Engineering Electromagnetics Lab 2 Field of Continuously Distributed Line Charge

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***Abstract*—In this lab, we will calculate the distribution of electric field built by continuous line charge, and plot the relevant figures on MATLAB environment, and study the difference between integration and infinitesimal methods on analyzing electric field. Finally, quantitative analysis the relationship between error and segments we used in calculation.**

***Index Terms*—Electric fields, Infinitesimal methods, Numerical analysis, MATLAB.**

I. Introduction

THIS report is the second lab report of the course Engineering Electromagnetic Theory. Aiming at showcasing the usage and applicability of the numerical analysis of electric fields.

II. System Setup

The system is a segment of line distributed charge lie on the x-axis, from $(-\frac{l}{2}, 0)$ to $(\frac{l}{2}, 0)$. The charge density of the line charge is ρ_l. The figure below illustrates this setup which will be discussed later in this paper.

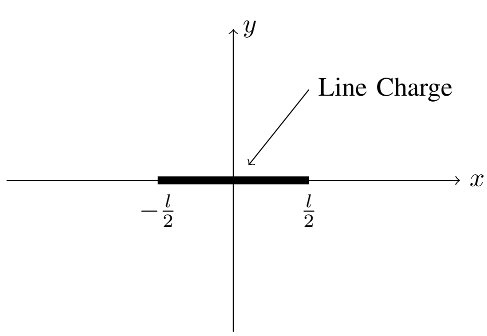

Fig. 1. System setup

In this lab, we consider that $l = 2\mathrm{cm}$, the line charge density is $\rho_l = 1 \times 10^{-9}\mathrm{C/m}$. We visualize the electric potential by equipotential lines and electric field by stream lines, both in 2-dimension coordinates, or $x - y$ plane.

III. Notations

TABLE I
NOTATIONS FOR CONSTANTS AND PHYSICAL QUANTITIES

Symbols	Description	Unit
k	electrostatic constant	$N \cdot m^2/C^2$
V	E-Potential	V
Q	electric charge of a point charge	C
Q_i	electric charge of a point charge i	C
ρ_l	line charge density	C/m
l	length of the line charge	m
$\mathbf{E}$	E-Field	V/m
$\mathbf{R}$	displacement	m
$\mathbf{a_R}$	unit vector on direction of $\mathbf{R}$	1
$\mathbf{R}_i$	displacement of i^{th} charge	m
s_i	seperation of point charges	m
ΔX	Grid spacing of numerical calculation	m
ΔY	Grid spacing of numerical calculation	m

IV. Theoretical Calculation Method

Theoretically, we can use calculation to compute the potential distribution in space, the take negative gradient of potential will obtain electric field.

Then we calculate electric potential at field point (X_0, Y_0).

$$V(X_0, Y_0) = \int_{-\frac{l}{2}}^{\frac{l}{2}} k \cdot \frac{\rho_l \cdot \mathrm{d}x}{|\mathbf{R}|} \tag{1}$$

where

$$|\mathbf{R}| = \sqrt{(X_0 - x)^2 + Y_0^2} \tag{2}$$

Then, we evaluate the integral, substituting $l = 2$.

$$V(X_0, Y_0) = k\rho_l \ln\left(\frac{1 - X_0 + \sqrt{(1 - X_0)^2 + Y_0^2}}{-1 - X_0 + \sqrt{(1 + X_0)^2 + Y_0^2}}\right) \tag{3}$$

Then according to the relationship between $\mathbf{E}$ and V

$$\mathbf{E} = -\nabla V \tag{4}$$

Thus, after theoretical calculation, we numerically present this result by Matlab.

