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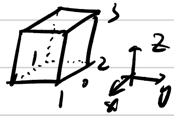
2. Given the current density $\mathbf{J} = -10^4[\sin(2x)e^{-2y}\mathbf{a}_x + \cos(2x)e^{-2y}\mathbf{a}_y]\ \mathrm{kA/m^2}$: a) Find the total current crossing the plane $y = 1$ in the a_y direction in the region $0<x<1$, $0<z<2$; b) Find the total current leaving the region $0<x<1$, $0<y<1$, $2<z<3$ by integrating $\mathbf{J}\cdot d\mathbf{S}$ over the surface of the cube; c) Repeat part b, but use the divergence theorem.

(a) Total current



$$I = \iint \vec{J}(y=1)\cdot \vec{a}_y\, dx\, dz$$

$$= \int_0^2\int_0^1 -10^4\cdot \cos(2x) e^{-2}\, dx\, dz$$

$$= -e^{-2}\cdot 10^4\cdot \sin(2) = -1.23\times 10^3\ kA$$

(b) Total current leaving

$$I = \iint \vec{J}\cdot d\vec{S} = \iint \left(J_x(x=1) - J_x(x=0)\right) dy\, dz$$

$$+ \iint \left(J_z(z=3) - J_z(z=2)\right) dx\, dy$$

$$+ \iint \left(J_y(y=1) - J_y(y=0)\right) dx\, dz$$

$$= -10^4\cdot\left[\int_2^3\int_0^1 \sin(2) e^{-2y}\, dy\, dz + \int_2^3\int_0^1 \cos(2x)(e^{-2}-1)\, dx\, dz\right]$$

$$= -10^4\cdot\left(\frac{1-e^{-2}}{2}\sin(2) + \frac{\sin(2)}{2}\cdot(e^{-2}-1)\right)$$

$$= 0$$

(c) By divergence theorem

$$\nabla\cdot\vec{J} = \frac{\partial}{\partial x}J_x + \frac{\partial}{\partial y}J_y + \frac{\partial}{\partial z}J_z$$

$$= -10^4\left(2\cos(2x)e^{-2y} - 2\cos(2x)e^{-2y}\right) = 0$$

Thus, $I_{out} = \iiint \nabla\cdot\vec{J}\, dV = 0$

4. Two perfectly-conducting cylindrical surfaces of length l are located at $\rho = 3$ cm and $\rho = 5$ cm. The total current passing radially outward through the medium between the cylinders is 3 A dc (direct current, 直流电流). Find the voltage and resistance between the cylinders, and **E** in the region between the cylinders, if a conducting material having $\sigma = 0.05$ S/m is present for $3 < \rho < 5$ cm.

As $3 < \rho < 5$

$$I = 3A = j \cdot S = 2\pi\rho l \cdot j$$

$$\Rightarrow \vec{J}(\rho) = \frac{3}{2\pi\rho L}\vec{a}_\rho$$

Thus

$$\vec{E} = \frac{\vec{J}(\rho)}{\sigma} = \frac{3}{2\pi L\sigma}\frac{1}{\rho}\vec{a}_\rho$$

Then the voltage given by

$$V = \int_{\rho_1}^{\rho_2} \vec{E} \cdot d\rho = \frac{3}{2\pi l\sigma}\ln\frac{\rho_2}{\rho_1}$$

Resistance

$$R = \int_{\rho_1}^{\rho_2} \frac{d\rho}{\sigma \cdot 2\pi\rho l} = \frac{1}{2\pi\sigma l}\ln\frac{\rho_2}{\rho_1}$$

Numerically

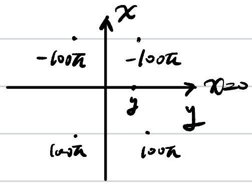
$$V = \frac{4.878}{L} \text{ Volts}$$

$$\vec{E}(\rho) = \frac{9.549}{L}\frac{1}{\rho}\vec{a}_\rho$$

$$R = \frac{1.626}{l}\ \Omega$$

5. Two point charges of -100π μC are located at (2,-1,0) and (2,1,0). The surface x=0 is a conducting plane. a) Determine the surface charge density at the origin; b) Determine ρ_S at $P(0, h, 0)$.

Use charge mirroring method. Two mirrored charge is shown on the figure.
At point $(0, y, 0)$, E-Field given by



$$\vec{E}\cdot\vec{a}_x = \frac{q_0}{4\pi\varepsilon_0}\left(\frac{2}{[(y+1)^2+2^2]^{1.5}} + \frac{2}{[(y-1)^2+2^2]^{1.5}} + \frac{2}{[(y+1)^2+2^2]^{1.5}} + \frac{2}{[(y-1)^2+2^2]^{1.5}}\right)$$

where $q_0 = 100\pi\ \mu C$

Due to the E-field above the conductor plane is

$$E_\perp = \frac{\rho_s}{\varepsilon_0}$$

Thus

$$\rho_s(h) = \frac{q_0}{\pi}\left(\frac{1}{[(y+1)^2+4]^{3/2}} + \frac{1}{[(y-1)^2+4]^{3/2}}\right)\Bigg|_{y=h}$$

$$= \left(\frac{10^{-4}}{[(h+1)^2+4]^{3/2}} + \frac{10^{-4}}{[(h-1)^2+4]^{3/2}}\right)\ C/m^2$$

At $h=0$,

$$\rho_s(0) = 1.79\times10^{-5}\ C/m^2$$

6. Atomic hydrogen contains 5.5×10^{25} atoms/m^3 at a certain temperature and pressure. When an electric field of 4 kV/m is applied, each dipole formed by the electron and positive nucleus has an effective length of 7.1×10^{-19} m. a) Find **P** (Dipole moment per unit volume); b) Find the relative permittivity ϵ_r. 提示：氢原子含有一个质子和一个电子。质子、电子的电荷量在第二次课件中可以找到。

a) dipole moment

$$p = ed = 1.1375 \times 10^{-37} \ C\cdot m$$

Then

$$\vec{P} = \Sigma \vec{p} = \vec{p}\cdot N = -6.256 \times 10^{-12} \ \vec{a}_E \ C/m^2$$

b) Due to

$$\vec{P} = \varepsilon_0(\varepsilon_r - 1)\vec{E}$$

$$\Rightarrow \varepsilon_r = \frac{|\vec{P}|}{|\vec{E}|}\frac{1}{\varepsilon_0} + 1 = 1 + 1.766 \times 10^{-4}$$