Robust Ranking of Happiness Outcomes: A Median Regression Perspective *†

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Abstract

The mean rank of happiness outcomes between groups has often been estimated using ordered probit and logit models. However, it has recently been highlighted that such ranking is not identified in most applications. Can we then learn anything from a mean rank between groups that is reported by standard statistical softwares such as STATA? We argue it can instead be interpreted as the median rank, which is identified even when the mean rank is not. We thus suggest focusing on ranking happiness outcomes (and other ordinal data) by the median rather than the mean. The median ranking can also be performed semiparametrically and we provide a new constrained mixed integer optimization procedure for implementation. To illustrate, we use General Social Survey data to show the well-known Easterlin Paradox in the happiness literature holds for the US over the period 1972 to 2006.

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1 Introduction

The study of human happiness has been cited as one of the fastest growing research fields in economics over the last two decades (Kahneman and Krueger (2006); Clark et al. (2008); Stutzer and Frey (2013)). By looking at what socioeconomic and other factors predict (or cause) people to report higher or lower scores on a subjective well-being (SWB) scale, researchers have been able to add new insights to what have become standard views in economics. For example, studies of job and life satisfaction have shown that people tend to care far more about relative income rather than absolute income (Clark and Oswald (1996); Ferrer-i-Carbonell (2005)), while unemployment is likely to hurt less when there is more of it around (Clark (2003); Powdthavee (2007)). The use of SWB data has therefore enabled economists to test many of the assumptions in conventional economic models that might have been untestable before the availability of proxy utility data (e.g., Di Tella et al. (2001); Stevenson and Wolfers (2013); Gruber and Mullainathan (2006); Boyce et al. (2013)). It has also led many policy makers to start redefining what it means to be successful as a community and as a nation (Kahneman et al. (2004); Stiglitz et al. (2009); De Neve and Sachs (2020)).

One main objective of happiness research is to understand the determinants of SWB and how they compare across groups. The predominant approach to the SWB data analysis is either through linear regression (OLS) or ordinal parametric methods (e.g. ordered probit or logit). See, e.g., Ferrer-i-Carbonell and Frijters (2004). Conclusions are then drawn, as is common in applied fields of economics and other social sciences, based on conditional or unconditional mean comparisons using these estimates. Recent studies, however, have highlighted there is more to these standard analyses than meets the eye.

The issues trace to the fact that SWB is an ordinal measure. For example, in the happiness context, consider the 3-point ordinal happiness scale in the General Social Survey (GSS). In the GSS, respondents were asked whether they were “1. not too happy”, “2. pretty happy”, or “3. very happy”. We know that a score of 3 is higher than a score of 2 or 1. But we cannot interpret the extent of which each category is higher than another without further assumptions. Suppose we are interested in comparing a particular statistic between two ordinal variables such as ranking the mean SWB of two different groups. The rank order of any statistic is identified if that order relation remains unchanged when the ordinal variables undergo any increasing transformation. This stability requirement makes it clear that such ranking may not always be identified. In contrast, the mean ranking between *cardinal* variables with finite first moment is always identified. Yet, the mean ranking of *ordinal* variables is identified if and only if there holds between them a first order stochastic dominance (FOSD) relation, which is a partial order relation.

When an ordinal variable is used as the dependent variable in a linear regression, Schröder and Yitzhaki (2017) provide conditions under which the unconditional mean ranking between groups or signs of OLS estimates can be arbitrarily changed by applying increasing transformation of the ordinal variable. While it is not surprising that such problem may occur when an ordinal variable is treated as a cardinal variable, an analogous problem also exists in the context of an ordered response model, which is independent of order preserving relabelling of the ordinal outcomes.

To see this, as we shall do throughout this paper, we interpret observed ordinal outcomes through an ordered response model characterized by latent variable threshold crossing conditions (cf. McFadden (1974)). Bond and Lang (2019, BL hereafter) point out that, for a pair of latent variables that follow the conventional continuous two-parameter distribution, such as normal or logistic, when their means differ, there is a FOSD relation if and only if their variances are identical. I.e., the mean ranking is not identified in commonly used parametric models with heteroskedasticity. The following example formally illustrates this point.

Example 1: Suppose we observe Y and D that respectively denote reported happiness from a 3-point scale and a female gender dummy so that $Y = \mathbf{1}\{H < 0\} + 2 \cdot \mathbf{1}\{0 \leq H \leq 1\} + 3 \cdot \mathbf{1}\{H > 1\}$ with latent happiness H satisfying $H|D \sim N(\beta_0 + \beta_1 D, \sigma^2(D))$. Suppose $\sigma^2(D)$ is known. Then we can identify $\beta_1 = E[H|D = 1] - E[H|D = 0]$ as long as $0 < \Pr[D = 1] < 1$. When $\sigma^2(1) \neq \sigma^2(0)$, BL show that there exists an exponential function τ_1 such that if $\beta_1 > 0$ then $E[\tau_1(H)|D = 1] < E[\tau_1(H)|D = 0]$; analogously, if $\beta_1 < 0$ then an exponential function τ_2 can always be found so that $E[\tau_2(H)|D = 1] > E[\tau_2(H)|D = 0]$. I.e., β_1 *can be identified but it cannot be used to rank the mean happiness between men and women.*

To demonstrate the practical implication of the non-identification of the mean ranking, BL take nine of the most well-known findings from the happiness literature, estimate each one using probit to replicate the conventional mean comparison result between groups and then reverse it by applying a certain exponential transformation to the latent happiness or SWB variable. By similar arguments, BL further point out that this non-identification issue is relevant to analysis of all ordinal variables in economic and social science applications.

