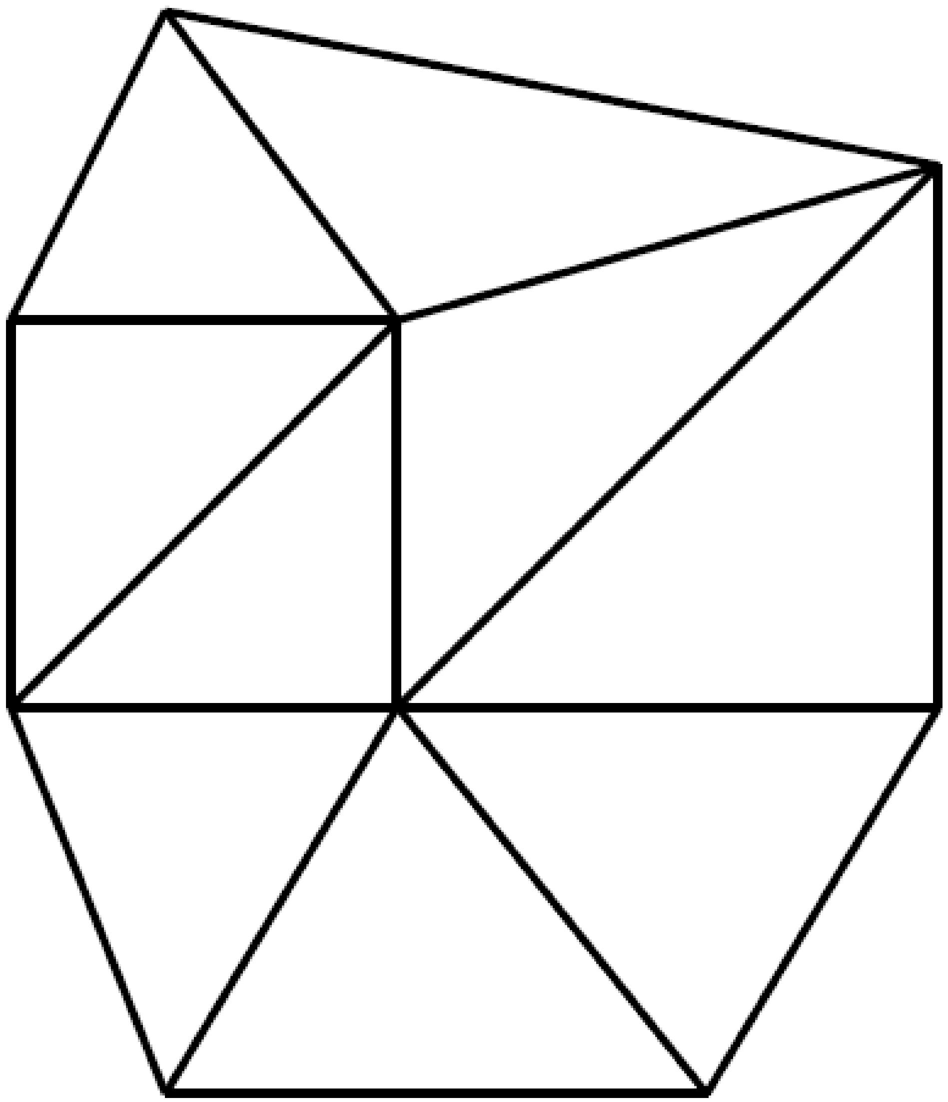
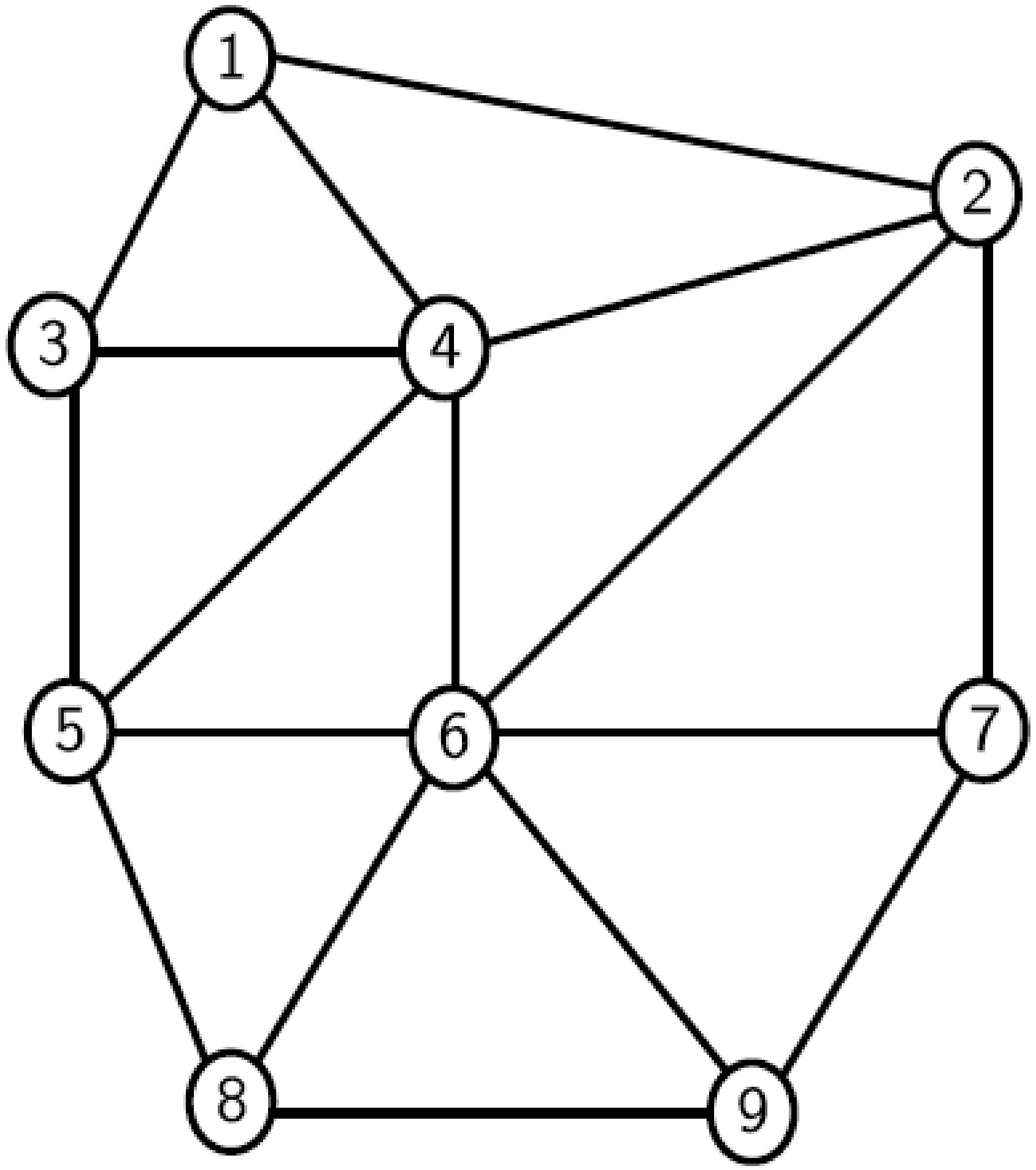
What is a mesh?



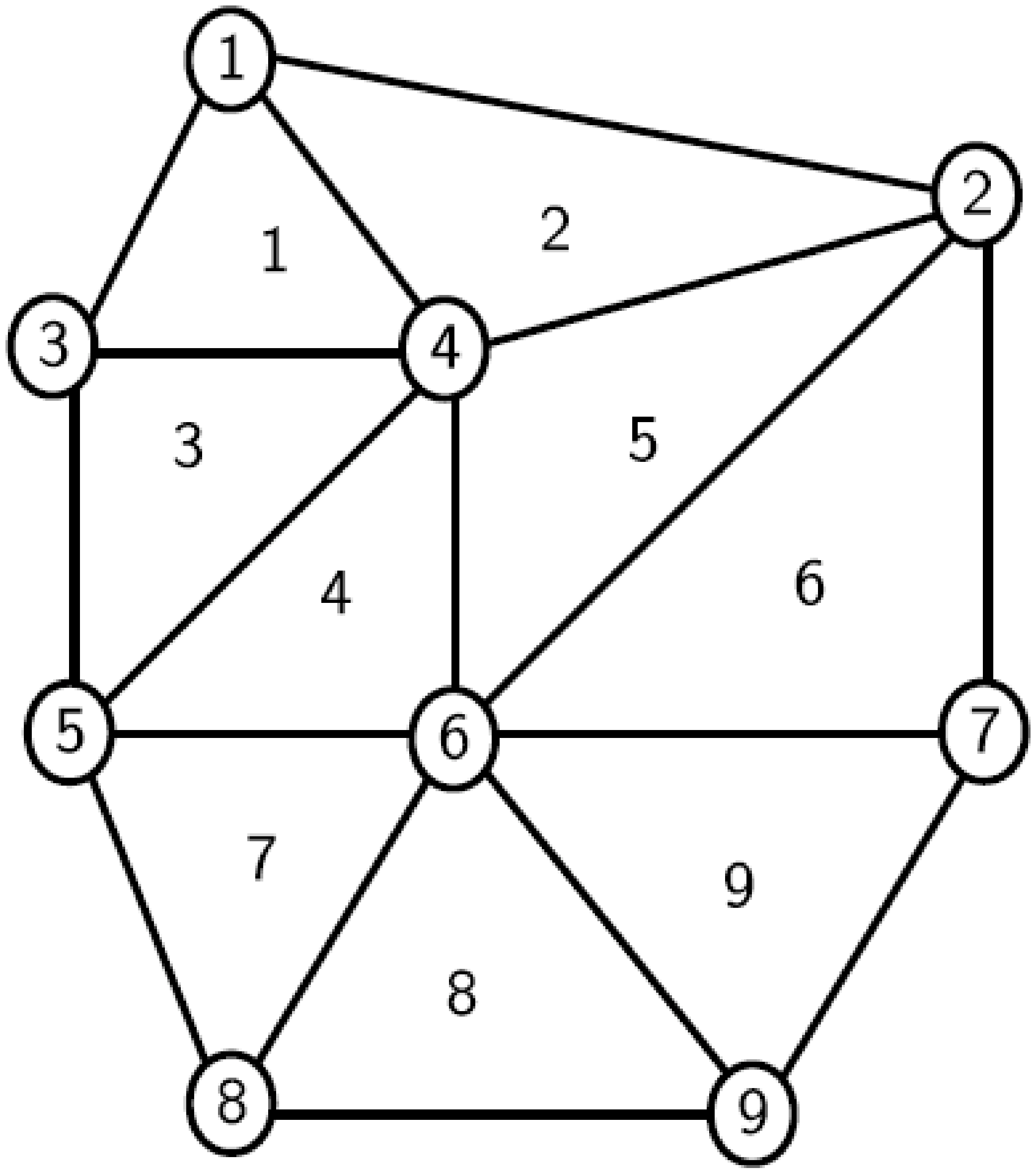
- A domain partition
- A graph

What is a mesh?



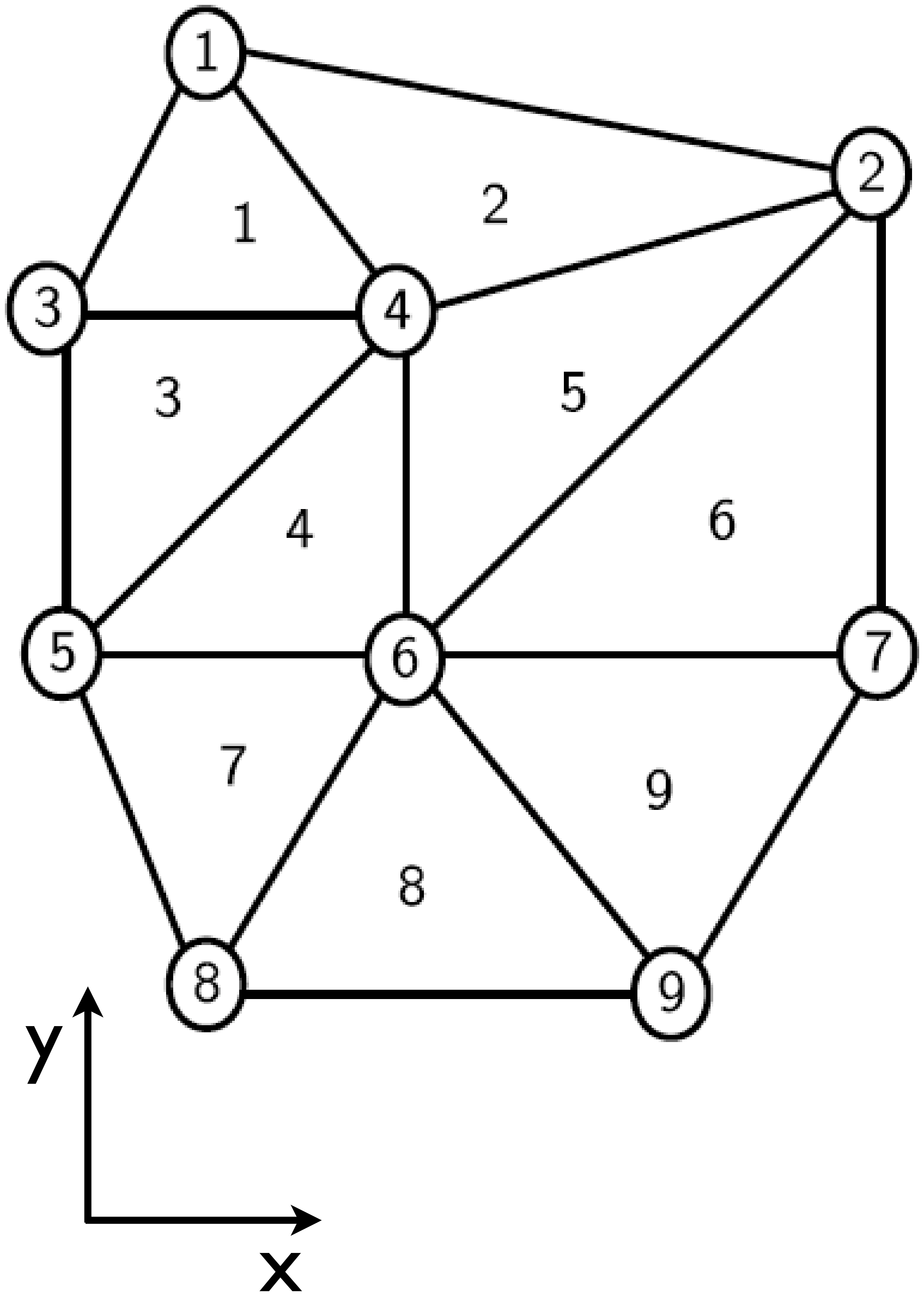
- A domain partition
- A graph
- A numbering of vertices

What is a mesh?



