离散(下)群码作业错题讲解

2018-10-16

编码11.1 16,18,20,26@412

16. Consider the (3, 9) encoding function e.

$$\begin{aligned} e(000) &= 000000000 & e(100) &= 010011010 \\ e(001) &= 011100101 & e(101) &= 111101011 \\ e(010) &= 010101000 & e(110) &= 001011000 \\ e(011) &= 110010001 & e(111) &= 110000111 \end{aligned}$$

(a) Find the minimum distance of e.

(b) How many errors will e detect?

a) minimum of distance of e is 3.
b) e can detect 2 or fewer errors.

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18. Show that the (3, 7) encoding function $e: B^3 \to B^7$ defined by

$$
\begin{array}{ll}
e(000) = 0000000 & e(100) = 1000101 \\
e(001) = 0010110 & e(101) = 1010011 \\
e(010) = 0101000 & e(110) = 1101101 \\
e(011) = 0111110 & e(111) = 1111011
\end{array}
$$

is a group code.

(1) e of B^7 is e(000). (2) $\forall a \in e(B^3)$, $a \oplus a = e$, reverse exist.

(3) list multiplication table, $(e(B^3), \oplus)$ is closure.

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⊕	e(000)	e(001)	e(010)	e(011)	e(100)	e(101)	e(110)	e(111)
e(000)	e(000)	e(001)	e(010)	e(011)	e(100)	e(101)	e(110)	e(111)
e(001)	e(001)	e(000)	e(011)	e(010)	e(101)	e(100)	e(111)	e(110)
e(010)	e(010)	e(011)	e(000)	e(001)	e(110)	e(111)	e(100)	e(101)
e(011)	e(011)	e(010)	e(001)	e(000)	e(111)	e(110)	e(101)	e(100)
e(100)	e(100)	e(101)	e(110)	e(111)	e(000)	e(001)	e(010)	e(011)
e(101)	e(101)	e(100)	e(111)	e(110)	e(001)	e(000)	e(011)	e(010)
e(110)	e(110)	e(111)	e(100)	e(101)	e(010)	e(011)	e(000)	e(001)
e(111)	e(111)	e(110)	e(101)	e(100)	e(011)	e(010)	e(001)	e(000)

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20. Find the minimum distance of the group code defined in Exercise 18.

The minimum distance of the e(B^3) is 2.

26. Let

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

be a parity check matrix. Determine the (3, 6) group code $e_H : B^3 \to B^6$.

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26. $$\begin{bmatrix} 000 \\ 001 \\ 010 \\ 011 \\ 100 \\ 101 \\ 110 \\ 111 \end{bmatrix} * \begin{bmatrix} 100100 \\ 010011 \\ 001111 \end{bmatrix} = \begin{bmatrix} 000000 \\ 001111 \\ 010011 \\ 011100 \\ 100100 \\ 101011 \\ 110111 \\ 111000 \end{bmatrix}$$

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8. e is the encoding function in Exercise 16 of Section 11.1.

16. Consider the (3, 9) encoding function e.

$$
\begin{aligned}
e(000) &= 000000000 & \qquad e(100) &= 010011010 \\
e(001) &= 011100101 & \qquad e(101) &= 111101011 \\
e(010) &= 010101000 & \qquad e(110) &= 001011000 \\
e(011) &= 110010001 & \qquad e(111) &= 110000111
\end{aligned}
$$

In Exercises 5 through 10, let e be the indicated encoding function and let d be an associated maximum likelihood decoding function. Determine the number of errors that (e, d) will correct.

(e,d) can correct 1 error.

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10. e is the encoding function in Exercise 18 of Section 11.1.

18. Show that the (3, 7) encoding function $e\colon B^3 \to B^7$ defined by

$$e(000) = 0000000 \qquad e(100) = 1000101$$
$$e(001) = 0010110 \qquad e(101) = 1010011$$
$$e(010) = 0101000 \qquad e(110) = 1101101$$
$$e(011) = 0111110 \qquad e(111) = 1111011$$

In Exercises 5 through 10, let e be the indicated encoding function and let d be an associated maximum likelihood decoding function. Determine the number of errors that (e, d) will correct.

(e,d) can correct 0 error.

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13. Consider the (3, 5) group encoding function $e : B^3 \to B^5$ defined by

$$e(000) = 00000 \qquad e(100) = 10011$$
$$e(001) = 00110 \qquad e(101) = 10101$$
$$e(010) = 01001 \qquad e(110) = 11010$$
$$e(011) = 01111 \qquad e(111) = 11100.$$

Decode the following words relative to a maximum likelihood decoding function.

