

Esercizio 1

$$T(n) = \begin{cases} 1, & \text{se } n \leq 5 \\ 2 \cdot T\left(\frac{n}{5}\right) + 3 \cdot T\left(\sqrt[5]{n}\right) + 4, & \text{altrimenti} \end{cases}$$

Livello	Nodi per livello	Dimensioni in input	Contributo per nodo	Contributo totale per livello
0	1	n	4	4
1	5	$\frac{n}{5}$ o $\sqrt[5]{n}$	4	20
2	25	$\frac{n}{25}$ o $\sqrt[5]{\frac{n}{5}}$ o $\sqrt[5]{\sqrt[5]{n}}$	4	100
i	5^i	/	4	$4 \cdot 5^i$

Calcoliamo le altezze:

$$\frac{n}{5^{h_{\max}}} \leq 5 \Leftrightarrow n \leq 5 \cdot 5^{h_{\max}} \Leftrightarrow 5^{h_{\max}} \geq \frac{n}{5} \Leftrightarrow h_{\max} \geq \log_5\left(\frac{n}{5}\right)$$

$$\frac{1}{5^{h_{\min}}} \leq 5 \Leftrightarrow \frac{1}{5^{h_{\min}}} \log_5(n) \leq 1 \Leftrightarrow \log_5(n) \leq 5^{h_{\min}} \Leftrightarrow h_{\min} \geq \log_5(\log_5(n))$$

Calcoliamo le sommatorie:

$$\sum_{i=0}^{h_{\min}} 4 \cdot 5^i \leq T(n) \leq \sum_{i=0}^{h_{\max}} 4 \cdot 5^i$$

$$4 \sum_{i=0}^{h_{\min}} 5^i \leq T(n) \leq 4 \sum_{i=0}^{h_{\max}} 5^i$$

$$4 \frac{5^{h_{\min}+1} - 1}{5 - 1} \leq T(n) \leq 4 \frac{5^{h_{\max}+1} - 1}{5 - 1}$$

$$5 \cdot 5^{\log_5(\log_5(n))} - 1 \leq T(n) \leq 5 \cdot 5^{\log_5\left(\frac{n}{5}\right)} - 1$$

$$5 \log_5(n) - 1 \leq T(n) \leq n - 1 \Rightarrow \Omega(\log_5(n)) \text{ e } O(n)$$

