MI3.22
Advanced Programming for HPC
Master ICT, USTH, 2nd year

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Lecture 2 – Pointer jumping & Reduce

- Pointer Jumping
- Reduce

Overview

- Pointer Jumping
- Reduce

List Ranking

Input: linked list of size n.

Output: for each element i the distance $d[i]$ to end of the list:

$$d[i] = \begin{cases} 0 & \text{if next[i]} = \text{ NIL} \\ \text{d[next[i]]} + 1 & \text{if next[i]} \neq \text{ NIL} \end{cases}$$

Sequential complexity : $O(n)$

With PRAM: complexity $O(\log(n))$ – base 2 –

Associate one PE to each list element i ...

```
1  RANK_LIST(L)
2  {Initialization}
3  FOR each PE i in parallel:
4     IF next[i] = NIL THEN d[i] ← 0 ELSE d[i] ← 1
5  { Main loop }
6  WHILE exists a node i such that next[i] ≠ NIL:
7        FOR each PE i in parallel { with synchronized access }
8              IF next[i] ≠ NIL THEN
9                    d[i] ← d[i] + d[next[i]] {Each PE read, THEN write}
10                   next[i] ← next[next[i]] {idem}
```

Line 6 :

- CRCW: write in a boolean "*ended*" + fusion
- CREW: $O(log(n))$! Better, with loop **FOR** s=1 **TO** $\lceil \log n \rceil$
- EREW: idem, writing into temporary variable

SCAN

Let $\oplus$ be a binary associative operation ...

Input: sequence $(x_1, x_2, \ldots, x_n)$ known as a linked list

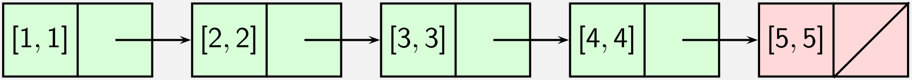
Output: sequence $(y_1, y_2, \ldots, y_n)$ where

$y_k = y_{k-1} \oplus x_k = x_1 \oplus x_2 \oplus \ldots \oplus x_k$

PRAM Algorithm

```
SCAN(L)
{Initialization}
FOR each PE i in parallel:
    y[i] ← x[i]
{Main loop}
WHILE exists node i such that next[i] ≠ NIL: {Or FOR loop ...}
    FOR each PE i in parallel
        IF next[i] ≠ NIL THEN
            y[next[i]] ← y[i] ⊕ y[next[i]]
            next[i] ← next[next[i]]
```

$[i, j]$ denotes $x_i \oplus x_{i+1} \oplus \ldots \oplus x_j$ for $i \leq j$ ... Example:



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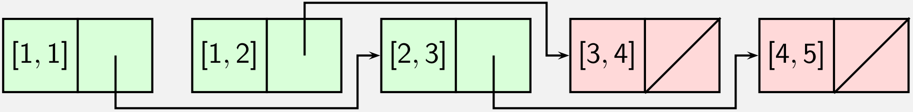

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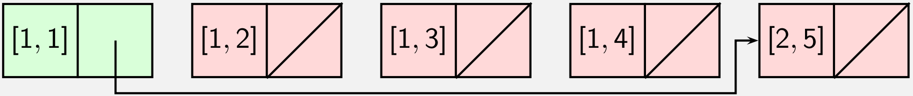

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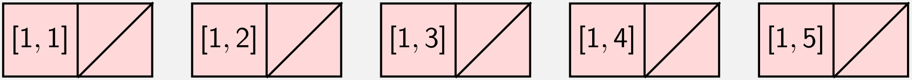


