LEXI Angles

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1 Introduction

Let $\mathscr{F}_b$ be the reference frame fixed to the lander body, and $\mathscr{F}_d$ be the reference frame fixed to the LEXI detector. At any point in time, we wish to write the rotation matrix $\mathbf{R}^{db}$:

$$\mathbf{v}_d = \mathbf{R}^{db}\mathbf{v}_b \tag{1}$$

that maps an arbitrary vector $\mathbf{v}_b$ expressed in $\mathscr{F}_b$ into the corresponding vector $\mathbf{v}_d$ expressed in $\mathscr{F}_b$. Note that $\mathbf{R}^{bd} = \mathbf{R}^{db^T}$. The matrix $\mathbf{R}^{db}$ is a function of three variables:

$$\mathbf{R}^{db} = f(\theta_1(t), \theta_2(t), \theta_3) \tag{2}$$

The angles θ_1 and θ_2 are explicitly time-dependent, and can be thought of as either the Az-El or the RA-DEC angles, but that distinction is not important for the present discussion. The angles θ_1 and θ_2 define the direction of the detector boresight, and the angle θ_3 defines the "roll" of the detector's horizontal and vertical axes about the boresight. Note that θ_3 does not vary in time, because the gimbal has no actuation about the boresight; the value of θ_3 is determined by the mechanical design of the detector plate relative to the gimbal. The purpose of this memo is to describe how to measure θ_3 from the LEXI CAD model.

The reference frames are defined by their unit basis vectors, $\mathscr{F}_b = \{\hat{\mathbf{b}}_1, \hat{\mathbf{b}}_2, \hat{\mathbf{b}}_3\}$ and $\mathscr{F}_d = \{\hat{\mathbf{d}}_1, \hat{\mathbf{d}}_2, \hat{\mathbf{d}}_3\}$. The matrix $\mathbf{R}^{db}$ is a Direction Cosine Matrix (DCM), which means that each element R_{ij} of $\mathbf{R}^{db}$ is the direction cosine between $\hat{\mathbf{d}}_i$ and $\hat{\mathbf{b}}_j$. Thus, the first step is to use CAD software to measure the angle between the unit basis vectors. As shown in Fig. 1, the frame $\mathscr{F}_b$ is defined using the same convention that FireFly uses for the Lander Body Frame, where $\hat{\mathbf{b}}_1$ points towards zenith out the top deck, $\hat{\mathbf{b}}_2$ points towards north during a nominal landing, and $\hat{\mathbf{b}}_3$ points west during a nominal landing. The detector frame $\mathscr{F}_d$ is defined with $\hat{\mathbf{d}}_1$ and $\hat{\mathbf{d}}_2$ aligned with Ramiz's convention (X goes positive to the right in an image, Y goes positive down in an image), and $\hat{\mathbf{d}}_3$ is parallel to the boresight with positive direction going from detector to observation target. Based on the measured angles within CAD, the DCM

