Lexu (Gavin) Zhao & Michael Zhang

Project Outline: Fourier Transform

Problem/Question

Given a complex sound wave simulated by a computer algorithm, we want to manually construct the same sound by choosing one specific frequency. However, there is no single frequency being able to depict it. **Why is the computer able to successfully generate a given square wave?** We hope it can be answered by Fourier transformation, a fundamental mathematical tool for various modern technologies benefiting everyone's life. It's widely used in processing images and sounds, designing circuits, analyzing any type of wave information by engineers, observing distant astronomical objects by physicists, solving differential equations by mathematicians, etc. The application we will investigate is sound decomposition, which uses Fourier transform to break complicated sound signals into the form of a superposition of characteristic waves. Noticing that Fourier transform doesn't require the subject to be periodic or fluctuating, a periodic wave is a good place to start with.

Hypothesis

We hypothesized that computer programs can use Fourier transform to compose the square wave since the benefit of the Fourier transform is to allow the translation of the complex changes of vibration with time into a linear combination of simple frequencies that are more characterizable.
Specifically, we give a Fourier Transform Formula:

$$F(k) = \int_{-\infty}^{\infty} f(x) e^{ikx} dx$$

And for a square wave, we have the following general formulation for its Fourier Transform, denoting the square wave function as rect(x),

$$F(k) = \int_{-\infty}^{\infty} rect(x) e^{ikx} dx = \sum_{k=-\infty}^{\infty} \frac{2sin(k\omega_0 T)}{k} \delta(\omega - \omega_0)$$

Experiment Outline

To test out our hypothesis that computer programs can construct the square wave by summing up multiple characteristic waves, we show: that by interpreting and constructing several sine functions from frequencies that result from the Fourier transform, a close-enough square wave could be reconstructed.

1. Generate a sound square wave by computer synthesizer and listen to it.
2. From the mathematical result obtained in the hypothesis, we can calculate multiple frequencies and amplitudes of different sinusoidal waves. (Visually, we can obtain a graph of frequency in which there are some "spikes" showing up at different frequency levels.)
3. To visualize, we use the frequencies and amplitudes we calculated from step two to graph a series of sinusoidal functions.
4. We overlap all the basic waves and obtain their superposition waveform, then check whether they lead to the same shape or sound compared to the initial square wave.

Experimental Results

If the resulting graph is consistent with the initial square wave generated, then the Fourier transform indeed provides the tools needed to decompose a wave function into simple sine waves, the superpositions of which could, as the proposed experiment would show, experimentally reconstruct the original wave successfully. Contrarily, if the result is inconsistent, the potential error might be the inadequate sine waves in forming the desired wave, then we can increase the number of sine waves and perform it again.

Conclusion

In this experiment, we are able to show that a square wave is made up of numerous sine waves by performing Fourier analysis. In general, Fourier transformation is essentially a method of taking a complex structure and breaking it into its normalized simple components, finding the "recipe" of a soup. Specifically, Fourier Transform maps a domain to its corresponding reciprocal space, which is useful in analyzing the properties of the original function.