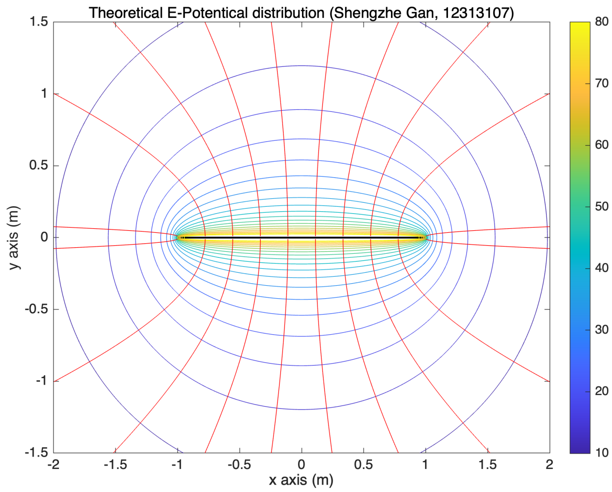

Fig. 2. Theoretical Calculation, black line represents the line charge

V. Infinitesimal Approximation Method

In this method, we consider use a finite number of single point charges to represent the line distributed charge.

E-field excited by single point charge is

$$\mathbf{E} = k\frac{Q}{|\mathbf{R}|^2}\mathbf{a_R} \tag{5}$$

Thus, considering a series of point charges, of amount N. These charge series is located on the position of the line charge, uniformly distributed, and the total charge is equal to the line charge.

$$\rho_l \cdot l = \sum_{i=1}^{N} Q_i \tag{6}$$

$$l = \sum_{i=1}^{N} s_i \tag{7}$$

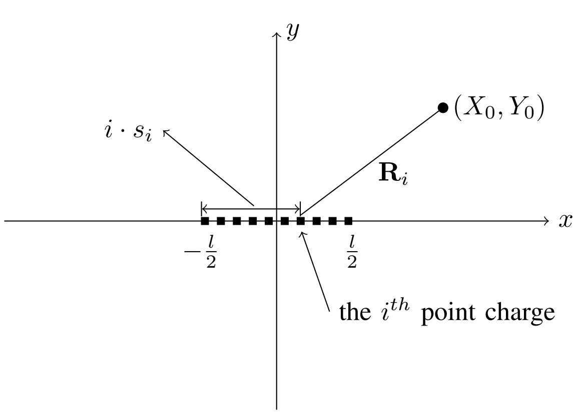

Fig. 3. System setup

Thus, for field point (X_0, Y_0), we evaluate the summation of E-potential excited by all separate charges.

$$\begin{aligned} V(X_0, Y_0) &= \sum_{i=1}^{N} k\frac{Q_i}{|\mathbf{R}_i|} \\ &= \sum_{i=1}^{N} \frac{kQ_i}{\sqrt{(X_0 + s_i \cdot i - l/2)^2 + Y_0^2}} \end{aligned} \tag{8}$$

Then, we replace differentiation with differential.

$$\begin{aligned} E_x(X_0, Y_0) &= -(V(X_0 + \Delta X) - V(X_0)) \\ E_y(X_0, Y_0) &= -(V(Y_0 + \Delta Y) - V(Y_0)) \end{aligned} \tag{9}$$

Therefore, we obtained the potential distribution and E-field distribution in the space by sum up a series of single charge points.

For comparison, we take different separation segments and visualize the result we obtained by the method introduced above.