Given that the mean ranking between groups based on probit and logit can only be identified under homoskedasticity, how should we then interpret empirical results based on these models? This is an important question because numerous conclusions have been drawn from such models where constant variance or equivalently FOSD is a priori assumed. Furthermore, and importantly, many previous empirical findings are intuitive and widely accepted in the literature. This suggest we may be able to learn something from the estimates of conditional means even if the mean ranking is fundamentally not identified.[1]

Our goal is to provide a pragmatic view on how to interpret results estimated from ordered logits and probits irrespective of whether the mean rank is identified, as well as on the method of comparing ordinal variables generally. Our argument focuses on using the median as a statistic for comparisons

[1] For example, we may expect "disability would reduce happiness" even if a transformation on latent happiness can be found to suggest otherwise.

instead of the mean. We have the following messages.

1. The median ranking of ordinal variables is identified under very weak conditions without requiring FOSD.

2. The median ranking is identified in probit and logit models. It can in fact be identified by the conditional means of latent variables of these models. The economic implications of prior results based on the mean ranking are therefore robust when interpreted as the median rank even when FOSD does not hold.

From a statistical perspective, it has long been documented that the median is the preferred summary statistic for describing central tendency of an ordinal outcome (e.g. see Stevens (1946)). Unlike the mean, the median respects the ordinal property because it is "equivariant" to all increasing transformations. I.e., letting $Med(Z)$ denote the median of a random variable Z and τ be an increasing function, then $\tau\left(Med(Z)\right) = Med(\tau\left(Z\right))$. Therefore the median rank of a pair of latent variables cannot be reversed by any monotone transformation, even if there is no FOSD relation between them.

Perhaps the most empirically relevant point of our paper is that well-known probit and logit methods, which can be performed e.g. with STATA's oglm command (Williams (2010)), automatically gives the median ranking between groups that is typically interpreted as the mean. This follows because the median of the latent variable from ordered probit and logit models are the same as the mean due to symmetry of the normal and logistic distributions. Researchers can therefore perform group comparison in the same way as previously, and interpret the ranking in terms of the median instead of the mean. Specifically, for example, consider again the illustrations in BL. There, BL find that all of their ordered probit estimates deliver qualitatively the same conclusions as in previous studies before they apply exponential transformations to reverse the results. When these estimates are interpreted as the median, they empirically support the economic intuition of all nine well-known conventions from the happiness literature rather than disputing them.

Can we relax the parametric framework when estimating the median of ordinal data? Firstly, suppose we maintain the probit or logit assumption, it is straightforward to estimate models with a very flexible form of heteroskedasticity that can even be nonparametrically specified (see, e.g., Chen and Khan (2003)). One can also relax the parametric distributional assumption. Specifically, Lee (1992) studies identification and estimation of median regression in a semiparametric ordered response model where the latent variable has an unknown distribution that admits a general form of heteroskedasticity. However, Lee's estimator is a generalization of the maximum score estimator (MSE) proposed by Manski (1985), which is well-known for its computational difficulty. Recently, Florios and Skouras (2008) have shown that one can feasibly compute Manski's MSE by reformulating it as a solution to a constrained mixed integer linear programming (MILP) problem.

We show the insights of Florios and Skouras (2008) can be extended to an ordered choice model with any finite number of outcomes. As an illustration, we revisit the Easterlin Paradox using the GSS data. The Paradox, named after Richard Easterlin, is an empirical observation that at any given point in time people's happiness correlates positively with income. Yet, despite the robust cross-sectional correlation, there does not seem to be a long-run relationship between economic growth and the aggregate happiness of people over time (Easterlin (1974), Easterlin et al. (2010)). Our semiparametric estimate supports the Easterlin Paradox empirically and shows its existence does not depend on symmetry or parametric distributional assumption.

An interesting question to ask is concerned with the usefulness of the median for economic applications as its role has appeared to be less prominent than the mean in economic theory. Specifically, in happiness economics, the latent variable is often viewed as utility (e.g. via happiness as a proxy) and focusing on the median deviates from the conventional expected utility paradigm. Nonetheless, from the decision theoretical perspective, median ranking of uncertain outcomes can be well justified through the quantile preference framework. We refer readers to Manski (1988b) and, for more recent developments, to Rostek (2010) and de Castro and Galvao (2019) where they consider agent making decisions by maximizing a quantile of her utility distribution. Relatedly, from the policy maker's standpoint, the median and other quantiles of happiness level within a population may also be of more interest than the mean from the sufficientarian welfare perspective (e.g., see Sechel (2019)).

We emphasize that our work is not a critique of BL. Their theoretical point that identification of the mean ranking of ordinal data in standard parametric models is possible only when homoskedasticity holds is important and cannot be disputed. Our aim is to mitigate the practical impact of this negative result by showing that economic intuition derived from the mean rank estimates based on these models remain informative under an alternative interpretation through the median perspective.[2] Relatedly, we also argue that median regression, which plays a parallel role to mean regression, can serve well as a econometric framework to analyze ordinal data with covariates.

BL also point out another challenge for comparing ordinal data, which concerns whether there is a common reporting function that puts the latent variable across groups on the same cardinal scale. In a threshold crossing model, this corresponds to the possibility that threshold values are heterogeneous across groups. In the present paper, we maintain the common threshold assumption but focus on the median ranking perspective in ordinal data analysis. One may relax this assumption by using a parametric compound hierarchical ordered response model (see King et al. (2004)). To the best of our knowledge, extensions to semiparametric ordinal median regression models with heterogeneous thresholds have not yet been developed and would be an important topic for further research.