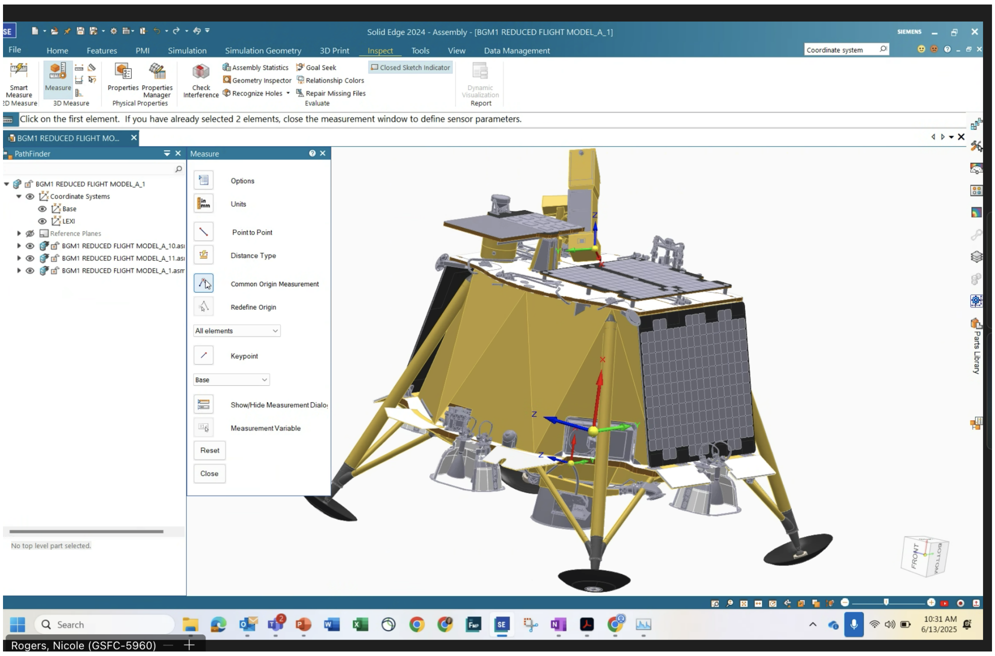

Fig. 1 LEXI and BGM1 CAD models with detector frame and lander body frames defined.

of $\mathbf{R}^{db}$ was found to be:

$$\mathbf{R}^{db} = \begin{bmatrix} \cos(180-55.36) & \cos(38.17) & \cos(75.96) \\ \cos(180-76.31) & \cos(116.02) & \cos(29.89) \\ \cos(180-142.0) & \cos(64.19) & \cos(64.19) \end{bmatrix} \tag{3}$$

$$= \begin{bmatrix} -0.56841826 & 0.78618058 & 0.24259923 \\ -0.23666858 & -0.43868486 & 0.86698374 \\ 0.78801075 & 0.43538823 & 0.43538823 \end{bmatrix} \tag{4}$$

where all the angles above are in degrees, and the terms having the 180 inside the cosine were to account for the correct sign, since the various CAD programs were inconsistent in how they measured angles between vectors. Note that another interpretation of the DCM is as a collection of column vectors of one set of basis vectors expressed in the other frame, namely:

$$\mathbf{R}^{db} = \left[\hat{\mathbf{b}}_1^d \ \hat{\mathbf{b}}_2^d \ \hat{\mathbf{b}}_3^d\right] \tag{5}$$

where the superscript on a basis vector denotes the frame $\mathscr{F}$ in which it is expressed. The understanding in Eq. 5 allowed us to check the sign of the individual terms R_{ij}

to decide whether the 180 needed to be added to Eq. 3. The screenshots below give convenient visual verification of the sign needed in the components in Eq. 5.

We can double check that the numerical values given in Eq. 3 represent a valid rotation matrix in a couple of ways. One property of a rotation matrix is that the norm of the columns and rows should equal 1. Performing this calculation results in

$$\text{Norms along rows} = \begin{bmatrix}1.00001681 & 1.00005861 & 1.00004338\end{bmatrix} \tag{6}$$

$$\text{Norms along columns} = \begin{bmatrix}1.00003614 & 1.00004361 & 1.00003905\end{bmatrix} \tag{7}$$

which are sufficiently close to 1. Another check is that any rotation matrix is orthogonal, and thus $\mathbf{R}^{-1} = \mathbf{R}^T$, which we can check with

$$\mathbf{R}^{-1} - \mathbf{R}^T = \begin{bmatrix} 1.30566263\text{e-}05 & 2.77672763\text{e-}05 & -7.39737298\text{e-}05 \\ -3.87599546\text{e-}05 & 8.35426602\text{e-}05 & -4.61712640\text{e-}05 \\ 1.76195339\text{e-}05 & -8.53548825\text{e-}05 & -1.92275805\text{e-}05 \end{bmatrix} \tag{8}$$

which is sufficiently close to zero. We therefore can have confidence that $\mathbf{R}^{db}$ given above is a valid rotation matrix.

We can also visually check that $\mathbf{R}^{db}$ is correct by doing a visual inspection of the $\mathscr{F}_d$ basis vectors after being transformed with $\mathbf{R}^{bd}$ into $\mathscr{F}_b$ and comparing the illustrations of the frames in the CAD drawings. In Figs. 5 to 7 below, the 3D plot on the left shows the predicted $\mathscr{F}_d$ relative to $\mathscr{F}_b$ given the $\mathbf{R}^{bd}$ computed in Eq. 3, and the right half of each figure shows the CAD view of the lander in the matching orientation; visual inspection allows us to confirm (within the accuracy allowed by this sanity check) that the frames are oriented in the proper direction.