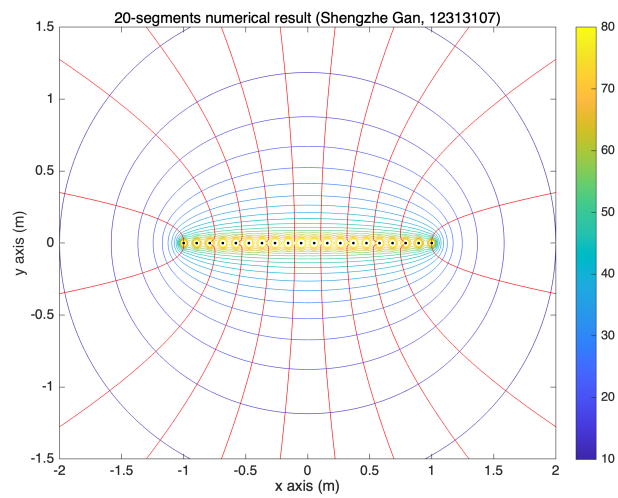

Fig. 4. 20 segments approximation, black dots represents point charges

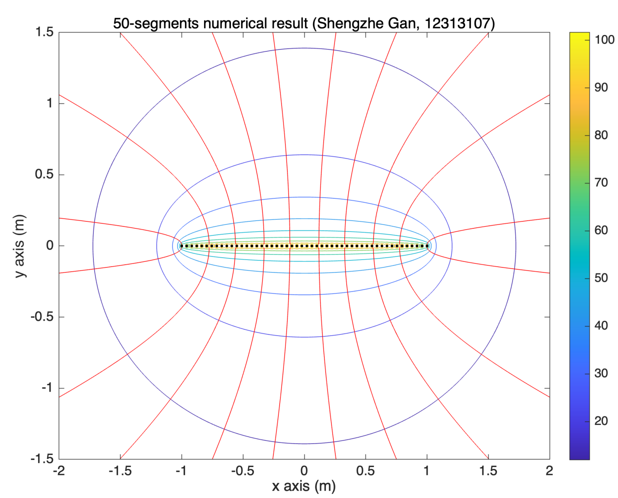

Fig. 5. 50 segments approximation, black dots represents point charges

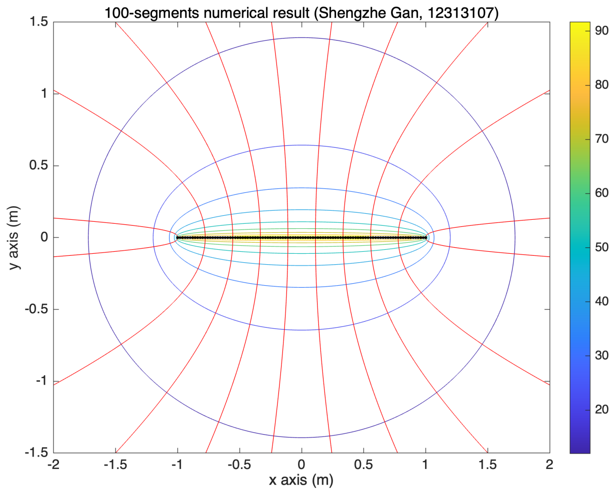

Fig. 6. 100 segments approximation, black dots represents point charges

From these images, we can find that the field distribution is approaching to the theoretically computed result as the separation segments increases.

VI. Method Evaluation

To quantitatively evaluate the error of the infinitesimal approximation and the theoretical calculation, we examine the difference of the potential calculated by both methods at all field points.

$$\text{Difference}(X_0, Y_0) = V_{\text{Theoretical}} - V_{\text{Numerical}} \tag{10}$$

Then, we visualize the result by heat map.

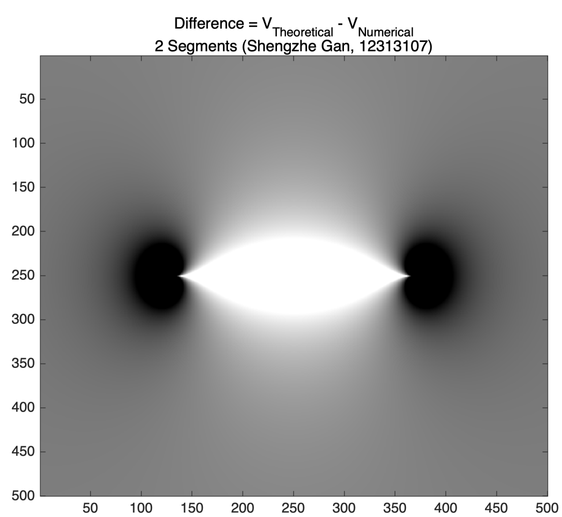

Fig. 7. 2 segments, it shows that huge deviation.

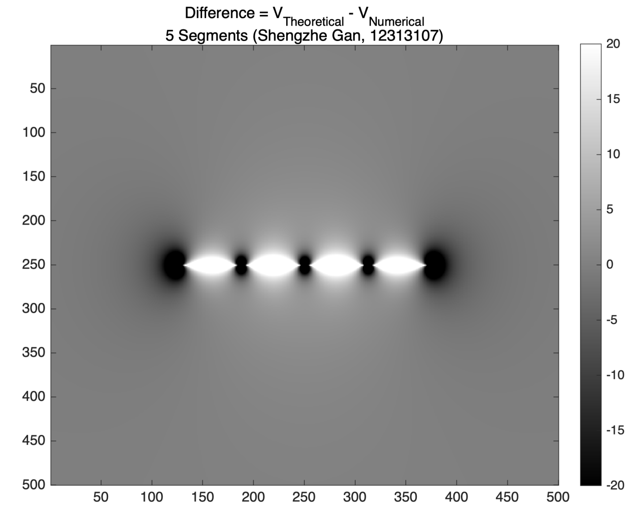

Fig. 8. 5 segments, the deviation becomes smaller, but still distinct.