[2]There are also other meaningful ways to compare distributions even if there does not hold a stochastic dominance relation. For example, one way to resolve the rank indeterminacy is to rank distributions with restricted stochastic dominance (Atkinson (1987)) where the dominance condition is considered over a restricted part of the distribution support. Another approach is to consider stability of the mean rank over a strict subclass of increasing transformation functions (see Bloem (2021) and Kaiser and Vendrik (2020)).

The remainder of this paper proceeds as follows. Section 2 gives an account on how statistical analysis for discrete ordinal data have been developed in economics and other disciplines, and some recent developments on estimating the median. Section 3 sets forth an ordered response model and formalizes our argument to identify the median rank. Section 4 present an MILP based approach to the implementation of a median estimator in a semiparametric ordered response model. Section 5 revisits the Easterlin Paradox using GSS data. Section 6 concludes.

2 Discrete ordinal data analysis: a brief review

We consider discrete ordinal outcome that represents an individual's ordered categorical response in the data. The defining property of an ordinal variable is that there is a *rank order* over values it can take but the distances between these values are arbitrary and carry no information. A discrete ordinal variable can therefore, in constrast to cardinal variables, be put on a scale like $\{1, 2, .., J\}$ without any loss of generality. Such data measurements are common in social and biomedical sciences. Examples include individual happiness (unhappy, neither happy nor unhappy, happy), severity of injury in the accident (fatal injury, incapacitating injury, non-incapacitating, possible injury, and non-injury), and lethality of an insecticide (unaffected, slightly affected, morbid, dead insects) among many others.

Ordinal data analysis in a regression framework is widely acknowledged to have been co-founded by two independent sources. One originates from the contribution of McKelvey and Zavoina (1975), who developed the well-known ordered probit model to study Congressional voting on the 1965 Medicare Bill. The other is due to McCullagh (1980), who focused on modelling proportional odds and proportional hazards that become prominent in the biomedical fields. Huge literature on ordinal data analysis has since grown from these influential works. We refer interested readers to Greene and Hensher (2010) and reference therein for the developments in social science, Agresti (1999) for the medical science, and Ananth (1997) for epidemiology.

Researchers from different fields take different approaches to analyzing ordinal data. Applied researchers in biomedical fields pay a great deal of attention to choosing an appropriate model for their data (goodness of fit) but place less importance on the interpretation of individual parameters (e.g. coefficients in a generalized linear model). On the other hand, researchers in social sciences often focus on the model parameters. Economists, for example, typically work with linear regression or ordered probit and logit models that are very convenient for interpreting parameters, especially in a setting with many covariates.

The linear regression method treats ordinal variables as if they were cardinal. Several studies have shown linear regression and ordered probit/logit models could deliver similar qualitative results empirically. For example, in a study of vehicle driver injury severity, Gebers (1998) compared both OLS and ordered logit estimation results and found that estimated coefficients of both models were of the same sign and generally agreed in magnitude and statistical significance. In an empirical analysis of the effect of economics sanctions, Major (2012) compared the OLS and ordered probit estimates and found that the results were similar across these two estimating models. See also Ferrer-i-Carbonell and Frijters (2004), who found both the OLS and ordered probit models produced comparable results in their study on the sources of individual well-being.

Theoretically, parameters in an OLS model can be interpreted through the arguments of linear projection. They identify the (partial) correlations between the dependent and independent variables. However, the signs of these correlations may not be robust to an order preserving transformation of the variable scale. See, e.g. Schröder and Yitzhaki (2017) for a theoretical analysis of this non-robustness issue in the approach of OLS for ordinal data.

For latent variable threshold crossing ordinal regression models, the regression coefficients are by construction invariant to any order preserving relabeling of the ordinal outcome. These parameters naturally have an interpretation of the conditional mean difference of latent variables of different groups. Their signs can thus be used to identify the mean ranking across groups whenever the ranking order is invariant for all increasing transformations of the latent variable. This requirement amounts to the condition of there being a FOSD relation between latent variable distributions of the groups. For the conventional parametric model, such as ordered logit or probit, it is straightforward to test for FOSD because, when the latent variables have the same variances, then FOSD follows from a difference in the means. However, if their variances differ then FOSD fails whenever their means also differ. This knife-edge condition makes parametric identification of the mean rank order very fragile as BL have illustrated. We refer readers to their paper for further discussions. More generally one can test FOSD directly in a semiparametric or nonparametric setting. See e.g. Carneiro et al. (2003), Cunha et al. (2007), Lewbel (1997, 2000), Lewbel and Schennach (2007), Honore and Lewbel (2002) and Kaplan and Zhuo (2020).

In this paper we propose that a natural alternative for ranking ordinal outcomes is to focus on the median instead of the mean. For commonly used ordered response models such as the ordered probit and logit, the median and mean are identical. Maximum likelihood estimation of the median in these models can therefore be performed as readily as the mean using standard statistical softwares. The median can also be estimated semiparametrically without distributional assumption. The seminal work of Manski (1975, 1985) develop the maximum score estimation in this setting for a binary choice model and Lee (1992) extends this to multiple ordered choice data. While theoretically appealing, performing maximum score estimation for the discrete choice model is computationally difficult. More specifically, the ordered response median regression estimator proposed by Lee (1992) is a solution to a non-smooth and non-covex least absolute deviation (LAD) optimization problem. The corresponding LAD objective function, which is akin to Manski's maximum score objective function, is piecewise constant with numerous local solutions. The computational challenges for solving the maximum score estimation problem is well noted in the econometrics literature (e.g. see Manski and Thompson (1986), Pinkse (1993), Skouras (2003)).