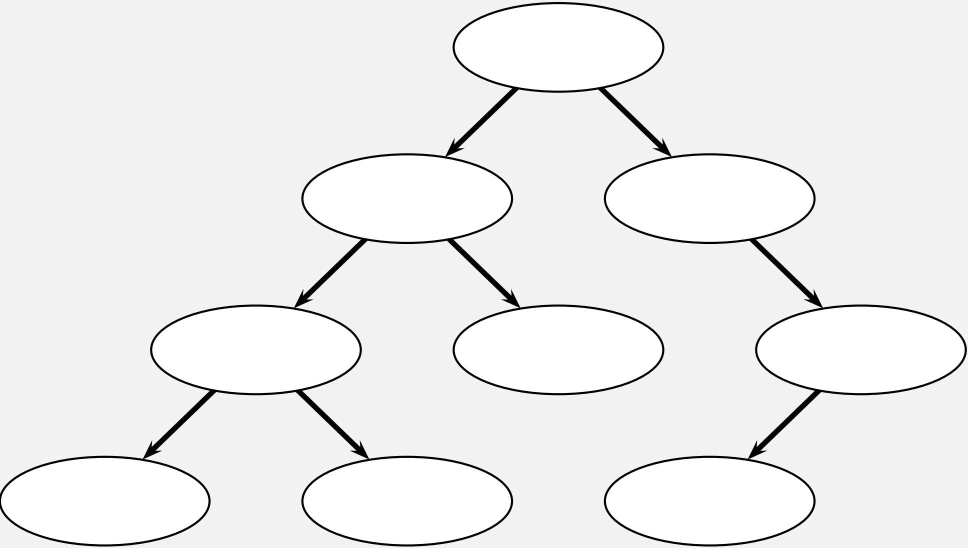
Euler Tower

Input: binary tree with n nodes (leaves included)
Output: depth of each node
Naive algorithm: breadth-first, with $O(d)$ steps where d is tree depth

$O(log(n))$ algorithm $\forall$ trees

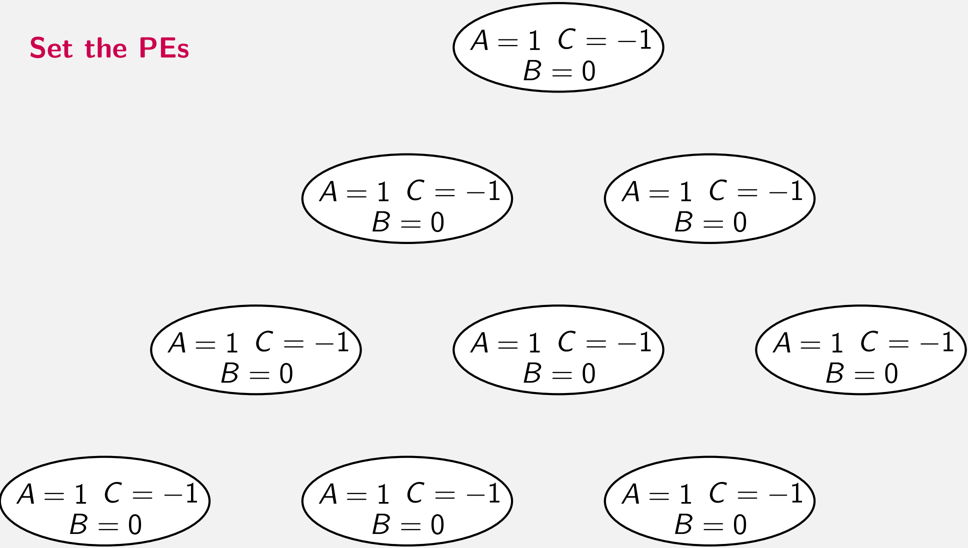
- Associate 3 PE (A, B, C) to each node, scan or prefix-scan
- Binary operation: addition in $\mathbb{Z}$
- Linking:
  - A on left child A if it exists, B else
  - B on right child A if it exists, C else
  - C on B –if left– or C –if right– of father (or NIL if root)
- Initialization :
  - $A = 1$, to go down
  - $B = 0$, to go right
  - $C = -1$, to go up

