Engineering Electromagnetics Theory Lab 3 Magnetic Field of Helmholtz Coils

GAN Shengzhe, 12313107
Department of Electronic and Electrical Engineering, SUSTech.

***Abstract*—In this lab, we will evaluate the distribution of magnetic field excited by Helmholtz Coils, with the load current in same and opposite direction.**

***Index Terms*—Magnetic fields, Biot-Savart Law, Helmholtz Coil, Numerical analysis, MATLAB.**

I. Introduction

THIS report is the third lab report of the course Engineering Electromagnetic Theory. Aiming at showcasing the Helmholtz Coil and it's fields.

II. System Setup

The system is two coils placed parallel to the $x-y$ plane and the origin of them are $(0,0,-1)$ and $(0,0,1)$ respectively. Both coils are load DC current I with same direction. Both of the coil's radius is a.

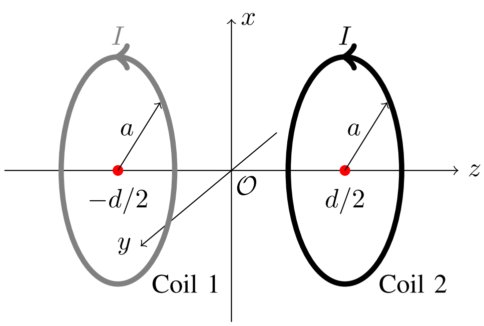

Fig. 1. System setup, with same current direction.

In this lab, we consider that $a = 2\text{m}$, the DC current load on the both coil is $I = 500\text{A}$, the separation of the two coils is $d = 2\text{m}$.

III. Notations

TABLE I
NOTATIONS FOR CONSTANTS AND PHYSICAL QUANTITIES

Symbols	Description	Unit
I	DC Current Magnitude	A
$\mathbf{H}$	Magnetic Field Vector	A/m
a	Radius of the Coil	m
d	Separation of the Two Coils	m
$I\mathrm{d}\mathbf{l}$	Current Element	$A \cdot m$
$\mathbf{R}$	Vector from the Current Element to Field Point	m

IV. Field Distribution Evaluation

We consider a current element $I\mathrm{d}\mathbf{l}$, from which the distance of field point P is $\mathbf{R}$. Thus, due to Biot-Savart law, we can obtain the magnetic field at point P

$$\mathrm{d}\mathbf{H} = \frac{I\mathrm{d}\mathbf{l} \times \mathbf{R}}{4\pi|\mathbf{R}|^3} \tag{1}$$

Then integrate we can obtain the magnetic field at point P.

$$\mathbf{H} = \int \frac{I\mathrm{d}\mathbf{l} \times \mathbf{R}}{4\pi|\mathbf{R}|^3} \tag{2}$$

We use MATLAB to numerically compute this integral. First, we express these vectors in coordinates form.

$$\mathrm{d}\mathbf{l} = \mathrm{d}l_x\mathbf{a}_x + \mathrm{d}l_y\mathbf{a}_y + \mathrm{d}l_z\mathbf{a}_z \tag{3}$$

$$\mathbf{R} = (x - x_l)\mathbf{a}_x + (y - y_l)\mathbf{a}_y + (z - z_l)\mathbf{a}_z \tag{4}$$

$$|\mathbf{R}| = \sqrt{(x - x_l)^2 + (y - y_l)^2 + (z - z_l)^2} \tag{5}$$

Where the corner marks x, y, z represent the x, y and z component, and the corner marker l represents the coordinates of l.

Then, due to the current is distributed on two circle, thus

$$\begin{aligned} x_l &= a \cdot \sin\theta \\ y_l &= a \cdot \cos\theta \\ z_l &= \pm\frac{d}{2} \end{aligned} \tag{6}$$

Coordinates of the micro elements $\mathrm{d}l$

$$\begin{aligned} \mathrm{d}l_x &= a \cdot \cos\theta\mathrm{d}\theta \\ \mathrm{d}l_y &= -a \cdot \sin\theta\mathrm{d}\theta \\ \mathrm{d}l_z &= 0 \end{aligned} \tag{7}$$

Therefore, after all these former calculation, we can easily with the help of matrix calculation in MATLAB to obtain the magnetic field in the mesh grid we defined.

Our target is to examine the H-field distribution on the $x-y$ plane. Then, we complete the codes, to plotting the field distribution we are interested in. All the MATLAB codes are appended in the appendix at the end of this report.

