The Arnoldi algorithm

– It turns out that $\{r^{(0)}, Ar^{(0)}, A^2r^{(0)}, \dots, A^{m-1}r^{(0)}\}$ is usually not a good basis, since it becomes ill conditioned (the basis vectors become too similar) as m increases. Alternatively, an orthonormal basis should be used. To construct that, we use the Arnoldi algorithm:

Arnoldi $(A, b, x^{(0)}, m)$

$v_1 = \dfrac{r^{(0)}}{\|r^{(0)}\|}$, where $r^{(0)} = b - Ax^{(0)}$

for $j = 1, \dots, m$

$\quad w = Av_j$

$\quad$ for $i = 1, \dots, j$

$\qquad h_{i,j} = v_i \cdot w$

$\qquad w \leftarrow w - h_{i,j} v_i$

$\quad$ end

} orthogonalisation

$\quad h_{j+1,j} = \|w\|$

$\quad v_{j+1} = \dfrac{w}{h_{j+1,j}}$

} normalisation

end

(condensed form)

$v_1 = \dfrac{r^{(0)}}{\|r^{(0)}\|}$

for $j = 1, \dots, m$

$\quad [v_{j+1}, h_j] = \text{getKrylov}(A, v_j)$

$\quad$ (short form for what's on the left → will be important later!)

end