What is the big picture ?

- understand modeling
- in order to deal with real problems, decompose the underlying equations in small simpler specific type
- well posedness study
- discretize equations
- solve linear systems
- progressive learning
  - from simple to difficult
  - from ODEs to PDEs
  - from 1 to 2, 3, 4, 5 or 6 dimensions
  - from finite differences to finite elements or finite volumes methods
  - from linear to nonlinear problems

ODEs vs PDEs

ODE : Ordinary Differential Equation
PDE : Partial Differential Equation

What are the differences between ODEs and PDEs ?

- PDEs include more than one differential formula or more than one partial differential formula when ODEs include only one type. ODEs are distinguished from PDEs, which involve partial derivatives of several variables.
- ODEs deals with simple modelings. PDEs allows to take both time and position into account.
- ODEs need initial conditions
- PDEs can need initial conditions and/or boundary conditions

Example of ODEs

Newton's second law

Example of PDEs

Heat equation, Schrödinger equation, Maxwell's equations, Navier Stokes equations, Euler equations, Boltzmann equations

$$\begin{aligned}
&\partial_t u(t,x) - \Delta u(t,x) = 0, \quad (t,x) \in [0,T] \times \Omega, \\
&u(0,x) = u_0(x), \quad x \in \Omega, \\
&u(t,x) = 0, \quad (t,x) \in [0,T] \times \partial\Omega.
\end{aligned}$$

- Δ is called Laplacian
- $u : \mathbb{R}_t \times R_x^d \longrightarrow \mathbb{R}$ lives in a space of infinite dimension (typically $L^2(0,T;H_0^1(\Omega))$
- After discretization (finite differences for example), the solution u at time t is approximated by a vector of finite dimension $\mathbf{u}(t)$ with components $u_j(t)$, $j \in [0,J]$.
- The Laplacian operator is replaced by a matrix A
- The Heat equation becomes a differential system : ODE

$$\frac{d}{dt}\mathbf{u} - A\mathbf{u} = 0$$

Aim of the course

- Identify various type of equations (ODE, elliptic, parabolic, hyperbolic equations)
- Give hints about well posedness (mainly for linear problem)
- Which numerical method is adapted ?
- Visualization and interpretation of the results