- A domain partition
- A graph
- A numbering of vertices
- But also of triangles

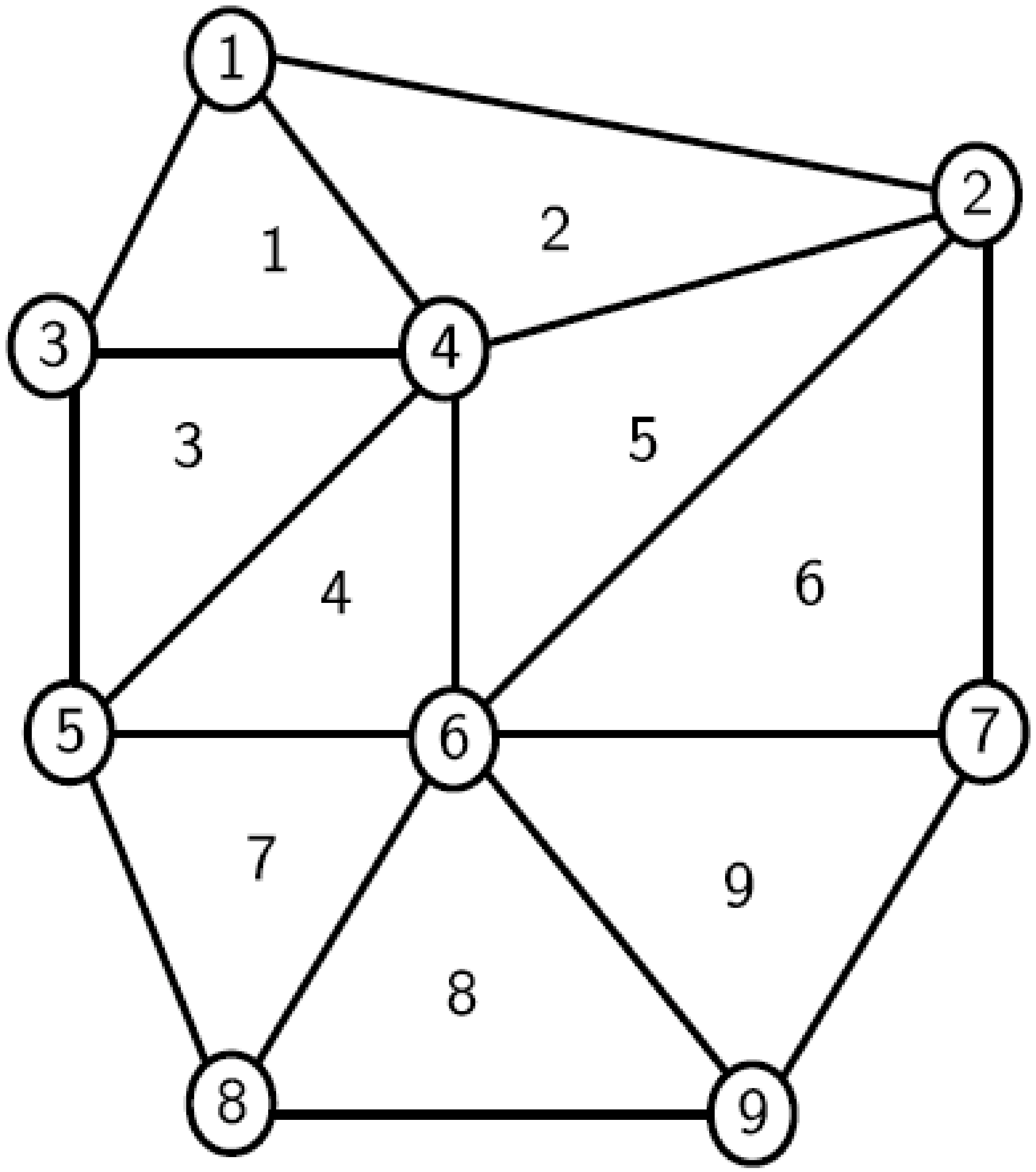
What is a mesh?



- A list of triangles coordinates

id	x	y
1	2	15
2	12	13
3	0	11
4	5	11
5	0	6
6	5	6
7	12	6
8	2	0
9	9	0

What is a mesh?

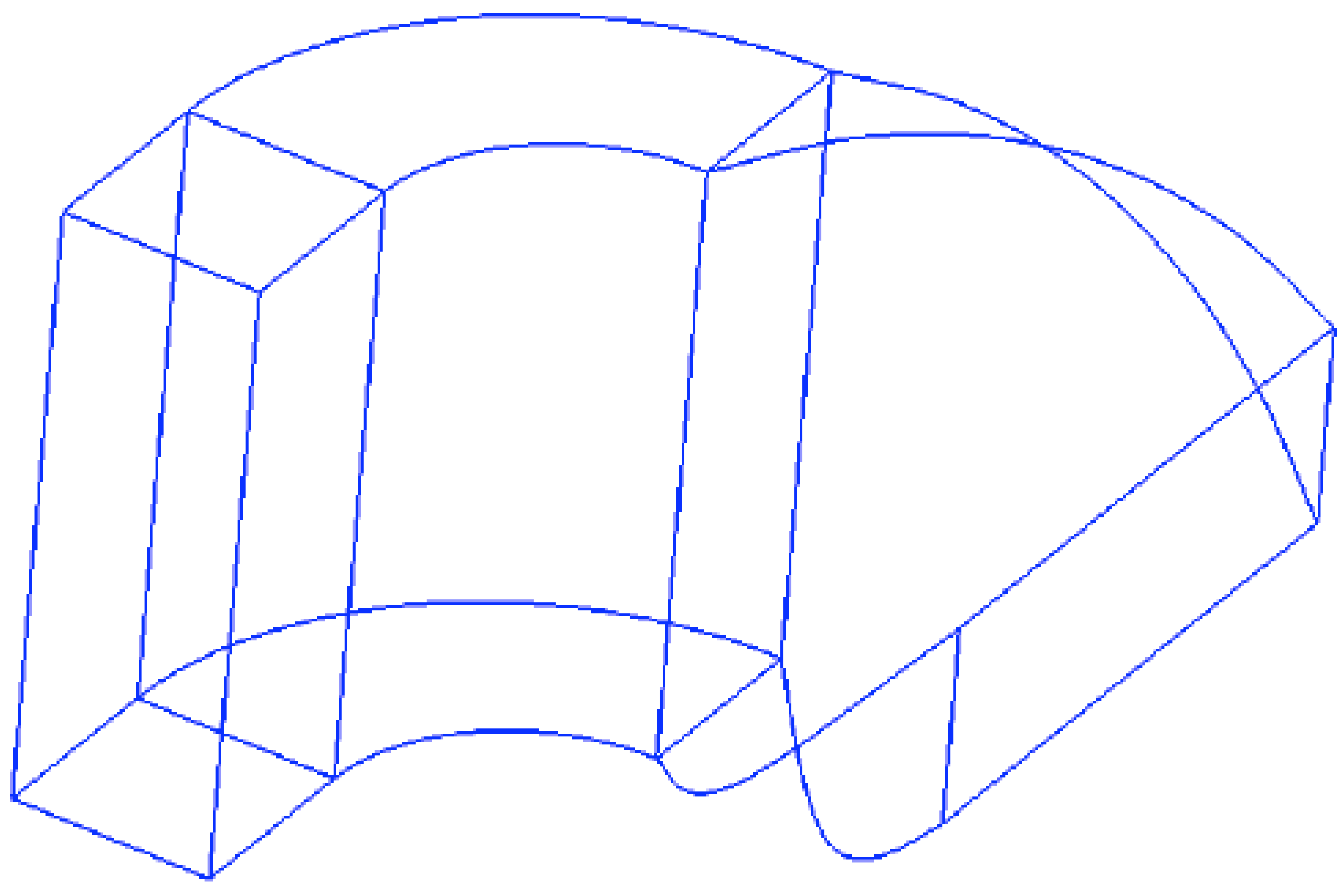


- A belonging tabular

tr	n1	n2	n3
1	1	3	4
2	1	4	2
3	3	5	4
4	4	5	6
5	4	6	2
6	2	6	7
7	5	8	6
8	6	8	9
9	6	9	7

How to design a mesh?

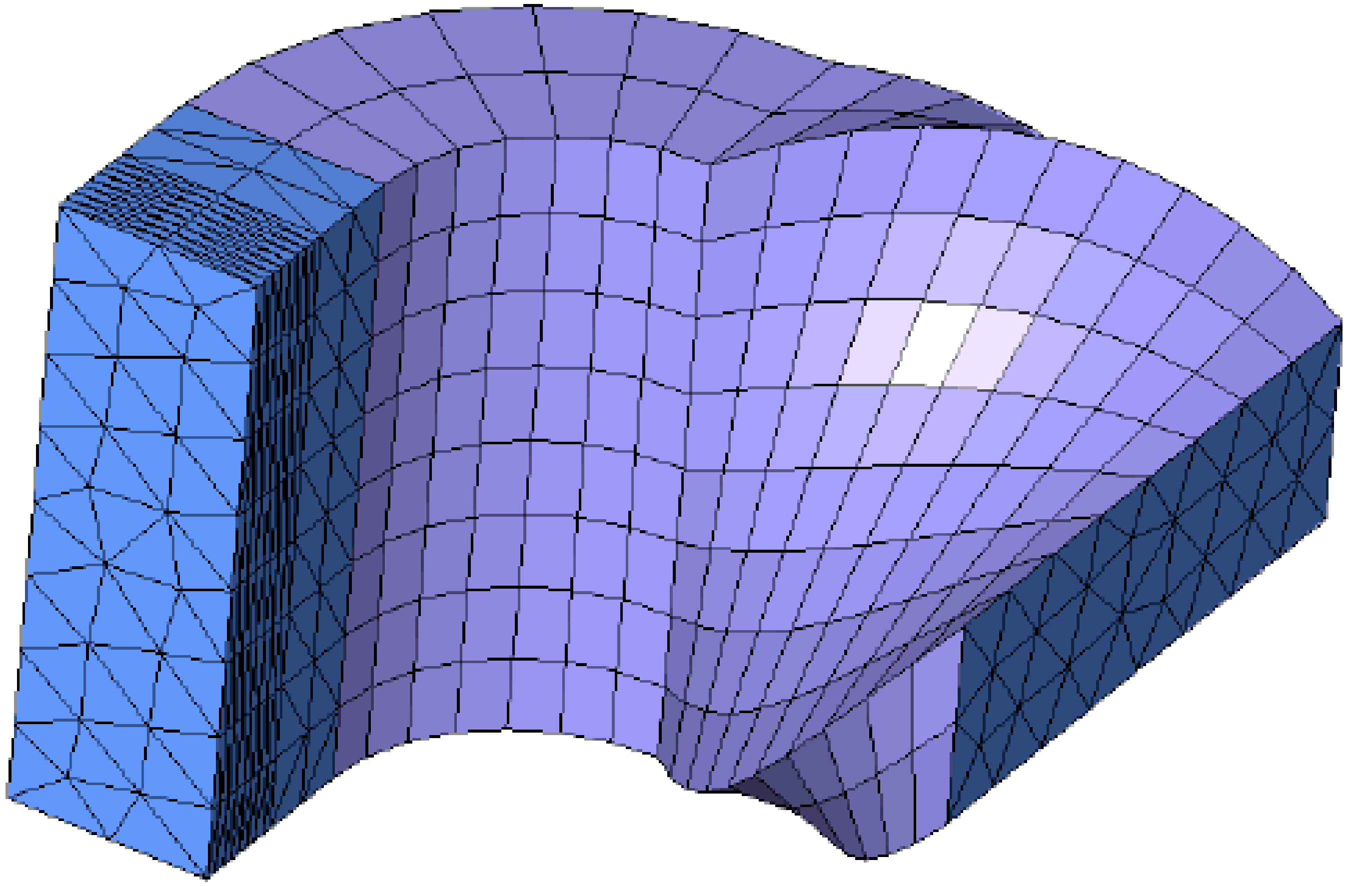
- By hand, for simple case (structured mesh)
- We a mesh generator, for example Gmsh



How to design a mesh?

- By hand, for simple case (structured mesh)
- We a mesh generator, for example Gmsh

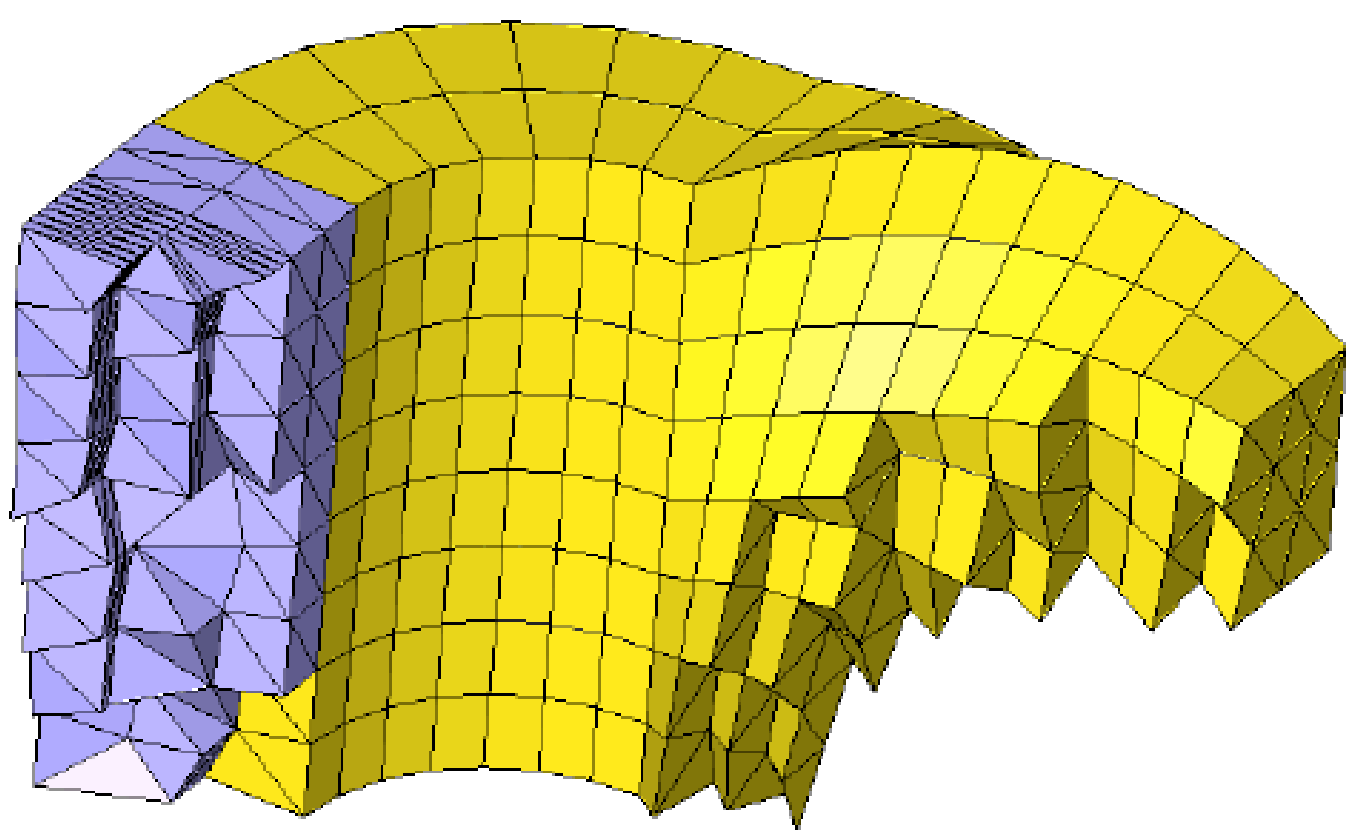
Surface mesh



How to design a mesh?

