HW-4 胡程哲 12313107

2. Two point charges, 1 nC at (0, 0, 0.1) and −1 nC at (0, 0, −0.1), are in free space. a) Calculate V at $P(0.3, 0, 0.4)$; b) Calculate $|\mathbf{E}|$ at P; c) Now treat the two charges as a dipole at the origin and find V at P.

a) From the definition of E-potential

$$V = \frac{q_1}{4\pi\varepsilon_0}\frac{1}{|\vec{r}-\vec{r}_1|} + \frac{q_1}{4\pi\varepsilon_0}\frac{1}{|\vec{r}-\vec{r}_1|}$$

where $q_1 = 1nC$, $r_1 = 0.1$, $q_2 = -1nC$, $r_2 = -0.1$

$$V = 5.77 \text{ Volts}$$

b) Vector form

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{|\vec{r}-\vec{r}_1|^3}\cdot(\vec{r}-\vec{r}_1) + \frac{q_2}{|\vec{r}-\vec{r}_2|^3}(\vec{r}-\vec{r}_2)\right)$$

in components

$$E_x = \vec{E}\cdot\vec{a}_x = 12.64 \text{ V/m}$$

$$E_y = \vec{E}\cdot\vec{a}_y = 21.71 \text{ V/m}$$

Then

$$|\vec{E}| = \sqrt{E_x^2 + E_y^2} = 25.12 \text{ V/m}$$

c) As we treat the set of charge as a dipole, then

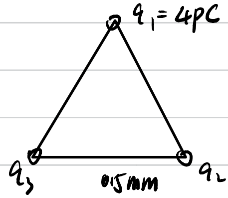
$$V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$

where $p = 2q\ell = 2\times10^{-10}$ C·m, $\theta = \arctan\frac{0.4}{0.3} = 0.9273$ rad

then

$$V(0.3, 0, 0.4) = 5.75 \text{ Volts}$$

4. Three **identical** point charges of 4 pC each are located at the corners of an **equilateral triangle** 0.5 mm on a side in free space. How much work must be done to move one charge to a point **equidistant** from the other two and on the line joining them?



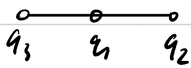
Calculate the energy of this system

$$U_{init} = \frac{1}{2}\sum_i q_i U_i$$

$$= \frac{1}{2}\left[q_1(U_{1,3}+U_{1,2}) + q_2(U_{2,3}+U_{2,1}) + q_3(U_{3,1}+U_{3,2})\right]$$

$$= 8.628\times10^{-10} \text{ Joule}$$

$$U_{final} = \frac{1}{2}\sum_i q_i U_i$$



$$= \frac{1}{2}\left[q_1(U_{1,3}+U_{1,2}) + q_2(U_{2,3}+U_{2,1}) + q_3(U_{3,1}+U_{3,2})\right]$$

$$= 1.438\times10^{-9} \text{ Joule}$$

Therefore

$$W = \Delta U = U_{final} - U_{init} = 5.752\times10^{-10} \text{ Joule}$$

5. Given the electric field $\mathbf{E} = (y+1)\mathbf{a}_x + (x-1)\mathbf{a}_y + 2\mathbf{a}_z$, find the potential difference between the points: a) (2,-2,-1) and (0,0,0); b) (3,2,-1) and (-2,-3,4).

a)

$$V = -\int_{(0,0,0)}^{(2,-2,-1)} \vec{E} \cdot d\vec{l} = -\int_{(0,0,0)}^{(2,-2,-1)} \vec{E}\cdot\vec{a_x}\,dx + \vec{E}\cdot\vec{a_y}\,dy + \vec{E}\cdot\vec{a_z}\,dz$$

$$= -\left[\int_0^2 (y+1)\Big|_{y=0} dx + \int_0^{-2} (x-1)\Big|_{x=2} dy + \int_0^{-1} 2\,dz\right]$$

$$= -(2 + (-2) + (-2))$$

$$= 2 \text{ Volts}$$

b)

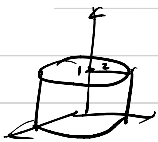
$$V = -\int_{(-2,-3,4)}^{(3,2,-1)} \vec{E} \cdot d\vec{l}$$

$$= -\left[\int_{-2}^{3} (y+1)\Big|_{y=-3} dx + \int_{-3}^{2} (x-1)\Big|_{x=3} dy + \int_4^{-1} 2\,dz\right]$$

$$= -(-2\times 5 + 2\times 5 + 2\times(-1-4))$$

$$= 10 \text{ Volts}$$

6. Within the cylinder $\rho = 2$, $0 < z < 1$, the potential is given by $V = 100 + 50\rho + 150\rho\sin\varphi$ V. a) Find V, $\mathbf{E}$, $\mathbf{D}$, and ρ_v at $P(1, 60^\circ, 0.5)$ in free space; b) How much charge lies within the cylinder?



a) at point P

$$V = V(1, 60^\circ, 0.5) = 279.9 \text{ Volts}$$

Use the relation

$$\vec{E} = -\nabla V$$

$$= -\left(\frac{\partial}{\partial \rho} V \vec{a}_\rho + \frac{1}{\rho}\frac{\partial}{\partial \varphi} V \vec{a}_\varphi + \frac{\partial}{\partial z} V \vec{a}_z\right)$$

$$= -\left[(50 + 150\sin\varphi)\vec{a}_\rho + \frac{1}{\rho}(150\rho\cos\varphi)\vec{a}_\varphi + 0\right]$$

$$= -50(1+3\sin\varphi)\vec{a}_\rho - 150\cos\varphi\,\vec{a}_\varphi$$

Therefore, at the point P

$$\vec{E} = (-179.9\,\vec{a}_\rho - 75\,\vec{a}_\varphi)\ V/m$$

This senario is in free space, thus

$$\vec{D} = \varepsilon_0\vec{E} = -1.593\times10^{-9}\,\vec{a}_\rho - 6.641\times10^{-10}\,\vec{a}_\varphi \quad C$$

b) Due to

$$\oint_S \vec{D}\cdot d\vec{s} = q_{enc}$$

and

$$\vec{D} = \varepsilon_0\vec{E} = -\varepsilon_0\nabla V = -50(1+3\sin\varphi)\varepsilon_0\vec{a}_\rho - 150\varepsilon_0\cos\varphi\,\vec{a}_\varphi$$

Perform the surface integral

due to no z-axis component, therefore

$$\iint \vec{D}\cdot d\vec{s} = \int_0^{2\pi} \rho\cdot z\cdot D_\rho\, d\varphi$$

$$= \int_0^{2\pi} 2\cdot 1\cdot[-50(1+3\sin\varphi)\cdot\varepsilon_0]\cdot d\varphi$$

$$= -5.56\times10^{-9}C$$

Thus

$$q_{enc} = -5.56\ nC$$

The charge density is

$$\rho_v = \nabla \cdot \vec{D} = \frac{1}{\rho}\frac{\partial(\rho D_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial D_\varphi}{\partial \varphi} + \frac{\partial (D_z)}{\partial z}$$

$$= -\frac{50}{\rho}(1+3\sin\varphi)\varepsilon_0 + \frac{150\varepsilon_0 \sin\varphi}{\rho}$$

$$= -\frac{50\varepsilon_0}{\rho}$$