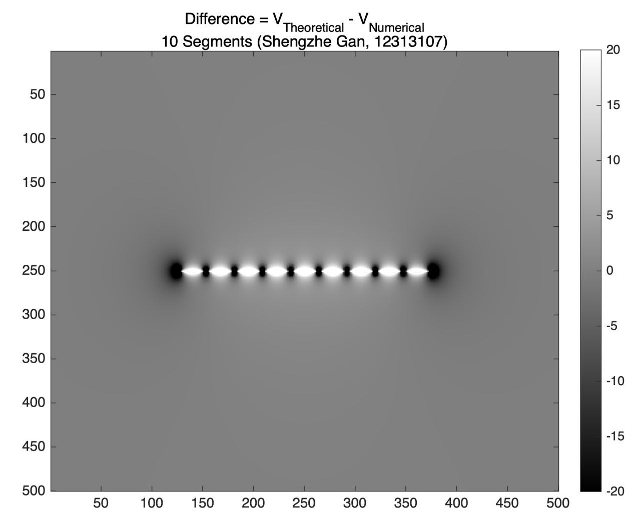

Fig. 9. 10 segments, not that distinctness.

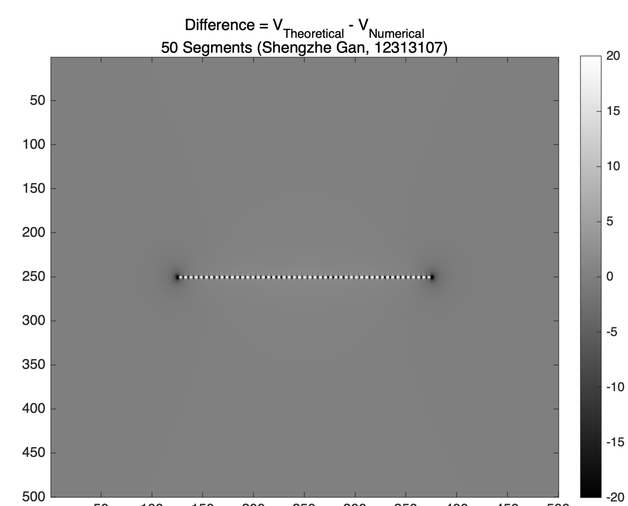

Fig. 10. 50 segments, much more smaller, almost neglectable.

From these images, we can intuitively find that as the number of segments increases, the deviation of the potential becomes smaller and smaller. To quantitatively analysis this phenomenon, we introduce the root mean squared value of the potential difference.

$$\text{RMS} = \sqrt{\sum_{\substack{i=1\\j=1}}^{K} \frac{\text{Difference}^2(X_i, Y_i)}{N^2}} \tag{11}$$

where K is the number of mesh grids in x and y axis.

Then for different separation segments, we plot the $\log \text{RMS}$-segments curve. The reason to take common logarithm is to better illustrate the change.

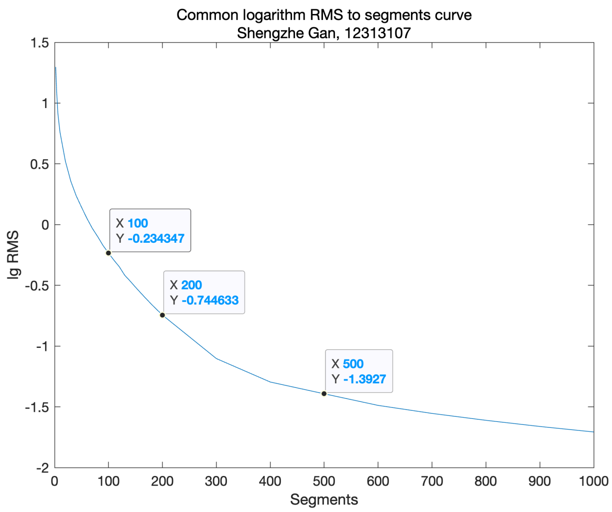

Fig. 11. Relation between $\log \text{RMS}$ and segments

From the figure, we can find that as the segments is 100, the error is reduced to about $0.58Volts$, and as the segments reached to 500, the deviation is about $0.041Volts$. Thus, the error is sufficiently small enough that we can apply this infinitesimal approximation by specific separation segments, which can be determined by real scenario we are facing at.

