Assignment for Chapter 9

2. Consider a coaxial capacitor having inner radius a, outer radius b, unit length, and filled with a material with dielectric constant, ϵ_r. Compare this to a parallel-plate capacitor having plate width, w, plate separation d, filled with the same dielectric, and having unit length. Express the ratio b/a in terms of the ratio d/w, such that the two structures will store the same energy. (Please also state the reasons!)

For parallel-plate capacitor

$$C = \frac{\varepsilon_0 \varepsilon_r w}{d} = \frac{\varepsilon_0 \varepsilon_r}{d/w}$$

For the coaxial capacitor

$$E = \frac{q_l}{2\pi \varepsilon_0 r}$$

The potential difference between the inner and outer conductor is

$$\Rightarrow \quad V = \frac{q_l}{2\pi \varepsilon_0} \ln \frac{b}{a}$$

Capacitor

$$C = \frac{\varepsilon_r q_l}{V} = \frac{2\pi \varepsilon_0 \varepsilon_r}{\ln b/a}$$

As they store same amount of energy

(1) With same amount of charge or potential difference

$$\frac{1}{2}\frac{Q^2}{C_1} = \frac{1}{2}\frac{Q^2}{C_2} \quad \text{or} \quad \frac{1}{2}C_1 V^2 = \frac{1}{2}C_2 V^2$$

$$\Rightarrow C_1 = C_2 \Rightarrow \frac{\varepsilon_0 \varepsilon_r}{d/w} = \frac{2\pi \varepsilon_0 \varepsilon_r}{\ln b/a}$$

$$\Rightarrow \frac{b}{a} = \exp\left(2\pi \frac{d}{w}\right)$$

3. Let $S = 100\text{mm}^2$, $d = 3$ mm, and $\epsilon_r = 12$ for a parallel-plate capacitor. a) Calculate the capacitance; b) After connecting a 6 V battery across the capacitor, calculate **E**, **D**, Q, and the total stored electrostatic energy; c) With the source still connected, the dielectric is carefully withdrawn from between the plates. With the dielectric gone, re-calculate **E**, **D**, Q, and the energy stored in the capacitor. d) If the charge and energy found in (c) are less than that found in (b) (which you should have discovered), what became of the missing charge and energy?

a) $C = \frac{\varepsilon_0 \varepsilon_r S}{d} = 3.54\ \text{pF}$

b) Neglect boundary effect

$$E = \frac{V}{d} = 2\ \text{kV/m}$$

homogenous dieletric

$$D = \varepsilon_0 \varepsilon_r E = 2.125 \times 10^{-7}\ \text{C/m}$$

$$Q = CV = 2.125 \times 10^{-11}\ \text{C}$$

$$\mathcal{E} = \frac{1}{2} C V^2 = 6.372 \times 10^{-11}\ \text{Joule}$$

c) E remains the same, but

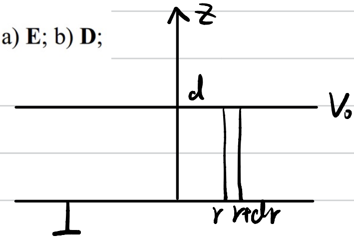
$$D' = \frac{D}{\varepsilon_r} = 0.177 \times 10^{-7}\ \text{C/m}$$

$$Q' = C'V = \frac{C}{\varepsilon_r} V = \frac{Q}{\varepsilon_r} = 0.177 \times 10^{-11}\ \text{C}$$

$$\mathcal{E}' = \frac{1}{2} C' V^2 = \frac{\mathcal{E}}{\varepsilon_r} = 0.531 \times 10^{-11}\ \text{Joule}$$

d) The charge are carried by the battery and vanished with the negetive-polar charge on the other plate of the capacitor, The missing energy are transformed into heat dispitated by the battery

4. A parallel-plate capacitor is made using two circular plates of radius a, with the bottom plate on the xy plane, centered at the origin. The top plate is located at $z = d$, with its center on the z axis. Potential V_0 is on the top plate; the bottom plate is grounded. Dielectric having radially-dependent permittivity fills the region between plates. The permittivity is given by $\epsilon(\rho) = \epsilon_0(1+\rho^2/a^2)$. Find: a) **E**; b) **D**; c) Q; d) C.



a) E is a constant

$$\vec{E} = -\frac{V_0}{d}\vec{a}_z$$

b) $\vec{D}$ is reletive to radius ρ

$$\vec{D}(r) = \varepsilon_0 \varepsilon_r(\rho)\vec{E} = -\varepsilon_0\left(1+\frac{\rho^2}{a^2}\right)\frac{V_0}{d}\vec{a}_z$$

c)

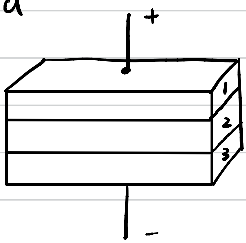
$$Q = \int_0^a -\vec{D}\cdot\vec{a}_z \cdot 2\pi\rho\, d\rho = \frac{3}{2}\pi\frac{\varepsilon_0 a^2 V_0}{d}$$

d)

$$C = \frac{Q}{V_0} = \frac{3\pi\varepsilon_0 a^2}{2d}$$

5. Two conducting plates, each 3 by 6 cm, and three slabs of dielectric, each 1 by 3 by 6 cm, and having dielectric constants of 1, 2, and 3 are assembled into a capacitor with $d = 3$ cm. Determine the two values of capacitance obtained by the two possible methods of assembling the capacitors.

First method



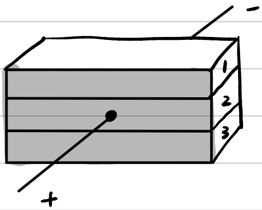
3-Capacitor connected in series

$$\frac{1}{C_1} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\Rightarrow C_1 = \frac{6}{11}\frac{\varepsilon_0 S}{d_i} = 0.87\ \text{pF}$$

Second method

3-Capacitor connecter in parallel



$$C_2 = C_1 + C_2 + C_3$$

$$\Rightarrow C_2 = \frac{6\varepsilon_0 S'}{d_i'} = 1.06\,pF$$

6. A potential field in free space is given in spherical coordinates as:

$$V(r) = \begin{cases} [\rho_0/(6\epsilon_0)]\,[3a^2 - r^2] & (r \le a) \\ (a^3\rho_0)/(3\epsilon_0 r) & (r \ge a) \end{cases}$$

where ρ_0 and a are constants. a) Use Poisson's equation to find the volume charge density everywhere; b) find the total charge present.

a) poisson's equation

$$\rho = -\nabla^2 V(r) \cdot \varepsilon_0$$

Where

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}$$

plugging in, we obtain

$$\rho(r) = \begin{cases} \rho_0, & r < a \\ 0, & r \geq a \end{cases}$$

Total charge

$$Q = \rho_0 \cdot \frac{4}{3}\pi a^3 = \frac{4}{3}\pi a^3 \rho_0$$