Recently Florios and Skouras (2008) propose a mixed integer optimization (MIO) based approach for the computation of maximum score estimators. In particular, they show that Manski's binary choice maximum score estimation problem can be equivalently reformulated as a mixed integer linear programming problem (MILP). Chen and Lee (2018) provide an alternative MILP formulation that complements the approach of Florios and Skouras (2008) for solving the maximum score estimation problem. These reformulations enable exact computation of maximum score estimators through modern efficient MIO solvers. Well-known numerical solvers such as CPLEX and Gurobi can be used to effectively solve the MIO problems. We refer readers to Bertsimas and Weismantel (2005) and Conforti et al. (2014) for recent and comprehensive texts on the MIO methodology and applications.

The estimators of Manski (1985) and Lee (1992) have been shown to be consistent under very weak conditions. On the other hand, maximum score type estimators converge at a cube-root rate and have non-standard asymptotic distributions. See Kim and Pollard (1990) and Seo and Otsu (2018). When there are continuous covariates, one can use the smoothed maximum score (SMS) estimator proposed by Horowitz (1992) that employs a smooth approximation of the original maximum score objective function. The SMS estimator is asymptotically normally distributed and can have a faster rate of convergence than the unsmoothed maximum score estimator. However, for implementation of the SMS estimator, users have to choose tuning parameters that might induce a smoothing bias which could be difficult to correct (Kotlyarova and Zinde-Walsh (2009)). We refer the reader to Horowitz (2009, Chapter 4) for a detailed review on the theoretical aspects of the maximum score and SMS estimators.

3 An empirical model and parameters of interest

For concreteness, we present an empirical model in the context of happiness application. Suppose we are interested in using SWB data taken from two groups, say A and B, to draw conclusions on whether people in group A are happier than those in group B. Examples of group identities include, gender, martial status, country, time etc. Previous analyses have been focusing on the mean as a statistic to compare happiness across groups. We will focus on the median. For brevity, we conduct analysis for the case with two groups. Our arguments below are general and can be straightforwardly extended to any finite number of groups.

Suppose we observe $\left(Y^l, X^l\right)$ for $l = A, B$, where Y^l denotes a reported happiness scale taking values from $\mathcal{Y} = \{1, \dots, J\}$ and X^l is a vector of covariates. We assume Y^l is derived from a threshold crossing model based on latent happiness index, H^l, such that

$$\begin{aligned} Y^l &= \sum\nolimits_{j=1}^{J} j \times \mathbf{1}\left\{\gamma_{j-1}^l < H^l \le \gamma_j^l\right\}, \\ H^l &= X^{l\top}\theta^l + U^l, \end{aligned} \tag{1}$$

for some strictly increasing real thresholds $\left\{\gamma_j^l\right\}_{j=1}^{J-1}$ with $\gamma_0^l = -\infty, \gamma_J^l = +\infty$, θ is a vector of parameters, and U^l is an unobserved scalar accounting for other factors.

Suppose we aim to rank medians of H^A and H^B. Since ordinal variables are scale free, a common approach in practice is to put H^A and H^B on the same scale by assuming the thresholds in (1) are the same for $l = A, B$. For notational simplicity suppose both groups contain the same set of covariates. Then we suppress the group index and pool the two threshold crossing models together in one unified framework:

$$\begin{aligned} Y &= \sum\nolimits_{j=1}^{J} j \times \mathbf{1}\left\{\gamma_{j-1} < H \leq \gamma_j\right\}, \\ H &= X^\top \pi^A + D \cdot X^\top \pi^B + U, \end{aligned} \tag{2}$$

where D is a dummy variable taking value 1 for group B and 0 otherwise, $\pi^A = \theta^A$, $\pi^B = \theta^B - \theta^A$ and $U = U^A + D(U^B - U^A)$. We emphasize here that using a common set of thresholds for both groups does not resolve the general non-identification of the mean ranking. Indeed, BL adopt this same framework in their empirical studies in order to reverse happiness results.

Our parameter of interest for median comparison is

$$\lambda(X) := Med\left(H|X, D = 1\right) - Med\left(H|X, D = 0\right). \tag{3}$$

By equivariance of the median, for any strictly increasing function $\tau(\cdot)$,

$$Med\left(\tau(H)|X, D = 1\right) \geq Med\left(\tau(H)|X, D = 0\right)$$

if and only

$$Med\left(H|X, D = 1\right) \geq Med\left(H|X, D = 0\right).$$

Therefore, the sign of $\lambda(X)$ enables us to rank the medians of latent happiness across groups and this ranking is robust against any increasing transformation of H.

We need to impose some assumptions on the distribution of $U|X, D$ in order to identify and estimate $\lambda(X)$.

Example 2: For median comparison based on an ordered probit model, we would assume U in (2) satisfies $U|X, D \sim N\left(0, \sigma^2(X, D)\right)$. Then $H|X, D \sim N\left(X^\top \pi^A + D \cdot X^\top \pi^B, \sigma^2(X, D)\right)$ and $\lambda(X) = X^\top \pi^B$.

If we set the support of Y in Example 2 to $\{1, 2, 3\}$, letting γ_1 and γ_2 be 0 and 1 respectively, and reducing X to a constant term, then we have the same setup as Example 1, where, by the current notation, the sign π^B determines the median happiness ranking between men and women.

In practice, we can estimate $\lambda(X)$ using parametric models. The simplest case would be to assume $U|X, D$ follows a normal or logistic distribution. Then it follows from distributional symmetry that the conditional mean and median of H coincide, and we can use standard statistical softwares for estimating the mean latent happiness in an ordered probit or logit model to estimate its median counterpart. We can also work with a less restrictive setting where the latent unobservable has an unknown distribution. In the next section we will focus on the general problem of estimating $Med\,(H|X,D)$ when the distribution of $U|X,D$ is nonparametrically specified.

