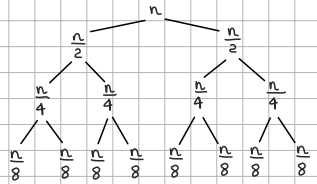


$$T(n) = 2T\left(\frac{n}{2}\right) + n^4$$

Livello	Nodi per livello	Dimensione di input	Contributo per nodo	Contributo totale per livello
0	1	$n$	$n^4$	$n^4$
1	2	$\frac{n}{2}$	$\frac{n^4}{16}$	$\frac{n^4}{8}$
2	4	$\frac{n}{4}$	$\frac{n^4}{256}$	$\frac{n^4}{64}$
3	8	$\frac{n}{8}$	$\frac{n^4}{4096}$	$\frac{n^4}{512}$
i	$2^i$	$\frac{n}{2^i}$	$\frac{n^4}{2^{4i}}$	$\frac{n^4}{2^{3i}}$



$$\begin{aligned} \sum_{i=0}^h \frac{n^4}{2^{3i}} &= n^4 \sum_{i=0}^h \frac{1}{2^{3i}} = n^4 \sum_{i=0}^h \left(\frac{1}{2^3}\right)^i = n^4 \cdot \frac{r^{h+1} - 1}{r - 1} = n^4 \cdot \frac{\left(\frac{1}{8}\right)^{h+1} - 1}{\frac{1}{8} - 1} = n^4 \cdot \frac{8}{7} \left(1 - \left(\frac{1}{8}\right)^{h+1}\right) = \\ &= n^4 \cdot \frac{8}{7} \left(1 - \frac{1}{8^{h+1}}\right) = n^4 \cdot \frac{8}{7} \left(1 - \frac{1}{8^{\log_2(n)}}\right) = n^4 \cdot \frac{8}{7} \left(1 - \frac{1}{2^{\log_2(n)}}\right) = n^4 \cdot \frac{8}{7} \left(1 - \frac{1}{8^{1/3}}\right) = \\ &= n^4 \cdot \frac{8}{7} \cdot \frac{8^{1/3} - 1}{8^{1/3}} = \frac{8n^4}{7} - \frac{n^4}{7} \end{aligned}$$

$$T(n) = 16T\left(\frac{n}{4}\right) + n^2$$

Livello	Nodi per livello	Dimensione di input	Contributo per nodo	Contributo totale per livello
0	1	$n$	$n^2$	$n^2$
1	16	$\frac{n}{4}$	$\frac{n^2}{16}$	$n^2$
2	256	$\frac{n}{16}$	$\frac{n^2}{256}$	$n^2$
3	4096	$\frac{n}{64}$	$\frac{n^2}{4096}$	$n^2$
i	$16^i$	$\frac{n}{4^i}$	$\frac{n^2}{16^i}$	$n^2$

$$\Theta(n^2)$$

$$T(n) = 7T\left(\frac{n}{3}\right) + n^2$$

Livello	Nodi per livello	Dimensione di input	Contributo per nodo	Contributo totale per livello
0	1	$n$	$n^2$	$n^2$
1	7	$\frac{n}{3}$	$\frac{n^2}{9}$	$7 \cdot \frac{n^2}{9}$
2	49	$\frac{n}{9}$	$\frac{n^2}{81}$	$49 \cdot \frac{n^2}{81}$
3	343	$\frac{n}{27}$	$\frac{n^2}{729}$	$343 \cdot \frac{n^2}{729}$
i	$7^i$	$\frac{n}{3^i}$	$\frac{n^2}{3^{2i}}$	$7^i \cdot \frac{n^2}{3^{2i}}$

$$\sum_{i=0}^h 7^i \cdot \frac{n^2}{3^{2i}} = n^2 \sum_{i=0}^h \frac{7^i}{3^{2i}} = n^2 \sum_{i=0}^h \left(\frac{7}{9}\right)^i = n^2 \cdot \frac{1 - \left(\frac{7}{9}\right)^{h+1}}{1 - \frac{7}{9}} = n^2 \cdot \frac{1 - \left(\frac{7}{9}\right)^{\log_2(n)}}{\frac{2}{9}} = \frac{9n^2}{2} \cdot \left(1 - \frac{7}{9} \left(\frac{7}{9}\right)^{\log_2(n)}\right) =$$

