ivp1-otchet-checkpoint

May 11, 2024

1

[9]:
```
import numpy as np
import matplotlib.pyplot as plt

def solve_ode(start, end, h, max_calls, eps, fs, initial_conditions):
    t = start
    v = np.array(initial_conditions)
    kounter = [0]
    print(f"{t_0:13.6f}{h:13.6f}{0:13d}{0:13d}", *[f"{x:12.6f}" for x in v])

    def heun_step(t, v, h):
        k1 = fs(t, v, kounter)
        k2 = fs(t + h, v + h * k1, kounter)
        return v + (h / 2) * (k1 + k2)

    steps = []
    min_steps = []
    num_steps = []
    solutions = []

    while t < end and kounter[0] < max_calls:

        k1 = fs(t, v, kounter)
        k2 = fs(t + h, v + h * k1, kounter)
        v1 = v + (h / 2) * (k1 + k2)

        k2 = fs(t + h/2, v + h/2 * k1, kounter)
        v2 = v + (h / 4) * (k1 + k2)

        v2 = heun_step(t + h/2, v2, h/2)

        r = np.linalg.norm(v2 - v1) / 3

        if r > eps:
            h /= 2
            steps.append(h)
        elif r < eps / 64:
            h *= 2
            steps.append(h)
        else:
            if t + h > end:
                h = end - t
            t += h
            v = v1
            steps.append(h)
            solutions.append(v)
            num_steps.append(kounter[0])
            min_steps.append(h)
            print(f"{t:13.6f}{h:13.6f}{r:13.5e}{kounter[0]:13d}", *[f"{x:12.
 ↪6f}" for x in v])

    return steps, min_steps, num_steps, solutions
```

2

[10]:
```
t_0 = 1.5
T = 2.5
h_0 = 0.1
N_x = 10000
eps = 0.0001
n = 3
```

[11]:
```
def fs(t, v, kounter):
    A = np.array([[-0.4, 0.02, 0], [0, 0.8, -0.1], [0.003, 0, 1]])
    kounter[0] += 1
    return np.dot(A, v)

initial_conditions = [1, 1, 2]
```

3

[12]:
```
steps, min_steps, num_steps, solutions = solve_ode(t_0, T, h_0, N_x, eps, fs,␣
 ↪initial_conditions)
```

```
     1.500000     0.100000            0            0     1.000000     1.000000
2.000000
     1.600000     0.100000  8.45154e-05            5     0.962820     1.061398
2.210309
     1.700000     0.100000  9.33737e-05           10     0.927221     1.125613
2.442690
     1.750000     0.050000  1.28171e-05           20     0.909992     1.158775
```

```
2.568019
        1.800000     0.050000  1.34742e-05           25      0.893138      1.192634
2.699768
        1.850000     0.050000  1.41654e-05           30      0.876652      1.227187
2.838267
        1.900000     0.050000  1.48925e-05           35      0.860527      1.262426
2.983862
        1.950000     0.050000  1.56573e-05           40      0.844756      1.298342
3.136916
        2.000000     0.050000  1.64617e-05           45      0.829333      1.334924
3.297812
        2.050000     0.050000  1.73079e-05           50      0.814253      1.372157
3.466951
        2.100000     0.050000  1.81979e-05           55      0.799508      1.410026
3.644757
        2.150000     0.050000  1.91341e-05           60      0.785092      1.448511
3.831672
        2.200000     0.050000  2.01189e-05           65      0.771001      1.487590
4.028165
        2.250000     0.050000  2.11548e-05           70      0.757227      1.527236
4.234726
        2.300000     0.050000  2.22444e-05           75      0.743766      1.567420
4.451871
        2.350000     0.050000  2.33906e-05           80      0.730612      1.608110
4.680143
        2.400000     0.050000  2.45962e-05           85      0.717758      1.649267
4.920111
        2.450000     0.050000  2.58644e-05           90      0.705200      1.690849
5.172377
        2.500000     0.050000  2.71984e-05           95      0.692932      1.732810
5.437568
        2.500000     0.000000  2.86017e-05          100      0.680948      1.775097
5.716349
```

