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3. The cylindrical surface $\rho = 8$ cm contains the surface charge density, $\rho_s = 5e^{-20|z|}$ nC/m². a) What is the total amount of charge present? b) How much flux leaves the surface $\rho = 8$ cm, 1 cm < z < 5cm, 30° < φ < 90°?

a) Charge given by

$$Q = \int_V \rho_s \cdot dv = \int_0^{2\pi}\int_{-\infty}^{\infty} \rho \cdot \rho_s \cdot dz\,d\varphi$$

$$= 0.125\,nC$$

b) $\Psi = Q_{\text{enclosed in the region}} = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\int_{0.01}^{0.05} \rho_s \cdot \rho \; dz\,d\varphi = 9.45\,pC$

4. Let $\mathbf{D} = 4xy\mathbf{a}_x + 2(x^2+z^2)\mathbf{a}_y + 4yz\mathbf{a}_z$ C/m² and evaluate surface integrals to find the total charge enclosed in the rectangular parallelepiped $0 < x < 2$, $0 < y < 3$, $0 < z < 5$ m.

$$\oint \vec{D}\cdot d\vec{S} = \int_0^5\int_0^3 D_x(x=2)\,dy\,dz - \int_0^5\int_0^3 D_x(x=0)\,dy\,dz$$

$$+ \int_0^5\int_0^2 D_y(y=3)\,dx\,dz - \int_0^5\int_0^2 D_y(y=0)\,dx\,dz$$

$$+ \int_0^3\int_0^2 D_z(z=5)\,dx\,dy - \int_0^3\int_0^2 D_z(z=0)\,dx\,dy$$

$$= \int_0^5\int_0^3 4\cdot 2y\,dy\,dz + \int_0^3\int_0^2 20y\,dx\,dy$$

$$= 360\ C$$

5. An infinitely long cylindrical dielectric of radius b contains charge within its volume of density $\rho_v = a\rho^2$, where a is a constant. Find the electric field strength, $\mathbf{E}$, both inside and outside the cylinder.

Use Gauss' law

$$\oint \vec{E}\cdot d\vec{S} = \frac{Q}{\varepsilon_0}$$

Symmetry tells us $\vec{E}$ is not dependent on φ. then

$$E\cdot 2\pi\rho = \begin{cases} \frac{1}{\varepsilon_0}\int_0^\rho \rho_v \cdot 2\pi\rho\,d\rho & \rho \le b \\ \frac{1}{\varepsilon_0}\int_0^b \rho_v\cdot 2\pi\rho\,d\rho & \rho > b \end{cases} \Rightarrow E = \begin{cases} \frac{a\rho^3}{4\varepsilon_0}, & \rho \le b \\ \frac{ab^4}{4\varepsilon_0\rho}, & \rho > b \end{cases}$$