$$\frac{9n^2}{2} \cdot \left(1 - \frac{7}{9} \left(\frac{2^{\log_2(n)}}{2^{\log_2(n)}}\right)^{\log_2(n)}\right) = \frac{9n^2}{2} \cdot \left(1 - \frac{7}{9} \left(\frac{2^{\log_2(n)}}{2^{\log_2(n)}}\right)^{\log_2(n)}\right) = \frac{9n^2}{2} \cdot \left(1 - \frac{7}{9} \frac{n^{\log_2(n)}}{n^{\log_2(n)}}\right) =$$

$$\frac{9n^2}{2} - \frac{7}{2} n^2 \frac{n^{\log_2(n)}}{n^{\log_2(n)}} = \frac{9n^2}{2} - \frac{7}{2} n^2 \cdot n^{\log_2(n)} = \frac{9n^2}{2} - \frac{7}{2} n^{2 + \log_2(n)} \Rightarrow \Theta(n^2)$$

$$T(n) = 7T\left(\frac{n}{2}\right) + n^2$$

Livello	Nodi per livello	Dimensione di input	Contributo per nodo	Contributo totale per livello
0	1	n	$n^2$	$n^2$
1	7	$\frac{n}{2}$	$\frac{n^2}{4}$	$7 \cdot \frac{n^2}{4}$
2	49	$\frac{n}{4}$	$\frac{n^2}{16}$	$49 \cdot \frac{n^2}{16}$
3	343	$\frac{n}{8}$	$\frac{n^2}{64}$	$343 \cdot \frac{n^2}{64}$
i	$7^i$	$\frac{n}{2^i}$	$\frac{n^2}{2^{2i}}$	$7^i \cdot \frac{n^2}{2^{2i}}$

$$\begin{aligned} \sum_{i=0}^h 7^i \cdot \frac{n^2}{2^{2i}} &= n^2 \sum_{i=0}^h \frac{7^i}{2^{2i}} = n^2 \sum_{i=0}^h \left(\frac{7}{4}\right)^i = n^2 \cdot \frac{1 - \left(\frac{7}{4}\right)^{h+1}}{1 - \frac{7}{4}} = n^2 \cdot \frac{1 - \left(\frac{7}{4}\right)^{\log_2(n)}}{-\frac{3}{4}} = -\frac{4}{3} n^2 \left(1 - \frac{7}{4} \left(\frac{7}{4}\right)^{\log_2(n)}\right) = \\ &= -\frac{4n^2}{3} \left(1 - \frac{7}{4} \left(\frac{2^{\log_2(n)}}{2^{\log_2(n)}}\right)^{\log_2(n)}\right) = -\frac{4n^2}{3} \left(1 - \frac{7}{4} \left(\frac{2^{\log_2(n)}}{2^{\log_2(n)}}\right)^{\log_2(n)}\right) = -\frac{4n^2}{3} \left(1 - \frac{7}{4} \frac{n^{\log_2(n)}}{n^{\log_2(n)}}\right) = \\ &= -\frac{4}{3} n^2 + \frac{7}{3} n^{\log_2(n)} \Rightarrow \Theta(n^{\log_2(n)}) \end{aligned}$$

$$T(n) = 2T\left(\frac{n}{2}\right) + \sqrt{n}$$

Livello	Nodi per livello	Dimensione di input	Contributo per nodo	Contributo totale per livello
0	1	n	$\sqrt{n}$	$\sqrt{n}$
1	2	$\frac{n}{4}$	$\frac{\sqrt{n}}{2}$	$\sqrt{n}$
2	4	$\frac{n}{16}$	$\frac{\sqrt{n}}{4}$	$\sqrt{n}$
3	8	$\frac{n}{64}$	$\frac{\sqrt{n}}{8}$	$\sqrt{n}$
i	$2^i$	$\frac{n}{4^i}$	$\frac{\sqrt{n}}{2^i}$	$\sqrt{n}$

$$\Rightarrow \Theta(\sqrt{n} \log_2(n))$$

$$T(n) = T(n-2) + n^2$$

Livello	Nodi per livello	Dimensione di input	Contributo per nodo	Contributo totale per livello
0	1	$n$	$n$	$n$
1	1	$n-2$	$(n-2)^2$	$(n-2)^2$
2	1	$n-4$	$(n-4)^2$	$(n-4)^2$
3	1	$n-8$	$(n-8)^2$	$(n-8)^2$
$i$	1	$n-2^i$	$(n-2^i)^2$	$(n-2^i)^2$