The next step is to find the roll angle given $\mathbf{R}^{db}$. Let the sequence of rotations implied by Eq. 2 be given by the intrinsic 1-2-3 Euler angle sequence:

$$\mathbf{R}^{db} = \mathbf{R}_3(\theta_3)\mathbf{R}_2(\theta_2)\mathbf{R}_1(\theta_1) \tag{9}$$

where θ_3 is the roll angle we wish to find. The equation for this 1-2-3 sequence is

$$\mathbf{R}^{db} = \begin{bmatrix} c_3c_2 & c_3s_2s_1 + s_3c_1 & -c_3s_2c_1 + s_3s_1 \\ -s_3c_2 & -s_3s_2s_1 + c_3c_1 & s_3s_2c_1 + c_3s_1 \\ s_2 & -c_2s_1 & c_2c_1 \end{bmatrix} \tag{10}$$

where $c_1 = \cos\theta_1$, $s_1 = \sin\theta_1$, etc. We can solve for θ_3 using

$$\theta_3 = \text{atan2}\left(-R_{2,1}, R_{1,1}\right) \tag{11}$$

which, given the numerical values for $\mathbf{R}^{db}$ above, give $\theta_3 = 157.3949$ degrees.

A few more steps are required to compute the correct $\mathbf{R}^{db}$ given the RA and DEC angles measured at a given time. That works out to:

$$\mathbf{R}^{db} = \mathbf{R}_3(\theta_3)\mathbf{R}_2(\theta_2)\mathbf{R}_1(\theta_1) \tag{12}$$

where θ_3 is the value given just above, and θ_1 and θ_2 are computed from the actual boresight RA (α) and DEC (δ) angles. Given the RA and DEC angles expressed in the J2000 frame (or ITRF frame, since they are nearly identical for our purposes), the boresight vector expressed in $\mathscr{F}_{\text{J2000}}$ is

$$\mathbf{v}_{\text{J2000}} = \begin{bmatrix} \cos\delta\cos\alpha \\ \cos\delta\sin\alpha \\ \sin\delta \end{bmatrix} \tag{13}$$

This vector is then expressed in the actual lander body frame (i.e. reflecting how the real lander was oriented on the surface according to post-landed telemetry) with

$$\mathbf{v}_{\text{BODYACT}} = \mathbf{R}^{BODYACT \leftarrow J2000}\mathbf{v}_{\text{J2000}} \tag{14}$$

where $\mathbf{R}^{BODYACT \leftarrow J2000}$ is provided in the quaternion data files provided by Mike, based on the post-landed telemetry and FreeFlyer simulation. Then, we can compute the angles θ_1 and θ_2 using

$$\theta_1 = \text{atan2}(v_y, v_z) \tag{15}$$

$$\theta_2 = \text{asin}(v_x) \tag{16}$$

where $\mathbf{v}_{\text{BODYACT}} = (v_x, v_y, v_z)$. Thus, we now know the roll angle θ_3 from earlier (which is constant), and θ_1 and θ_2 can be found at any point in time from the boresight RADEC pointing history data, and we can compute $\mathbf{R}^{db}$ using Eq. 12. If we want to rotate LEXI images between the detector frame to the J2000 frame (and vice versa), that is done with:

$$\mathbf{R}^{d \leftarrow \text{J2000}} = \mathbf{R}^{db}\mathbf{R}^{\text{BODYACT} \leftarrow J2000} \tag{17}$$

or the transpose of the above, noting that the "b" in $\mathbf{R}^{db}$ is treated as the "actual" lander body (versus the "nominal" lander body). When I compute these angles near the time of sunset using the python script, I get approximately $\theta_1 = -0.007$ deg, and $\theta_2 = 29.003$ deg, which makes sense for LEXI pointed westward in a default configuration.

