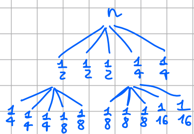
Esercizio 1



$$T(n) = \begin{cases} 1, & \text{se } n \leq 2 \\ 3\,T(\sqrt{n}) + 2\,T(\sqrt[4]{n}) + \log(n), & \text{altrimenti} \end{cases}$$

Livello	Nodi per livello	Dimensione in input	Contributo per nodo	Contributo totale per livello
0	1	n	$\log(n)$	$\log(n)$
1	5	$n^{\frac{1}{2}}$ o $n^{\frac{1}{4}}$	$\frac{1}{2}\log(n)$ o $\frac{1}{4}\log(n)$	$2\log(n)$
2	25	$n^{\frac{1}{4}}$ o $n^{\frac{1}{8}}$ o $n^{\frac{1}{16}}$	$\frac{1}{4}\log(n)$ o $\frac{1}{8}\log(n)$ o $\frac{1}{16}\log(n)$	$4\log(n)$
i	5^i	/	/	$2^i\log(n)$

$\Rightarrow 3 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4}$

$\Rightarrow 9 \cdot \frac{1}{4} + 12 \cdot \frac{1}{8} + 4 \cdot \frac{1}{16}$

$$n^{\frac{1}{4^{hmin}}} \leq 2 \iff \frac{1}{4^{hmin}}\log(n) \leq 1 \iff 2^{2hmin} \geq \log(n) \iff hmin \geq \frac{1}{2}\log(\log(n))$$

$$n^{\frac{1}{2^{hmax}}} \leq 2 \iff \frac{1}{2^{hmax}}\log(n) \leq 1 \iff 2^{hmax} \geq \log(n) \iff hmax \geq \log(\log(n))$$

Calcolo della sommatoria:

$$\sum_{i=0}^{hmin} 2^i \log(n) \leq T(n) \leq \sum_{i=0}^{hmax} 2^i \log(n)$$

$$\log(n) \sum_{i=0}^{hmin} 2^i \leq T(n) \leq \log(n) \sum_{i=0}^{hmax} 2^i$$

$$\log(n) \cdot \frac{2^{hmin+1} - 1}{2-1} \leq T(n) \leq \log(n)\, \frac{2^{hmax+1} - 1}{2-1}$$

$$\log(n)\left(2^{\frac{1}{2}\log(\log(n))+1} - 1\right) \leq T(n) \leq \log(n)\left(2^{\log(\log(n))+1} - 1\right)$$

$$\log(n)\left(2 \cdot 2^{\frac{1}{2}\log(\log(n))} - 1\right) \leq T(n) \leq \log(n)\left(2 \cdot 2^{\log(\log(n))} - 1\right)$$

$$\log(n)\left(2 \cdot \frac{1}{2}\log(n) - 1\right) \leq T(n) \leq \log(n)\left(2\log(n) - 1\right)$$

$$\log(n)^2 - \log(n) \leq T(n) \leq 2\log(n)^2 - \log(n) \quad \Rightarrow \Theta(\log(n)^2)$$