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Informatics II
Exercise 12

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Graphs

Task 1

1. For an undirected graph G with n vertices and m edges, which statement is correct?

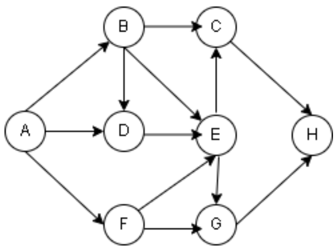
   A. if m<n, G is unconnected

   B. if m>=n, there is a loop in G

   C. if m>n, G is connected

   D. if m<n, there is no loop in G

2. A possible sequence of Depth-First search on the graph is



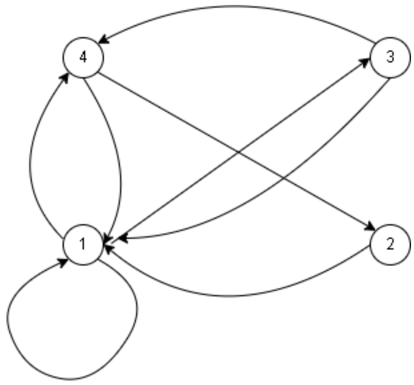
   A. ABDFCEGH

   B. ABCHDEGF

   C. ADECHBFG

   D. AFBDCEGH

3. Show adjacency matrix of the graph below

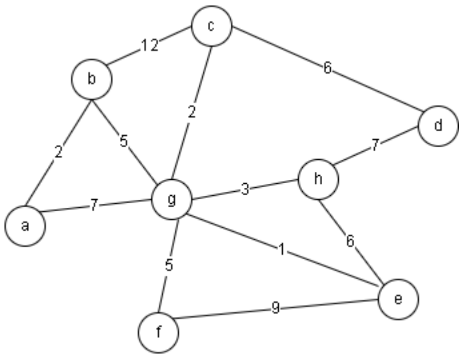


4. For an undirected weighted graph, its minimum spanning tree may not exist, but if it exists, it might not be unique.

   A. True

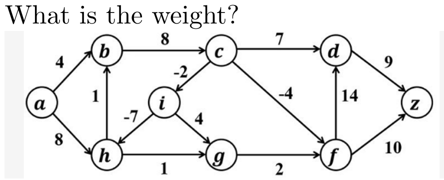
   B. False

5. Use the Prim-Jarnik algorithm to compute to compute a Minimum Spanning Tree for the graph below (start from node 'a').



Task 2

1. Which algorithm can help us get the minimum weight from 'a' to 'z'? What is the weight?



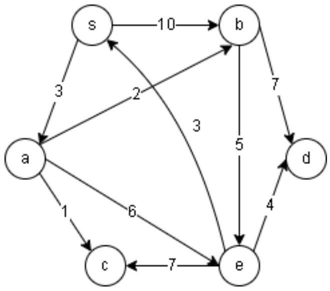
   A. Bellman-Ford 21

   B. Bellman-Ford 16

   C. Dijkstra 21

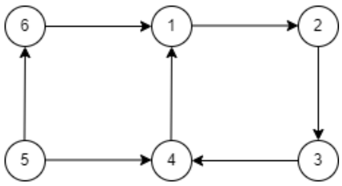
   D. Dijkstra 16

2. Use Dijkstra algorithm to compute shortest path from start node 's' for the graph.



Task 3

A root vertex of a directed graph is a vertex u with a directed path from u to v for every pair of vertices (u, v) in the graph. In other words, all other vertices in the graph can be reached from the root vertex. Given a graph, write C code that finds the root vertex using Breadth First Search as well as Depth First Search approach. A graph can have multiple root vertices. In such cases, the solution should find all the root vertices.
For example, the root vertex is 5 since it has a path to every other vertex in the following graph:

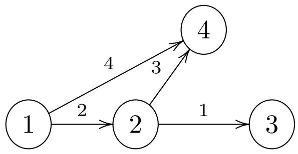


A framework code with the Queue implementation is provided. Use an adjacency matrix to represent the graph in the code.

Task 4

Consider a directed weighted graph $G = (V, E)$. Let $n = |V|$ and $m = |E|$. Each vertex of V has a unique integer label between 1 and n. Each edge of E has a distinct weight between 1 and m.

The edges of E are stored in an array A. Each array element is a record with three fields: f (from), t (to), and w (weight). The array elements are *sorted* by the edge weight in ascending order. Note that position in A starts at 1.



$G = (V, E)$

	1	2	3	4
f	2	1	2	1
t	3	2	4	4
w	1	2	3	4

Array A representing G

Example. A[1] is for the directed edge $(2, 3)$, so $A[1].f = 2$, $A[1].t = 3$, and $A[1].w = 1$.

A **path** $p =< v_0, v_1, ..., v_k >$ is a sequence of vertices where adjacent vertices v_i and v_{i+1} are connected by an edge. A path can visit the same vertex *multiple* times. The **length** of a path p is the number of vertices of p.

Task 4.1:

A path $p =< v_0, v_1, ..., v_k >$ is a **weight-incremented path** if and only if the weights of two neighboring edges in p are strictly increased, *i.e.*, $w(v_{i-1}, v_i) < w(v_i, v_{i+1})$ where $1 \leq i < k$. A path with only one edge is also consider as a weight-incremented path.

Determine the maximum length of weight-incremented paths in G_2.

Task 4.2:

The path $p =< v_0, v_1, ..., v_k >$ terminates at the vertex v_k. Consider an array dp of size n. Let $dp[v]$ store the **maximum length** of weight-incremented paths that are terminated at vertex v in G with m edges. Note that position in dp starts at 1.

When edges with weights $\leq i$ are considered, we use dp_i to denote the states of dp.

Consider G and the array dp with size $4 (= |V_2|)$ for G.

Fill in $dp_1 =$ | | | | | when edges with weights ≤ 1 are considered.
Fill in $dp_2 =$ | | | | | when edges with weights ≤ 2 are considered.
Fill in $dp_3 =$ | | | | | when edges with weights ≤ 3 are considered.
Fill in $dp_4 =$ | | | | | when edges with weights ≤ 4 are considered.

Task 4.3:Determine the recursive problem formulation for $dp_i[v]$. Assume that the directed edge (a, b) has the weight i.

Task 4.4: Write the pseudocode algorithm `maximum_len(`A`, `m`)` that returns the maximum length of weight-incremented paths in $G = (V, E)$ with m ($m = |E|$) edges.

Requirement: The asymptotic complexity of your solution should be $O(m)$.