V. Result of Same Current Direction

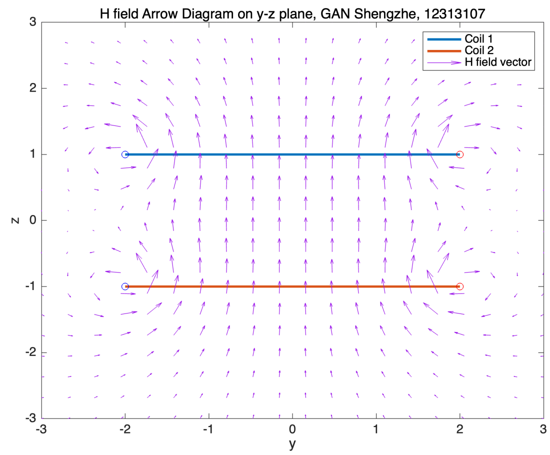

Fig. 2. Arrow Diagram of H Field

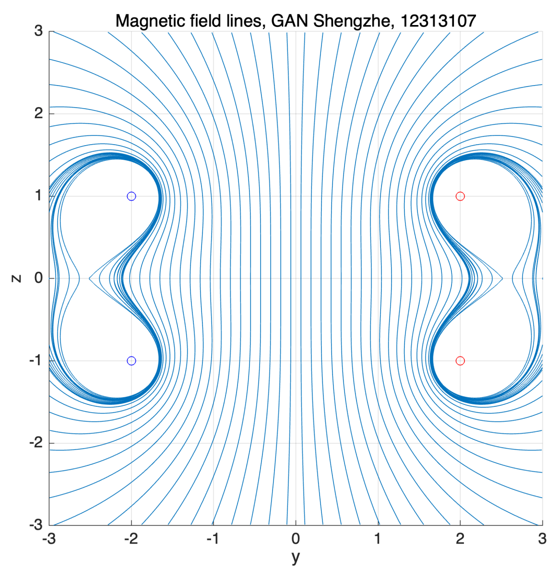

Fig. 3. Magnetic Lines of H field

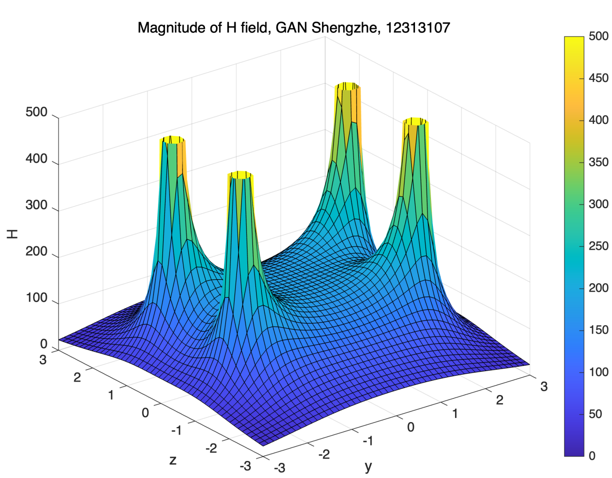

Fig. 4. Magnitude of H Field

From the three diagrams, we can clearly see that the magnetic field at the origin excited by the two coils is super uniform, which can be almost seen as uniform distributed field.

VI. Result of Opposite Current Direction

The setup of the system is shown in the diagram below.

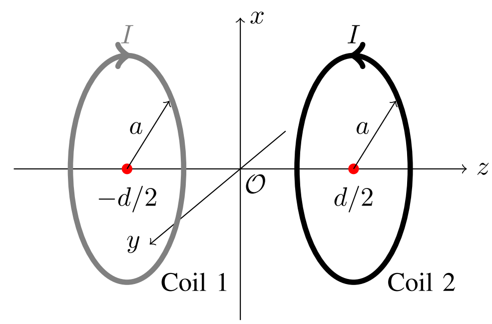

Fig. 5. System setup, with opposite current direction.