$$T(n) = 4T\left(\frac{n}{3}\right) + n \log_2(n)$$

Livello	Nodi per livello	Dimensione di input	Contributo per nodo	Contributo totale per livello
0	1	$n$	$n \log_2(n)$	$n \log_2(n)$
1	4	$\frac{n}{3}$	$\frac{n}{3} \log_2\left(\frac{n}{3}\right)$	$\frac{4n}{3} \log_2\left(\frac{n}{3}\right)$
2	16	$\frac{n}{9}$	$\frac{n}{9} \log_2\left(\frac{n}{9}\right)$	$\frac{16n}{9} \log_2\left(\frac{n}{9}\right)$
3	64	$\frac{n}{27}$	$\frac{n}{27} \log_2\left(\frac{n}{27}\right)$	$\frac{64n}{27} \log_2\left(\frac{n}{27}\right)$
$i$	$4^i$	$\frac{n}{3^i}$	$\frac{n}{3^i} \log_2\left(\frac{n}{3^i}\right)$	$\frac{4^i n}{3^i} \log_2\left(\frac{n}{3^i}\right)$

$$\sum_{i=0}^h \frac{4^i n}{3^i} \cdot \log_2\left(\frac{n}{3^i}\right) =$$

$$T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n}$$

Livello	Nodi per livello	Dimensione di input	Contributo per nodo	Contributo totale per livello
0	1	$n$	$\sqrt{n}$	$\sqrt{n}$
1	2	$\frac{n}{4}$	$\frac{\sqrt{n}}{2}$	$\sqrt{n}$
2	4	$\frac{n}{16}$	$\frac{\sqrt{n}}{4}$	$\sqrt{n}$
3	8	$\frac{n}{64}$	$\frac{\sqrt{n}}{8}$	$\sqrt{n}$
$i$	$2^i$	$\frac{n}{4^i}$	$\frac{\sqrt{n}}{2^i}$	$\sqrt{n}$

$$\Rightarrow \Theta(\sqrt{n} \log_2(n))$$

$$T(n) = 2T\left(\frac{n}{4}\right) + n$$

Livello	Nodi per livello	Dimensione di input	Contributo per nodo	Contributo totale per livello
0	1	$n$	$n$	$n$
1	2	$\frac{n}{4}$	$\frac{n}{4}$	$\frac{n}{2}$
2	4	$\frac{n}{16}$	$\frac{n}{16}$	$\frac{n}{4}$
3	8	$\frac{n}{64}$	$\frac{n}{64}$	$\frac{n}{8}$
$i$	$2^i$	$\frac{n}{4^i}$	$\frac{n}{4^i}$	$\frac{n}{2^i}$

$$\sum_{i=0}^h \frac{n}{2^i} = n \sum_{i=0}^h \left(\frac{1}{2}\right)^i = n \cdot \frac{1 - \left(\frac{1}{2}\right)^{h+1}}{1 - \frac{1}{2}} = 2n \left(1 - \frac{1}{2} \left(\frac{1}{2}\right)^{\log_2(n)}\right) = 2n \left(1 - \frac{1}{2} \left(2^{-\log_2(n)}\right)\right) = 2n \left(1 - \frac{1}{2n}\right) = 2n - 1 \Rightarrow \Theta(n)$$

$$T(n) = 2T\left(\frac{n}{4}\right) + n^2$$

Livello	Nodi per livello	Dimensione di input	Contributo per nodo	Contributo totale per livello
0	1	$n$	$n^2$	$n^2$
1	2	$\frac{n}{4}$	$\frac{n^2}{16}$	$\frac{n^2}{8}$
2	4	$\frac{n}{16}$	$\frac{n^2}{256}$	$\frac{n^2}{64}$
3	8	$\frac{n}{64}$	$\frac{n^2}{4096}$	$\frac{n^2}{512}$
$i$	$2^i$	$\frac{n}{4^i}$	$\frac{n^2}{4^{2i}}$	$\frac{n^2}{8^i}$

$$\sum_{i=0}^h \frac{n^2}{8^i} = n^2 \sum_{i=0}^h \left(\frac{1}{8}\right)^i = n^2 \cdot \frac{1 - \left(\frac{1}{8}\right)^{h+1}}{1 - \frac{1}{8}} = \frac{8n^2}{7} \cdot \left(1 - \frac{1}{8} \left(\frac{1}{8}\right)^{\log_2(n)}\right) = \frac{8n^2}{7} \left(1 - \frac{1}{8} \cdot 2^{-3 \log_2(n)}\right) =$$