If we wish to find the angle between the LEXI boresight and the local horizontal, ϕ, we can use the following procedure. Let the LEXI boresight in the detector frame $\mathscr{F}_d$ be $\mathbf{v}_d = [0, 0, 1]^T$. Then this boresight vector can be expressed in the "Body Nominal" frame (equivalent to a local topocentric frame) with

$$\mathbf{v}_{\text{BODYNOM}} = \mathbf{R}^{\text{BODYNOM} \leftarrow J000}\mathbf{R}^{J2000 \leftarrow \text{BODYACT}}\mathbf{R}^{bd}\mathbf{v}_d \tag{18}$$

where $\mathbf{R}^{bd}$ comes from evaluating the transpose of Eq. 10, given θ_1 from Eq. 15, θ_2 from Eq. 16, and $\theta_3 = 157.3949$ deg. Recognizing that the x component is the vertical component of $\mathbf{v}_{\text{BODYNOM}}$, we find ϕ with

$$\phi = \text{asin}\left(\mathbf{v}_{\text{BODYNOM}}[0]\right) \tag{19}$$

which works out to be a nearly constant value of around $\phi = 24.825$ deg for the time span at sunset when the gimbal was stationary. Comparing this value with $\theta_2 =$

29.003 deg, these two values are in agreement, because the actual lander body was tilted 4.197 deg from local vertical in a direction that was less than 1 degree from due west. In other words, as a sanity check, starting from the as-landed orientation, the boresight direction needs to be tilted up by approximately 4.2 deg to reach the local horizontal, then another 24.8 deg to reach the final value of 29 deg, thus completing the sanity check.

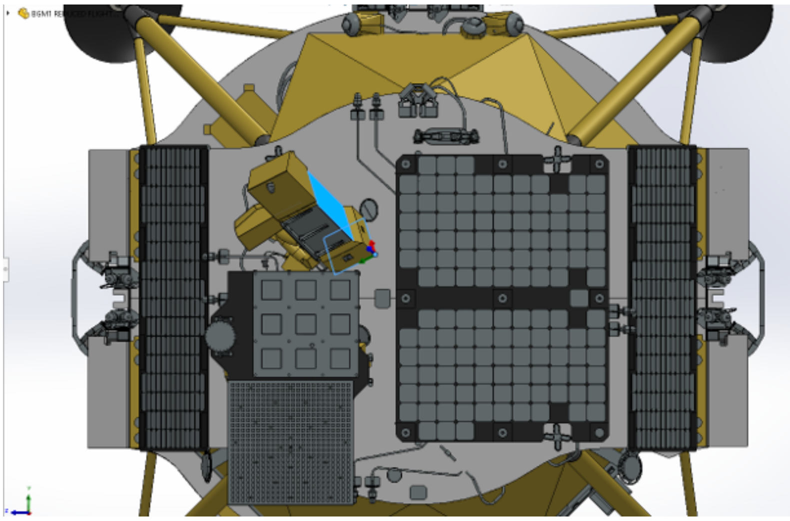

Fig. 2 View of Lander Y-Z frame.