4 Estimating a semiparametric ordinal median regression model using MILP

We consider the following semiparametric median regression for ordinal outcomes.

$$Y = \sum\nolimits_{j=1}^{J} j \times \mathbf{1}\left\{\gamma_{j-1} < H \leq \gamma_j\right\}, \tag{4}$$

$$H = X^\top \theta + U, \tag{5}$$

where the covariate vector X subsumes the group dummy variables. Let $\gamma := \{\gamma_j\}_{j=1}^{J-1}$ denote the vector of threshold parameters. We make the following assumptions.

Assumption I

(i) X is a $(p+1)$ dimensional vector that does not contain a constant term.

(ii) $|\theta_1| = 1$.

(iii) $Med(U|X) = 0$ almost surely.

As the regression intercept cannot be separately identified from the threshold parameters, I(i) normalizes the regression intercept to be zero. Since we only observe outcomes from the events $\gamma_{j-1} < H \leq \gamma_j$, $j \in \{1,\ldots,J\}$, scale of the parameters (θ,γ) cannot be identified. I(ii) settles the parameter scale by normalizing magnitude of the first element of θ to be 1. I(iii) imposes the zero median restriction. The last assumption is fairly mild, for example, it allows for nonparametric specification of the distribution of latent unobservable U, which admits general unknown form of heteroskedasticity conditional on the covariates.

It is straightforward to deduce from I(iii) that, for $j \in \{1,\ldots,J\}$,

$$P\,(Y \leq j|X) \geq 0.5 \Longleftrightarrow \gamma_j \geq X^\top \theta. \tag{6}$$

The sign-matching relations above yield the conditional median for Y:

$$Med(Y|X) = \sum\nolimits_{j=1}^{J} j \times \mathbf{1}\left\{\gamma_{j-1} < X^\top \theta \leq \gamma_j\right\}.$$

We are interested in the estimation of $Med(H|X)$, which equals $X^\top\theta$ under Assumption I(iii). Let $X = (X_1, \widetilde{X})$, where X_1 is a scalar random variable and $\widetilde{X}$ is the subvector of X that excludes the covariate X_1, and let $\theta = (\theta_1, \beta)$ so that $X^\top\theta = \theta_1 X_1 + \widetilde{X}^\top\beta$. Let $\Theta \subset \mathbb{R}^{p+J-1}$ denote the parameter space containing (β, γ). We use (b, c) to denote a generic point of Θ.

Given a random sample $(Y_i, X_i)_{i=1}^n$, letting $c_0 = -\infty$ and $c_J = \infty$, Lee (1992) showed that consistent estimator $(\widehat{\theta}_1, \widehat{\beta}, \widehat{\gamma})$ of $(\theta_1, \beta, \gamma)$ can be derived as a solution to the following LAD estimation problem:

$$\min_{(a,b,c)\in\{1,-1\}\times\Theta} \sum_{i=1}^n \left| Y_i - \sum_{j=1}^J j \times \mathbf{1}\left\{c_{j-1} < aX_{1i} + \widetilde{X}_i^\top b \le c_j\right\}\right|. \tag{7}$$

The LAD estimator through (7) is a generalization of the maximum score estimation approach of Manski (1985) for the binary response model to that for the discrete ordered response case. As in the problem of maximum score estimation, the objective function in (7) is piecewise constant with numerous local optima. In what follows, we provide an algorithm which enables exact computation of a global solution to the LAD problem (7).

Because the term $\left|Y - \sum_{j=1}^J j \times \mathbf{1}\left\{\gamma_{j-1} < X^\top\theta \le \gamma_j\right\}\right|$ is identical to

$$|Y - J| + \sum_{j=1}^{J-1} \left[|Y - j| - |Y - j - 1|\right] \times \mathbf{1}\left\{X^\top\theta \le \gamma_j\right\},$$

the problem (7) above is therefore equivalent to the following minimization problem:

$$\min\{\min_{(b,c)\in\Theta} S_n(1, b, c), \min_{(b,c)\in\Theta} S_n(-1, b, c)\}$$

where

$$S_n(a, b, c) := \sum_{i=1}^n \sum_{j=1}^{J-1} \left[|Y_i - j| - |Y_i - j - 1|\right] \times \mathbf{1}\left\{aX_{1i} + \widetilde{X}_i^\top b \le c_j\right\}.$$

We now present our computational algorithm for solving the sub-problem:

$$\min_{(b,c)\in\Theta} S_n(a, b, c) \tag{8}$$

for a given value of a.

Our approach is based on the method of mixed integer optimization. Specifically, we can reformulate the LAD sub-problem (8) as the following mixed integer linear programming (MILP) problem:

$$\min_{(b,c)\in\Theta,(d_{i,1},\ldots,d_{i,J-1})_{i=1}^n} \sum_{i=1}^n \sum_{j=1}^{J-1} \left[|Y_i - j| - |Y_i - j - 1|\right] \times d_{i,j} \tag{9}$$

subject to

$$(d_{i,j} - 1)\, M_{i,j} \le c_j - aX_{1i} - \widetilde{X}_i^\top b < d_{i,j}(M_{i,j} + \delta),\ \ (i,j) \in \{1, ..., n\} \times \{1, ..., J-1\}, \tag{10}$$

$$c_j < c_{j+1},\ \ j \in \{1, ..., J-2\}, \tag{11}$$

$$d_{i,j} \le d_{i,j+1},\ \ (i,j) \in \{1, ..., n\} \times \{1, ..., J-2\}, \tag{12}$$

$$d_{i,j} \in \{0, 1\},\ \ (i,j) \in \{1, ..., n\} \times \{1, ..., J-1\}, \tag{13}$$

where $\delta > 0$ is a small positive scalar (e.g. $\delta = 10^{-6}$ as in our numerical study), and

$$M_{i,j} \equiv \max_{(b,c)\in\Theta} \left|c_j - aX_{1i} - \widetilde{X}_i^\top b\right|,\ \ (i,j) \in \{1, ...n\} \times \{1, ..., J-1\}. \tag{14}$$

