

Lecture 7: the conjugate gradient method

- The conjugate gradient (CG) method is another Krylov method, but designed for SPD matrices. Starting from an initial guess $x_0 \in \mathbb{R}^n$, we do

$$r_0 = b - A x_0$$

$$p_0 = r_0$$

for $m = 0, 1, \dots$ (number of iterations)

$$\alpha_m = \frac{r_m^T r_m}{p_m^T (A p_m)} \rightarrow \text{store!}$$

$$x_{m+1} = x_m + \alpha_m p_m$$

$$r_{m+1} = r_m - \alpha_m (A p_m)$$

$$\beta_{m+1} = \frac{r_{m+1}^T r_{m+1}}{r_m^T r_m}$$

$$p_{m+1} = r_{m+1} + \beta_m p_m$$

end

→ cost per iteration:

- 1x matrix-vector product: $A p_m$

- 2x scalar products: $r_{m+1}^T r_{m+1}$, $p_m^T (A p_m)$