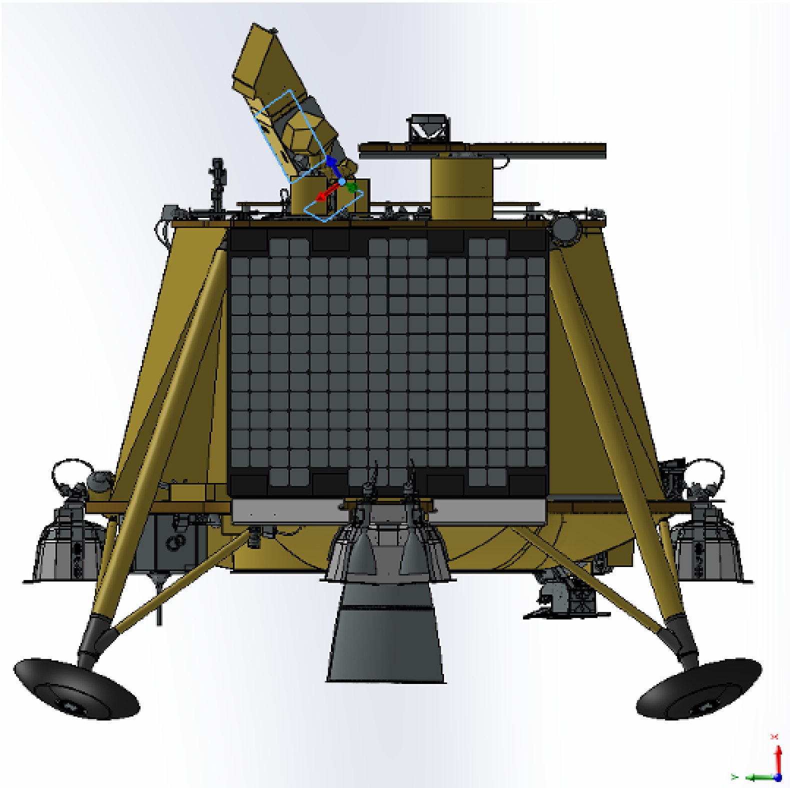

Fig. 3 View of Lander X-Y frame.

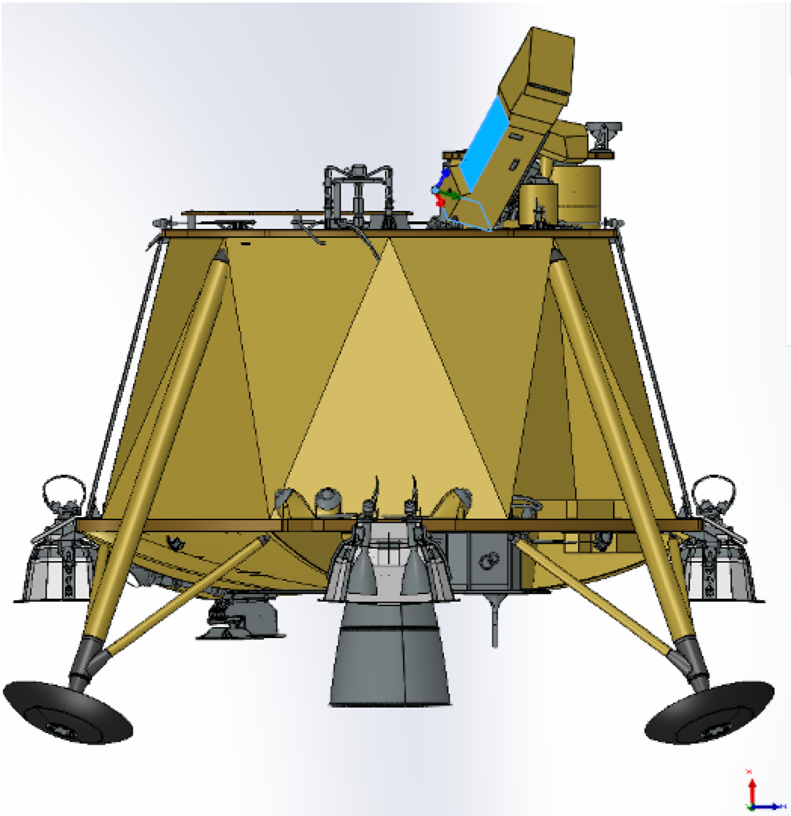

Fig. 4 View of Lander X-Z frame.