Solving the constrained MILP problem (9) is equivalent to solving the minimization problem (8). To see this, for any $(b, c) \in \Theta$, the sign constraints (10) and the dichotomization constraints (13) ensure that $d_{i,j} = \mathbf{1}\left\{aX_{1i} + \widetilde{X}_i^\top b \leq c_j\right\}$ for $(i, j) \in \{1, ...n\} \times \{1, ..., J-1\}$. We also enforce monotonicity of the threshold parameters through inequality constraints (11). Note that (10), (11) and (13) together imply (12), which we explicitly impose so as to further tighten the MILP problem.

The equivalence between (8) and (9) enables us to employ the modern MIO solvers to exactly compute the LAD estimator $(\widehat{\theta}_1, \widehat{\beta}, \widehat{\gamma})$. For numerical implementation, note that the values $(M_{i,1}, ..., M_{i,J-1})_{i=1}^n$ in the inequality constraints (10) can be computed by formulating the maximization problem in (14) as linear programming problems, which can be efficiently solved by modern optimization solvers. Hence these values can be easily computed and stored as the input to the MILP problem (9).

5 Revisiting the Easterlin Paradox

The Easterlin Paradox is one of the most well-known results in the happiness economics literature (Easterlin (1974, 1995, 2005)). It is due to Richard Easterlin, as he examined the relationship between happiness and income for various countries. He finds that, despite the economic growth, reported happiness stays stable over time. In contrast, there is strong evidence that within a given time period those with high income are happier than those with low income. The Paradox leads to a widespread belief that increasing the income of all does not improve well-being of all, and it is the relative, not absolute, income that is important for individual well-being (e.g. see Layard (2005)).

We used a subset of the General Social Survey (GSS) from the years 1972 to 2006, which is a dataset that has been extensively used for analyzing happiness in the United States (see e.g. Stevenson and Wolfers (2008)). GSS happiness variable comes from respondents being asked whether they are "1. not too happy", "2. pretty happy", or "3. very happy". So we estimated the semiparametric ordered response model (4) with $J = 3$ using our MILP approach of Section 4. We used the MATLAB implementation of the Gurobi Optimizer to solve the MILP problem (9).[3] To mitigate the computational cost, we selected a random sample of 500 responders who answered the happiness question in the GSS bi-annually from the years 1974 to 1990, 1991, 1993, and bi-annually again from 1994 to 2006. The total size of our estimating sample was 9500 from 19 different years. We standardized the income variable to have zero mean and unity variance, and normalized the magnitude of coefficient for this variable to be one. We note that our income variable has quite a rich support[4],

[3]The MATLAB codes for estimating the semiparametric ordered response model of Section 4 are available from the authors via the website https://github.com/LeyuChen/ordinal-LAD. This implementation requires the Gurobi solver, which is freely available for academic purposes.

[4]The GSS collected income data in 12 bands, computed their mid-points and debased them accordingly. So the income variable in our sample could take 228 possible values.

which would facilitate parameter identification in our semiparametric model.[5] We performed the median regression of reported happiness on the income variable along with age, age-squared, degree dummy, female dummy, marriage status dummy, and time fixed effects. There were 26 parameters in our empirical model including 18 time effect parameters and 2 latent thresholds.

We also estimated the medians from generalized ordered probit and logit models that allow for parametric forms of heteroskedasticity using the `oglm` command (Williams (2010)) in STATA. We divided the parametric estimates by their respective estimated coefficient of income so as to make comparisons with the semiparametric estimates. Table 1 presents estimates of parameters and their confidence intervals of all models. The confidence intervals for the parametric and semiparametric estimators are computed using normal approximation and m out of n bootstrap respectively.

Table 1 shows that parametric and semiparametric estimates associated with socioeconomic factors are qualitatively the same and similar in magnitude. The sign of semiparametric estimate of the income effect is positive, which coincides with those estimated in the parametric models. The other effects also conform with the convention in the happiness literature that a person with a graduate degree and/or being a female and/or being married tends to be happier than those with respective counterparts. The estimated age effect indicates some convexity that has often been attributed to the midlife nadir (e.g. see Cheng et al. (2017)). The parametric and semiparametric estimates of time effects and threshold parameters are also qualitatively similar. Overall it appears that the parametric and semiparametric models of happiness deliver qualitatively the same economic intuition. We now turn our attention to the relevance of our empirical findings to the Easterlin Paradox.

There are two components to the Easterlin Paradox. One is that income is positively correlated with people's happiness within any time period. The other is that people's happiness does not increase as the nation becomes more prosperous over time. Figure 1 plots the estimated time fixed effects from the semiparametric and ordered probit models. We include fitted lines through the estimates and super-impose in the graph of how the logarithm of real per capita GDP grows with time. Figure 2 provides analogous plots of the semiparametric estimates against the ordered logit ones. Figure 3 plots the estimated time fixed effects from both the parametric and semiparametric models against the logarithm of real per capita GDP directly with their corresponding fitted lines. Figures 1 – 3 empirically support the Easterlin Paradox along the timespan dimension. Our empirical study therefore supports the predictions of Easterlin Paradox for the US during the period 1972 to 2006.