$$= \frac{8n^2}{7} \left(1 - \frac{1}{8n^3}\right) = \frac{8n^2}{7} - \frac{1}{7n} \Rightarrow \Theta(n^2)$$

$$T(n) = 3T\left(\frac{n}{3}\right) + \sqrt{n}$$

Livello	Nodi per livello	Dimensione di input	Contributo per nodo	Contributo totale per livello
0	1	n	$\sqrt{n}$	$\sqrt{n}$
1	3	$\frac{n}{3}$	$\sqrt{\frac{n}{3}}$	$3\sqrt{\frac{n}{3}}$
2	9	$\frac{n}{9}$	$\sqrt{\frac{n}{9}}$	$3\sqrt{n}$
3	27	$\frac{n}{27}$	$\sqrt{\frac{n}{27}}$	$27\sqrt{\frac{n}{27}}$
i	$3^i$	$\frac{n}{3^i}$	$\sqrt{\frac{n}{3^i}}$	$3^i \sqrt{\frac{n}{3^i}}$

$$\sum_{i=0}^h 3^i \sqrt{\frac{n}{3^i}} = \sum_{i=0}^h 3^i \cdot \left(\frac{n}{3^i}\right)^{\frac{1}{2}} = \sqrt{n} \sum_{i=0}^h \left(\frac{3}{3^{\frac{1}{2}}}\right)^i = \sqrt{n} \sum_{i=0}^h \left(\frac{3}{\sqrt{3}}\right)^i =$$

$$= \sqrt{n} \cdot \frac{1 - \left(\frac{3}{\sqrt{3}}\right)^{h+1}}{1 - \frac{3}{\sqrt{3}}} = \sqrt{n} \cdot \frac{1 - (\sqrt{3})^{h+1}}{1 - \sqrt{3}} \cdot \frac{1 + \sqrt{3}}{1 + \sqrt{3}} = \sqrt{n} \cdot \frac{1 + \sqrt{3} - (\sqrt{3})^{h+1} - (\sqrt{3})^{h+2}}{-2}$$

$$= -2\sqrt{n} \left(1 + \sqrt{3} - \sqrt{3}(\sqrt{3})^{\log_2(n)} - 3(\sqrt{3})^{\log_2(n)}\right) = \frac{\sqrt{n}}{2} \cdot 1 + \sqrt{3} - (\sqrt{3}^{\log_2(n)})(\sqrt{3} + 3)$$

$$T(n) = 3T\left(\frac{n}{3}\right) + n$$

Livello	Nodi per livello	Dimensione di input	Contributo per nodo	Contributo totale per livello
0	1	n	n	n
1	3	$\frac{n}{3}$	$\frac{n}{3}$	n
2	9	$\frac{n}{9}$	$\frac{n}{9}$	n
3	27	$\frac{n}{27}$	$\frac{n}{27}$	n
i	$3^i$	$\frac{n}{3^i}$	$\frac{n}{3^i}$	n

$$\Rightarrow \Theta(n \log_2(n))$$

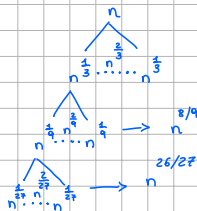
$$T(n) = 3T\left(\frac{n}{3}\right) + n^2$$

Livello	Nodi per livello	dimensione di input	Contributo per nodo	Contributo totale per livello
0	1	$n$	$n^2$	$n^2$
1	3	$\frac{n}{3}$	$\frac{n^2}{9}$	$\frac{n^2}{3}$
2	9	$\frac{n}{9}$	$\frac{n^2}{81}$	$\frac{n^2}{9}$
3	27	$\frac{n}{27}$	$\frac{n^2}{729}$	$\frac{n^2}{27}$
$i$	$3^i$	$\frac{n}{3^i}$	$\frac{n^2}{3^{2i}}$	$\frac{n^2}{3^i}$