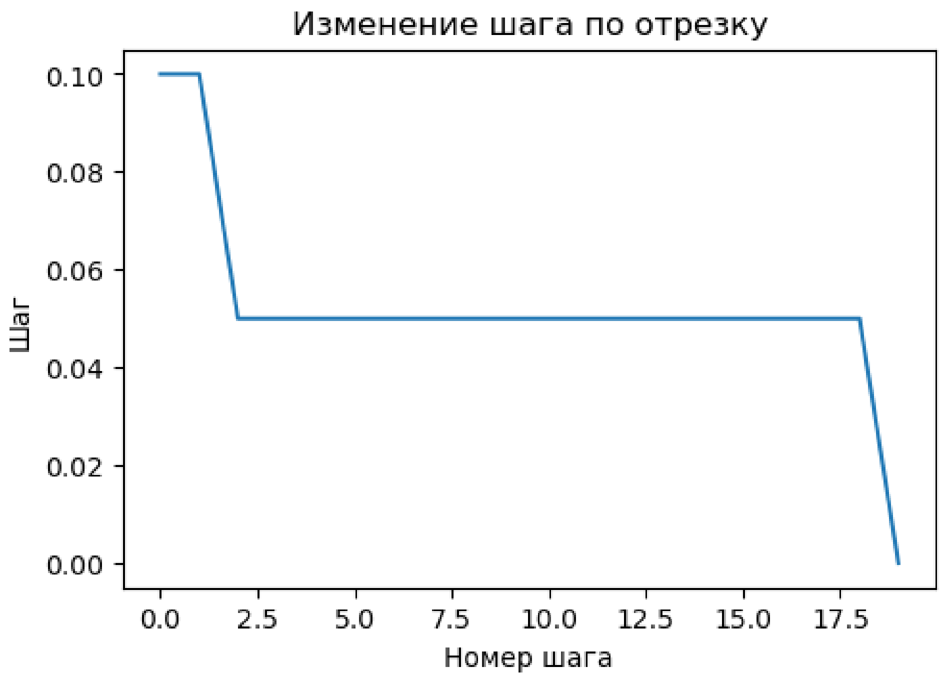
4

4.0.1

[13]:
```python
plt.figure(figsize=(12, 8))

plt.subplot(2, 2, 1)
plt.plot(range(len(steps)), steps)
plt.xlabel('         ')
plt.ylabel('  ')
plt.title('                    ')
```

[13]:
```
Text(0.5, 1.0, '                    ')
```

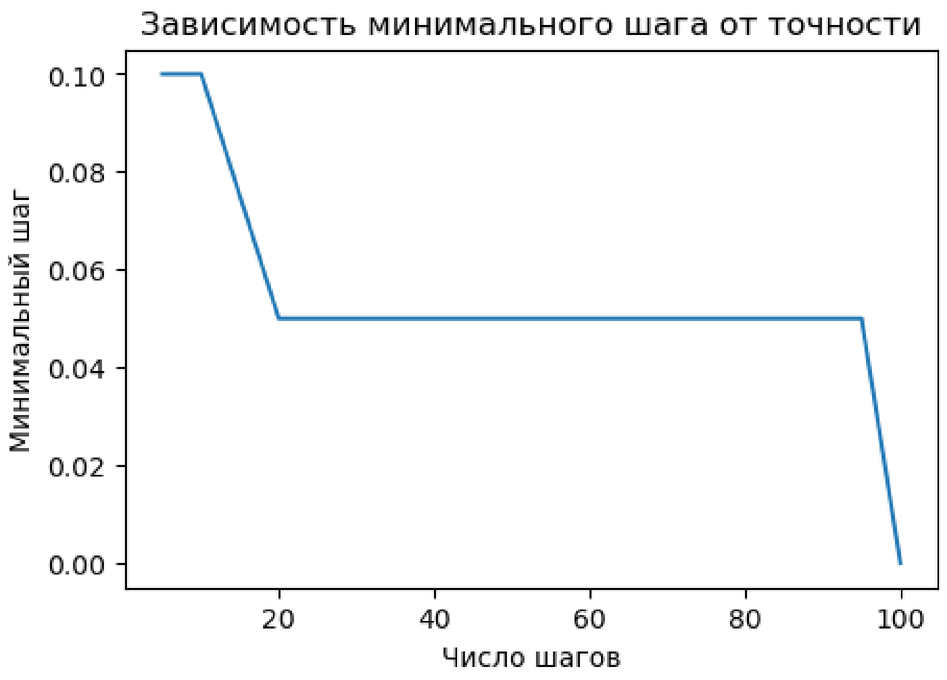


4.0.2

```
[14]: plt.figure(figsize=(12, 8))

      plt.subplot(2, 2, 2)
      plt.plot(num_steps, min_steps)
      plt.xlabel('          ')
      plt.ylabel('             ')
      plt.title('                                ')
```

```
[14]: Text(0.5, 1.0, '                                ')
```