- By hand, for simple case (structured mesh)
- We a mesh generator, for example Gmsh

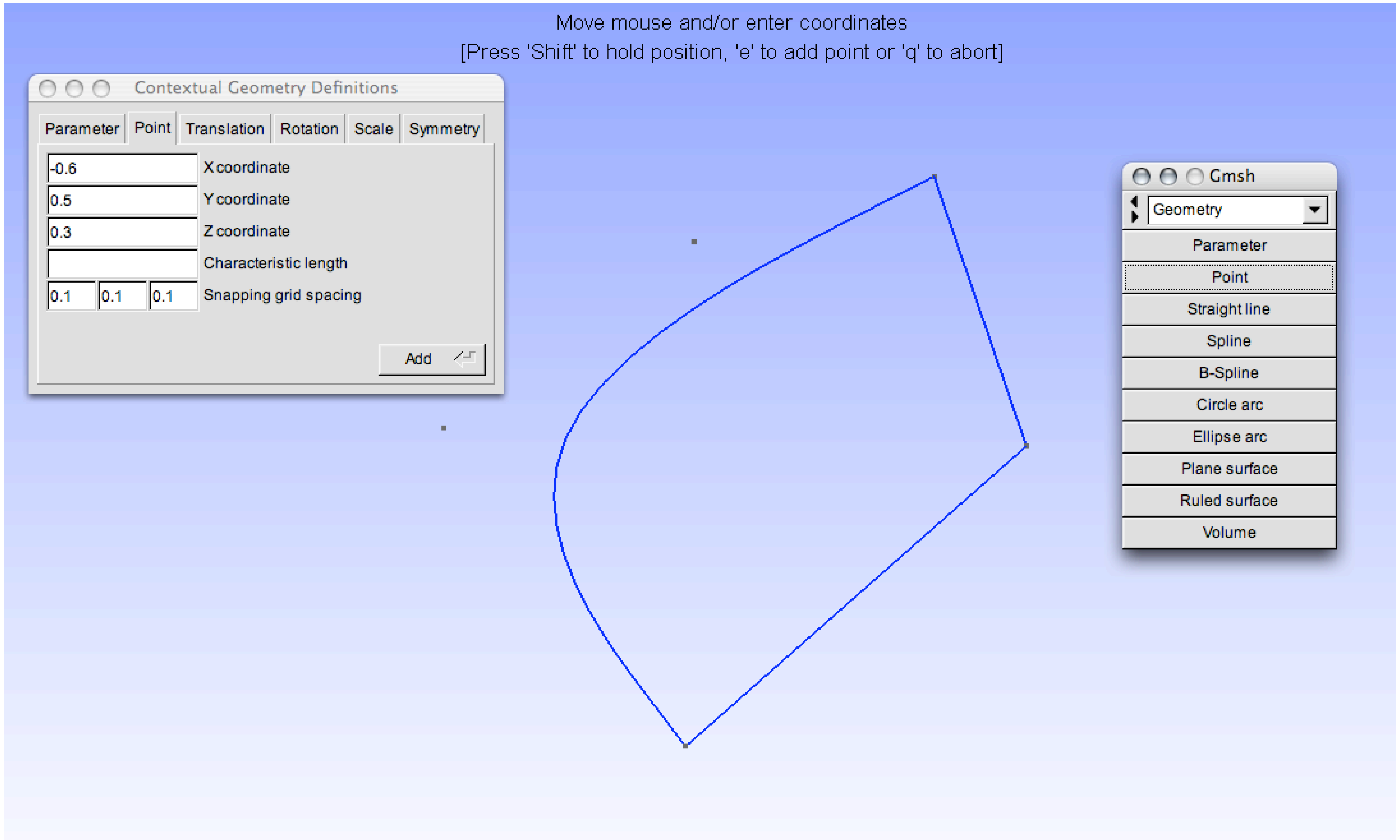
Volume mesh



Basic cycle of mesh generation
with Gmsh

Define problem geometry

- with Gmsh GUI



Basic cycle of mesh generation

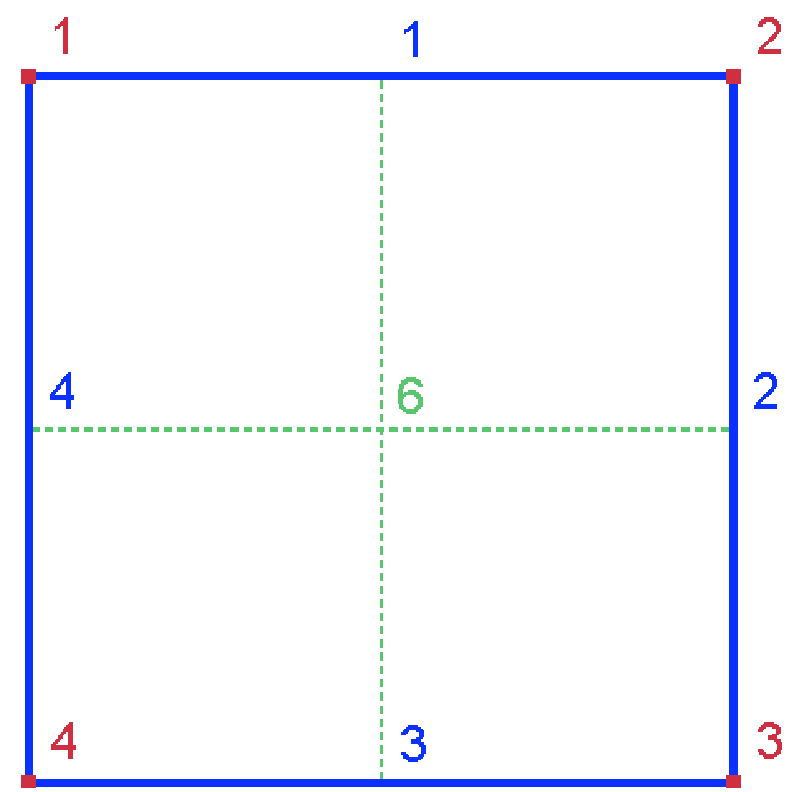
with Gmsh

Define problem geometry

- write a file describing the geometry

carre.geo

```
// Commentaires

lc = 0.1;   // longueur caractéristique

// Définition des Points

Point(1) = {-1,1,0,lc};
Point(2) = {1,1,0,lc};
Point(3) = {1,-1,0,lc};
Point(4) = {-1,-1,0,lc};

// Définition des Lignes

Line(1) = {1,2};
Line(2) = {2,3};
Line(3) = {3,4};
Line(4) = {4,1};

// Définition de la Surface

Line Loop(5) = {1,2,3,4};

Plane Surface(6) = {5};
```



Points

Lines

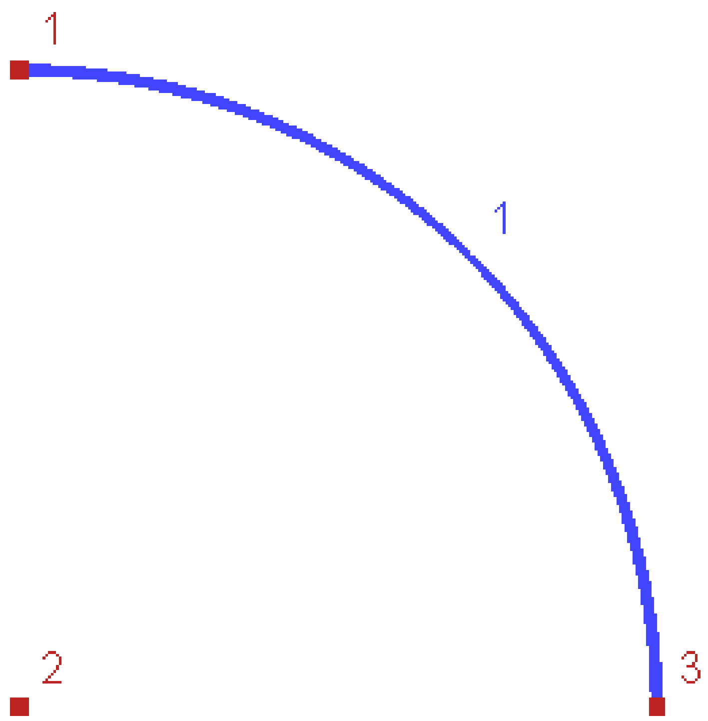
Surfaces

Basic cycle of mesh generation
with Gmsh

Other curves

- Example: define a circle arc

```
lc=0.1;

// Circle

r=1; // radius
c1=0; c2=0; // center of the circle

Point(1)={c1,c2+r,0,lc}; // Start point of the arc
Point(2)={c1,c2,0,lc};  // Center of the circle
Point(3)={c1+r,c2,0,lc}; // End point of the arc

Circle(1)={1,2,3};
```



But also

- Ellipsoid, Splines, B-splines

Basic cycle of mesh generation

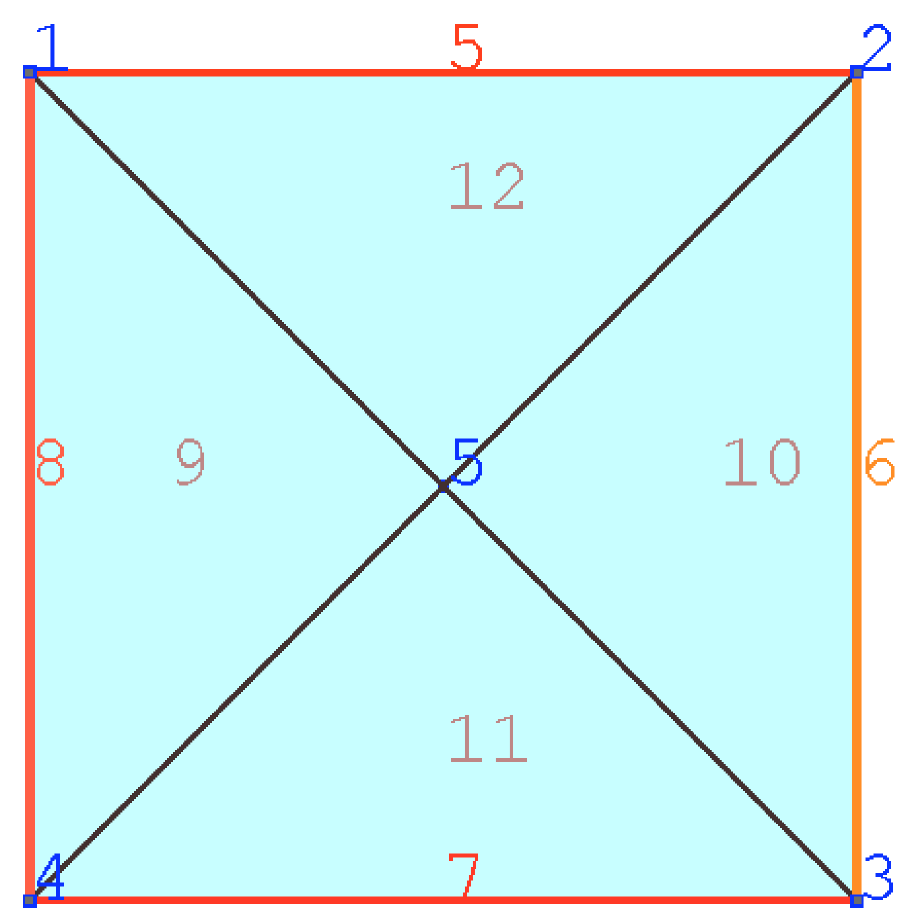
with Gmsh

Mesh the domain

- Trough this GUI
- Or write a file describing the mesh

carre.msh

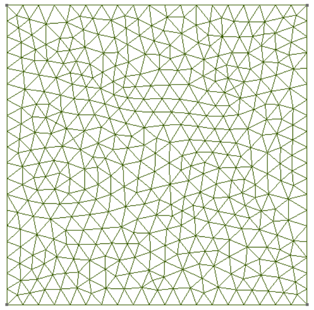
```
$MeshFormat
2 0 8
$EndMeshFormat
$Nodes
5
1 -1 1 0
2 1 1 0
3 1 -1 0
4 -1 -1 0
5 0 0 0
$EndNodes
$Elements
12
1 15 3 0 1 0 1
2 15 3 0 2 0 2
3 15 3 0 3 0 3
4 15 3 0 4 0 4
5 1 3 0 1 0 1 2
6 1 3 0 2 0 2 3
7 1 3 0 3 0 3 4
8 1 3 0 4 0 4 1
9 2 3 0 6 0 1 5 4
10 2 3 0 6 0 3 5 2
11 2 3 0 6 0 5 3 4
12 2 3 0 6 0 5 1 2
$EndElements
```



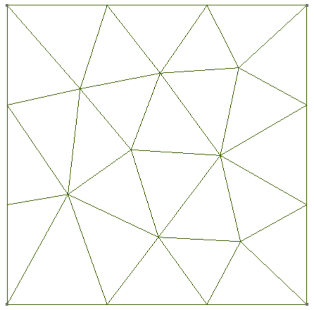
Basic cycle of mesh generation
with Gmsh

Mesh the domain

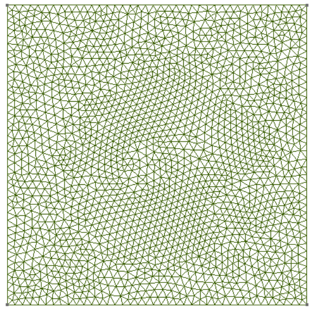
- With the help of the command line
    - `gmsh -2 carre.geo`



    - `gmsh -2 carre.geo -clscale 5`



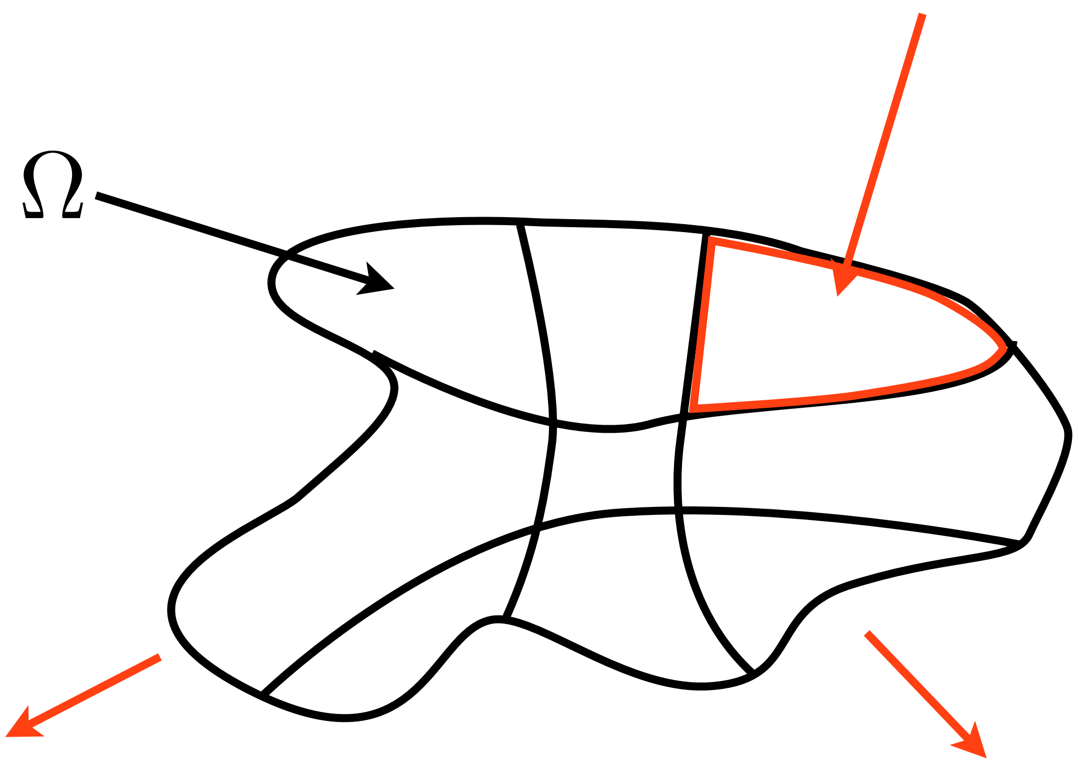
    - `gmsh -2 carre.geo -o carre_fine.msh -clscale 0.05`