VII. Conclusion

To be concluded, we use both integration method and infinitesimal method to calculate the electric distribution of a continuous line charge in the 2-dimensional coordinates. We adjust the number of separation segments in infinitesimal method to analyze the differences between the results, and draw the conclusion that the error decreases rapidly with increasing number of segments, and finally it is negligible as the separation segments is large enough.

Thus, it is effective to use the infinitesimal method in the electric field analysis, where the continuous charge is transferred into a sufficient number of small segments and considered as a set of separate point charges to obtain a result very similar to the analytical result. This sufficient number of segments is not too large and relevant to the specific situation we are facing to solve.

Acknowledgments

Thanks to our professor for offering us the template and the theoretical part of the integration method.

In this lab, the instructor gave us almost one month to finish this formal scientific paper. Thanks to this chance which greatly helped me to dive into the formal paper composing.

APPENDIX
MATLAB CODES

This part of Matlab codes is applied for plotting the theoretical potential distribution by equipotential lines and E-field distribution by stream lines.

```matlab
% Theoretical model

%% Figure 1
clear; clc;
k = 9e9;
rho = 1e-9;

% define the meshgrid for the scenario
xm = 2;
ym = 1.5;
x = linspace(-xm,xm,200);
y = linspace(-ym,ym,200);

[X,Y] = meshgrid(x,y);

% According to the theoretical calculation, we can obtain the potential

V = k * rho * log( (1-X+((1-X).^2+Y.^2).^0.5) ./ (-1-X+((-1-X).^2+Y.^2).^0.5) );
Vmin = 10;
Vmax = 80;
Veq = linspace(Vmin,Vmax,20);

[Ex,Ey]=gradient(-V);

[start_x, start_y] = meshgrid(linspace(-1,1,10), 0.01*ones(1,10));
[start_x1, start_y1] = meshgrid(linspace(-1,1,10), -0.01*ones(1,10));

figure;
plot([-1,1],[0, 0],'k', 'LineWidth', 1.5, 'DisplayName', 'Charge')
hold on

colorbar();
contour(X,Y,V,Veq, 'DisplayName', 'Equipotential line');
hold on
h = streamline(X,Y,Ex,Ey,start_x, start_y);
h1 = streamline(X,Y,Ex,Ey,start_x1, start_y1);
set(h, 'Color', 'r');
set(h1, 'Color', 'r');

title('Theoretical E-Potential distribution (Shengzhe Gan, 12313107)');
xlabel('x axis (m)');
ylabel('y axis (m)');
exportgraphics(gca, 'theoretical.png', 'Resolution', 320);
```

