

Esercizio 1

$$S_K = \begin{cases} X_2 - X_3 + X_5 = -1 \\ -X_1 + 2X_2 + 2X_4 + X_5 = 2 \\ -X_1 + 2X_3 + KX_4 + (K-2)X_5 = 3K-1 \\ -X_1 + 2X_3 + 2X_4 + (K-3)X_5 = K+2 \end{cases} \sim \begin{pmatrix} 0 & 1 & -1 & 0 & 1 & -1 \\ -1 & 2 & 0 & 2 & 1 & 2 \\ -1 & 0 & 2 & K & K-2 & 3K-1 \\ -1 & 0 & 2 & 2 & K-3 & K+2 \end{pmatrix} \sim \begin{pmatrix} -1 & 2 & 0 & 2 & 1 & 2 \\ 0 & 1 & -1 & 0 & 1 & -1 \\ -1 & 0 & 2 & K & K-2 & 3K-1 \\ -1 & 0 & 2 & 2 & K-3 & K+2 \end{pmatrix}$$

$$\sim \begin{pmatrix} -1 & 2 & 0 & 2 & 1 & 2 \\ 0 & 1 & -1 & 0 & 1 & -1 \\ 0 & -2 & 2 & K-2 & K-3 & 3K-3 \\ 0 & -2 & 2 & 0 & K-4 & K \end{pmatrix} \xrightarrow{\cdot \frac{1}{2}} \begin{pmatrix} -1 & 2 & 0 & 2 & 1 & 2 \\ 0 & 1 & -1 & 0 & 1 & -1 \\ 0 & -1 & 1 & \frac{K-2}{2} & \frac{K-3}{2} & \frac{3K-3}{2} \\ 0 & -1 & 1 & 0 & \frac{K-4}{2} & \frac{K}{2} \end{pmatrix} \sim \begin{pmatrix} -1 & 2 & 0 & 2 & 1 & 2 \\ 0 & 1 & -1 & 0 & 1 & -1 \\ 0 & 0 & 0 & \frac{K-2}{2} & \frac{K-1}{2} & \frac{-1}{2} \\ 0 & 0 & 0 & 0 & \frac{K-2}{2} & \frac{K-2}{2} \end{pmatrix}$$

$$\sim \begin{pmatrix} -1 & 2 & 0 & 2 & 1 & 2 \\ 0 & 1 & -1 & 0 & 1 & -1 \\ 0 & 0 & 0 & K-2 & K-1 & 3K-5 \\ 0 & 0 & 0 & 0 & K-2 & K-2 \end{pmatrix}$$

• Per $K=2$

$$\begin{pmatrix} -1 & 2 & 0 & 2 & 1 & 2 \\ 0 & 1 & -1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \sim \begin{cases} -X_1 + 2X_2 + 2X_4 + X_5 = 2 \\ X_2 - X_3 + X_5 = -1 \\ X_5 = 1 \end{cases} \sim \begin{cases} X_1 = 2X_3 + 2X_4 - 5 \\ X_2 = X_3 - 2 \\ X_5 = 1 \end{cases} \quad \begin{matrix} X_3 = s \\ X_4 = t \end{matrix}$$

$$V(s) = \left\{ \begin{pmatrix} 5 \\ -2 \\ 0 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} s + \begin{pmatrix} 2 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} t \right\}$$

Per $K \neq 2$:

$$\begin{pmatrix} -1 & 2 & 0 & 2 & 1 & 2 \\ 0 & 1 & -1 & 0 & 1 & -1 \\ 0 & 0 & 0 & K-2 & K-1 & 3K-5 \\ 0 & 0 & 0 & 0 & K-2 & K-2 \end{pmatrix} \sim \begin{cases} -X_1 + 2X_2 + 2X_4 + X_5 = 2 \\ X_2 - X_3 + X_5 = -1 \\ (K-2)X_4 + (K-1)X_5 = 3K-5 \\ (K-2)X_5 = K-2 \end{cases} \sim \begin{cases} X_1 = 2X_3 + \frac{4-K}{K-2} \\ X_2 = X_3 - 2 \\ X_4 = \frac{2K-3}{K-2} \\ X_5 = 1 \end{cases} \quad X_3 = t$$

$$V(s) = \left\{ \begin{pmatrix} \frac{4-K}{K-2} \\ -2 \\ 0 \\ \frac{2K-3}{K-2} \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} t \right\}$$