Euler Tower Example



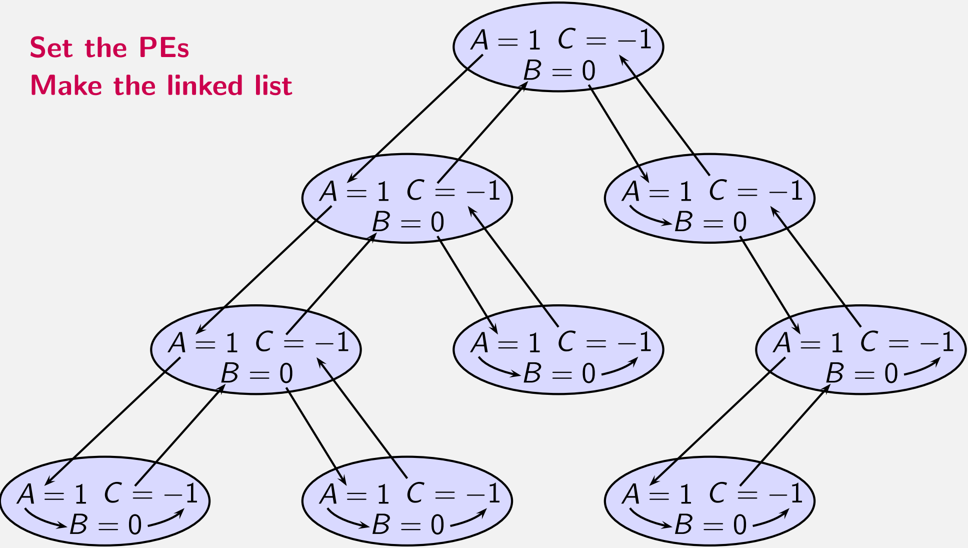
Euler Tower Example

Set the PEs



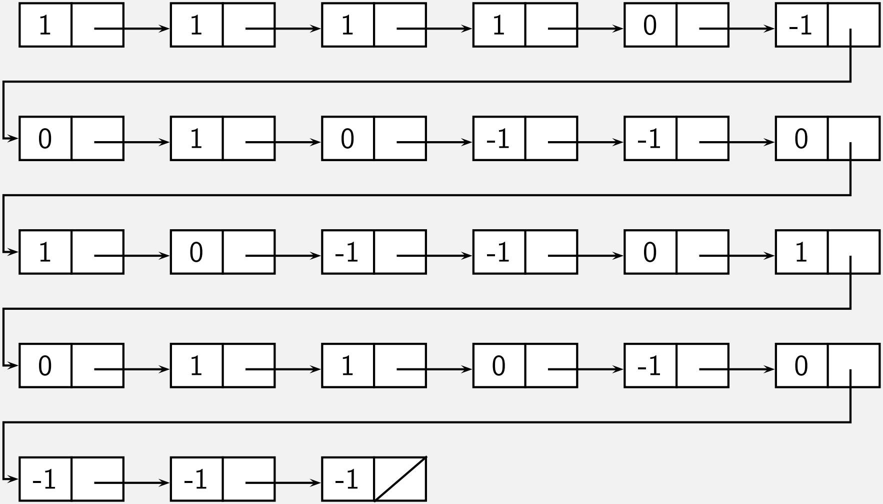
Euler Tower Example

Set the PEs
Make the linked list



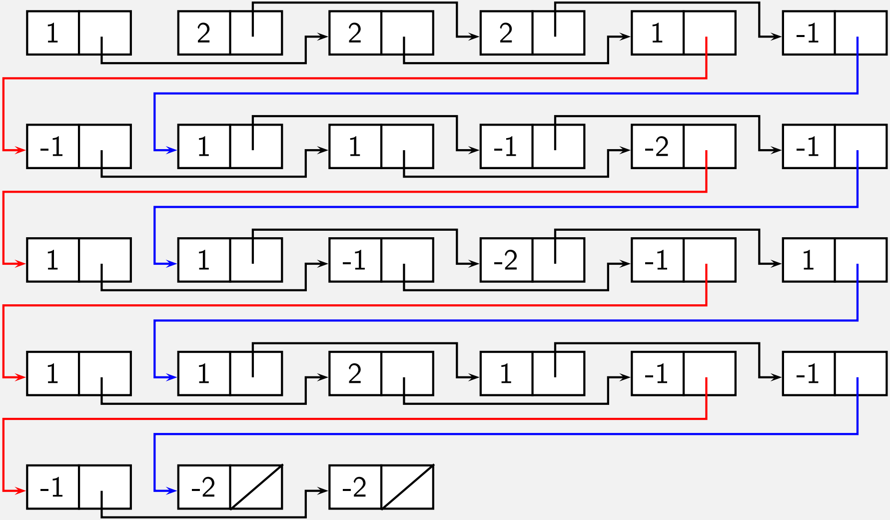
Euler Tower Example