The 2 basic ideas of the F.E.M

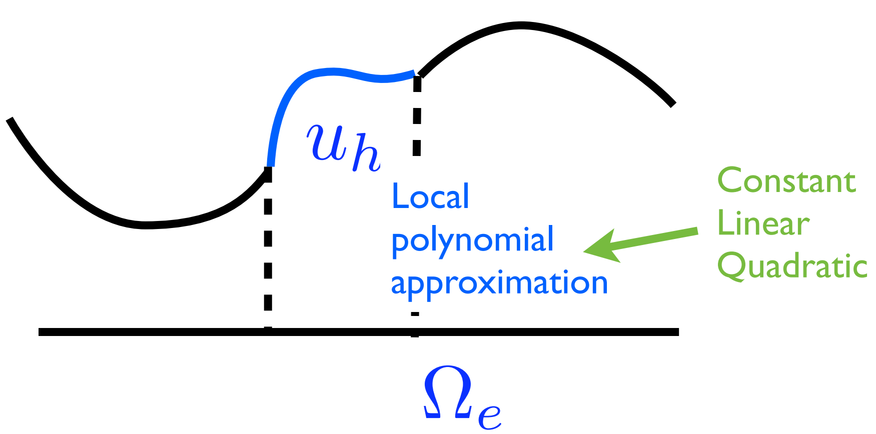
$$\overline{\Omega} = \cup_{i=1}^{N} \{\overline{\Omega_e}\}$$
$$\Omega_e \cap \Omega_f = \emptyset \text{ if } e \neq f$$

Element Ω_e



The problem geometry is divided into small elements

1 Piecewise approximation of the unknown function



2 Summability of integrals

$$\int_{\Omega} = \sum_{e} \underbrace{\int_{\Omega_e}}$$

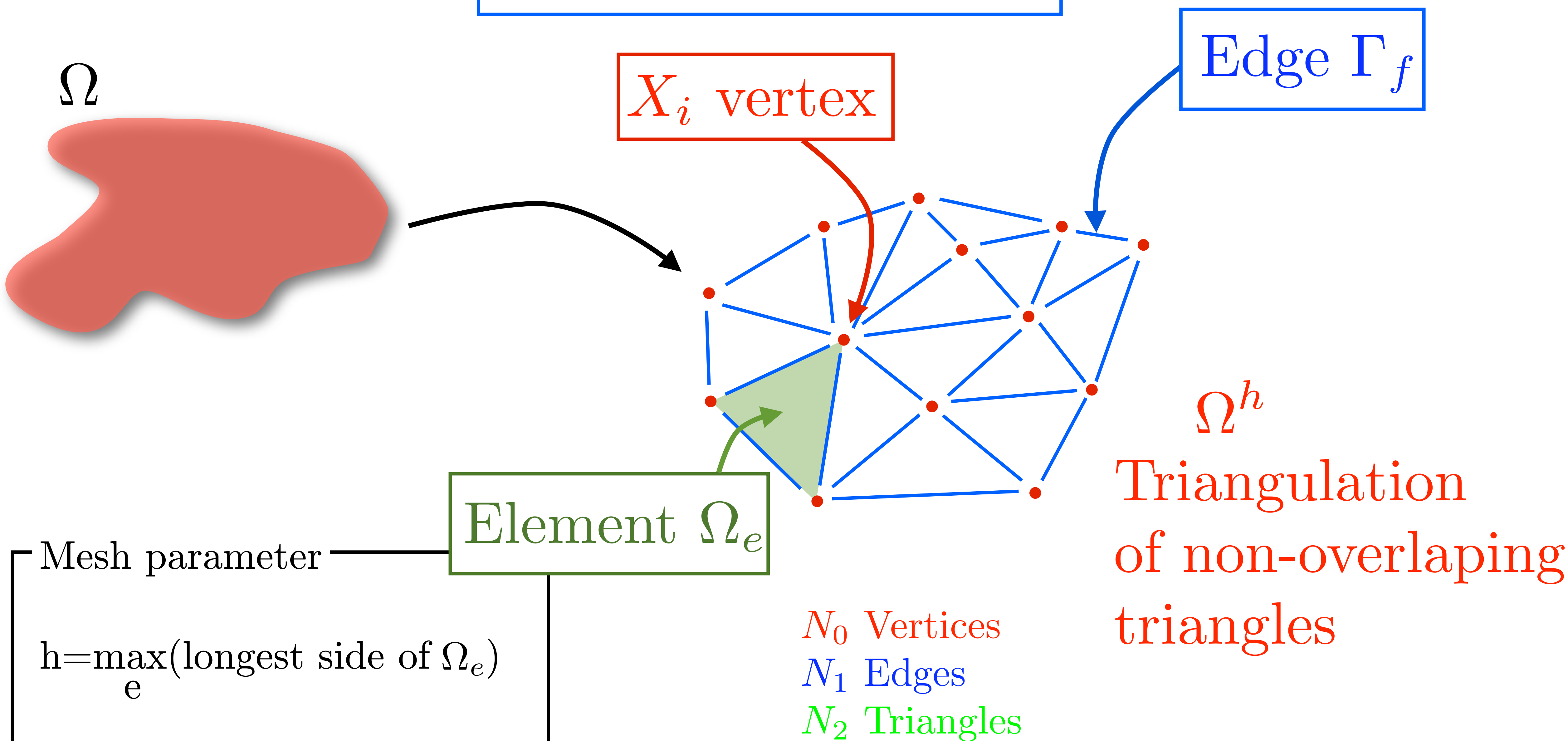
$$\int_{\widehat{\Omega}} \quad \text{Parent Element}$$

2 bis

Poisson Equation

?u such that

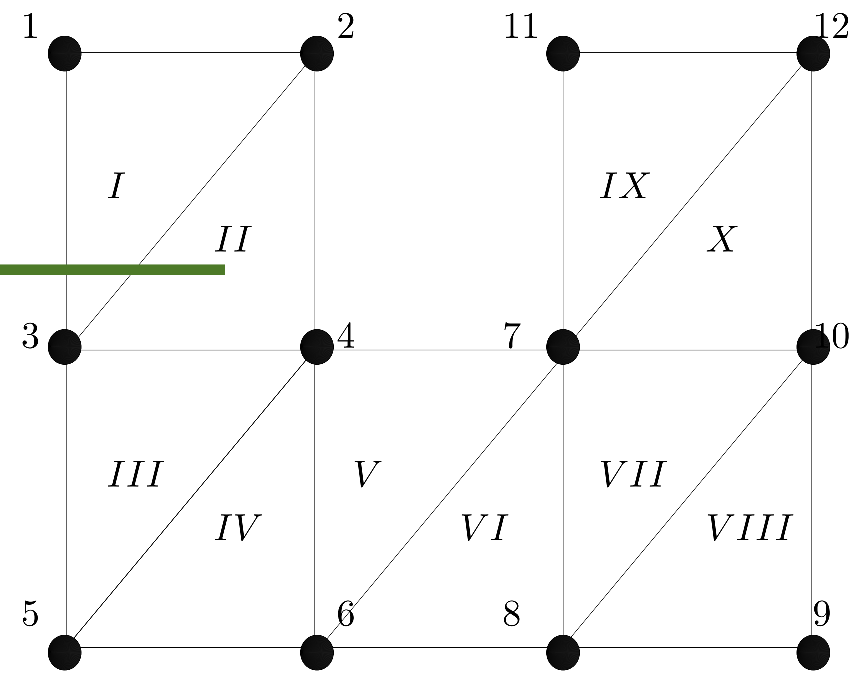
$$\Delta u + f = 0 \quad \text{in } \Omega$$
$$u = 0 \quad \text{in } \partial\Omega$$



Mesh parameter

$$h = \max_e(\text{longest side of } \Omega_e)$$

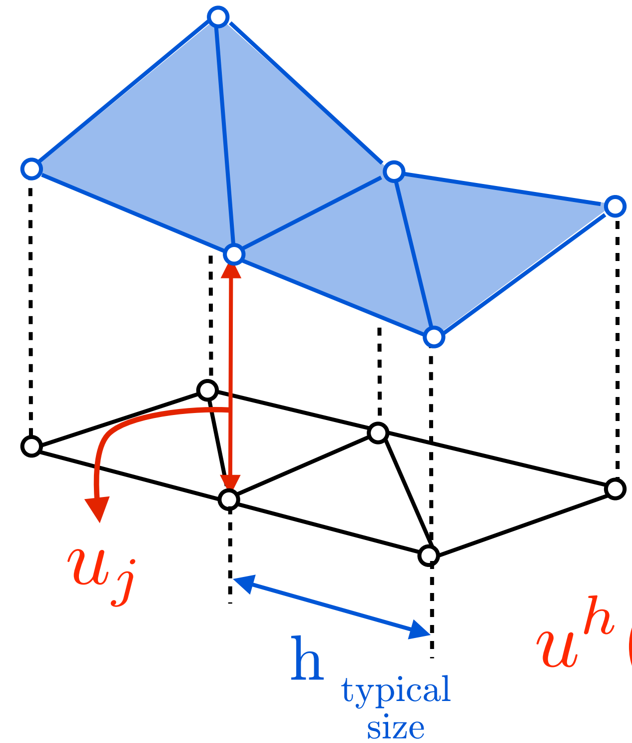
Mesh example



Triangle	Sommets		
1	1	3	2
2	3	4	2
3	5	4	3
4	5	6	4
5	6	7	4
6	6	8	7
7	8	10	7
8	8	9	10
9	7	12	11
10	10	12	7

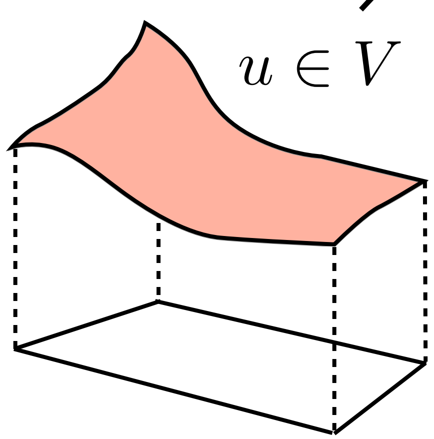
Sommet	X_i	Y_i
1	0.0	2.0
2	1.0	2.0
3	0.0	1.0
4	1.0	1.0
5	0.0	0.0
6	1.0	0.0
7	2.0	1.0
8	2.0	0.0
9	3.0	0.0
10	3.0	1.0
11	2.0	2.0
12	3.0	2.0

