甘圣哲 12313107 Week 1

1. Done.

2.

1) $\vec{a}_1 = \dfrac{\vec{A}-\vec{B}}{|\vec{A}-\vec{B}|} = \dfrac{-\vec{a}_x-\vec{a}_y+5\vec{a}_z}{\sqrt{1+1+25}} = -\dfrac{1}{\sqrt{27}}\vec{a}_x - \dfrac{1}{\sqrt{27}}\vec{a}_y + \dfrac{5}{\sqrt{27}}\vec{a}_z = \boxed{-0.192\vec{a}_x - 0.192\vec{a}_y + 0.962\vec{a}_z}$

2) Mid-point: $\left(\dfrac{1+2}{2}, \dfrac{2+3}{2}, \dfrac{3-2}{2}\right) = \left(\dfrac{3}{2}, \dfrac{5}{2}, \dfrac{1}{2}\right) = \boxed{(1.5, 2.5, 0.5)}$

Then

$$\vec{a}_2 = \frac{\frac{3}{2}\vec{a}_x + \frac{5}{2}\vec{a}_y + \frac{1}{2}\vec{a}_z}{\sqrt{(\frac{3}{2})^2+(\frac{5}{2})^2+(\frac{1}{2})^2}} = \frac{3}{\sqrt{35}}\vec{a}_x + \frac{5}{\sqrt{35}}\vec{a}_y + \frac{1}{\sqrt{35}}\vec{a}_z = \boxed{0.507\vec{a}_x + 0.845\vec{a}_y + 0.169\vec{a}_z}$$

3. Knowing that

$$\left.\begin{aligned} &|\vec{OA}-\vec{OB}| = 10 \\ &\vec{OA} = 6\vec{a}_x - 2\vec{a}_y - 4\vec{a}_z \\ &\vec{OB} = k\cdot\tfrac{2}{3}\vec{a}_x - k\cdot\tfrac{2}{3}\vec{a}_y + k\cdot\tfrac{1}{3}\vec{a}_z \end{aligned}\right\} \Rightarrow \left(6-\frac{2k}{3}\right)^2 + \left(-2+\frac{2k}{3}\right)^2 + \left(-4-\frac{k}{3}\right)^2 = 10^2$$

Solve the equation, get

$$k^2 - 8k - 44 = 0$$

$$\Rightarrow \quad k_1 = 4+2\sqrt{15}, \quad k_2 = 4-2\sqrt{15}$$

Therefore, coordinate of B

$$\left(\frac{8+4\sqrt{15}}{3}, -\frac{8+4\sqrt{15}}{3}, \frac{4+2\sqrt{15}}{3}\right) \text{ or } \left(\frac{8-4\sqrt{15}}{3}, -\frac{8-4\sqrt{15}}{3}, \frac{4-2\sqrt{15}}{3}\right)$$

In decimal form

$$\boxed{(7.83, -7.83, 3.92) \text{ or } (-2.50, 2.50, -1.25)}$$

4.

1) At P

$\vec{G}(P) = 24\times1\times2\,\vec{a}_x + 12\cdot(1^2+2)\vec{a}_y + 18\cdot(-1)^2\vec{a}_z = \boxed{48\vec{a}_x + 36\vec{a}_y + 18\vec{a}_z}$

2) G at Q is

$\vec{G}(Q) = 24\times(-2)\times1\,\vec{a}_x + 12((-2)^2+2)\vec{a}_y + 18\cdot3^2\vec{a}_z = \boxed{-48\vec{a}_x + 72\vec{a}_y + 162\vec{a}_z}$

$$\vec{a}_Q = \frac{\vec{G}(Q)}{|\vec{G}(Q)|} = -\frac{8}{\sqrt{937}}\vec{a}_x + \frac{12}{\sqrt{937}}\vec{a}_y + \frac{27}{\sqrt{937}}\vec{a}_z = \boxed{-0.26\,\vec{a}_x + 0.39\,\vec{a}_y + 0.88\,\vec{a}_z}$$

