Esercizio 1

$$T(n) = \begin{cases} 1, & \text{se } n \leq 2 \\ 3T(\sqrt{n}) + \log(n), & \text{altrimenti} \end{cases}$$

Livello	Nodi per livello	Dimensione in input	Contributo per nodo	Contributo totale per livello
0	1	n	$\log(n)$	$\log(n)$
1	3	$n^{\frac{1}{2}}$	$\frac{1}{2}\log(n)$	$\frac{3}{2}\log(n)$
2	9	$n^{\frac{1}{4}}$	$\frac{1}{4}\log(n)$	$\frac{9}{4}\log(n)$
3	27	$n^{\frac{1}{8}}$	$\frac{1}{8}\log(n)$	$\frac{27}{8}\log(n)$
i	3^i	$n^{\frac{1}{2^i}}$	$\frac{1}{2^i}\log(n)$	$\frac{3^i}{2^i}\log(n)$

Calcoliamo l'altezza:

$$n^{\frac{1}{2^h}} \leq 2 \iff \log_2\left(n^{\frac{1}{2^h}}\right) \leq 1 \iff \frac{1}{2^h}\log(n) \leq 1 \iff 2^h \geq \log(n) \iff h \geq \log_2(\log_2(n))$$

Calcoliamo la sommatoria:

$$\sum_{i=0}^{h} \frac{3^i}{2^i}\log(n) = \log_2(n)\sum_{i=0}^{h}\left(\frac{3}{2}\right)^i = \log_2(n) \cdot \frac{1-\left(\frac{3}{2}\right)^{h+1}}{1-\frac{3}{2}} = -2\log_2(n) \cdot \left(1 - \frac{3}{2}\left(\frac{3}{2}\right)^{\log_2(\log_2(n))}\right) =$$

$$= -2\log_2(n) \cdot \left(1 - \frac{3}{2}\left(\frac{3^{\log_2(\log_2(n))}}{2^{\log_2(\log_2(n))}}\right)\right) = -2\log_2(n) \cdot \left(1 - \frac{3}{2}\left(\frac{2^{\log_2(3)^{\log_2(\log_2(n))}}}{\log_2(n)}\right)\right) =$$

$$= -2\log_2(n)\left(1 - \frac{3}{2}\left(\frac{2^{\log_2(\log_2)^{\log_2(3)}}}{\log_2(n)}\right)\right) = -2\log_2(n)\left(1 - \frac{3}{2}\left(\frac{\log_2(n)^{\log_2(3)}}{\log_2(n)}\right)\right) =$$

$$= -2\log_2(n)\left(1 - \frac{3\log_2(n)^{\log_2(3)}}{2\log_2(n)}\right) = -2\log_2(n) + 3\log_2(n)^{\log_2(3)} \Rightarrow \Theta\left(\log_2(n)^{\log_2(3)}\right)$$