2D Finite Element Approximation



$$u^h(X_j) = u_j$$

$$\boxed{u \approx u^h = \sum_{i=1}^{n} U_i \ \phi_i(x)}$$



$u \in V$

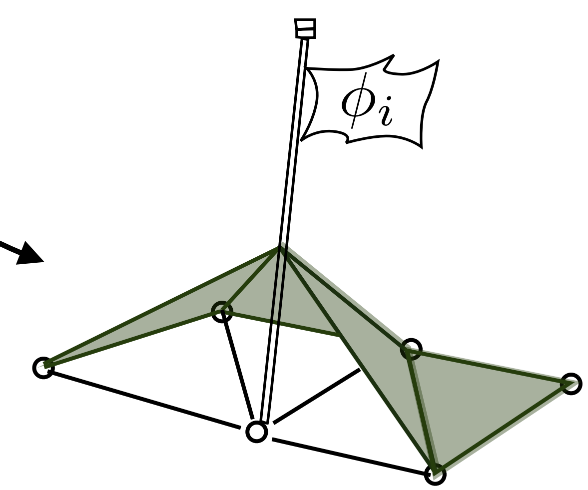
$\in V^h \subset U$

$\dim(V^h) = n$

unknown nodal values

known shape functions

$$\phi_j(X_i) = \delta_{ij}$$



Global ... and local shape functions

element index

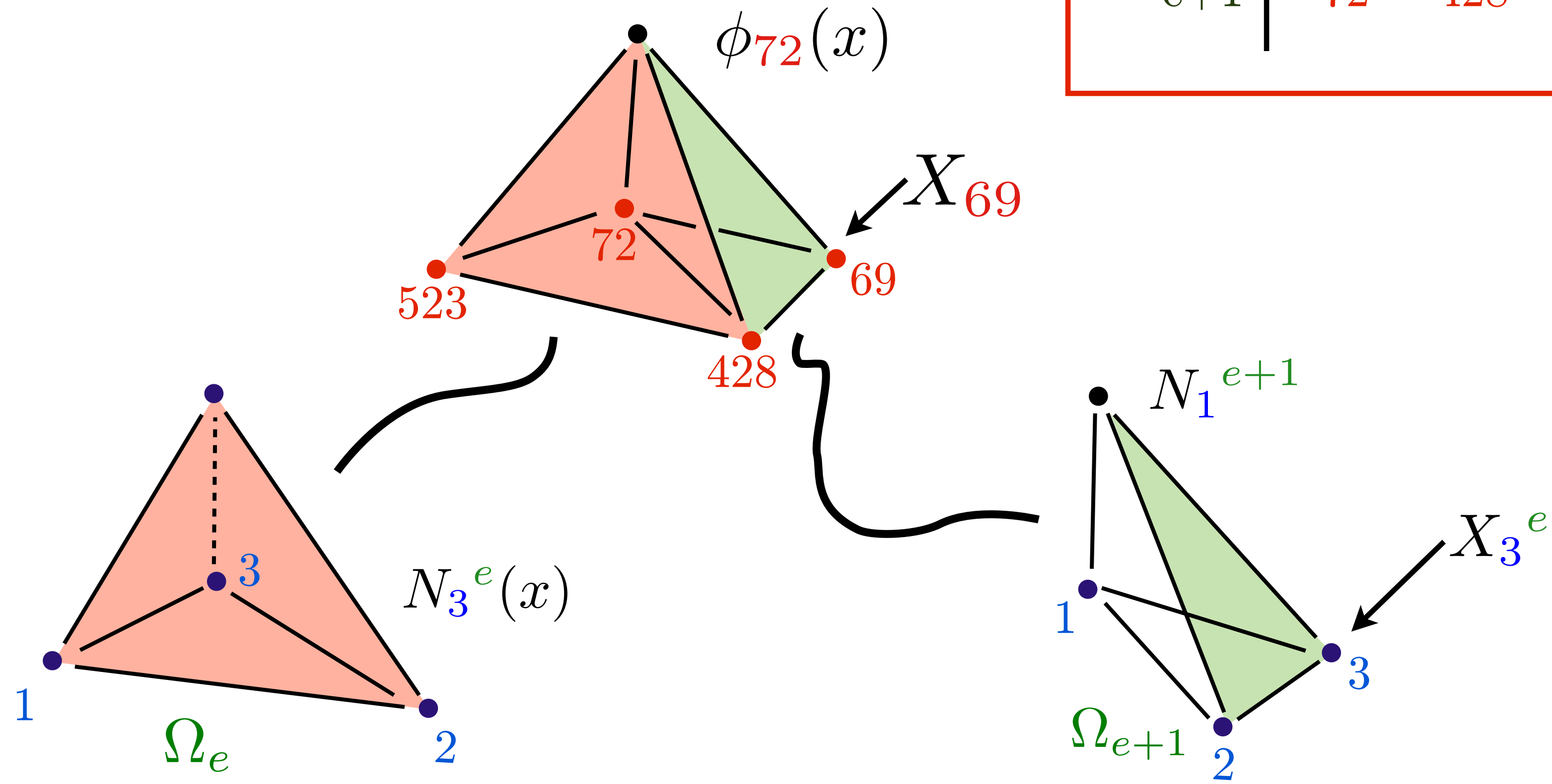
$$\phi_{72}(x) = N_3{}^{e}(x)$$

global label

only on Ω_e

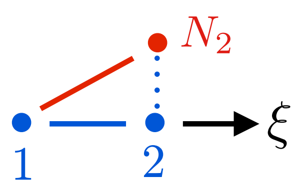
local label

	1	2	3
e	523	428	72
e+1	72	428	69



‘Linear’ Shape Functions

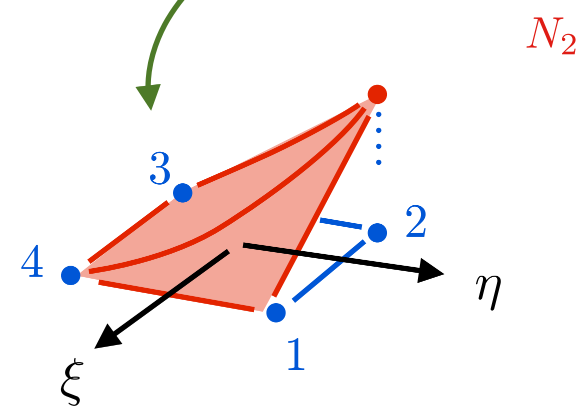
P$_1$ - C$_0$



$$N_1(\xi) = (1-\xi)/2$$
$$N_2(\xi) = (1+\xi)/2$$

Ref : [-1,1]

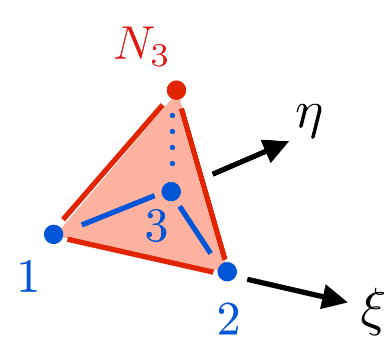
Bilinear Shape Functions



$$N_1(\xi,\eta) = (1+\xi)(1+\eta)/4$$
$$N_2(\xi,\eta) = (1-\xi)(1+\eta)/4$$
$$N_3(\xi,\eta) = (1-\xi)(1-\eta)/4$$
$$N_4(\xi,\eta) = (1+\xi)(1-\eta)/4$$

Ref : [-1,1]x[-1,1]

Q$_1$ - C$_0$



$$N_1(\xi,\eta) = 1-\xi-\eta$$
$$N_2(\xi,\eta) = \xi$$
$$N_3(\xi,\eta) = \eta$$

Ref : [0,1]x[0,1]

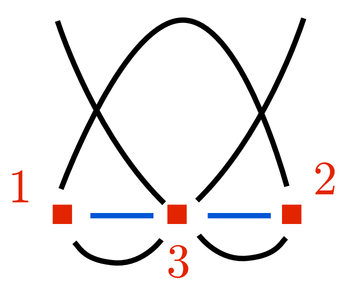
Turner Triangles

P$_1$ - C$_0$

u^h continuous piecewise ‘linear’

Continuous Quadratic Shape Functions

Ref : [-1,1]

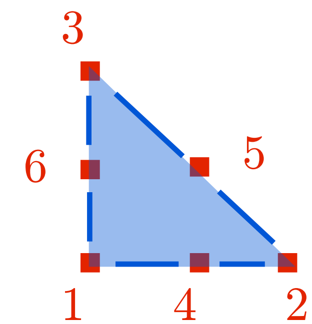


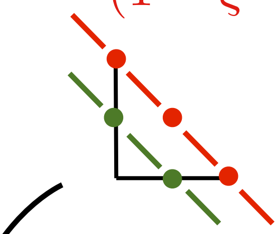
$$N_1(\xi) = -\xi(1-\xi)/2$$

$$N_2(\xi) = \xi(1+\xi)/2$$

$$N_3(\xi) = (1-\xi)(1+\xi)$$

P_2 - C_0





$$N_1(\xi,\eta) = \overbrace{(1-\xi-\eta)(1-2\xi-2\eta)}^{1-3(\xi+\eta)+2(\xi+\eta)^2}$$

$$N_2(\xi,\eta) = \xi(2\xi-1)$$

$$N_3(\xi,\eta) = \eta(2\eta-1)$$

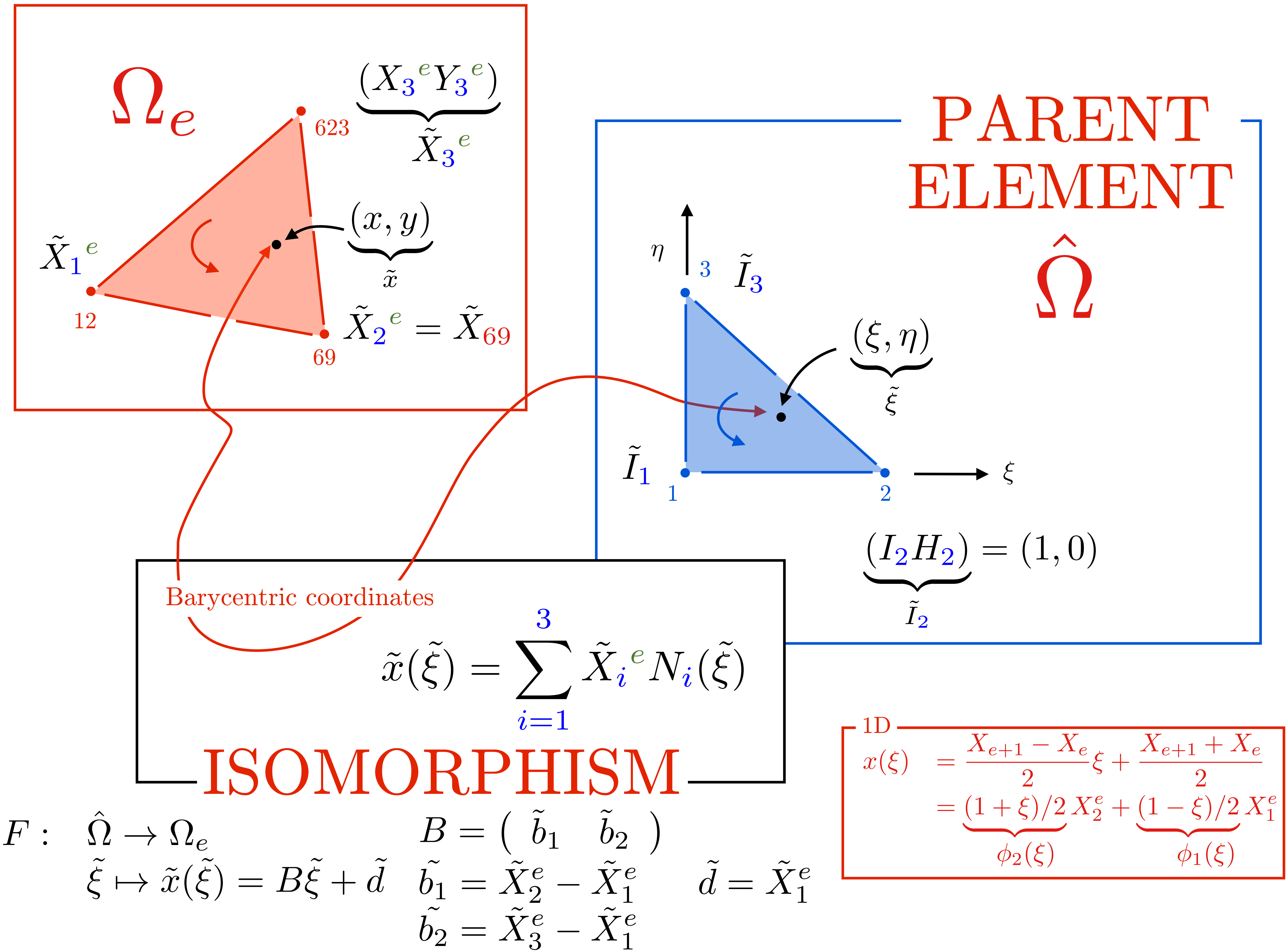
$$N_4(\xi,\eta) = 4\xi(1-\xi-\eta)$$

$$N_5(\xi,\eta) = 4\xi\eta$$

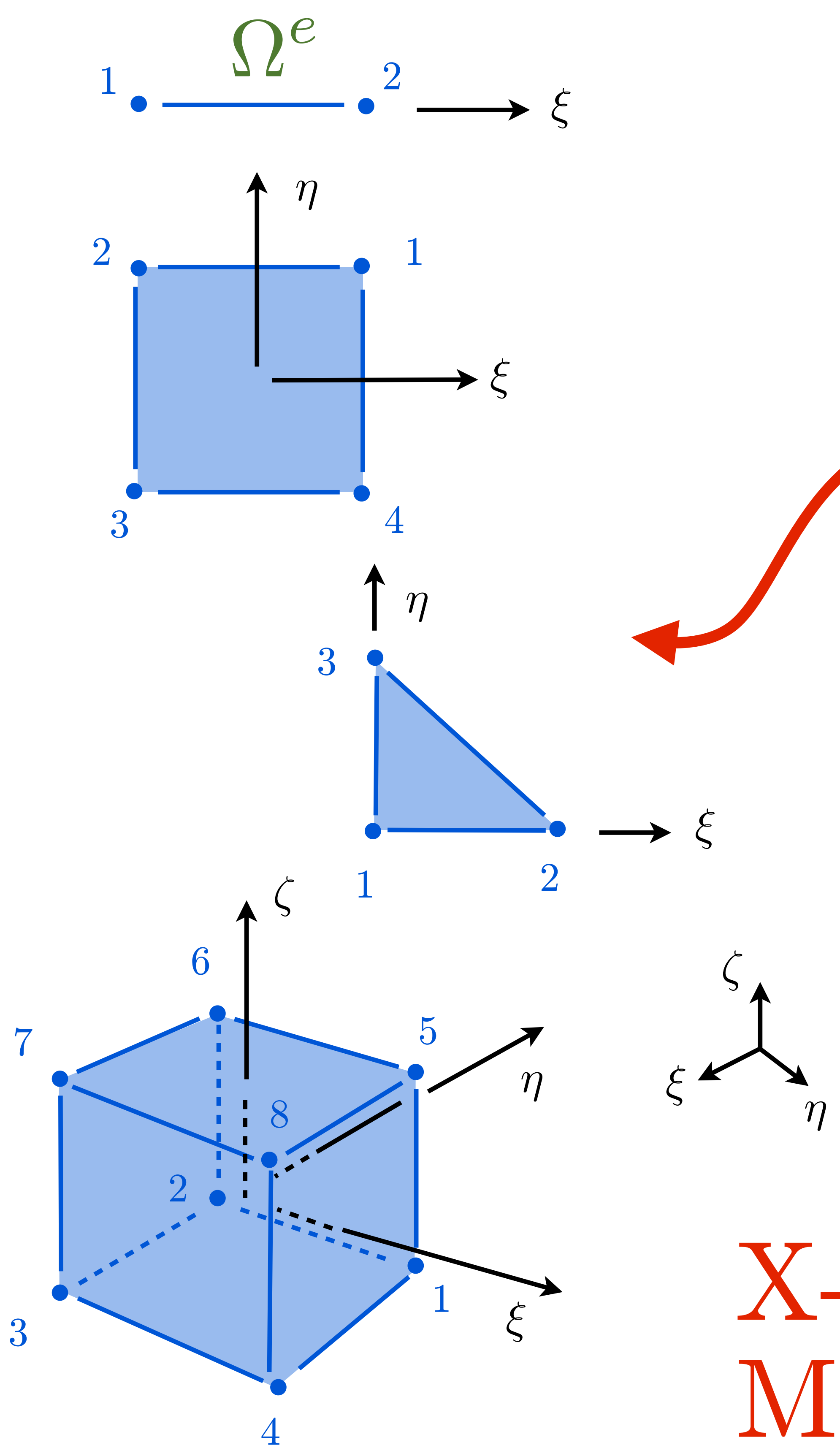
$$N_6(\xi,\eta) = 4\eta(1-\xi-\eta)$$

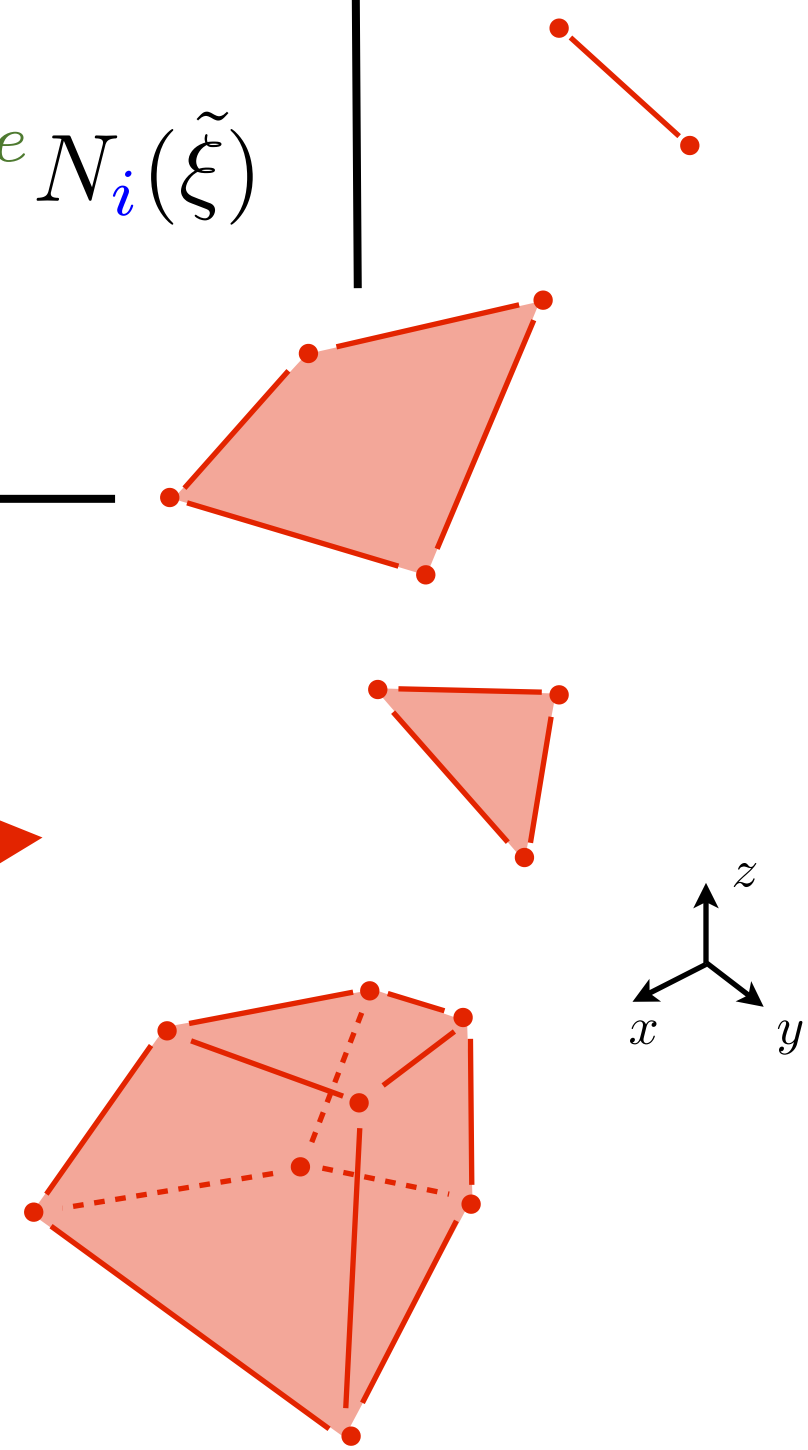
Ref : [0,1]x[0,1]