$$\begin{aligned} \sum_{i=0}^h \frac{n^2}{3^i} &= n^2 \sum_{i=0}^h \left(\frac{1}{3}\right)^i = n^2 \cdot \frac{1 - \left(\frac{1}{3}\right)^{h+1}}{1 - \frac{1}{3}} = \frac{3n^2}{2} \cdot 1 - \frac{1}{3} \left(\frac{1}{3}\right)^{\log_2(n)} = \\ &= \frac{3n^2}{2} \cdot 1 - \frac{1}{3} \left(2^{\log_2\left(\frac{1}{3}\right)}\right)^{\log_2(n)} = \frac{3n^2}{2} \cdot 1 - \frac{1}{3} \left(2^{\log_2(n)}\right)^{\log_2\left(\frac{1}{3}\right)} = \frac{3n^2}{2} \cdot \left(1 - \frac{n^{\log_2\left(\frac{1}{3}\right)}}{3}\right) = \\ &= \frac{3n^2}{2} - \frac{3n^2 \cdot n^{\log_2\left(\frac{1}{3}\right)}}{2} \Rightarrow \Theta(n^2) \end{aligned}$$

$$T(n) = \begin{cases} 1, & \text{se } n \leq 2 \\ \sqrt[3]{n^2} T(\sqrt[3]{n}) + n & \text{altrimenti} \end{cases}$$

Livello	Nodi per livello	dimensione di input	Contributo per nodo	Contributo totale per livello
0	1	$n$	$n$	$n$
1	$n^{\frac{2}{3}}$	$n^{\frac{1}{3}}$	$n^{\frac{1}{3}}$	$n$
2	$n^{\frac{4}{9}}$	$n^{\frac{1}{9}}$	$n^{\frac{1}{9}}$	$n$
3	$n^{\frac{26}{27}}$	$n^{\frac{1}{27}}$	$n^{\frac{1}{27}}$	$n$
$i$	$n^{\frac{3^i - 1}{3^i}}$	$n^{\frac{1}{3^i}}$	$n^{\frac{1}{3^i}}$	$n$



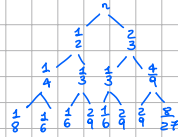
Calcolo l'altezza:

$$\frac{1}{3^h} \leq 2 \Leftrightarrow \frac{1}{3^h} \log_2(n) \leq 1 \Leftrightarrow 3^h \geq \log_2(n) \Leftrightarrow h \geq \log_3(\log_2(n))$$

Il risultato è  $\Theta(n \log_3(\log_2(n)))$

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{3}\right) + n$$

Livello	Nodi per livello	dimensione di input	Contributo per nodo	Contributo totale per livello
0	1	n	n	n
1	2	$\frac{n}{2} \circ \frac{2n}{3}$	$\frac{n}{2} \circ \frac{2n}{3}$	$\frac{5}{6}n$
2	4	$\frac{n}{4} \circ \frac{2n}{6} \circ \frac{4n}{9}$	$\frac{n}{4} \circ \frac{2n}{6} \circ \frac{4n}{9}$	$\frac{49}{36}n$
i	$2^i$	/	/	$\frac{7^i}{6^i}n$



$$T(n) = \begin{cases} 1, & \text{se } n \leq 1 \\ T\left(\frac{3n}{4}\right) + T\left(\frac{n}{2}\right) + n, & \text{altrimenti} \end{cases}$$

Livello	Nodi per livello	Dimensione di input	Contributo per nodo	Contributo totale per livello
0	1	n	n	n
1	2	$\frac{3n}{4} \cup \frac{n}{2}$	$\frac{3n}{4} \cup \frac{n}{2}$	$\frac{5}{4}n$
2	4	$\frac{9n}{16} \cup \frac{3n}{8} \cup \frac{1n}{4}$	$\frac{9n}{16} \cup \frac{3n}{8} \cup \frac{1n}{4}$	$\frac{25}{16}n$
i	$2^i$	/	/	$\frac{5^i}{4^i}n$



Calcolo le altezze:

$$\frac{3}{4^{h_{\min}}} n \leq 1 \Leftrightarrow 4^{h_{\min}} \geq 3n \Leftrightarrow h_{\min} \geq \log_4(3n)$$

$$\frac{n}{2^{h_{\max}}} \leq 1 \Leftrightarrow 2^{h_{\max}} \geq n \Leftrightarrow \log_2(n)$$