Print as a list: loop 0



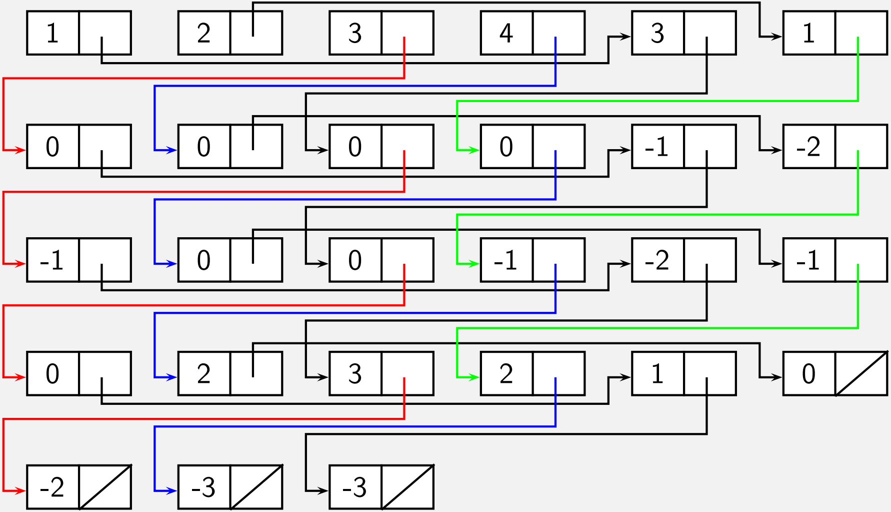
Euler Tower Example

Print as a list: loop 1



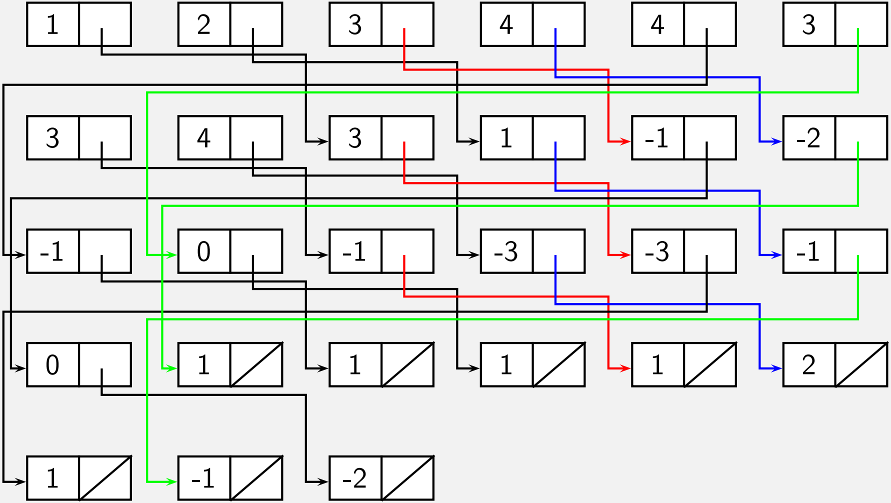
Euler Tower Example

Print as a list: loop 2



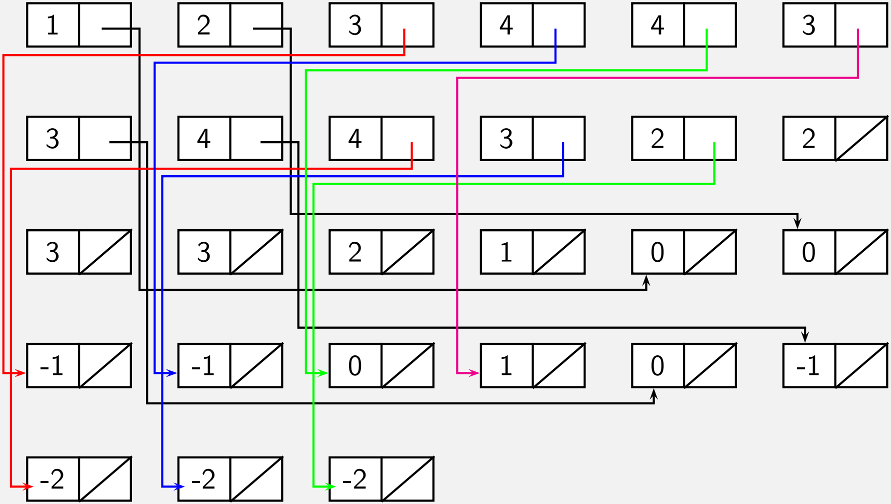
Euler Tower Example