P_2 - C_0


Ω_e
$\underbrace{(X_3{}^e Y_3{}^e)}_{\tilde{X}_3{}^e}$
623
$\underbrace{(x,y)}_{\tilde{x}}$
$\tilde{X}_1{}^e$
12
69
$\tilde{X}_2{}^e = \tilde{X}_{69}$
PARENT ELEMENT
$\hat{\Omega}$
η
3
$\tilde{I}_3$
$\underbrace{(\xi,\eta)}_{\tilde{\xi}}$
$\tilde{I}_1$
1
2
ξ
$\underbrace{(I_2 H_2)}_{\tilde{I}_2} = (1,0)$
Barycentric coordinates
$\tilde{x}(\tilde{\xi}) = \sum_{i=1}^{3} \tilde{X}_i{}^e N_i(\tilde{\xi})$
ISOMORPHISM
1D
$x(\xi) = \frac{X_{e+1} - X_e}{2}\xi + \frac{X_{e+1} + X_e}{2}$
$= \underbrace{(1+\xi)/2}_{\phi_2(\xi)} X_2^e + \underbrace{(1-\xi)/2}_{\phi_1(\xi)} X_1^e$
$F: \quad \hat{\Omega} \to \Omega_e$
$\tilde{\xi} \mapsto \tilde{x}(\tilde{\xi}) = B\tilde{\xi} + \tilde{d}$
$B = \begin{pmatrix} \tilde{b}_1 & \tilde{b}_2 \end{pmatrix}$
$\tilde{b}_1 = \tilde{X}_2^e - \tilde{X}_1^e$
$\tilde{b}_2 = \tilde{X}_3^e - \tilde{X}_1^e$
$\tilde{d} = \tilde{X}_1^e$

$$\tilde{x}(\tilde{\xi}) = \sum_{i=1}^{n} \tilde{X}_i{}^e N_i(\tilde{\xi})$$





X-LINEAR MAPPING

$$N_i(\tilde{I}_j) = \delta_{ij}$$

N_i REDUCES TO A LINEAR FUNCTION ON ALL EDGES

FINITE ELEMENT = VARIATIONAL METHOD

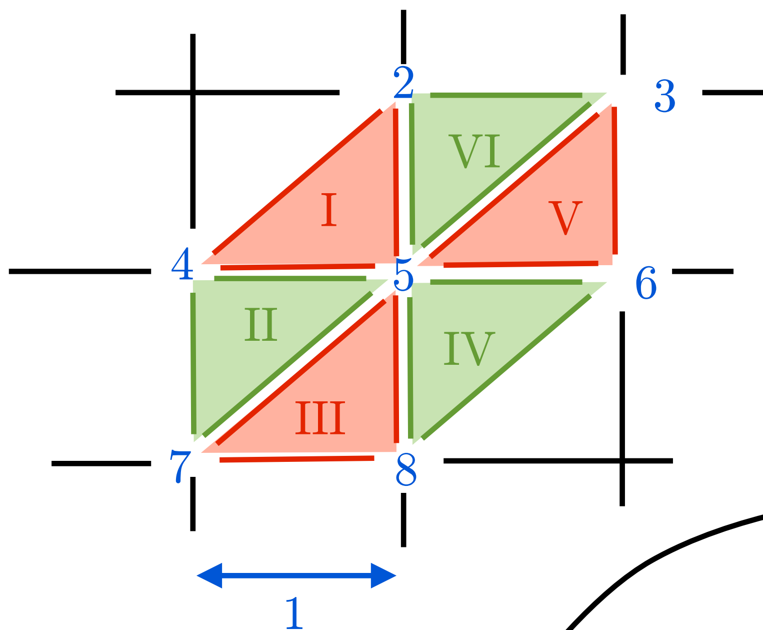
$$
\boxed{\begin{array}{l} ?\ u^h \in \mathcal{U}^h \\ a(u^h, \hat{u}^h) = b(\hat{u}^h) \quad \forall \hat{u}^h \in \mathcal{U}^h \end{array}}
$$

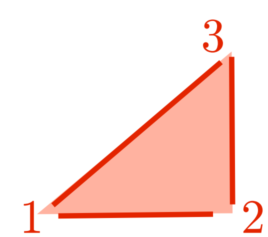
GALERKIN METHOD

STIFFNESS MATRIX CALCULATION

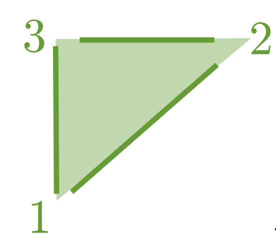
$$\underbrace{\left\langle \frac{\partial N_i^e}{\partial x}\frac{\partial N_j^e}{\partial x} + \frac{\partial N_i^e}{\partial y}\frac{\partial N_j^e}{\partial y}\right\rangle_{\Omega_e}}$$

$$A_{ij} = \bigoplus A_{ij}^e$$





$$\begin{array}{c|cc} & N_{i,x} & N_{i,y} \\ \hline 1 & -1 & 0 \\ 2 & 1 & -1 \\ 3 & 0 & 1 \end{array} \longrightarrow \frac{1}{2}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$



$$\begin{array}{c|cc} & N_{i,x} & N_{i,y} \\ \hline 1 & 0 & -1 \\ 2 & 1 & 0 \\ 3 & -1 & 1 \end{array} \longrightarrow \overbrace{\frac{1}{2}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ -1 & -1 & 2 \end{bmatrix}}^{A_{ij}^e}$$

CHECK !

$\sum N_i = 1$

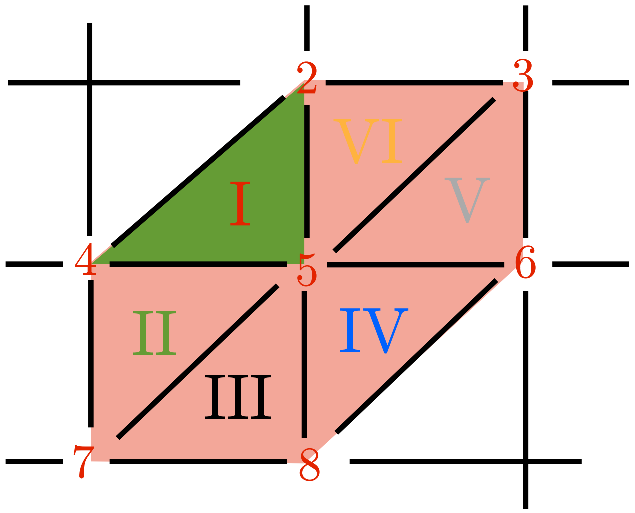
$\sum N_{i,x} = 0$

$\sum N_{i,y} = 0$

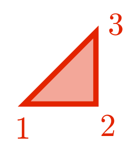
$N_1 = (1 - x) + \text{cst}$

$N_{1,x} = -1$

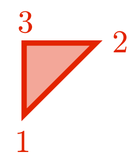
$N_{1,y} = 0$



ASSEMBLING LOCAL MATRICES



$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \frac{1}{2}$$

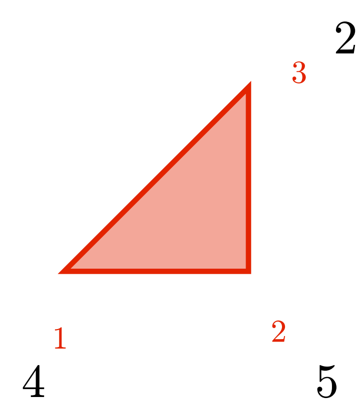


$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ -2 & -1 & 2 \end{bmatrix} \frac{1}{2}$$

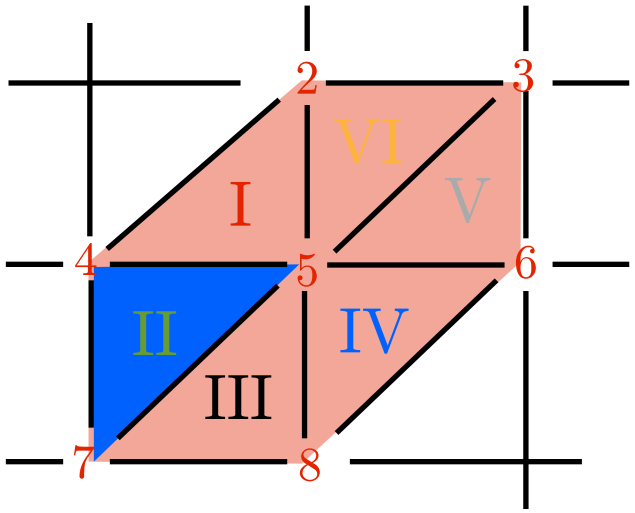
LOCATION TABLE

I	4	5	2
II	7	5	4
III	7	8	5
IV	8	6	5
V	5	6	3
VI	5	3	2

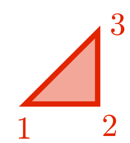
	2	3	4	5	6	7	8
2	1			-1			
3							
4			1	-1			
5	-1		-1	2			
6							
7							
8							



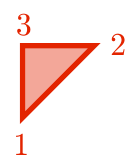
$$\begin{array}{cc} & \begin{array}{ccc} 4 & 5 & 2 \\ 1 & 2 & 3 \end{array} \\ \begin{array}{c} 4\ \ 1 \\ 5\ \ 2 \\ 2\ \ 3 \end{array} & \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \frac{1}{2} \end{array}$$



ASSEMBLING LOCAL MATRICES



$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \frac{1}{2}$$

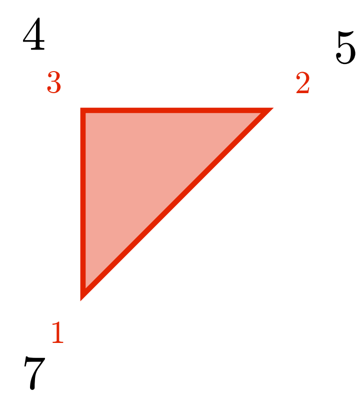


$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ -2 & -1 & 2 \end{bmatrix} \frac{1}{2}$$

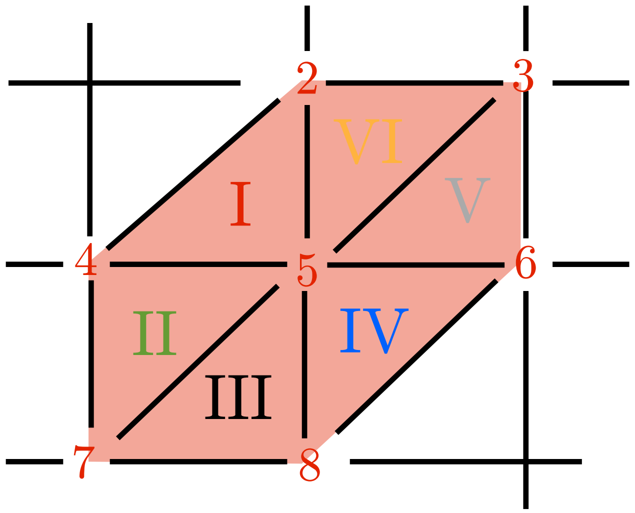
LOCATION TABLE

I	4	5	2
II	7	5	4
III	7	8	5
IV	8	6	5
V	5	6	3
VI	5	3	2

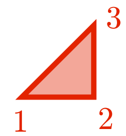
	2	3	4	5	6	7	8
2	1			-1			
3							
4			1 2	-1 -1		-1	
5	-1		-1-1	2 1			
6							
7			-1			1	
8							



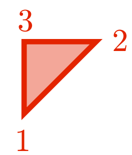
$$\begin{array}{cc} & \begin{array}{ccc} 7 & 5 & 4 \\ {\scriptstyle 1} & {\scriptstyle 2} & {\scriptstyle 3} \end{array} \\ \begin{array}{c} 7 \;{\scriptstyle 1} \\ 5 \;{\scriptstyle 2} \\ 4 \;{\scriptstyle 3} \end{array} & \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ -2 & -1 & 2 \end{bmatrix} \frac{1}{2} \end{array}$$



ASSEMBLING LOCAL MATRICES



$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \frac{1}{2}$$

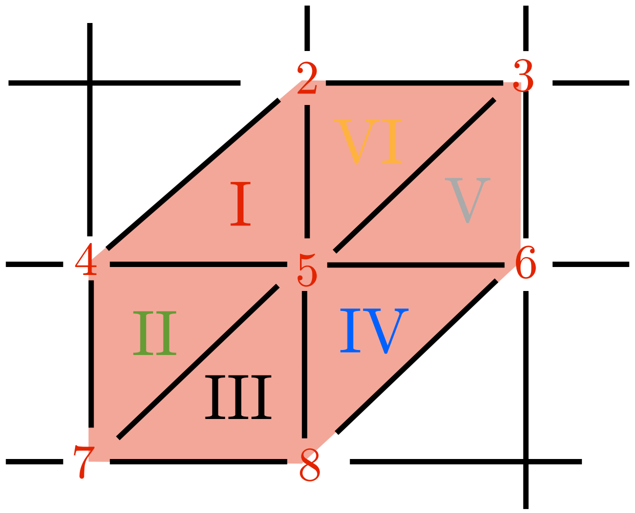


$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ -2 & -1 & 2 \end{bmatrix} \frac{1}{2}$$

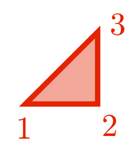
$\underbrace{\text{LOCATION TABLE}}$

I	4	5	2
II	7	5	4
III	7	8	5
IV	8	6	5
V	5	6	3
VI	5	3	2

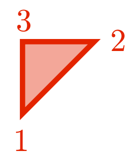
	2	3	4	5	6	7	8
2	1 2	-1		-1 -1			
3	-1	1 1			-1		
4			1 2	-1 -1		-1	
5	-1 -1		-1 -1	2 1 1 2 1 1	-1 -1		-1 -1
6		-1		-1 -1	1 2		
7			-1			1 1	-1
8				-1 -1		-1	2 1



