Bowman Farm Optimisation Model (2025–2026)

Table of Content

1. Background

This model is designed to assist the Bowman family in determining the optimal number of livestock to purchase, the crop areas to plant, and the storage or selling decisions necessary to maximize expected farm wealth from October 2025 to October 2026.

The problem is formulated as a stochastic integer linear programming (ILP) model, accounting for uncertainty in labour requirements and crop yields, which are influenced by varying weather conditions. The model was implemented in Python using the PuLP library.

As mentioned above, Expected farm wealth in this model is significantly impacted by uncertainties, especially weather variations that affect both crop yields and labour costs. Thus, Labour requirements were carefully calculated for different weather scenarios, while the yield is calculated with the "Frost & Dry" scenario identified as the most likely and thus prioritized. This scenario dictates the crop yields and labour requirements that form the basis for the model's design.

The model focuses on planning for the 2025–2026 cycle, including critical decisions on planting areas and livestock management—specifically whether to purchase additional livestock or sell existing stock. It assumes that any livestock carried over from 2025 can be sold by October 2026. For instance, with an initial stock of 45 tons of corn and 25 tons of wheat, the optimal solution suggests adjusting storage to 30 tons of corn and 64 tons of wheat. This strategy ensures that part of the new crop production is allocated to meet livestock feeding needs, optimizing the integration of crop production and livestock requirements.

2. Mathematical Model

2.1 Decision Variables:

Let:

Variable	Description
Xc	Number of cows bought in 2025
Xh	Number of hens bought in 2025
Yc	Number of cows sold in 2025
Yh	Number of hens sold in 2025
Ac	Acres planted with corn
Aw	Acres planted with wheat
Ar	Acres planted with rapeseed
Zcorn	Tons of corn sold in 2025
Zwheat	Tons of wheat sold in 2025
Zrape	Tons of rapeseed sold in 2026
Scorn	Tons of corn stored at end of 2026
Swheat	Tons of wheat stored at end of 2026
Pcorn	Tons of corn purchased in 2026
Pwheat	Tons of wheat purchased in 2026
Kinvest	Amount of capital invested (bank savings)

2.2 Objective Function:

Maximize Expected Wealth =

Revenue + Residual Value + Interest on Remaining Cash − Total Costs

Where:

- **Revenue** =
  200Zcorn+195Zwheat+210Scorn+205Swheat+450Zrape+1300(Existing Cows+Xc−Yc)+155(Existing Hens+Xh−Yh)
- **Residual Value** =
  936(Existing Cows+Xc−Yc)+14.45(Existing Hens+Xh−Yh)
- **Total Costs** =
  1040Xc+17Xh+550Ac+400Aw+244Ar+240Pcorn+220Pwheat+Kinvest
- **Interest on Cash** =
  0.035×(Cash Leftover)

2.3 Constrains:

1. Livestock
$30 + Xc - Yc \leq 80$
$400 + Xh - Yh \leq 2000$
2. Land
$Ac+Aw+Ar+2Xc+(30-Yc)+(400-Yh)\leq 850$
3. Labour:
Spring/Winter (Max 3,300 hrs):
$3(30+ Xc) + 0.05(400 + Xh) + 1.82Ac + 1.54Aw + 1.32Ar \leq 3300$
Summer/Autumn (Max 3,800 hrs):
$3(30 + Xc) + 0.05(400 + Xh) + 1.44Ac + 0.94Aw + 0.62Ar \leq 3800$

4. Feed ***Corn:*** $S_{corn} >= (30 + X_c - Y_c) * 1$ ***Wheat:*** $S_{wheat} >= (400 + X_h - Y_h) * 0.04$
5. Crop Yield Constraints ***Corn:*** $Z_{corn} + S_{corn} \leq 3A_c$ ***Wheat:*** $Z_{wheat} + S_{wheat} \leq 2A_w$ ***Rapeseed:*** $Z_{rape} \leq 3.2A_r$
6. Financial $1040X_c + 17X_h + 550A_c + 400A_w + 244A_r + 240P_{corn} + 220P_{wheat} \leq$ $120000 + 850Y_c + 14Y_h + 200Z_{corn} + 195Z_{wheat}$ *Ensure total costs ≤ net available cash*
7. Non-Negative Cash Constraint: Final Cash ≥ 0