Esercizio 2

$$S_K : \begin{cases} X_1 + KX_2 + X_3 = K \\ KX_2 + X_4 = K+1 \\ X_1 + KX_2 + KX_3 + (K+1)X_4 = K \end{cases} \sim \begin{pmatrix} 1 & K & 1 & 0 & K \\ 0 & K & 0 & 1 & K+1 \\ 1 & K & K & K+1 & K \end{pmatrix} \sim \begin{pmatrix} 1 & K & 1 & 0 & K \\ 0 & K & 0 & 1 & K+1 \\ 0 & 0 & K-1 & K+1 & 0 \end{pmatrix}$$

Per $K=1$:

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \sim \begin{cases} X_1 + X_2 + X_3 = 1 \\ X_2 + X_4 = 0 \end{cases} \sim \begin{cases} X_1 = 1 - X_3 - X_4 \\ X_2 = X_4 \end{cases} \quad \begin{matrix} X_3 = s \\ X_4 = t \end{matrix}$$

$$V(s) = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix} s + \begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix} t \right\}$$

Per $\kappa = 0$

$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{array} \right) \sim \left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{array} \right) \sim \begin{cases} X_1 + X_3 = 0 \\ X_3 + X_4 = 0 \\ X_4 = 1 \end{cases} \sim \begin{cases} X_1 = 1 \\ X_3 = 1 \\ X_4 = 1 \end{cases} \quad X_2 = t$$

$$V(s) = \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} t \right\}$$

Esercizio 3

$$S_{\kappa} \begin{cases} X_1 - X_2 + (1-\kappa)X_3 + (1-\kappa)X_4 = -2 \\ -X_1 + X_2 + 2X_3 + 2X_4 = 2 \\ -2X_2 + (1-\kappa)X_3 + (2-\kappa)X_4 = -1 \end{cases} \sim \left(\begin{array}{cccc|c} 1 & -1 & 1-\kappa & 1-\kappa & -2 \\ -1 & 1 & 2 & 2 & 2 \\ 0 & -2 & 1-\kappa & 2-\kappa & 1 \end{array} \right) \sim \left(\begin{array}{cccc|c} 1 & -1 & 1-\kappa & 1-\kappa & -2 \\ 0 & 0 & 3-\kappa & 3-\kappa & 0 \\ 0 & -2 & 1-\kappa & 2-\kappa & 1 \end{array} \right) \sim$$

$$\left(\begin{array}{cccc|c} 1 & -1 & 1-\kappa & 1-\kappa & -2 \\ 0 & -2 & 1-\kappa & 2-\kappa & 1 \\ 0 & 0 & 3-\kappa & 3-\kappa & 0 \end{array} \right) \sim \left(\begin{array}{cccc|c} 1 & -1 & 1-\kappa & 1-\kappa & -2 \\ 0 & -2 & 1-\kappa & 2-\kappa & 1 \\ 0 & 0 & 3-\kappa & 3-\kappa & 0 \end{array} \right)$$

Per $\kappa = 3$:

$$\left(\begin{array}{cccc|c} 1 & -1 & -2 & -2 & -2 \\ 0 & -2 & -2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \Rightarrow \text{Compatibile, presenta due VL} \Rightarrow \dim(V(s)) = 2$$

Per $\kappa \neq 3$:

$$\left(\begin{array}{cccc|c} 1 & -1 & 1-\kappa & 1-\kappa & -2 \\ 0 & -2 & 1-\kappa & 2-\kappa & 1 \\ 0 & 0 & 3-\kappa & 3-\kappa & 0 \end{array} \right) \Rightarrow \text{compatibile, presenta 1 VL} \Rightarrow \dim(V(s)) = 1$$

Per $\kappa = 2$

$$\left(\begin{array}{cccc|c} 1 & -1 & -1 & -1 & -2 \\ 0 & -2 & -1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{array} \right) \sim \begin{cases} X_1 - X_2 - X_3 - X_4 = -2 \\ -2X_2 - X_3 = 1 \\ X_3 + X_4 = 0 \end{cases} \sim \begin{cases} X_1 = \frac{X_4 - 5}{2} \\ X_2 = \frac{X_4 - 1}{2} \\ X_3 = -X_4 \end{cases} \quad X_4 = t$$

$$V(s) = \left\{ \begin{pmatrix} -5/2 \\ -1/2 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1/2 \\ 1/2 \\ -1 \\ 1 \end{pmatrix} t \right\}$$