ASSEMBLING LOCAL MATRICES



$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \frac{1}{2}$$



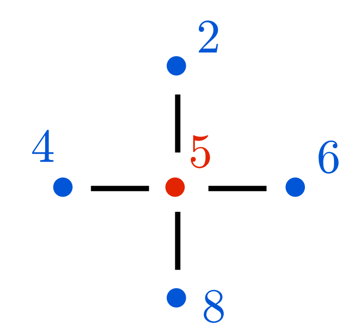
$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ -2 & -1 & 2 \end{bmatrix} \frac{1}{2}$$

LOCATION TABLE

I	4	5	2
II	7	5	4
III	7	8	5
IV	8	6	5
V	5	6	3
VI	5	3	2

	2	3	4	5	6	7	8
2	1 2	-1		-1 -1			
3	-1	1 1			-1		
4			1 2	-1 -1		-1	
5	-1 -1		-1 -1	2 1 1 2 1 1	-1 -1		-1 -1
6		-1		-1 -1	1 2		
7			-1			1 1	-1
8				-1 -1		-1	2 1

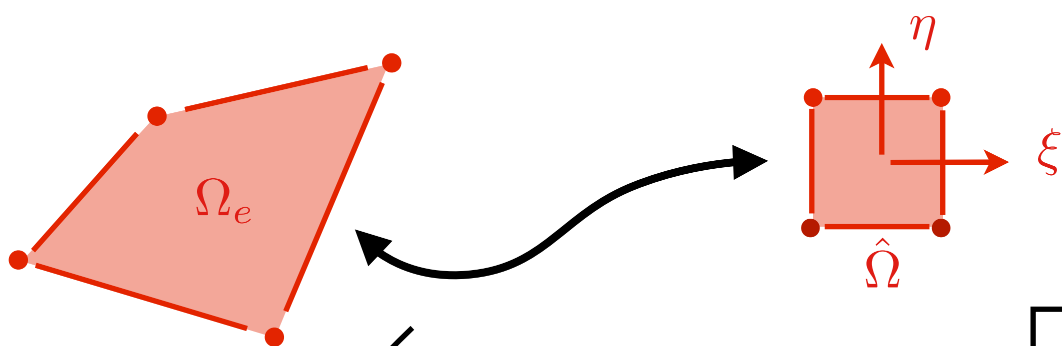
$$\frac{1}{2}\left(8U_5 \; -2U_2 - 2U_4 \; -2U_8 - 2U_6\right)$$



REGULAR LATTICE OF TURNER TRIANGLES = CENTRAL FINITE DIFFERENCES

COMPUTATION OF THE ELEMENT STIFFNESS MATRIX

$$A^e_{ij} = \int_{\Omega_e} \nabla N^e_i \cdot \nabla N^e_j \; d\Omega$$



$$\boxed{\int_{\Omega} \longrightarrow \int_{\hat{\Omega}}}$$

$$\boxed{\frac{\partial}{\partial \tilde{x}} \longrightarrow \frac{\partial}{\partial \tilde{\xi}}}$$

$$\boxed{\int_{\hat{\Omega}} f(\xi) \longrightarrow \sum_k w_k f(\xi_k)}$$

3 DIFFICULTIES TO HANDLE !

MORE DIFFICULT IN THE GENERAL 2D-3D CASE

EASY IN 1D

EASY FOR 2D LINEAR MAPPING

$$\tilde{x}(\tilde{\xi}) = \sum_{i=1}^{4} \tilde{X}^e_i N_i(\tilde{\xi})$$

$$\int_{\Omega_e} \longrightarrow \int_{\hat{\Omega}}$$

$$G(\tilde{x}) = G(F(\tilde{\xi})) = (GoF)(\tilde{\xi}) = \hat{G}(\tilde{\xi})$$

$$\left(\nabla_{\tilde{\xi}}\hat{G}\right)(\tilde{\xi}) = \begin{pmatrix} \frac{\partial F_1}{\partial \xi} & \frac{\partial F_2}{\partial \xi} \\ \frac{\partial F_1}{\partial \eta} & \frac{\partial F_2}{\partial \eta} \end{pmatrix} \cdot \left(\nabla_{\tilde{x}} G\right)(\tilde{x})$$

$$= B^T \left(\nabla_{\tilde{x}} G\right)(\tilde{x})$$

$$\left(\nabla_{\tilde{x}} G\right)(\tilde{x}) = B^{-T}\left(\nabla_{\tilde{\xi}}\hat{G}\right)(\tilde{\xi})$$

$$\int_{\Omega_e} \psi(\tilde{x})d\tilde{x} = \int_{\Omega_e} \psi(F(\tilde{\xi}))d\tilde{x} = \int_{\hat{\Omega}=F^{-1}(\Omega_e)} \hat{\psi}(\tilde{\xi})|det\ J_F|d\tilde{\xi}$$

$$det\ J_F = det\ B = 2\text{Area}(\Omega_e)$$

$$\int_{\hat{\Omega}} \nabla_{\tilde{x}}\phi_{r_j}(F(\tilde{\xi})) \cdot \nabla_{\tilde{x}}\phi_{r_i}(F(\tilde{\xi}))|det\ B|d\tilde{\xi} =$$

$$\int_{\hat{\Omega}} B^{-T}\nabla_{\tilde{\xi}} N_j^e(\tilde{\xi}) \cdot B^{-T}\nabla_{\tilde{x}} N_i^e(\tilde{\xi})|det\ B|d\tilde{\xi}$$

$$\int_{\hat{\Omega}} f(\tilde{\xi}) \longrightarrow \sum_k w_k f(\tilde{\xi}_k)$$

Numerical
integration
required !

Weights

$\sum w_k = 1!$
if $\int_{\hat{\Omega}} = 1$

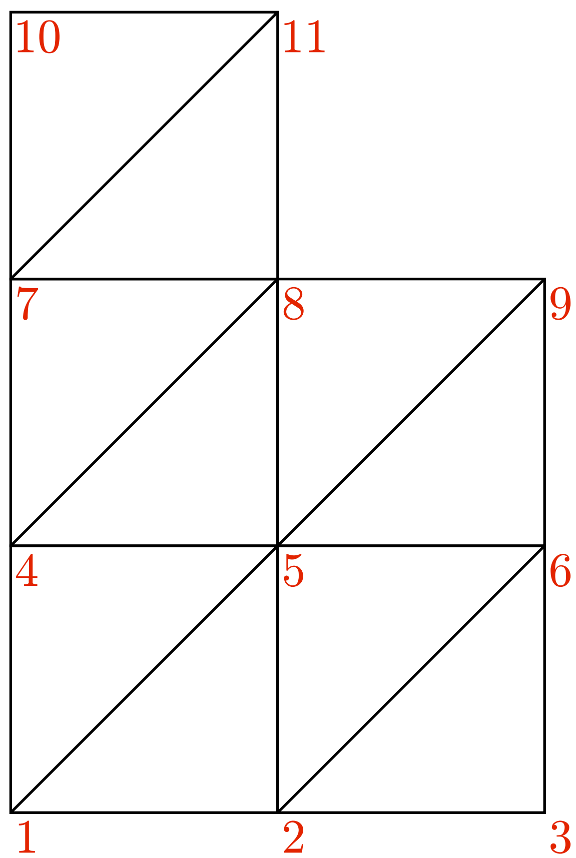
Integration
points

To be
selected
in a clever way

Analytical
way is
impossible !

$$\underbrace{\tilde{\nabla}\phi_i \cdot \tilde{\nabla}\phi_j \quad J_e(\tilde{\xi})}_{f(\tilde{\xi})}$$

GAUSS
LEGENDRE
QUADRATURE !



Connectivity table

	1	2	3	4	5	6	7	8	9	10	11
1	x	x		x	x						
2	x	x	x		x	x					
3		x	x			x					
4	x			x	x		x	x			
5	x	x		x	x	x		x	x		
6		x	x		x	x			x		
7				x			x	x		x	x
8				x	x		x	x	x		x
9					x	x		x	x		
10							x			x	x
11							x	x		x	x

How to implement it in a computer ?

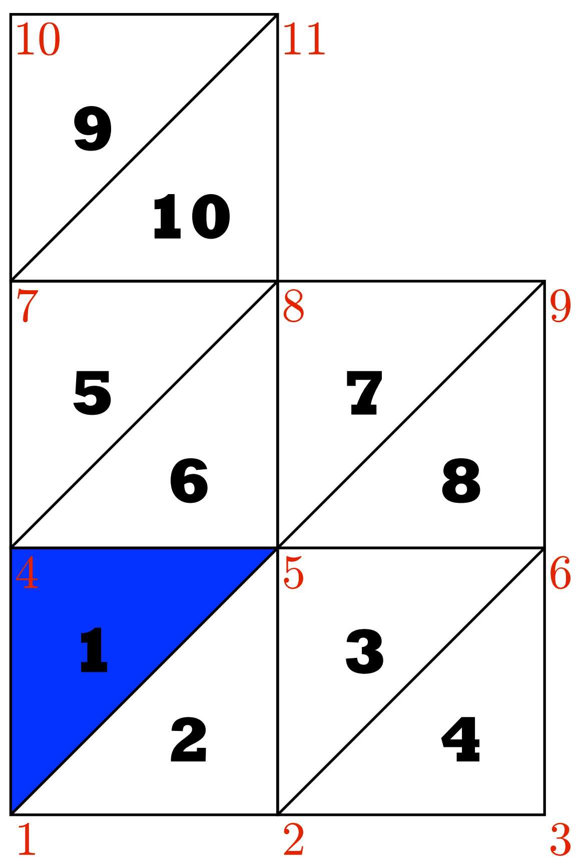
We assume that every node has at most 10 neighbouring nodes

Simultaneous assembly procedure

Connectivity table

Number of elements per lines

	1	2	3	4	5	6	7	8	9	10	
1	1	5	4								3
2											0
3											0
4	1	5	4								3
5	1	5	4								3
6											0
7											0
8											0
9											0
10											0
11											0



Triangle 1 : (1 5 4)

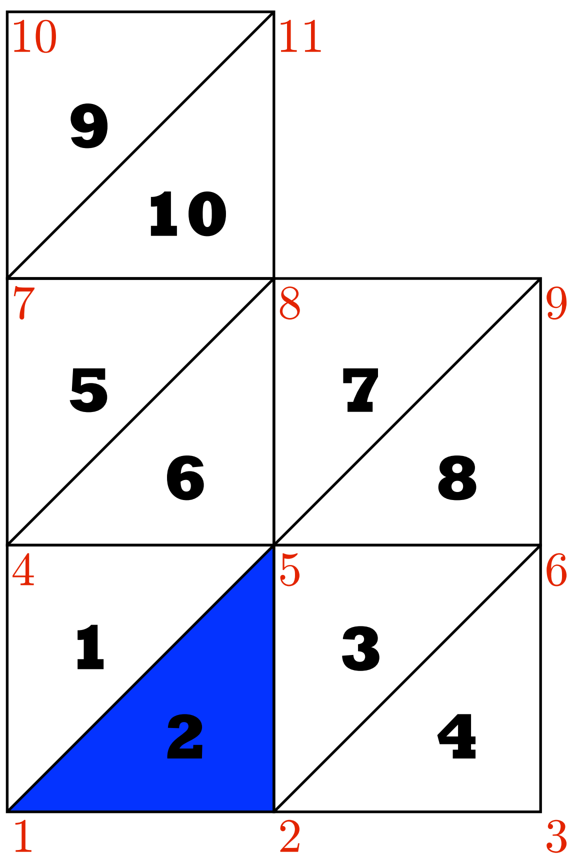
Node 1 was never be encoutered: put all nodes of the triangles on row 1
Node 5 was never be encoutered: put all nodes of the triangles on row 5
Node 4 was never be encoutered: put all nodes of the triangles on row 4

We assume that every node has at most 10 neighbouring nodes

Number of elements per lines

Connectivity table

	1	2	3	4	5	6	7	8	9	10	
1	1	5	4	2							4
2	1	2	5								3
3											0
4	1	5	4								3
5	1	5	4	2							4
6											0
7											0
8											0
9											0
10											0
11											0



Triangle 2 : (1 2 5)

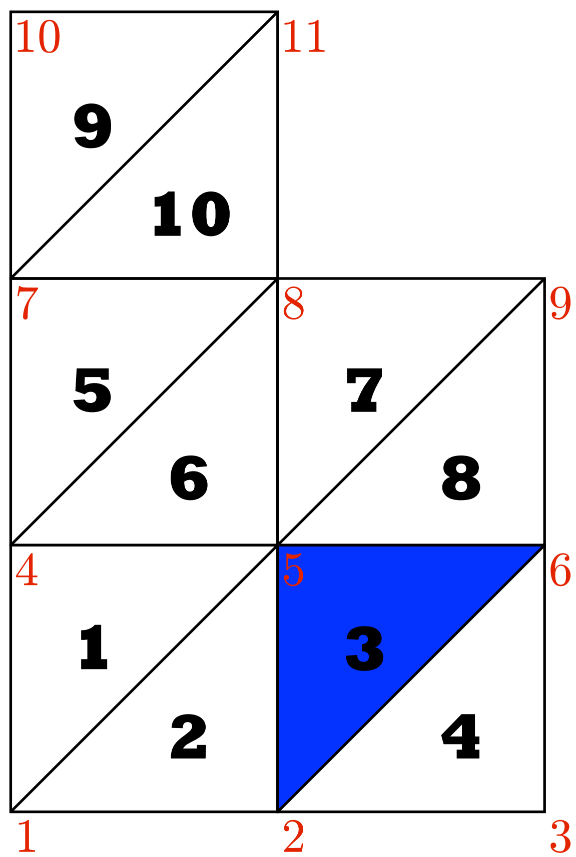
Node 1 was already be encoutered: 1 and 5 exist in row 1, not 2
Node 2 was never be encoutered: put all nodes of the triangles on row 2
Node 5 was already be encoutered: 1 and 5 exist in row 5, not 2

We assume that every node has at most 10 neighbouring nodes

Number of elements per lines

Connectivity table

	1	2	3	4	5	6	7	8	9	10	
1	1	5	4	2							4
2	1	2	5	6							4
3											0
4	1	5	4								3
5	1	5	4	2	6						5
6	2	6	5								3
7											0
8											0
9											0
10											0
11											0



Triangle 3 : (2 6 5)

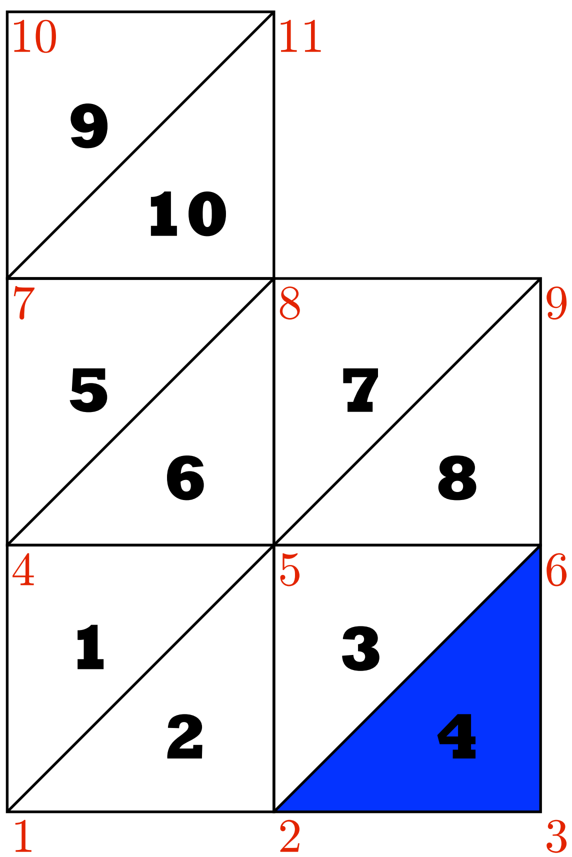
Node 2 was already be encoutered: 2 and 5 exist in row 2, not 6
Node 6 was never be encoutered: put all nodes of the triangles on row 6
Node 5 was already be encoutered: 2 and 5 exist in row 5, not 6