Calcolo le sommatorie:

$$\sum_{i=0}^{h_{\min}} \frac{5^i}{4^i} n \leq T(n) \leq \sum_{i=0}^{h_{\max}} \frac{5^i}{4^i} n \Leftrightarrow n \sum_{i=0}^{h_{\min}} \left(\frac{5}{4}\right)^i \leq T(n) \leq n \sum_{i=0}^{h_{\max}} \left(\frac{5}{4}\right)^i \Leftrightarrow$$

$$n \left( \frac{\left(\frac{5}{4}\right)^{h_{\min}+1}}{\frac{5}{4}-1} - 1 \right) \leq T(n) \leq n \left( \frac{\left(\frac{5}{4}\right)^{h_{\max}+1}}{\frac{5}{4}-1} - 1 \right) \Leftrightarrow 4n \left( \frac{\left(\frac{5}{4}\right)^{\log_4(3n)}}{\frac{5}{4}-1} - 1 \right) \leq T(n) \leq 4n \left( \frac{\left(\frac{5}{4}\right)^{\log_2(n)}}{\frac{5}{4}-1} - 1 \right) \Leftrightarrow$$

$$4n \left( \frac{\frac{5}{4} \left( \frac{5^{\log_4(3n)}}{4^{\log_4(3n)}} \right) - 1}{\frac{5}{4}-1} \right) \leq T(n) \leq 4n \left( \frac{\frac{5}{4} \left( \frac{5^{\log_2(n)}}{4^{\log_2(n)}} \right) - 1}{\frac{5}{4}-1} \right) \Leftrightarrow$$

$$4n \left( \frac{\frac{5}{4} \left( \frac{(4^{\log_4(3n)})^{\log_4(3n)}}{3n} \right) - 1}{\frac{5}{4}-1} \right) \leq T(n) \leq 4n \left( \frac{\frac{5}{4} \left( \frac{(2^{\log_2(n)})^{\log_2(n)}}{n^2} \right) - 1}{\frac{5}{4}-1} \right) \Leftrightarrow$$

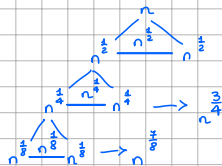
$$4n \left( \frac{\frac{5}{4} \frac{(3n)^{\log_4(3n)}}{3n} - 1}{\frac{5}{4}-1} \right) \leq T(n) \leq 4n \left( \frac{\frac{5}{4} \frac{n^{\log_2(5)}}{n^2} - 1}{\frac{5}{4}-1} \right)$$

$$\frac{5}{3} (3n)^{\log_4(3n)} - 4n \leq T(n) \leq \frac{5n^{\log_2(5)}}{n^2} - 4n \Rightarrow \Omega((3n)^{\log_4(3n)}) \text{ e } O(n^{\log_2(5)})$$



$$T(n) = \sqrt{n} \cdot T(\sqrt{n}) + n$$

Livello	Nodi per livello	Dimensione di input	Contributo per nodo	Contributo totale per livello
0	1	$n$	$n$	$n$
1	$n^{\frac{1}{2}}$	$n^{\frac{1}{2}}$	$n^{\frac{1}{2}}$	$n$
2	$n^{\frac{3}{4}}$	$n^{\frac{1}{4}}$	$n^{\frac{1}{4}}$	$n$
i	$n^{\frac{2^i-1}{2^i}}$	$n^{\frac{1}{2^i}}$	$n^{\frac{1}{2^i}}$	$n$



$$T(n) = 15T\left(\frac{n}{4}\right) + n^2 \log(n)$$

Livello	Nodi per livello	Dimensione di input	Contributo per nodo	Contributo totale per livello
0	1	$n$	$n^2 \log(n)$	$n^2 \log(n)$
1	15	$\frac{1}{4}n$	$\frac{1}{16}n^2 \log\left(\frac{1}{4}n\right)$	$\frac{15}{16}n^2 \log\left(\frac{1}{4}n\right)$
2	225	$\frac{1}{16}n$	$\frac{1}{256}n^2 \log\left(\frac{1}{16}n\right)$	$\frac{225}{256}n^2 \log\left(\frac{1}{16}n\right)$
i	$15^i$	$\frac{1}{4^i}n$	$\frac{1}{4^{2i}}n^2 \log\left(\frac{1}{4^i}n\right)$	$\frac{15^i}{4^{2i}}n^2 \log\left(\frac{1}{4^i}n\right)$

$$\frac{1}{4^h} n \leq 1 \iff 4^h \geq n \iff h \geq \log_4(n)$$