Esercizio 1

- $T_{\mathcal{E}}^{\mathcal{E}} = \begin{pmatrix} 1 & 3 \\ 2 & 2 \end{pmatrix}$. Calcoliamo il polinomio caratteristico: $|P_T(\lambda)| = 0 \Rightarrow$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 3 \\ 2 & 2-\lambda \end{vmatrix} = \lambda^2 - 3\lambda - 4 \Rightarrow \underset{ma = 1}{\lambda_1 = -1}, \; \underset{ma = 1}{\lambda_2 = 4}$$

$\lambda_1 \to mg = 1$ e $\lambda_2 \to mg = 1$. Poiché $ma = mg$ per entrambi gli autovalori, allora l'endomorfismo é semplice $\Rightarrow$ é diagonalizzabile.

- Per $\lambda = -1$:

$$\left(\begin{array}{cc|c} 2 & 3 & 0 \\ 2 & 3 & 0 \end{array}\right) \sim \{ 2x + 3y = 0 \sim \{ x = -\tfrac{3}{2} y \Rightarrow V_{-1} = \text{Span}\left\{ \begin{pmatrix} -3/2 \\ 1 \end{pmatrix} \right\}$$

- Per $\lambda = 4$

$$\left(\begin{array}{cc|c} -3 & 3 & 0 \\ 2 & -2 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} -3 & 3 & 0 \\ 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} -1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right) \sim \{ x = y \Rightarrow V_4 = \text{Span}\left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$$

Quindi $B = \left\{ \begin{pmatrix} -3/2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$ e $T_B^B = \begin{pmatrix} -1 & 0 \\ 0 & 4 \end{pmatrix}$

- $T_{\mathcal{E}}^{\mathcal{E}} = \begin{pmatrix} -1 & -1 \\ 1 & -3 \end{pmatrix}$. Calcolo il polinomio caratteristico $|P_T(\lambda)| = 0$

$$\begin{vmatrix} -1-\lambda & -1 \\ 1 & -3-\lambda \end{vmatrix} = \lambda^2 + 4\lambda + 4 = 0 \Rightarrow \lambda = -2 \text{ con } ma = 2$$

Calcoliamo l'autospazio per trovare mg:

$$\left(\begin{array}{cc|c} 1 & -1 & 0 \\ 1 & -1 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right) \sim \{ x = y \Rightarrow V_{-2} = \text{Span}\left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$$

$mg(-2) = 1 \Rightarrow ma(-2) \neq mg(-2) \Rightarrow$ non é diagonalizzabile.

- Calcolo il polinomio caratteristico $|P_A(\lambda)| = 0$

$$\begin{vmatrix} 1-\lambda & -2 & 2 \\ 0 & 1-\lambda & 0 \\ 1 & 0 & -\lambda \end{vmatrix} = -(\lambda-1)(\lambda+1)(\lambda-2) = 0 \Rightarrow \lambda = 1, -1, 2$$

Gli autovalori sono distinti $\Rightarrow$ endomorfismo semplice $\Rightarrow$ è diagonalizzabile
Calcolo gli autospazi:

- Per $\lambda = 1$:

$$\left(\begin{array}{ccc|c} 0 & -2 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & -2 & 2 & 0 \end{array}\right) \sim \begin{cases} x = z \\ y = z \end{cases} \Rightarrow V_1 = \text{Span}\left\{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right\}$$

- Per $\lambda = -1$:

$$\left(\begin{array}{ccc|c} 2 & -2 & 2 & 0 \\ 0 & 2 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right) \sim \begin{cases} x = -z \\ y = 0 \end{cases}$$

$$\Rightarrow V_{-1} = \text{Span}\left\{\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}\right\}$$

- Per $\lambda = 2$

$$\left(\begin{array}{ccc|c} -1 & -2 & 2 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & -2 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} -1 & -2 & 2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} -1 & -2 & 2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \sim \begin{cases} x = 2z \\ y = 0 \end{cases}$$

$$\Rightarrow V_2 = \text{Span}\left\{\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}\right\}$$

$$B = \left\{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}\right\} \quad e \quad (L_A)_B^B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

• Calcolo il polinomio caratteristico $|P_{LA}(\lambda)| = 0$:

$$\begin{vmatrix} 2-\lambda & 1 & 1 \\ 1 & 3-\lambda & 0 \\ 3 & 1 & -\lambda \end{vmatrix} = -\lambda^3 + 5\lambda^2 - 2\lambda - 8 = -(\lambda+1)(\lambda-4)(\lambda-2) = 0 \Rightarrow$$
$$\Rightarrow \lambda = -1, 4, 2$$

Gli autovalori sono distinti => endomorfismo semplice => è diagonalizzabile
Calcolo gli autospazi :

- Per $\lambda = -1$

$$\left(\begin{array}{ccc|c} 3 & 1 & 1 & 0 \\ 1 & 4 & 0 & 0 \\ 3 & 1 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 4 & 0 & 0 \\ 0 & -11 & 1 & 0 \\ 0 & -11 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 4 & 0 & 0 \\ 0 & -11 & 1 & 0 \end{array}\right) \sim \begin{cases} x = -\frac{4}{11} z \\ y = \frac{1}{11} z \end{cases}$$

$$\Rightarrow V_{-1} = \text{Span}\left\{\begin{pmatrix} -4/11 \\ 1/11 \\ 1 \end{pmatrix}\right\}$$

- Per $\lambda = 4$:

$$\left(\begin{array}{ccc|c} -2 & 1 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 3 & 1 & -4 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 4 & -4 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \sim \begin{cases} x = z \\ y = z \end{cases} \Rightarrow$$

$$\Rightarrow V_4 = \text{Span}\left\{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right\}$$

- Per $\lambda = 2$

$$\left(\begin{array}{ccc|c} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 3 & 1 & -2 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -2 & -2 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \sim \begin{cases} x = z \\ y = -z \end{cases}$$

$$\Rightarrow V_2 = \text{Span}\left\{\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}\right\}$$

$$B = \left\{\begin{pmatrix} -4/11 \\ 1/11 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}\right\} \quad e \quad (LA)^B_B = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

- Calcolo il polinomio caratteristico $|P_{LA}(\lambda)| = 0$:

$$\begin{vmatrix} 3-\lambda & 2 & -1 \\ 0 & -3-\lambda & 0 \\ 1 & 1 & 1-\lambda \end{vmatrix} = (-3-\lambda)(\lambda-2)(\lambda-2) = 0 \Rightarrow \lambda = -3, 2$$

Non é un endomorfismo semplice dato che $ma(-3) = 1$ e $ma(2) = 2$.
Calcoliamo quindi $mg(2)$ per vedere se é diagonalizzabile :

- Per $\lambda = 2$:

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & -5 & 0 & 0 \\ 1 & 1 & -1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & -5 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \sim \begin{cases} x = z \\ y = 0 \end{cases}$$

$V_2 = \text{Span}\left\{\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}\right\} \Rightarrow mg(2) = 1 \neq ma(2) \Rightarrow$ NO diagonalizzabile

Esercizio 2