Esercizio 1

$$\log_2(n^{2n}) + n - \log_2(n) = \Theta(\log_2(n)^n)$$

1) Semplifico

$$2n\log_2(n) + n - \log_2(n) = \Theta(n\log_2(n))$$

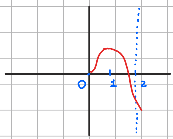
2) Limite

$$\lim_{n\to\infty} \frac{2n\log_2(n)+n-\log_2(n)}{n\log_2(n)} = \lim_{n\to\infty} \overset{2}{\frac{2n\log_2(n)}{n\log_2(n)}} + \overset{0}{\frac{n}{n\log_2(n)}} - \overset{0}{\frac{\log_2(n)}{n\log_2(n)}} = 2$$

3) Derivata

$$\frac{d}{dn}\frac{2n\log_2(n)+n-\log_2(n)}{n\log_2(n)} = \frac{d}{dn}\left(2+\frac{1}{\log_2(n)}-\frac{1}{n}\right) = 0 - \frac{n\ln(2)}{2\log_2(n)} + \frac{1}{n^2} = -\frac{n^3\ln(2)-2\log_2(n)}{2n^2\log_2(n)}$$

$$\frac{2\log_2(n) - n^3\ln(2)}{2n^2\log_2(n)} > 0$$



$2\log_2(n) - n^3\ln(2) > 0 \Rightarrow$ Numeratore sempre negativo

↳ per cui $0 \le n \le 1$ (per denominatore)

Quindi $C_1 = 2$ e $C_2 = \dfrac{2\,2\log_2(2)+2-\log_2(2)}{2\log_2(2)} = \dfrac{5}{2}$