(a) 11001 (b) 01010 (c) 00111

(a) min{δ(11001,e(b))}=
δ(11001,01001)=1,
d(11001)=010.

(b) min{δ(01010,e(b))}=
δ(01010,11010) or
δ(01010,01001) =1,
d(01010)=110 or 010.

(c) min{δ(00111,e(b))}=
δ(00111,01111) or
δ(00111,00110) =1,
d(00111)=011 or 001.

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In Exercises 16 through 18, determine the coset leaders for $N = e_H(B^m)$ for the given parity check matrix **H**.

18. $\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

m=3,r=3,n=6.

$$\begin{matrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{matrix}$$

$a_4 = a_1 \oplus a_2$

$a_5 = a_2 \oplus a_3$

$a_6 = a_3$

$$\begin{bmatrix} 000 \\ 001 \\ 010 \\ 011 \\ 100 \\ 101 \\ 110 \\ 111 \end{bmatrix} * \begin{bmatrix} 100100 \\ 010110 \\ 001011 \end{bmatrix} = \begin{bmatrix} 000000 \\ 001011 \\ 010110 \\ 011101 \\ 100100 \\ 101111 \\ 110010 \\ 111001 \end{bmatrix}$$

对!

000000	001011	010110	011101	100100	101111	110010	111001
000001	001010	010111	011100	100101	101110	110011	111000
000010	001001	010100	010011	100110	101101	110000	111011
000100	001111	010010	011001	100000	101011	110110	111101
001000	000011	011110	010101	101100	100111	111010	110001
010000	011011	000110	001101	110100	111111	100010	101001
000101	001110	010011	011000	100001	101010	110111	111100
001100	000111	011010	010001	101000	100011	111110	110101

这个有3

取不在上面的任意一个

coser leader={000000,000001,000010,000100,001000,010000,

000101 or 011000 or 100001, 001100 or 010001 or 101000}

共7个

陪集头

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In Exercises 19 through 21, compute the syndrome for each coset leader found in the specified exercise.

21. Exercise 18.

a3

$$\varepsilon * H = \begin{bmatrix} 000000 \\ 000001 \\ 000010 \\ 000100 \\ 001000 \\ 010000 \\ 000101 \\ 001100 \end{bmatrix} * \begin{bmatrix} 100 \\ 110 \\ 011 \\ 100 \\ 010 \\ 001 \end{bmatrix} = \begin{bmatrix} 000 \\ 001 \\ 010 \\ 100 \\ 011 \\ 110 \\ 101 \\ 111 \end{bmatrix}$$

检验子

000100

100 100

{000,001,010,100,011,110,101,111}

陪集头的检验子

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23. Let

$$\mathbf{H} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

be a parity check matrix. Decode the following words relative to a maximum likelihood decoding function associated with e_H.

01

(a) 10100 (b) 01101 (c) 11011

他们就是

00000	01101	10011	11110
00001	01100	10010	11111
00010	01111	10001	11100
00100	01001	10111	11010
01000	00101	11011	10110
10000	11101	00011	01110
00110	01011	10101	11000
01010	00111	11001	10100

瞎写的

这两个就是

$a_3 = a_2$

$a_4 = a_1$

$a_5 = a_1 \oplus a_2$

m=2, r=3,n=5.

直接相乘!

太麻烦了

$$B^2 * H = \begin{bmatrix} 00 \\ 01 \\ 10 \\ 11 \end{bmatrix} * \begin{bmatrix} 10011 \\ 01101 \end{bmatrix} = \begin{bmatrix} 00000 \\ 01101 \\ 10011 \\ 11110 \end{bmatrix}$$

$B^2 * H$ 就行

$10100 * H$

$$\begin{matrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{matrix}$$

陪集头 {00000, 01101, 10011, 11110}

- (a) (10100)*H=(111), => ε = 01010
- xt⊕ε=11110, => 11.

  (b) (01101)*H=(000),=> ε = 00000
- xt⊕ε=01101, => 01.

  (c) (11011)*H=(101),=> ε = 01000
- xt⊕ε=10011, =>10.

$$\varepsilon * H = \begin{bmatrix} 00000 \\ 00001 \\ 00010 \\ 00100 \\ 01000 \\ 10000 \\ 00110 \\ 01010 \end{bmatrix} * \begin{bmatrix} 011 \\ 101 \\ 100 \\ 010 \\ 001 \end{bmatrix} = \begin{bmatrix} 000 \\ 001 \\ 010 \\ 100 \\ 101 \\ 011 \\ 110 \\ 111 \end{bmatrix}$$

$10011 \Rightarrow 10$.

原来是这样

这是啥.