Esercizio 1

$$T(n)\begin{cases} 1, & \text{se } n \leq 2 \\ 2 \cdot T(\sqrt[4]{n}) + \log(2n), & \text{altrimenti} \end{cases}$$

Livello	Nodi per livello	Dimensione in input	Contributo per nodo	Contributo totale per livello
0	1	n	$\log(2n)$	$\log(2n)$
1	2	$n^{\frac{1}{4}}$	$1+\frac{1}{4}\log(n)$	$2+\frac{1}{2}\log(n)$
2	4	$n^{\frac{1}{16}}$	$1+\frac{1}{16}\log(n)$	$4+\frac{1}{4}\log(n)$
3	8	$n^{\frac{1}{64}}$	$1+\frac{1}{64}\log(n)$	$8+\frac{1}{8}\log(n)$
i	2^i	$n^{\frac{1}{4^i}}$	$1+\frac{1}{4^i}\log(n)$	$2^i+\frac{1}{2^i}\log(n)$

Dobbiamo calcolare l'altezza:

$$n^{\frac{1}{4^h}} \leq 2 \Rightarrow \log_2\left(n^{\frac{1}{4^h}}\right) \leq 1 \Rightarrow \frac{1}{4^h}\log_2(n) \leq 1 \Rightarrow 4^h \geq \log_2(n) \Rightarrow h \geq \log_2(\log_2(n))$$

$$\sum_{i=0}^{h} 2^i + \frac{1}{2^i}\log_2(n) = \sum_{i=0}^{h} 2^i + \log_2(n)\sum_{i=0}^{h}\left(\frac{1}{2}\right)^i = \frac{1-2^{h+1}}{1-2} + \log_2(n)\cdot\frac{1-\left(\frac{1}{2}\right)^{h+1}}{1-\frac{1}{2}} =$$

$$=\left(2(2)^{\log_2(\log_2(n))} - 1\right) + \left(2\log_2(n)\cdot 1 - \frac{1}{2}\left(\frac{1}{2}\right)^{\log_2(\log_2(n))}\right) = 2\log_2(n) - 1 + \left(2\log_2(n)\left(1 - \frac{1}{2\log_2(n)}\right)\right) =$$

$$= 2\log_2(n) - 1 + 2\log_2(n) - 1 = 4\log_2(n) - 2 \Rightarrow \Theta(\log_2(n))$$