Engineering Electromagnetics Theory Lab 4 Magnetic Focusing

GAN Shengzhe, 12313107
Department of Electronic and Electrical Engineering, SUSTech.

***Abstract*—This is the report for Engineering Electromagnetic Theory Lab 4. In this lab, we will evaluate the effect of magnetic focusing.**

***Index Terms*—Magnetic fields, Magnetic focusing, Lorentz force, numerical analysis, MATLAB.**

I. INTRODUCTION

THIS report is the fourth lab report of the course Engineering Electromagnetic Theory. The purpose was to showcase the magnetic focusing effect.

For a beam of charged particles with a small angle of divergence, given the same velocity component at the direction of the magnetic field B, their trajectory will have the same screw pitch. After a period, they will converge at another point. The phenomenon that the diverged charged particles focus at one point is similar to the phenomenon that lens can let the light beam focus at one point. Therefore, it is called as magnetic focusing.

II. SYSTEM SETUP

The system is a free space with no electric field and static magnetic field $\mathbf{B} = 8\ \mathbf{a}_z\ Wb/m^2$.

16 positive charges are located at the origin, with different initial velocity, but the velocity projection on z-axis are all the same.

The velocity for the i^{th} charge is

$$\begin{aligned} v_x(i) &= 0.1 \cdot \sin(\frac{i\pi}{8})\ \mathrm{m/s} \\ v_y(i) &= 0.1 \cdot \cos(\frac{i\pi}{8})\ \mathrm{m/s} \\ v_z(i) &= 10\ \mathrm{m/s} \end{aligned} \tag{1}$$

III. NOTATIONS

TABLE I
NOTATIONS FOR CONSTANTS AND PHYSICAL QUANTITIES

Symbols	Description	Unit
$\mathbf{B}$	Magnetic Field	Wb/m^2
$\mathbf{E}$	Electric Field	N/C
$\mathbf{F}$	Force	N
q	amount of charge	C
$\mathbf{r}$	displacement vector	m
r_i	projection of displacement vector on i axis	m
$\mathbf{v}$	velocity of charge	m/s
v_i	velocity projection on i axis	m/s
$\mathbf{a}$	acceleration of charge	m/s
a_i	acceleration projection on i axis	m/s
$\mathbf{i}$	unit vector of x axis	1
$\mathbf{j}$	unit vector of y axis	1
$\mathbf{k}$	unit vector of z axis	1
m	mass of the charge	kg

IV. THEORETICAL CALCULATION

The force exerts on the charge is given by

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \tag{2}$$

Then we use rectangular coordinates, and we projections to illustrate this equation for one charge.

$$\mathbf{E} = E_x\mathbf{i} + E_y\mathbf{j} + E_z\mathbf{k} \tag{3}$$

$$\mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k} \tag{4}$$

$$\mathbf{r} = r_x\mathbf{i} + r_y\mathbf{j} + r_z\mathbf{k} \tag{5}$$

$$\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k} \tag{6}$$

$$\mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k} \tag{7}$$

Then, we calculate the cross product by a determinant

$$\mathbf{v} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix} \tag{8}$$

We use numerical calculation by Matlab, and the integral is transferred into summation.

$$\mathbf{r}(i+1) = \mathbf{r}(i) + \mathbf{v}(i) \cdot \delta t \tag{9}$$

$$\mathbf{v}(i+1) = \mathbf{v}(i) + \mathbf{a}(i) \cdot \delta t \tag{10}$$

$$\mathbf{a}(i) = \mathbf{F}(i)/m \tag{11}$$

Thus, split into x, y and z components, select proper time resolution δt, we can program Matlab codes to evaluate the trace of the moving charge.

The Matlab codes are attached to appendix.

V. Result

First, we choose the time resolution $\delta t = 0.001s$, and retain for 3 seconds, the diagrams are shown as follows.

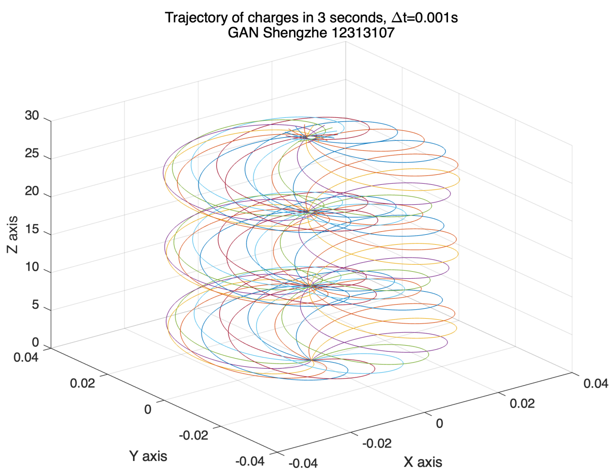

Fig. 1. Default view, time resolution 0.001 s

Then examine the side view.

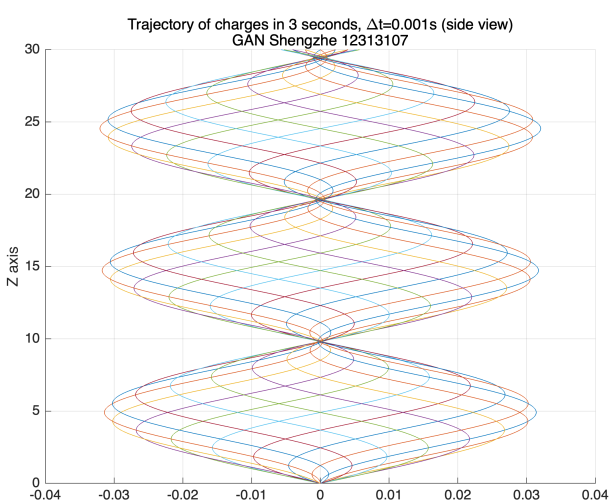

Fig. 2. Side view, time resolution 0.001 s

Also examine the top view.

