Esercizio 1

• $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2, \quad \begin{pmatrix} x \\ y \end{pmatrix} \longmapsto \begin{pmatrix} x-y \\ xy \end{pmatrix}$

Controlliamo se vale la prima proprietà:

$$T\begin{pmatrix} x+x' \\ y+y' \end{pmatrix} = \begin{pmatrix} x+x'-y-y' \\ (x+x')(y+y') \end{pmatrix} = \begin{pmatrix} x+x'-(y+y') \\ (x+x')(y+y') \end{pmatrix}$$

$$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x-y \\ xy \end{pmatrix} \qquad T\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x'-y' \\ x'-y' \end{pmatrix}$$

$$T\begin{pmatrix} x \\ y \end{pmatrix} + T\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x-y \\ xy \end{pmatrix} + \begin{pmatrix} x'-y' \\ x'y' \end{pmatrix} = \begin{pmatrix} x-y+x'-y' \\ xy+x'y' \end{pmatrix} \quad ✗$$

Possiamo dunque dire che T NON è lineare.

• $T: \mathbb{R}^4 \longrightarrow \mathbb{R}^2, \quad \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \longmapsto \begin{pmatrix} x-w \\ z+w \end{pmatrix}$

i) $$T\begin{pmatrix} x+x' \\ y+y' \\ z+z' \\ w+w' \end{pmatrix} = \begin{pmatrix} x+x'-(w+w') \\ z+z'+w+w' \end{pmatrix}$$

$$T\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} x-w \\ z+w \end{pmatrix} \qquad T\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} x'-w' \\ z'+w' \end{pmatrix}$$

$$T\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} + T\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} x-w \\ z+w \end{pmatrix} + \begin{pmatrix} x'-w' \\ z'+w' \end{pmatrix} = \begin{pmatrix} x-w+x'-w' \\ z+w+z'+w \end{pmatrix} = \begin{pmatrix} x+x'-(w+w') \\ z+z'+w+w' \end{pmatrix} \quad ✓$$

ii) $$T\begin{pmatrix} \lambda x \\ \lambda y \\ \lambda z \\ \lambda w \end{pmatrix} = \begin{pmatrix} \lambda x - \lambda w \\ \lambda z + \lambda w \end{pmatrix} = \begin{pmatrix} \lambda(x-w) \\ \lambda(z+w) \end{pmatrix} = \lambda \begin{pmatrix} x-w \\ z+w \end{pmatrix} = \lambda T \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \quad ✓$$

Possiamo dunque dire che T è lineare.

• $T: \mathbb{M}_{2,2}(\mathbb{R}) \longrightarrow \mathbb{M}_{2,2}(\mathbb{R}), \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \longmapsto \frac{1}{2}\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix} - \begin{pmatrix} a & c \\ b & d \end{pmatrix} \right)$

i)