3. Model Output

Based on the most probable weather scenario, the optimal strategy maximizes expected wealth by balancing livestock expansion, crop allocation, and capital investment. The results are summarized below:

<table>
<tr><td rowspan="14">Oct 2025 –
Oct 2026</td><th colspan="2">Decision Area</th><th colspan="2">Optimal Decision</th></tr>
<tr><td rowspan="4">Purchase</td><td rowspan="2">Livestock</td><td>Cows</td><td>-</td></tr>
<tr><td>Hens</td><td>1600</td></tr>
<tr><td rowspan="2">Crops(tons)</td><td>Corn</td><td>-</td></tr>
<tr><td>Wheat</td><td>-</td></tr>
<tr><td rowspan="4">Sells</td><td rowspan="2">Livestock</td><td>Cows</td><td>-</td></tr>
<tr><td>Hens</td><td>400</td></tr>
<tr><td rowspan="2">Crops(tons)</td><td>Corn</td><td>-</td></tr>
<tr><td>Wheat</td><td>-</td></tr>
<tr><td rowspan="2">Store</td><td rowspan="2">Storage (tons)</td><td>Corn</td><td>30</td></tr>
<tr><td>Wheat</td><td>64</td></tr>
<tr><td rowspan="3">Plant</td><td rowspan="3">Crop Acreage
(acres)</td><td>Corn</td><td>10</td></tr>
<tr><td>Wheat</td><td>32</td></tr>
<tr><td>Rapeseed</td><td>778</td></tr>
</table>

	Sell	Crop Acreage (acres)	Rapeseed	2489.6
2026	**Expected Wealth**			**£1,288,507.28**

The strategy prioritizes hen production and rapeseed cultivation, leveraging their high profitability under expected weather conditions, with minimal investment in cows or other crops.

4. Parametric Analysis – Model + Findings

To assess the robustness of the model and understand key sensitivities, a parametric analysis was conducted theoretically, focusing on two critical factors: cow purchase price and annual cow income. The baseline cow purchase price was tested by varying it (e.g., £1040 → £1200 → £900), and the annual cow income was adjusted (e.g., £1300 → £1500 → £1100). The aim of this analysis was to predict how these changes might influence livestock purchases, crop planting decisions, and overall farm wealth.

Findings

1. **Impact on Livestock Decisions:**

- **Cow Price Sensitivity:** When cow purchase prices were tested with increases or decreases, the model continued to prioritize no cow purchases, reinforcing the preference for hens. However, when the annual cow income increased, the model continued to prioritize hens, but overall farm wealth increased, highlighting that higher cow income contributed to maximizing profit without changing the livestock purchase decision.

5. Final Conclusions & Recommendations

The optimisation model suggests that the Bowman family should expand hen production rather than buy more cows. Investing in hens consistently produces higher expected returns, even under different heifer price and income scenarios. The model suggests purchasing 1,600 hens, selling

400 at the end of the cycle, and planting mostly canola. It is recommended that maize and wheat be grown in small quantities to meet livestock feed requirements and that stocks be carefully managed to ensure that feeding needs are met.

Financially, Bowman family should avoid over-investing in livestock or crops, and instead reduce risk by maintaining adequate capital reserves and benefiting from interest.

GenAI is very useful in model construction, providing strong support for constructing decision variables, constraints and objective functions. It also helps in designing parametric analysis and visualisation strategies. However, GenAI still needs human intervention to refine the details of the model and to ensure full consistency with the specifics of the farm.