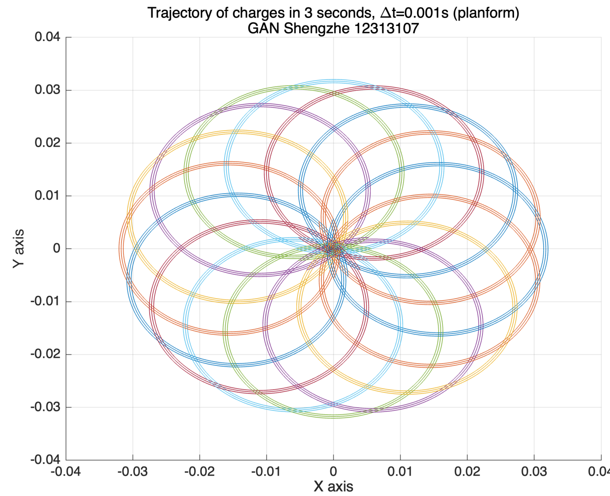

Fig. 3. Top view time resolution 0.001 s

From these diagrams, we can clearly find that these charges are focused with a specific interval. With corresponds to our assumptions.

VI. Conclusion

From this experiment, we have learned the way to conduct numerical calculation of the equation of motion of the charged particles moving in the electromagnetic field, and plot the trajectory of the motion in space in Matlab, which greatly helped me to explore complex situation of this type of problem.

In the situation stated in this lab, I visually learned the magnetic focusing effect which is widely used in physics science.

Acknowledgments

Thanks to our professor for offering us the template and the theoretical part of the calculation.

Appendix
MATLAB Codes

This part of Matlab codes is applied for numerical calculation and visualization.

```
clear;
N=16;
m=0.02;
q=0.016;
dt=0.001;
t=0:dt:3;

vx=zeros(N,length(t));
vy=vx;
vz=vx;
for i = 1:N
    vx(i,1)=0.1*sin((i-1)*pi/8);
    vy(i,1)=0.1*cos((i-1)*pi/8);
    vz(i,1)=10;
end

rx=zeros(N,length(t));
ry=rx;
rz=rx;
Bx=0; By=0; Bz=8;
Fx=zeros(N,length(t));
Fy=Fx;Fz=Fx;
ax=zeros(N,length(t));
ay=ax;az=ax;
for j=1:N
    for i=1:(length(t)-1)
        Fx(j,i)=q*(vy(j,i)*Bz-vz(j,i)*By);
        Fy(j,i)=q*(vz(j,i)*Bx-vx(j,i)*Bz);
        Fz(j,i)=q*(vx(j,i)*By-vy(j,i)*Bx);
        ax(j,i)=Fx(j,i)/m;
        ay(j,i)=Fy(j,i)/m;
        az(j,i)=Fz(j,i)/m;
        vx(j,i+1)=vx(j,i)+ax(j,i)*dt;
        vy(j,i+1)=vy(j,i)+ay(j,i)*dt;
        vz(j,i+1)=vz(j,i)+az(j,i)*dt;
        rx(j,i+1)=rx(j,i)+vx(j,i)*dt;
        ry(j,i+1)=ry(j,i)+vy(j,i)*dt;
        rz(j,i+1)=rz(j,i)+vz(j,i)*dt;
    end
end

figure;
plot3(rx(1,:),ry(1,:),rz(1,:));
hold on
for i = 2:N
    plot3(rx(i,:),ry(i,:),rz(i,:));
end
grid;
axis([-0.04 0.04 -0.04 0.04 0 30])
title({'Trajectory of charges in 3 seconds, \Deltat=0.001s'},{'GAN Shengzhe 12313107'});
xlabel('X axis', 'fontsize', 12);
ylabel('Y axis', 'fontsize', 12);
zlabel('Z axis', 'fontsize', 12);
exportgraphics(gca, 'T=3_t=001.png', 'Resolution', 320);

view(2)
axis([-0.04 0.04 -0.04 0.04 0 30])
title({'Trajectory of charges in 3 seconds, \Deltat=0.001s(Planform)'},{'GAN Shengzhe 12313107'});
xlabel('X axis', 'fontsize', 12);
ylabel('Y axis', 'fontsize', 12);
zlabel('Z axis', 'fontsize', 12);
exportgraphics(gca, 'T=3_t=001_xz.png', 'Resolution', 320);

view(0,0)
axis([-0.04 0.04 -0.04 0.04 0 30])
title({'Trajectory of charges in 3 seconds, \Deltat=0.001s(Side view)'},{'GAN Shengzhe 12313107'});
xlabel('X axis', 'fontsize', 12);
ylabel('Y axis', 'fontsize', 12);
zlabel('Z axis', 'fontsize', 12);
exportgraphics(gca, 'T=3_t=001_xy.png', 'Resolution', 320);
```

Listing 1. Theoretical plot codes

References

[1] William H. Hayt, Jr., John A. Engineering electromagnetics—8th ed.
[2] Youwei JIA., Engineering electromagnetic theory-4.
[3] Insert codes block in LaTeX https://blog.csdn.net/wxd1233/article/details/127196149.