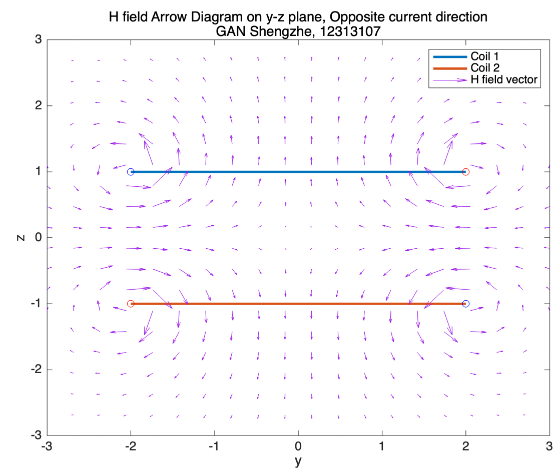

Fig. 6. Arrow Diagram of H Field, opposite current direction

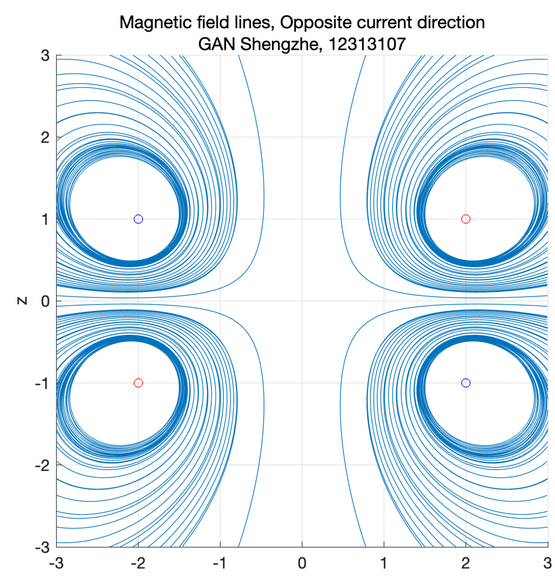

Fig. 7. Magnetic Lines of H field, opposite current direction

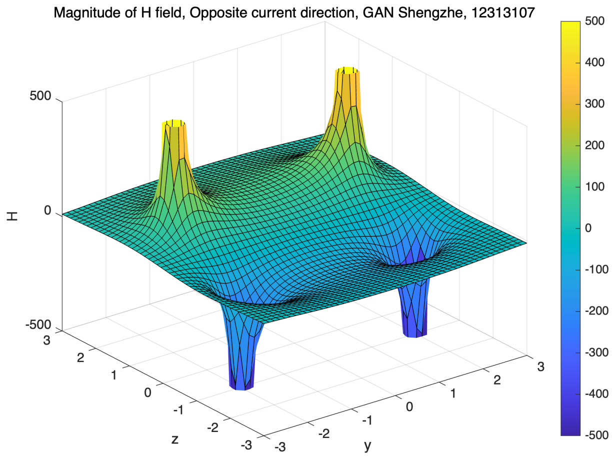

Fig. 8. Magnitude of H Field, opposite current direction

Then we take the absolute value of the H field, present the result as shown in the following diagram.

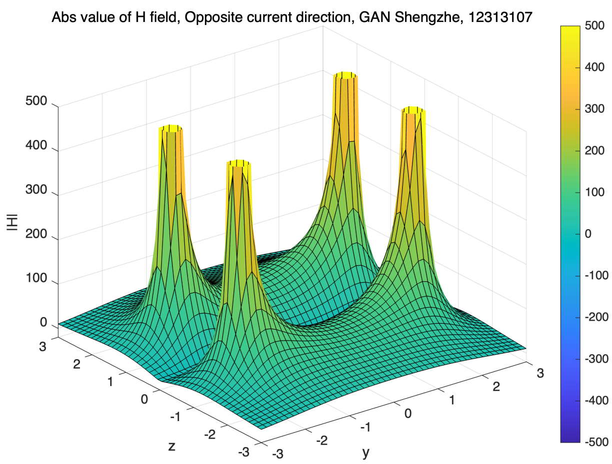

Fig. 9. Absolute value of H Field, opposite current direction

From these diagrams, we can find that there is a distinct gradient magnetic field along z axis near the origin. Or say, we obtained a 1-dimensional potential well along y axis as z approximately near zero.

VII. Conclusion

From this experiment, we've learned the way to plot the properties of the magnetic fields generated by current loop pairs, or Helmholtz Coils. And we got a better understanding and dived perspective physical diagram of this form of magnetic field.

Acknowledgments

Thanks to our professor for offering us the template and the theoretical part of the calculation.