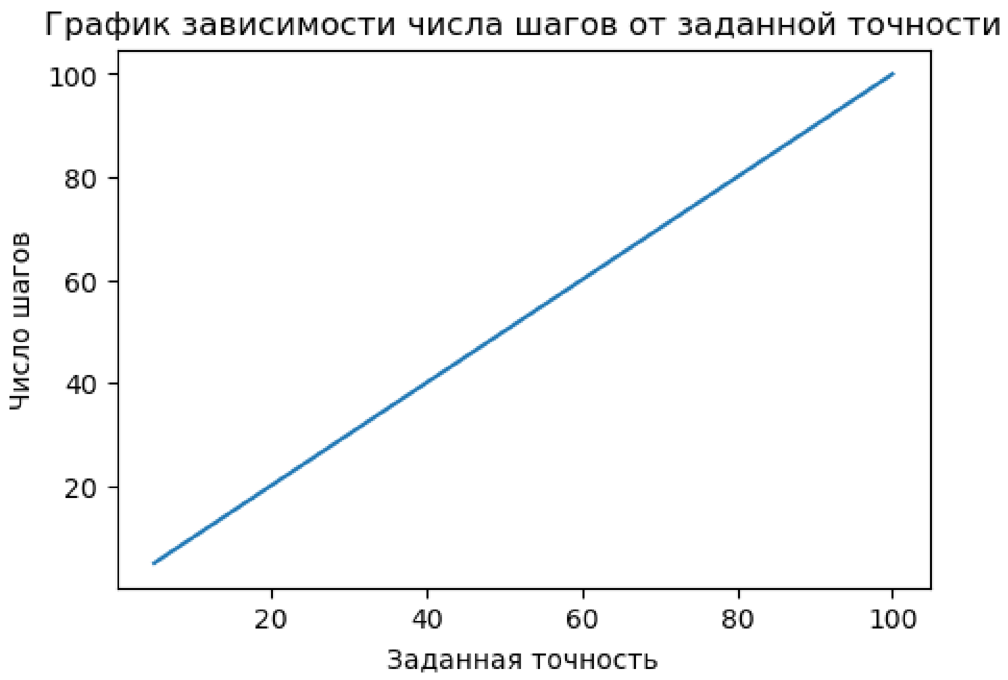


4.0.3

```
[15]: plt.figure(figsize=(12, 8))

      plt.subplot(2, 2, 3)
      plt.plot(num_steps, num_steps)
      plt.xlabel('          ')
      plt.ylabel('       ')
      plt.title('                              ')
```

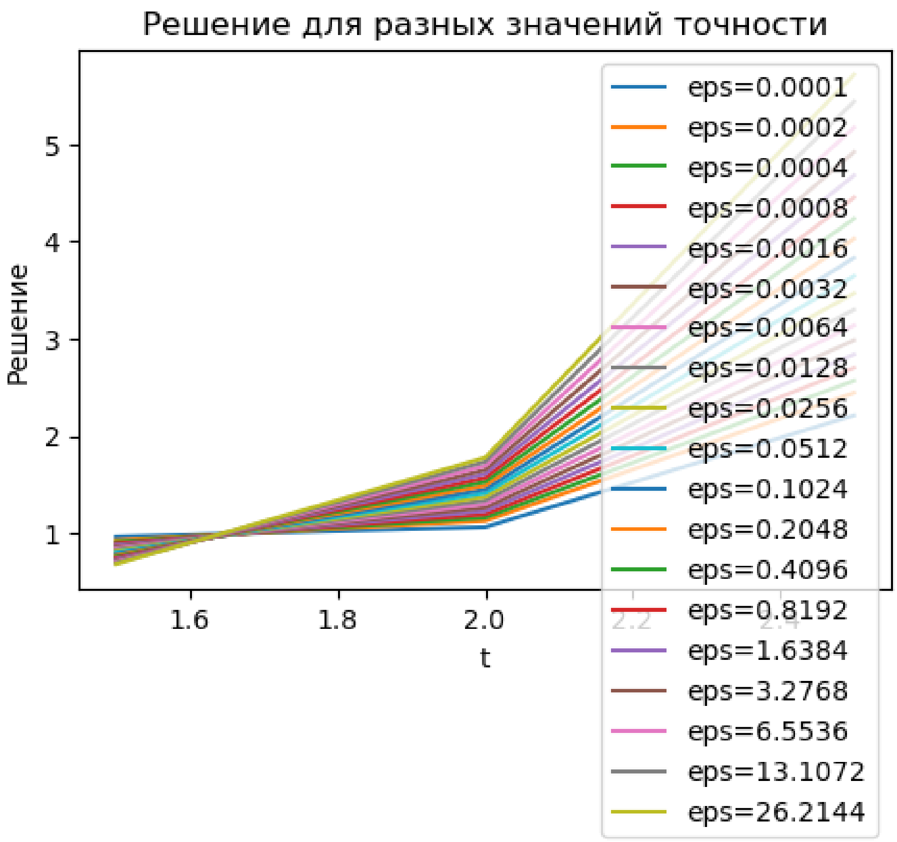
```
[15]: Text(0.5, 1.0, '                              ')
```



4.0.4

[16]:
```
plt.figure(figsize=(12, 8))

plt.subplot(2, 2, 4)
t = np.linspace(t_0, T, len(solutions[0]))
for i, sol in enumerate(solutions):
    plt.plot(t, sol, label=f"eps={eps*(2**i):.4f}")
    plt.xlabel('t')
    plt.ylabel('    ')
    plt.title('                     ')
    plt.legend()
```


Решение для разных значений точности
Решение
t
1.6
1.8
2.0
2.2
2.4
1
2
3
4
5
eps=0.0001
eps=0.0002
eps=0.0004
eps=0.0008
eps=0.0016
eps=0.0032
eps=0.0064
eps=0.0128
eps=0.0256
eps=0.0512
eps=0.1024
eps=0.2048
eps=0.4096
eps=0.8192
eps=1.6384
eps=3.2768
eps=6.5536
eps=13.1072
eps=26.2144