## Esercizio 1

$$T(n) = \begin{cases} 1, \text{ se } n \leq 4 \\ 8 \cdot T\left(\frac{n}{4}\right) + \sqrt{n}, \text{ altrimenti} \end{cases}$$

| Livello | Nodi per livello | Dimensione in input | Contributo per nodo | Contributo totale per livello |
|---|---|---|---|---|
| 0 | 1 | $n$ | $\sqrt{n}$ | $\sqrt{n}$ |
| 1 | 8 | $\frac{n}{4}$ | $\frac{\sqrt{n}}{2}$ | $4\sqrt{n}$ |
| 2 | 64 | $\frac{n}{16}$ | $\frac{\sqrt{n}}{4}$ | $16\sqrt{n}$ |
| 3 | 512 | $\frac{n}{64}$ | $\frac{\sqrt{n}}{8}$ | $64\sqrt{n}$ |
| $i$ | $8^i$ | $\frac{n}{4^i}$ | $\frac{\sqrt{n}}{2^i}$ | $4^i\sqrt{n}$ |

$$\sum_{i=0}^{h} 4^i\sqrt{n} = \sqrt{n}\sum_{i=0}^{h} 4^i = \sqrt{n}\cdot\frac{4^{h+1}-1}{4-1} = 3\sqrt{n}\cdot\left(4(4^{\log_4(n)})-1\right) = 3\sqrt{n}\cdot(4n-1) =$$

$$= 12n^{\frac{3}{2}} - 3\sqrt{n} \Rightarrow \Theta\left(n^{\frac{3}{2}}\right)$$