dr}}{2}}_{r_0} + d_{Dr}\cdot \text{round}_\downarrow\left(\frac{N_{Wk}(t)}{N_{Wk,L}}\right) = r_0 + d_{Dr}\cdot\text{round}_\downarrow\left(\frac{\frac{L_{Wk}(t)}{U_{Dr,Wk}(t)}}{\frac{h_{sp}}{d_{Dr}}}\right)$$

$$= r_0 + d_{Dr}\cdot\text{round}_\downarrow\left(\frac{v(t)\cdot t\cdot d_{Dr}}{2\pi\cdot r(t)\cdot h_{sp}}\right) = r_0 + d_{Dr}\,\text{round}_\downarrow\left(\frac{v(t)\cdot t}{2\pi r(t)}\cdot\frac{2\pi}{N_{Wk,L}}\right)$$

$$\text{round}_\downarrow(c\cdot x) \overset{?}{=} c\cdot\text{round}_\downarrow(x)$$

$$\underset{\text{const}}{v(t)}\cdot T_{Line} = U_{Dr,Line}(t) = \underset{\text{const}}{N_{Wk,L}}\cdot 2\pi r(t)$$

$$\tau_{Line} = k\cdot r(t)$$

$$r(t) = \frac{v(t)}{N_{Wk,L}\cdot 2\pi}\cdot\tau_{Line}$$

$$r(t) = r_{min} + \frac{d_{Dr}}{2} + d_{Dr}\cdot n_{Line}(t)$$

$$n_{Line}(t) = \text{floor}\left(\frac{t}{\tau_{Line}(t)}\right) = \text{floor}\left(\frac{t\cdot N_{Wk,L}\,2\pi r(t)}{v(t)}\right)$$

$$r(t) = c_1 + d_{Dr}\cdot\text{floor}\left(c_2\cdot t\cdot r(t)\right)$$

1. calc $\tau(n) = \frac{N_{Wk,L}\cdot 2\pi\cdot r(n)}{v}$
2. $n(t) \overset{?}{=}$

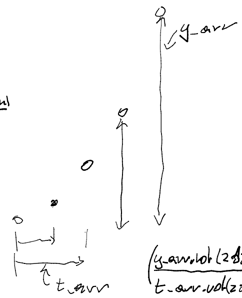
floor	$r(t)$
0	c_1
1	$c_1 + d_{Dr}$
2	$c_2 + 2d_{Dr}$
3	⋮
4	?

$\rightarrow$

$n(t)$	$r(t)$
0	$c_1 + n(t)\,d_{Dr}$
1	
2	
3	
4	

Quick n' dirty Winkelauflösung:
r über ganze Lage konstant
r in 1 wicklung mit

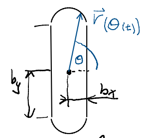
$$n_{Wk,tot}(t) = U_{tot}(t) \Rightarrow \frac{\partial n_{Wk,tot}(t)}{\partial t} = f(t) = \frac{\omega(t)}{2\pi}$$



$$d_{ix} = \frac{d_{it}}{2} \overset{!}{=} [int]$$

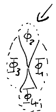
$$\left(\frac{y\_arr.rot(2d_{ix}) - y\_arr}{t\_arr.rot(2d_{ix}) - t\_arr}\right).rot(-d_{ix})$$

↳ then set $\dot{y}[0]$ and $\dot{y}[-1] = 0$



Winkelbereiche

$$\Phi_1 = \left\{\text{atan}\left(\frac{-b_y}{b_x}\right), \text{atan}\left(\frac{+b_y}{b_x}\right)\right\} \Rightarrow$$



$$\Phi_3 = \left\{\text{atan}\left(\frac{b_y}{b_x}\right), \text{atan}\left(\frac{b_y}{-b_x}\right)\right\}$$

⋮

$$x(\theta) = \begin{cases} b_x, & \theta\in\Phi_1 \\ -b_x, & \theta\in\Phi_3 \\ ?, & \theta\in\Phi_2,\Phi_4\end{cases}$$

Koordinaten von Kabel auf Spule abh. von θ

$$\vec{r}(\theta(t)) = \begin{pmatrix} x(\theta(t)) \\ y(\theta(t))\end{pmatrix}$$

$$y(\theta) = \begin{cases} b_x\cdot\tan(\theta), & \theta\in\Phi_1,\Phi_3 \\ ?, & \theta\in\Phi_2,\Phi_4\end{cases}$$

$$v_{Dr}^2 = \left[\dot{\vec{r}}(\theta(t))\right]^2 \leftarrow$$ DGL für Drehzahlberechnung mit v_{Dr} ... const. Drahtgeschw.

↳ DGL nach $\theta(t)$ auflösen (geht wahrscheinlich nur nummerisch

Approx:

$$\vec{r}(\theta(t)) = r_0^{n_{Li}}\cdot\begin{pmatrix}\frac{1}{2}\sin(2\theta(t)) \\ \sin^2(\theta(t))\end{pmatrix}$$

$$\dot{\vec{r}}(\theta)^2 \overset{!}{=} \text{const.} = v_{Dr}^2 \quad \text{Bedingung}$$

$$\partial_t \sin(2\theta(t)) = \cos(2\theta(t))\cdot 2\dot{\theta}(t)$$

$$\partial_t \sin^2(\theta(t)) = \underbrace{2\sin(\theta(t))\cdot\cos(\theta(t))}_{\frac{1}{2}\sin(2\theta(t))}\cdot\dot{\theta}(t)$$

$$\dot{\vec{r}}^2(\theta(t)) = r_0^{n_{Li}^2}\cdot\left(\cos^2(2\theta(t))\,\dot{\theta}^2(t) + \sin^2(2\theta(t))\,\dot{\theta}^2(t)\right)$$

$$= r_0^{n_{Li}^2}\cdot\dot{\theta}^2(t) = v_{Dr}^2$$

$$r_0\cdot\dot{\theta}^2(t) - v_{Dr}^2 = 0$$

From <https://www.wolframalpha.com/input?i=Runge-Kutta+method%2C+dy%2Fdx+%3D+sqrt%280.5%5E2+%2F+%28++%280.075*cos%28y%29%29%5E2+%2B+%280.0075*sin%28y%29%29%5E2+%2B+%28%280.0075%5E2-0.075%5E2%29*sin%282*y%29%29%5E2%2F%284*%28%280.075*cos%28y%29%29%5E2+%2B+%280.0075*sin%28y%29%29%5E2%29%29++%29%29%2C+y%280%29+%3D+0%2C+from+0+to+2%2C+h+%3D+.0125>