[5] The standard sufficient conditions for identification of maximum score type estimators require one of the covariates has support on $\mathbb{R}$ conditional on other explanatory variables. The full support condition is not necessary. It can be reduced to bounded or even finite support as long as the support is rich enough to guarantee that the sign-matching conditions (6) only hold at the true parameter value. See Manski (1988a) and Horowitz (2009, Chapter 4).

	Paramteric est.						Semiparametric est.		
	Probit	95% Conf. Int.		Logit	95% Conf. Int.			95% Conf. Int.	
income	1.000			1.000			1.000		
age	-0.273	-0.332	-0.214	-0.283	-0.343	-0.223	-0.275	-0.369	-0.211
age_sq	0.003	0.002	0.004	0.003	0.002	0.004	0.003	0.002	0.004
degree	1.453	0.996	1.910	1.475	1.014	1.935	1.471	0.824	2.177
female	0.688	0.422	0.954	0.718	0.440	0.996	0.733	0.309	1.156
married	5.210	4.306	6.114	5.579	4.648	6.509	5.765	5.393	7.066
year dummies									
1974	-0.122	-1.071	0.827	-0.119	-1.106	0.868	-2.523	-4.027	-1.652
1976	-0.414	-1.335	0.507	-0.435	-1.398	0.527	1.662	0.195	3.129
1978	0.063	-0.811	0.936	0.082	-0.824	0.988	2.218	0.837	3.598
1980	-0.137	-1.078	0.804	-0.164	-1.143	0.815	0.405	-1.087	1.897
1982	-0.438	-1.354	0.479	-0.477	-1.436	0.483	-0.953	-2.392	0.390
1984	0.163	-0.778	1.103	0.168	-0.810	1.146	1.682	0.416	3.173
1986	0.263	-0.641	1.166	0.202	-0.735	1.139	0.583	-0.844	2.010
1988	0.633	-0.271	1.538	0.666	-0.269	1.602	0.721	-0.705	2.146
1990	0.514	-0.395	1.423	0.576	-0.367	1.519	1.029	-0.408	2.466
1991	-0.191	-1.088	0.706	-0.266	-1.200	0.668	1.371	-0.031	2.794
1993	-0.460	-1.341	0.421	-0.497	-1.416	0.421	-1.972	-3.372	-0.829
1994	-0.980	-1.787	-0.173	-1.034	-1.877	-0.192	-1.984	-3.268	-0.867
1996	-0.419	-1.220	0.382	-0.448	-1.284	0.388	-0.484	-1.758	0.791
1998	-0.267	-1.070	0.536	-0.283	-1.121	0.556	-0.478	-1.709	0.799
2000	0.180	-0.631	0.990	0.165	-0.679	1.009	-1.952	-3.238	-0.937
2002	-0.447	-1.389	0.496	-0.540	-1.527	0.447	0.400	-0.928	1.905
2004	-0.337	-1.296	0.621	-0.358	-1.358	0.641	-0.585	-2.108	0.921
2006	-0.321	-1.147	0.504	-0.328	-1.189	0.534	-0.405	-1.718	0.908
cut1	-13.668	-16.047	-11.288	-14.391	-16.828	-11.954	-9.027	-11.425	-7.407
cut2	1.939	0.619	3.258	2.065	0.683	3.447	3.303	2.021	5.408
N obs.	31,180			31,180			9,500		

income is the standardized family income; *age* is the respondent's age; *age_sq* is age squared; *degree*, *female*, and *married* are dummy variables indicating respectively whether the respondent has obtained bachelor or graduate degree, is female, and is currently married.

Table 1: Parametric and semiparametric estimates

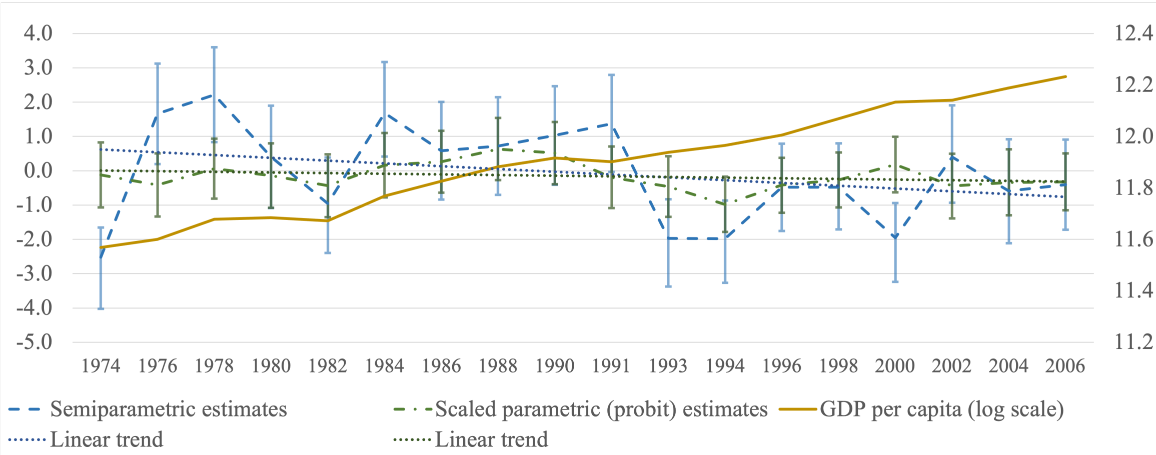

Figure 1: Parametric (probit) and semiparametric estimates. Estimated coefficients for time dummies (left axis) and Logarithm of GDP per capita (right axis)