Print as a list: loop 3



Euler Tower Example

Print as a list: loop 4



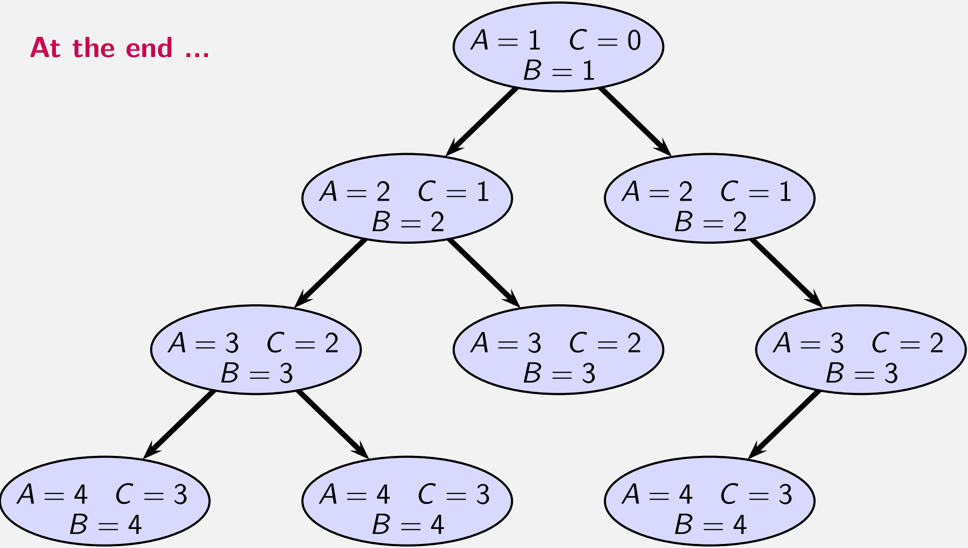
Euler Tower Example

Print as a list: loop 5

1	2	3	4	4	3
3	4	4	3	2	2
3	3	2	1	1	2
2	3	4	4	3	3
2	1	0			

Euler Tower Example

At the end ...