3) $\vec{QP} = \vec{OQ} - \vec{OP} = -96\vec{a}_x - 36\vec{a}_y + 144\vec{a}_z$

$$\vec{a}_{QP} = \frac{\vec{QP}}{|\vec{QP}|} = -\frac{8}{\sqrt{217}}\vec{a}_x - \frac{3}{\sqrt{217}}\vec{a}_y + \frac{12}{\sqrt{217}}\vec{a}_z = \boxed{-0.59\,\vec{a}_x - 0.20\vec{a}_y + 0.78\vec{a}_z}$$

4) The equation is

$$(24xy)^2 + 144(x^2+2)^2 + (18z^2)^2 = 3600$$

$$\Rightarrow \boxed{16x^2y^2 + 4x^4 + 16x^2 + 9z^4 = 84}$$

5.

a) Suppose $\vec{a}_p = x\vec{a}_x + y\vec{a}_y + z\vec{a}_z$, $x^2+y^2+z^2=1$

Then according to the restition

$$\begin{cases} \vec{a}_p \cdot \vec{r}_1 = 0 \\ \vec{a}_p \cdot \vec{r}_2 = 0 \end{cases} \Rightarrow \begin{cases} 7x+3y-2z=0 \\ -2x+7y-3z=0 \end{cases}$$

$$\Rightarrow \begin{cases} x = t \\ y = 5t \\ z = 11t \end{cases} \Rightarrow \text{Normalize} \Rightarrow \sqrt{1+5^2+11^2}\cdot k = 1 \Rightarrow k = \frac{1}{\sqrt{147}}$$

$$\Rightarrow \vec{a}_p = \frac{1}{7\sqrt{3}}\vec{a}_x + \frac{5}{7\sqrt{3}}\vec{a}_y + \frac{11}{7\sqrt{3}}\vec{a}_z = \boxed{0.082\vec{a}_x + 0.412\vec{a}_y + 0.907\vec{a}_z}$$

b) $\vec{u} = \vec{r}_1 - \vec{r}_2 = 9\vec{a}_x - 4\vec{a}_y + \vec{a}_z$ $\qquad \vec{v} = \vec{r}_2 - \vec{r}_3 = -2\vec{a}_x + 5\vec{a}_y - 6\vec{a}_z$

Then $\vec{a}_{p2} = x\vec{a}_x + y\vec{a}_y + z\vec{a}_z$, $x^2+y^2+z^2=1$

$$\begin{cases} \vec{a}_{p2}\cdot\vec{u} = 0 \\ \vec{a}_{p2}\cdot\vec{v} = 0 \end{cases} \Rightarrow \begin{cases} 9x-4y+z=0 \\ -2x+5y-6z=0 \end{cases} \Rightarrow \begin{cases} x = \frac{19}{\sqrt{4434}} \\ y = 26\cdot\sqrt{\frac{2}{2217}} \\ z = \frac{37}{\sqrt{4434}} \end{cases}$$

Therefore $\vec{a}_{p2} = \frac{19}{\sqrt{4434}}\vec{a}_x + 26\sqrt{\frac{2}{2217}}\vec{a}_y + \frac{37}{\sqrt{4434}}\vec{a}_z = \boxed{0.285\,\vec{a}_x + 0.781\,\vec{a}_y + 0.556\,\vec{a}_z}$

(c) $A = \frac{1}{2}|\vec{r}_1 \times \vec{r}_2| = \frac{1}{2}\cdot\left\|\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 7 & 3 & -2 \\ -2 & 7 & -3 \end{vmatrix}\right\| = \frac{35\sqrt{3}}{2} = \boxed{30.31}$

(d) $A_2 = \frac{1}{2}|(\vec{r}_1-\vec{r}_2)\times(\vec{r}_2-\vec{r}_3)| = \frac{1}{2}\left\|\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 9 & -4 & 1 \\ -2 & 5 & -6 \end{vmatrix}\right\| = \frac{\sqrt{4434}}{2} = \boxed{33.3}$