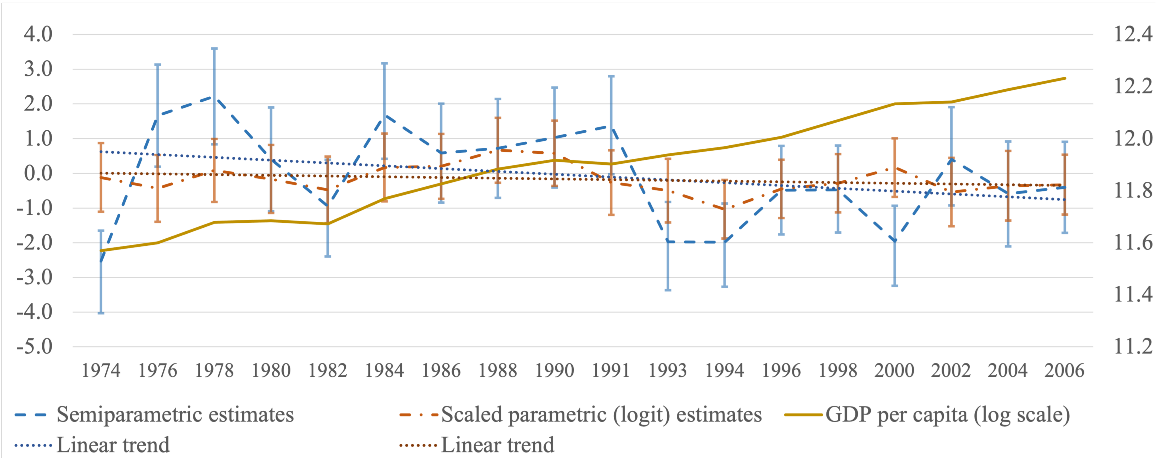

Figure 2: Parametric (logit) and semiparametric estimates. Estimated coefficients for time dummies (left axis) and Logarithm of GDP per capita (right axis)

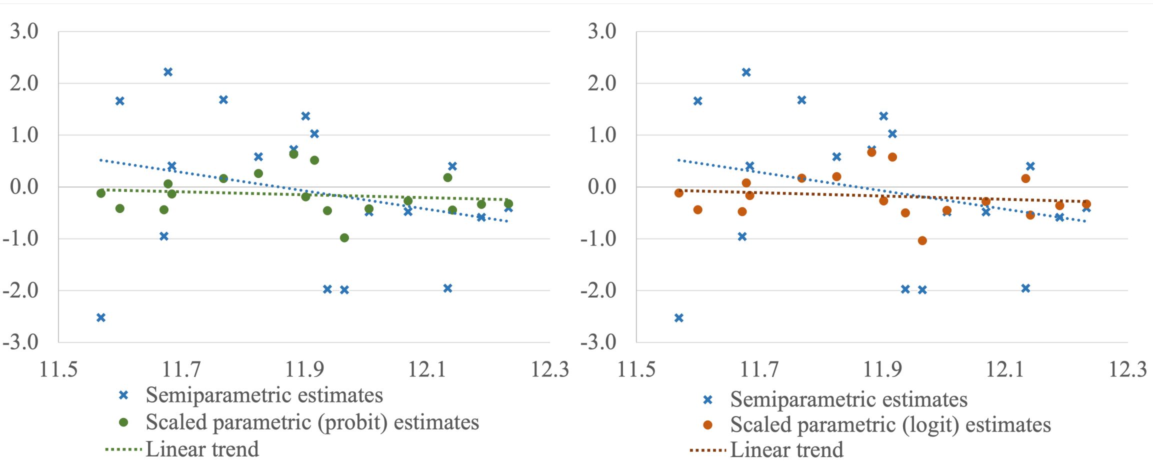

Figure 3: Parametric (probit and logit) and semiparametric estimates. Estimated coefficients for time dummies (vertical axis) and Logarithm of GDP per capita (horizontal axis)

6 Conclusion

A group ranking of ordinal outcomes is identified only when the ranking order is invariant across all increasing transformation on the ordinal variables. For the mean ranking, this invariance is equivalent to there being a first order stochastic dominance (FOSD) relation between the variables across groups. The usefulness of the probit and logit based mean ranking for discrete ordinal outcomes has in particular been put under question, as illustrated by Bond and Lang (2019), because FOSD in this setting requires the model to be homoskedastic, otherwise the mean rank is not identified.

In this paper we propose focusing on the median as a pragmatic alternative to the mean. Firstly, the median rank of ordinal outcomes can be identified even when the mean rank is not. Secondly, probit and logit based median ranks can be identified by the conditional means of latent variables of these parametric models and can hence be easily estimated using standard statistical softwares. Thirdly, we also propose a mixed integer optimization procedure to perform median regression in a semiparametric ordered response model that can accommodate unknown distribution of the latent unobservable.

In our empirical study, we apply the median ranking approach to investigate the Easterlin Paradox for the US using the GSS data. We find the Paradox exists empirically through the lens of both parametric and semiparametric models. This suggests the Easterlin Paradox is not an artefact from symmetry or other parametric distributional assumptions.

We conclude with further remarks on related aspects to the median regression approach. Our paper focuses on the median as a natural counterpart to the mean. Other quantiles also possess the equivariance property and are identified under weak conditions analogous to the median. A policy maker may be interested in how ranking between groups differ across quantiles, or targeting the ranking at some specific quantile levels such as those corresponding to individuals with lower levels of wellbeing (Flèche and Layard (2017), Clark et al. (2018)). Operationally, for parametric models, it is straightforward to estimate conditional quantiles of latent happiness and perform quantile ranking under the postulated distributional specification of the latent unobservable. One can also generalize our MILP algorithm for the semiparametric median estimation of Section 4 to the quantile case by replacing the absolute loss in (7) with the quantile check loss and modifying the MILP problem (9) accordingly. These extensions are interesting directions for further research.

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