Overview

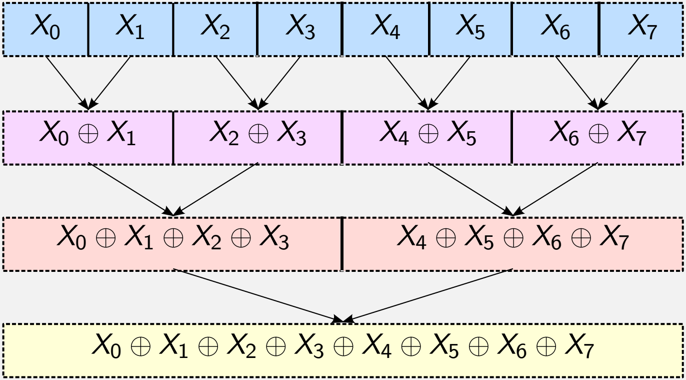
- Pointer Jumping
- Reduce

REDUCE

REDUCE consists to apply a binary associative commutative operator to all the elements of a given input of size n:

$$\bigoplus_{i=0}^{n-1} X_i$$

You have already seen the parallel version of this computation:



REDUCE using PRAM

Many solutions exists! The following corresponds to the figure displayed on previous slide ...

```
REDUCE(X, N, op)
  { X is an array of size N, op a binary operator }
  jump ← 1
  WHILE jump < N DO:
    FOR each PE i in parallel DO:
      IF (i MOD 2×jump) == 0 THEN
        next ← X_{i+jump}
        { implicit barrier }
        X_i ← op(X_i, next)
      END-IF
    END-FOR
    jump ← 2×jump
  END-WHILE
  RETURN X_0
END { REDUCE }
```

Thrust

We have two versions!

Classical REDUCE

```
template<typename InputIterator , typename T , typename BinaryFunction >
T thrust::reduce ( InputIterator  first ,     // data iterator
                   InputIterator  last ,      // end of data
                   T              init ,      // 0 by default
                   BinaryFunction binary_op   // thrust::plus<T> by default
) ;
```

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```

SEGMENTED-REDUCE (or reduce-by-key)

```
template<typename InputIterator1 , typename InputIterator2 ,
         typename OutputIterator1 , typename OutputIterator2 ,
         typename BinaryPredicate , typename BinaryFunction>
thrust::pair<OutputIterator1,OutputIterator2> thrust::reduce_by_key
    ( InputIterator1   keys_first ,
      InputIterator1   keys_last ,
      InputIterator2   values_first ,
      OutputIterator1  keys_output ,
      OutputIterator2  values_output ,
      BinaryPredicate  binary_predicate ,
      BinaryFunction   binary_function
    ) ;
```

Reduce and reduce-by-key example

```
#include <thrust/reduce.h>
// This example runs on HOST ...
int main() {
  const int N = 10;

  int iV[N] = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10 }; // input values

  thrust::plus<int> binary_op;
  int sum = thrust::reduce(iV, iV+N, 0, binary_op);
  // sum == (10*11)/2 == 55 ...

  int iK[N] = {0, 0, 0, 0, 1, 1, 1, 1, 2,  2 }; // input keys
  int oK[N];                                      // output keys
  int oV[N];                                      // output values
  thrust::equal_to<int>   binary_pred; // for keys

  thrust::pair<int*,int*> new_end =
      thrust::reduce_by_key(iK, iK + N, iV, oK, oV, binary_pred, binary_op);
  // Now, (new_end.first - oK) == (new_end.second - oV) == 3 !
  // The first three keys in oK are now {0, 1, 2}
  // The first three values in oV are now {10, 26, 19}
  return 0;
}
```

REDUCE-BY-KEY using PRAM

Need to consider the Keys: when they differ, cannot do the binary operation; egality means the operation is valid

```
REDUCE–BY–KEY(X, K, N, op)
  { X and K are arrays of size N, op a binary operator }
  jump ← 1
  WHILE jump < N DO:
    FOR each PE i in parallel DO:
      IF (i MOD 2×jump) == 0 THEN
        next ← X_{i+jump}
        { implicit barrier }
        IF K_i == K_next THEN
          X_i ← op(X_i, next)
        END–IF
      END–IF
    END–FOR
    jump ← 2×jump
  END–WHILE
END { REDUCE–BY–KEY }
```