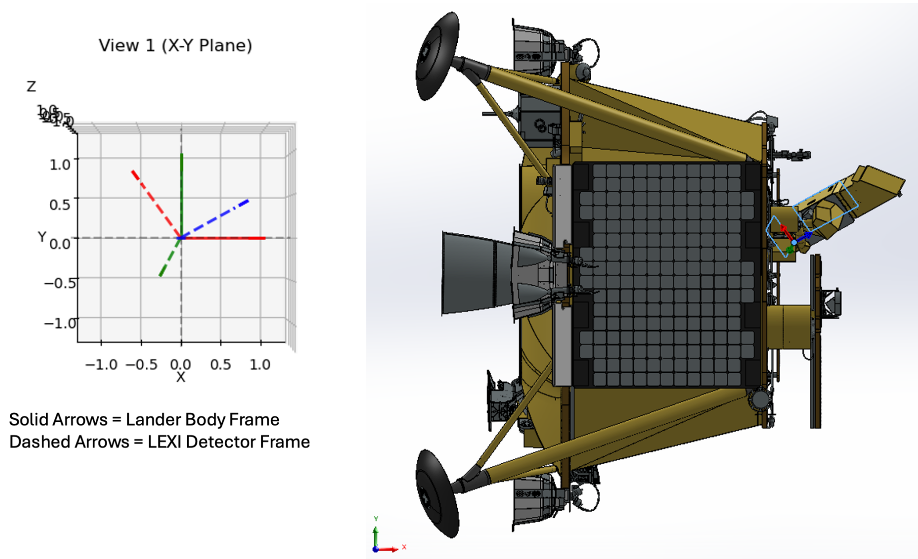

Fig. 5 View of Lander X-Y frame with check of rotated basis vectors.

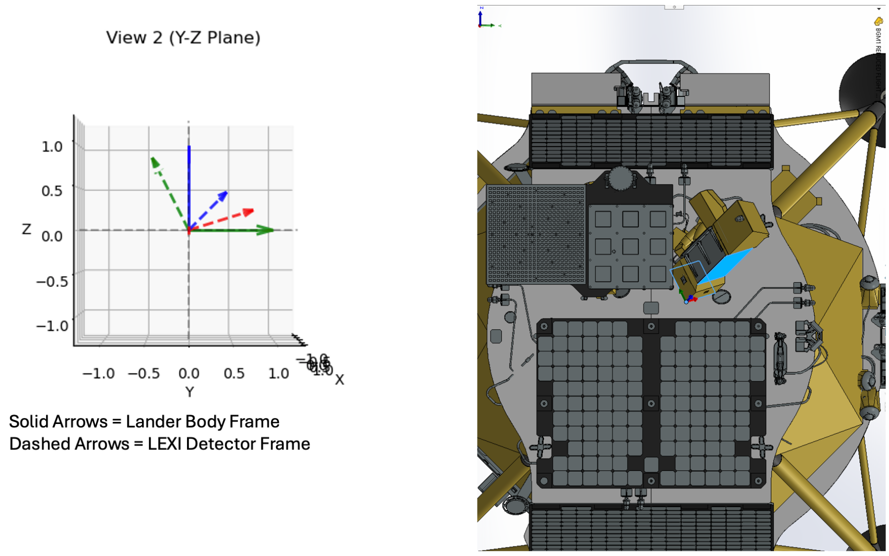

Fig. 6 View of Lander Y-Z frame with check of rotated basis vectors.

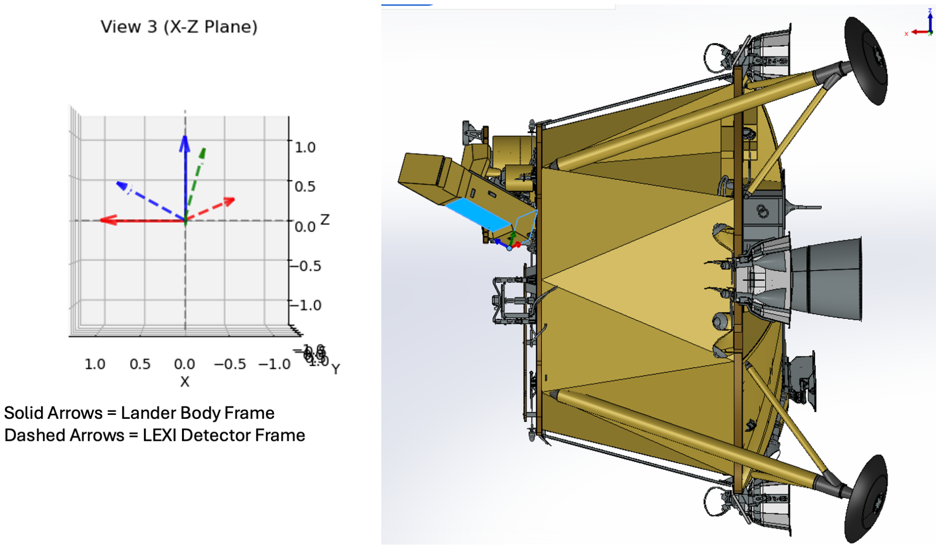

Fig. 7 View of Lander X-Z frame with check of rotated basis vectors.