Appendix
MATLAB Codes

This part of Matlab codes is applied for plotting the three diagrams with same direction of current.

```
clear;

a=2;
I=500;
C=I/(4*pi);
ym=3;
zm=3;

N=50;

y=linspace(-ym,ym,N);
z=linspace(-zm,zm,N);

theta0=linspace(0,2*pi,N+1);
theta1=theta0(1:N);

Hy1=zeros(N);
Hz1=zeros(N);

Hy2=zeros(N);
Hz2=zeros(N);

H1=zeros(N);
H2=zeros(N);

x11=a*cos(theta1); y11=a*sin(theta1);
theta2=theta0(2:N+1);
x12=a*cos(theta2); y12=a*sin(theta2);
zc=1; xc=(x12+x11)./2; yc=(y12+y11)./2;
dlz1=0;dlx1=x12-x11;dly1=y12-y11;

for i=1:N
    for j=1:N
    rx=0-xc; ry=y(j)-yc; rz=z(i)-zc;
    r3=sqrt(rx.^2+ry.^2+rz.^2).^3;
    dlXr_y=dlz1.*rx-dlx1.*rz;
    dlXr_z=dlx1.*ry-dly1.*rx;
    Hy1(i,j)=sum(C.*dlXr_y./r3);
    Hz1(i,j)=sum(C.*dlXr_z./r3);
    H1=(Hy1.^2+Hz1.^2).^0.5;
    end
end

x21=a*cos(theta1); y21=a*sin(theta1);
theta2=theta0(2:N+1);
x22=a*cos(theta2); y22=a*sin(theta2);
zc=-1; xc=(x22+x21)./2; yc=(y22+y21)./2;
dlz2=0;dlx2=x22-x21;dly2=y22-y21;

for i=1:N
    for j=1:N
    rx=0-xc; ry=y(j)-yc; rz=z(i)-zc;
    r3=sqrt(rx.^2+ry.^2+rz.^2).^3;
    dlXr_y=dlz2.*rx-dlx2.*rz;
    dlXr_z=dlx2.*ry-dly2.*rx;
    Hy2(i,j)=sum(C.*dlXr_y./r3);
    Hz2(i,j)=sum(C.*dlXr_z./r3);
    H2=(Hy2.^2+Hz2.^2).^0.5;
    end
end

Hy=Hy1+Hy2;
Hz=Hz1+Hz2;
H = H1+H2;
```

```

figure;
p1 = plot([-2 2], [1 1], 'LineWidth',2, 'DisplayName', 'Coil 1');
hold on
p2 = plot([-2 2], [-1 -1], 'LineWidth',2, 'DisplayName', 'Coil 2');
p3 = quiver(y,z,Hy,Hz, 'DisplayName', 'H field vector', 'Color',[0.62, 0.1,0.92]);
axis([-3,3,-3,3]);
plot(2,1,'ro',-2,1,'bo');
plot(2,-1,'ro',-2,-1,'bo');

title('H field Arrow Diagram on y-z plane, GAN Shengzhe, 12313107')
xlabel('y'),ylabel('z');
plotHandles = [p1, p2, p3];
legend(plotHandles);
exportgraphics(gca, 'quiver.png', 'Resolution', 320);


figure;
surf(y,z,H,'DisplayName','Magnitude of H Field');
clim([0,500])
colorbar()
axis([-3,3,-3,3,0,500])
xlabel('y'),ylabel('z'),zlabel('H');
title('Magnitude of H field, GAN Shengzhe, 12313107')
exportgraphics(gca, 'Magnitude.png', 'Resolution', 320);

figure;
ys = linspace(-3,3,50);
zs = zeros(1,50);
h1 = streamline(y,z,Hy,Hz,ys,zs);
hold on
h2 = streamline(y,z,-Hy,-Hz,ys,zs);
grid on
set(h1, 'Color', [0, 0.4470, 0.7410] );
set(h2, 'Color', [0, 0.4470, 0.7410] );
plot(2,1,'ro',-2,1,'bo');
plot(2,-1,'ro',-2,-1,'bo');
xlabel('y');
ylabel('z');
axis equal
axis([-3,3,-3,3])
title('Magnetic field lines, GAN Shengzhe, 12313107');
exportgraphics(gca, 'Hlines.png', 'Resolution', 320);
```