Listing 1. Theoretical plot codes

This part of Matlab codes is applied for plotting thepotential distribution by equipotential lines and E-field distribution by stream lines calculated by infinitesimal approximation method. Notice that with different separation segments, just adjust the variable `num_of_segments`

```matlab
l = 2;
total_charge = rho * l;

% divide the line distribuded charge in to an array of segments
num_of_segments = 20;
Qi = total_charge / num_of_segments;
position_of_point_charge = linspace(-1,1, num_of_segments);

% define the meshgrid for the scenario
xm = 2;
ym = 1.5;
x = linspace(-xm,xm,500);
y = linspace(-ym,ym,500);

[X,Y] = meshgrid(x,y);

% Then obtain the potential by summation
R_inverse = zeros(length(X),length(Y));
for i = 1 : num_of_segments
    R_inverse = R_inverse + 1./sqrt((X-position_of_point_charge(i)).^2+Y.^2);
end

V = k * Qi * R_inverse;
Vmin = 10;
Vmax = 80;
Veq = linspace(Vmin,Vmax,20);

[Ex,Ey]=gradient(-V);

[start_x, start_y] = meshgrid(linspace(-1,1,10), 0.01*ones(1,10));
[start_x1, start_y1] = meshgrid(linspace(-1,1,10), -0.01*ones(1,10));

figure;
c = contour(X,Y,V,Veq);
hold on
h = streamline(X,Y,Ex,Ey,start_x, start_y);
h1 = streamline(X,Y,Ex,Ey,start_x1, start_y1);
set(h, 'Color', 'r');
set(h1, 'Color', 'r', 'DisplayName', 'Stream line');

scatter(linspace(-1,1,num_of_segments),zeros(1,num_of_segments),'k','Marker','.', 'DisplayName', 'Charge')
colorbar();

% legend;
title('20-segments numerical result (Shengzhe Gan, 12313107)');
xlabel('x axis (m)');
ylabel('y axis (m)');
exportgraphics(gca, 'infinitesimal.png', 'Resolution', 320);
```

Listing 2. Infinitesimal plot codes

This part of Matlab codes is applied for plotting the difference of the potential calculated by both methods at all field points.

```matlab
k = 9e9;
rho = 1e-9;
l = 2;
total_charge = rho * l;

segs = [2 5 10 20 50];

```

```
% define the meshgrid for the scenario
xm = 2;
ym = 1.5;
x = linspace(-xm,xm,500);
y = linspace(-ym,ym,500);

[X,Y] = meshgrid(x,y);

% The tensor to save the results
V_layer = zeros(500,500,length(segs));

n = 0;

% calculate all numerical value
for num_of_segments = segs
    n = n + 1;
    Qi = total_charge / num_of_segments;
    position_of_point_charge = linspace(-1,1,num_of_segments);

    R_inverse = zeros(length(X),length(Y));
    for i = 1 : num_of_segments
        R_inverse = R_inverse + 1./sqrt((X-position_of_point_charge(i)).^2+Y.^2);
    end
    V_layer(:,:,n) = k * Qi * R_inverse;
end

% calculate the theoretical value
V_theory = k * rho * log((1-X+((1-X).^2+Y.^2).^0.5) ./ (-1-X+((-1-X).^2+Y.^2).^0.5));


for i = 1:length(segs)
    V1 = V_theory-V_layer(:,:,i);
    figure;
    imagesc(V1);
    colormap(gray);
    colorbar();
    clim([-20,20]);
    title({'Difference = V_{Theoretical} - V_{Numerical}',sprintf('%d Segments (Shengzhe Gan, 12313107)', segs(i))})
    exportgraphics(gca, sprintf('%d.png', segs(i)), 'Resolution', 300);
end
```

Listing 3. Difference potential value plot codes

In this part of Matlab codes, I plotted the RMS value. Notice that this part of codes are run after the former part, with a change of the `segs` vector, seen in the second line below.

```
% Change the segs as follows.
segs = [2:2:8 10:10:200 300:100:1000];

RMS = -1*ones(1,length(segs));

for i = 1:length(segs)
    Vd = V_theory-V_layer(:,:,i);
    V_rms = Vd.^2;
    RMS(i) = sqrt(sum(V_rms(:))/25e4);
end

figure;
plot(segs,log10(RMS))
title({'Common logarithm RMS to segments curve','Shengzhe Gan, 12313107'});
xlabel('Segments');
ylabel('lg RMS');

% Mark down the tips, generate automatically by Matlab
hDataTip = findobj(gca,"DataIndex",25);
delete(hDataTip)
hDataTip = findobj(gca,"DataIndex",28);
delete(hDataTip)
ax2 = gca;
chart2 = ax2.Children(1);
datatip(chart2,200,-0.7446);
datatip(chart2,500,-1.393);
datatip(chart2,100,-0.2343);

exportgraphics(gca, 'RMS_curve.png', 'Resolution', 300);
```

Listing 4. RMS value plotting

References

[1] William H. Hayt, Jr., John A. Engineering electromagnetics—8th ed.
[2] Youwei JIA., Engineering electromagnetic theory-2
[3] Insert codes block in LaTeX https://blog.csdn.net/wxd1233/article/details/127196149.