[illegible] 12313107 Engineering-Eletromagnetics

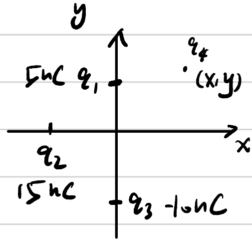
#2.

Suppose the 20nC Charge's position is (x, y)

Then, electric field at the origin with out 20nC Charge is

$$\vec{E}_0 = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{\ell^2}\cdot(-1)\vec{a}_y + \frac{q_2}{\ell^2}\cdot(-1)\vec{a}_x + \frac{q_3}{\ell^2}\vec{a}_y\right)$$

20nC Charge (q_4) is added



$$\vec{E} = \vec{E}_0 + \frac{q_4}{4\pi\epsilon_0}\frac{1}{[x^2+y^2]^{3/2}}[-x\vec{a}_x - y\vec{a}_y]$$

Our goal is to make

$$\vec{E}(0,0) = 0$$

Then, we get scalor equations

$$x: \quad -\frac{q_2}{\ell^2} - \frac{x \cdot q_4}{[x^2+y^2]^{3/2}} = 0$$

$$y: \quad -\frac{q_1}{\ell^2} + \frac{q_3}{\ell^2} - \frac{y \cdot q_4}{[x^2+y^2]^{3/2}} = 0$$

Solve the equation set, we can get

$$x = 3.43\,\text{cm}$$
$$y = -3.43\,\text{cm}$$

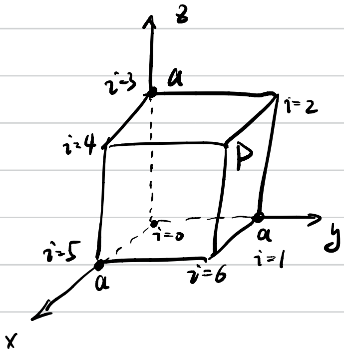
In summary, the required coordinate is

$$\boxed{x = 3.43\,\text{cm},\ y = -3.43\,\text{cm}}$$

#3.

E - field

$$\vec{E} = \frac{q}{4\pi\epsilon_0} \frac{\vec{r} - \vec{r_0}}{|\vec{r} - \vec{r_0}|^3}$$



Then E-field created by other charge excerted at point P is

$$\vec{E}_p = \sum_{i=0}^{6} \frac{Q}{4\pi\epsilon_0} \frac{\vec{r} - \vec{r_i}}{|\vec{r} - \vec{r_i}|^3} \quad , \quad \vec{r} = a(\vec{a_x} + \vec{a_y} + \vec{a_z})$$

$\vec{r_0} = 0$, $\quad \vec{r_1} = a\vec{a_y}$, $\quad \vec{r_2} = a(\vec{a_y} + \vec{a_z})$

$\vec{r_3} = a\vec{a_z}$, $\quad \vec{r_4} = a(\vec{a_x} + \vec{a_z})$, $\vec{r_5} = a\vec{a_x}$,

$\vec{r_6} = a(\vec{a_x} + \vec{a_y})$

Then calculate the summation, we can get

$$\vec{E}_p = \frac{Q}{4\pi\epsilon_0 a^2}\left(1 + \frac{1}{\sqrt{2}} + \frac{1}{3\sqrt{3}}\right) \cdot (\vec{a_x} + \vec{a_y} + \vec{a_z})$$

Or numerically

$$\boxed{\vec{E}_p = \frac{0.475\,Q}{4\pi\epsilon_0}(\vec{a_x} + \vec{a_y} + \vec{a_z})}$$

#4.

(i) E-field expression

$$\vec{E} = \frac{q}{4\pi\epsilon_0} \frac{\vec{r}-\vec{r}_0}{|\vec{r}-\vec{r}_0|^3}$$

Where

$\vec{r} = x\vec{a}_x + y\vec{a}_y + z\vec{a}_z$ is the position vector of P

$\vec{r}_0 = -\vec{a}_x + \vec{a}_y + 3\vec{a}_z$ is the position vec. of charge

Project on x, y, z axis, is

$$\vec{E} = \vec{E}\cdot\vec{a}_x\,\vec{a}_x + \vec{E}\cdot\vec{a}_y\,\vec{a}_y + \vec{E}\cdot\vec{a}_z\,\vec{a}_z$$

$$\vec{E}\cdot\vec{a}_x = \frac{q}{4\pi\epsilon_0}\frac{\vec{r}-\vec{r}_0}{|\vec{r}-\vec{r}_0|^3}\cdot\vec{a}_x = \frac{q}{4\pi\epsilon_0}\frac{x+1}{|\vec{r}-\vec{r}_0|^3}$$

$$\vec{E}\cdot\vec{a}_y = \frac{q}{4\pi\epsilon_0}\frac{\vec{r}-\vec{r}_0}{|\vec{r}-\vec{r}_0|^3}\cdot\vec{a}_y = \frac{q}{4\pi\epsilon_0}\frac{y-1}{|\vec{r}-\vec{r}_0|^3}$$

$$\vec{E}\cdot\vec{a}_z = \frac{q}{4\pi\epsilon_0}\frac{\vec{r}-\vec{r}_0}{|\vec{r}-\vec{r}_0|^3}\cdot\vec{a}_z = \frac{q}{4\pi\epsilon_0}\frac{z-3}{|\vec{r}-\vec{r}_0|^3}$$

Therefore, as $E_x = 500\ V/m$ which is a constant

$$E_x = \vec{E}\cdot\vec{a}_x = \frac{q}{4\pi\epsilon_0}\frac{x+1}{[(x+1)^2+(y-1)^2+(z-3)^2]^{3/2}} = \text{Const}$$

Then we can simplify the equation.

$$\boxed{\begin{array}{l} (x+1)^2 - A^2\left[(x+1)^2 + (y-1)^2 + (z-3)^2\right]^3 = 0 \\ A = \dfrac{4\pi\varepsilon_0 V}{q}, \quad V = 500\,V/m, \quad q = 100\,nC. \end{array}}$$

(ii) $P(-2, y_1, 3)$

Substitution

$$1 - A^2\left[1 + (y_1-1)^2 + 0\right]^3 = 0$$

$$\boxed{\Rightarrow y_1 = 0.308\,m \text{ or } y_1 = 1.692\,m}$$

#5.

a) Total charge

$$Q = \int \rho_v 4\pi r^2 dr$$

$$= \int_{0.03}^{0.05} \rho_v \cdot 4\pi r^2 \cdot dr$$

$$= \boxed{82.1\,pC}$$

b) Solve

$$\frac{Q}{2} = \int_{0.03}^{r_1} \rho_v \cdot 4\pi r^2 dr = \frac{4\pi}{3}\rho_v (r_1^3 - 0.03^3)$$

Solved that

$$\boxed{r_1 = 4.24\,cm}$$