Listing 1. Theoretical plot codes, Pt.1

This part of Matlab codes is applied for plotting the four diagrams with the opposite direction of current.

```
clear;
a=2;
I=500;
C=I/(4*pi);
ym=3;
zm=3;

N=50;

y=linspace(-ym,ym,N);
z=linspace(-zm,zm,N);

theta0=linspace(0,2*pi,N+1);
theta1=theta0(1:N);

Hy1=zeros(N);
Hz1=zeros(N);

Hy2=zeros(N);
Hz2=zeros(N);

H1=zeros(N);
H2=zeros(N);


x11=a*cos(theta1); y11=a*sin(theta1);
theta2=theta0(2:N+1);
x12=a*cos(theta2); y12=a*sin(theta2);
zc=1; xc=(x12+x11)./2; yc=(y12+y11)./2;
dlz1=0;dlx1=x12-x11;dly1=y12-y11;

for i=1:N
    for j=1:N
    rx=0-xc; ry=y(j)-yc; rz=z(i)-zc;
    r3=sqrt(rx.^2+ry.^2+rz.^2).^3;
    dlXr_y=dlz1.*rx-dlx1.*rz;
    dlXr_z=dlx1.*ry-dly1.*rx;
    Hy1(i,j)=sum(C.*dlXr_y./r3);
    Hz1(i,j)=sum(C.*dlXr_z./r3);
    H1=(Hy1.^2+Hz1.^2).^0.5;
    end
end

x21=a*cos(theta1); y21=a*sin(theta1);
theta2=theta0(2:N+1);
x22=a*cos(theta2); y22=a*sin(theta2);
zc=-1; xc=(x22+x21)./2; yc=(y22+y21)./2;
dlz2=0;dlx2=x22-x21;dly2=y22-y21;

for i=1:N
    for j=1:N
    rx=0-xc; ry=y(j)-yc; rz=z(i)-zc;
    r3=sqrt(rx.^2+ry.^2+rz.^2).^3;
    dlXr_y=dlz2.*rx-dlx2.*rz;
    dlXr_z=dlx2.*ry-dly2.*rx;
    Hy2(i,j)=sum(C.*dlXr_y./r3);
    Hz2(i,j)=sum(C.*dlXr_z./r3);
    H2=(Hy2.^2+Hz2.^2).^0.5;
    end
end

Hy=Hy1-Hy2;
Hz=Hz1-Hz2;
H = H1-H2;

figure;
p1 = plot([-2 2], [1 1], 'LineWidth',2, 'DisplayName', 'Coil 1');
hold on
p2 = plot([-2 2], [-1 -1], 'LineWidth',2, 'DisplayName', 'Coil 2');
p3 = quiver(y,z,Hy,Hz, 'DisplayName', 'H field vector', 'Color',[0.62, 0.1,0.92]);
axis([-3,3,-3,3]);
plot(2,1,'ro',-2,1,'bo');
plot(2,-1,'bo',-2,-1,'ro');
title({'H field Arrow Diagram on y-z plane, Opposite current direction'},{'GAN Shengzhe, 12313107'})
xlabel('y'),ylabel('z');
```

```
plotHandles = [p1, p2, p3];
legend(plotHandles);
exportgraphics(gca, 'quiver_inverse.png', 'Resolution', 320);

figure;
surf(y,z,abs(H),'DisplayName','Magnitude of H Field');
clim([-500,500])
colorbar()
axis([-3,3,-3,3,-500,500])
xlabel('y'),ylabel('z'),zlabel('H');
title('Magnitude of H field, Opposite current direction, GAN Shengzhe, 12313107')
exportgraphics(gca, 'Magnitude_inverse.png', 'Resolution', 320);

figure;
surf(y,z,abs(H),'DisplayName','Magnitude of H Field');
clim([-500,500])
colorbar()
axis([-3,3,-3,3,-20,500])
xlabel('y'),ylabel('z'),zlabel('|H|');
title('Abs value of H field, Opposite current direction, GAN Shengzhe, 12313107')
exportgraphics(gca, 'Magnitude_inverse_abs.png', 'Resolution', 320);

figure;
ys = 1.5*ones(1,50);
zs = linspace(-3,3,50);
[ys,zs]=meshgrid(ys,zs);
[y,z]=meshgrid(y,z);
h1 = streamline(y,z,Hy,Hz,ys,zs);
hold on
h2 = streamline(y,z,-Hy,-Hz,ys,zs);
ys2 = -1.5*ones(1,50);
zs2 = linspace(-3,3,50);
[ys2,zs2]=meshgrid(ys2,zs2);
h3 = streamline(y,z,Hy,Hz,ys2,zs2);
h4 = streamline(y,z,-Hy,-Hz,ys2,zs2);
grid on
set(h1, 'Color', [0, 0.4470, 0.7410] );
set(h2, 'Color', [0, 0.4470, 0.7410] );
set(h3, 'Color', [0, 0.4470, 0.7410] );
set(h4, 'Color', [0, 0.4470, 0.7410] );
plot(2,1,'ro',-2,1,'bo');
plot(2,-1,'bo',-2,-1,'ro');
hold off
xlabel('y');
ylabel('z');
axis equal
axis([-3,3,-3,3])
title({'Magnetic field lines, Opposite current direction'},{'GAN Shengzhe, 12313107'});
exportgraphics(gca, 'Hlines_inverse.png', 'Resolution', 320)
```

Listing 2. Theoretical plot codes Pt.2

References

[1] William H. Hayt, Jr., John A. Engineering electromagnetics—8th ed.
[2] Youwei JIA., Engineering electromagnetic theory-3.
[3] Insert codes block in LaTeX https://blog.csdn.net/wxd1233/article/details/127196149.