We assume that every node has at most 10 neighbouring nodes

Number of elements per lines

Connectivity table

	1	2	3	4	5	6	7	8	9	10	Number of elements per lines
1	1	5	4	2							4
2	1	2	5	6	3						5
3	2	3	6								3
4	1	5	4								3
5	1	5	4	2	6						5
6	2	6	5	3							4
7											0
8											0
9											0
10											0
11											0



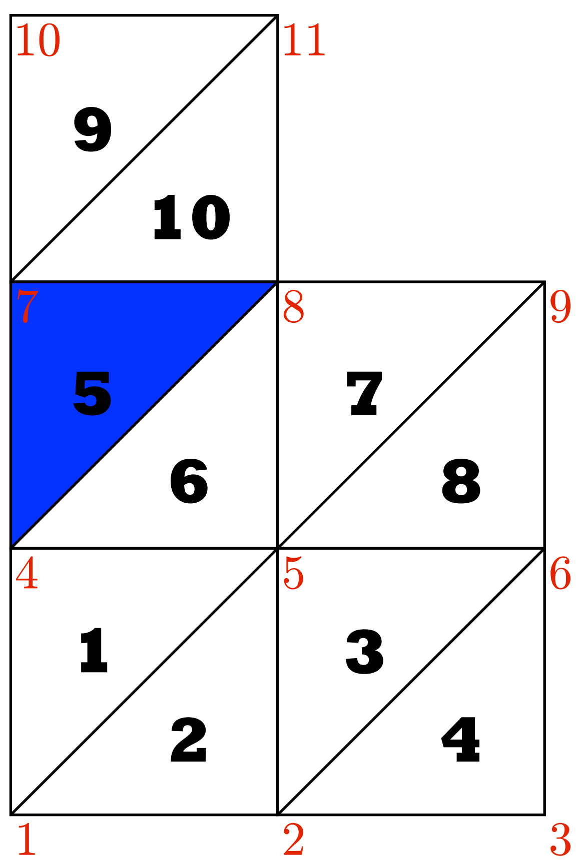
Triangle 4 : (2 3 6)

Node 2 was already be encoutered: 2 and 6 exist in row 2, not 3
Node 3 was never be encoutered: put all nodes of the triangles on row 3
Node 6 was already be encoutered: 2 and 6 exist in row 6, not 3

We assume that every node has at most 10 neighbouring nodes

Number of elements per lines

Connectivity table



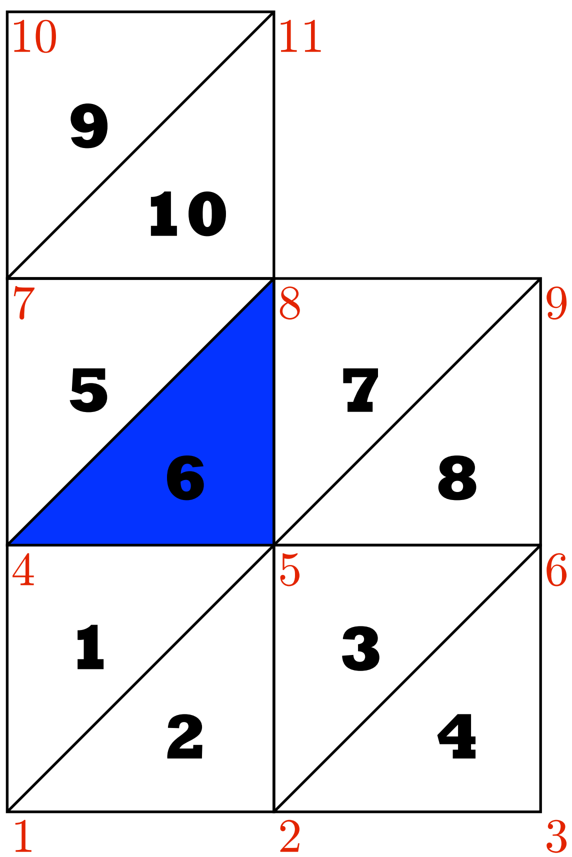
Triangle 5 : (4 8 7)

	1	2	3	4	5	6	7	8	9	10	
1	1	5	4	2							4
2	1	2	5	6	3						5
3	2	3	6								3
4	1	5	4	8	7						5
5	1	5	4	2	6						5
6	2	6	5	3							4
7	4	8	7								3
8	4	8	7								3
9											0
10											0
11											0

Node 4 was already be encoutered: 4 exists in row 4, not 8 and 7
Node 8 was never be encoutered: put all nodes of the triangles on row 8
Node 7 was never be encoutered: put all nodes of the triangles on row 7

We assume that every node has at most 10 neighbouring nodes

Number of elements per lines



Connectivity table

	1	2	3	4	5	6	7	8	9	10	
1	1	5	4	2							4
2	1	2	5	6	3						5
3	2	3	6								3
4	1	5	4	8	7						5
5	1	5	4	2	6	8					6
6	2	6	5	3							4
7	4	8	7								3
8	4	8	7	5							4
9											0
10											0
11											0

Triangle 6 : (4 5 8)

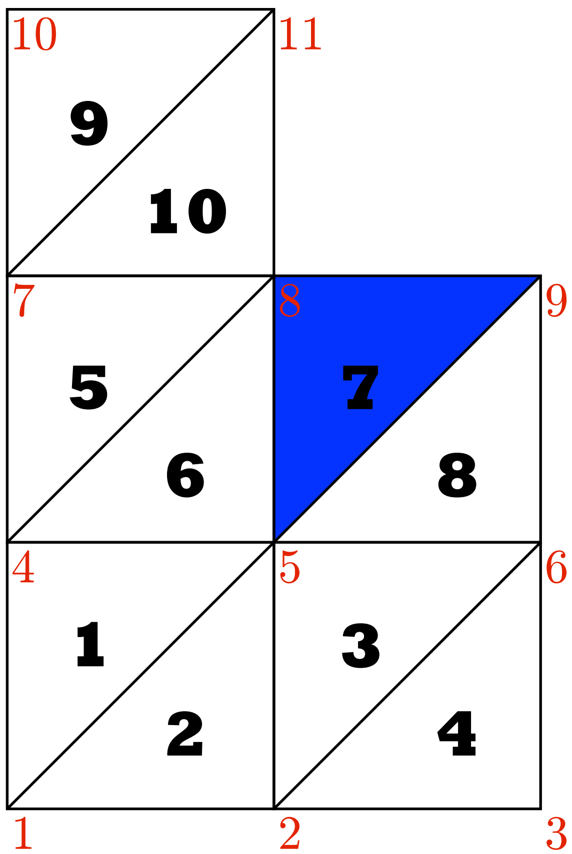
Node 4 was already be encoutered: 4, 5 and 8 exist in row 4
Node 5 was already be encoutered: 4 and 5 exist in row 5, not 8
Node 8 was already be encoutered: 4 and 8 exist in row 8, not 5

We assume that every node has at most 10 neighbouring nodes

Number of elements per lines

Connectivity table

	1	2	3	4	5	6	7	8	9	10	Number of elements per lines
1	1	5	4	2							4
2	1	2	5	6	3						5
3	2	3	6								3
4	1	5	4	8	7						5
5	1	5	4	2	6	8	9				7
6	2	6	5	3							4
7	4	8	7								3
8	4	8	7	5	9						5
9	5	9	8								3
10											0
11											0



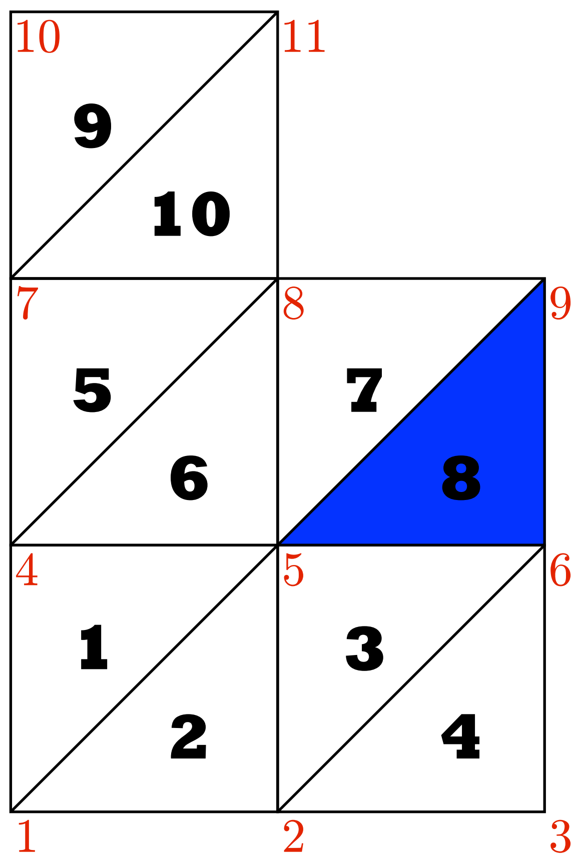
Triangle 7 : (5 9 8)

Node 5 was already be encoutered: 5 and 8 exist in row 5, not 9
Node 9 was never be encoutered: put all nodes of the triangles on row 9
Node 8 was already be encoutered: 5 and 8 exist in row 8, not 9

We assume that every node has at most 10 neighbouring nodes

Number of elements per lines

Connectivity table



	1	2	3	4	5	6	7	8	9	10	
1	1	5	4	2							4
2	1	2	5	6	3						5
3	2	3	6								3
4	1	5	4	8	7						5
5	1	5	4	2	6	8	9				7
6	2	6	5	3	9						5
7	4	8	7								3
8	4	8	7	5	9						5
9	5	9	8	6							4
10											0
11											0

Triangle 8 : (5 6 9)

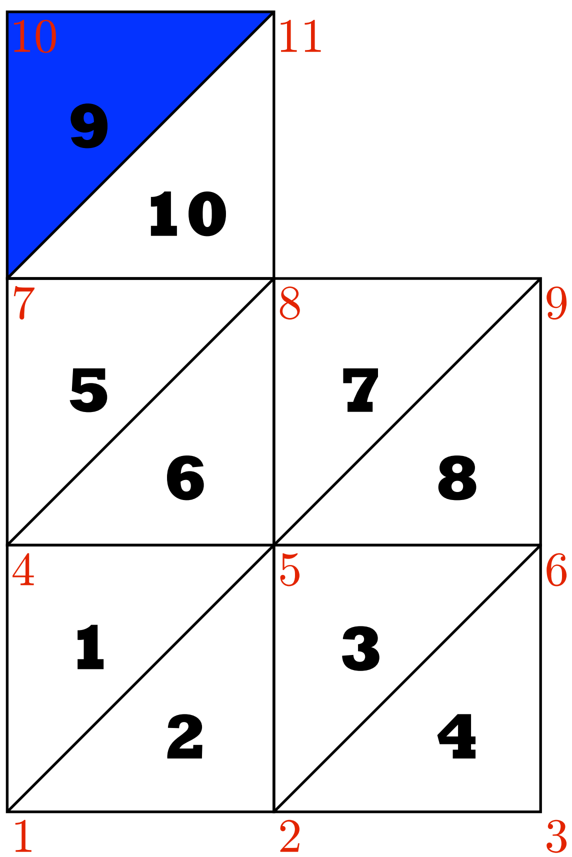
Node 5 was already be encoutered: 5, 6 and 9 exist in row 5
Node 6 was already be encoutered: 5 and 6 exist in row 6, not 9
Node 9 was already be encoutered: 5 and 9 exist in row 8, not 6

We assume that every node has at most 10 neighbouring nodes

Number of elements per lines

Connectivity table

	1	2	3	4	5	6	7	8	9	10	
1	1	5	4	2							4
2	1	2	5	6	3						5
3	2	3	6								3
4	1	5	4	8	7						5
5	1	5	4	2	6	8	9				7
6	2	6	5	3	9						5
7	4	8	7	11	10						5
8	4	8	7	5	9						5
9	5	9	8	6							4
10	7	11	10								3
11	7	11	10								3



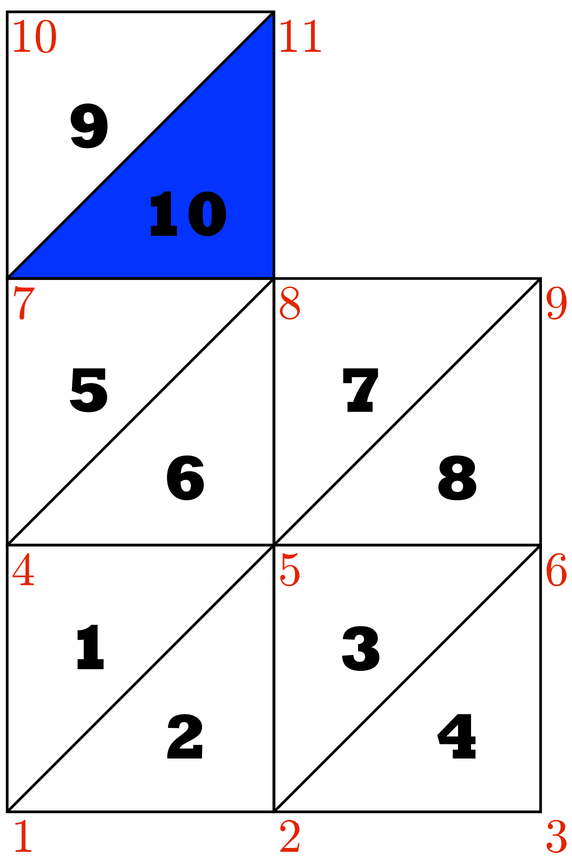
Triangle 9 : (7 11 10)

Node 7 was already be encoutered: 7 exists in row 7, not 11 and 10
Node 11 was never be encoutered: put all nodes of the triangles on row 11
Node 10 was never be encoutered: put all nodes of the triangles on row 10

We assume that every node has at most 10 neighbouring nodes

Number of elements per lines

Connectivity table



	1	2	3	4	5	6	7	8	9	10	
1	1	5	4	2							4
2	1	2	5	6	3						5
3	2	3	6								3
4	1	5	4	8	7						5
5	1	5	4	2	6	8	9				7
6	2	6	5	3	9						5
7	4	8	7	11	10						5
8	4	8	7	5	9	11					6
9	5	9	8	6							4
10	7	11	10								3
11	7	11	10	8							4

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Triangle 10 : (7 8 11)

Node 7 was already be encoutered: 7, 8 and 11 exist in row 7
Node 8 was already be encoutered: 7 and 8 exist in row 8, not 11
Node 11 was already be encoutered: 7 and 11 exist in row 11, not 8

Three arrays : values, ind, nbe

CSR storage : A(51), JA(51), IA(12)