离散の数学

Discrete mathematics

1. 这题估计是被某种神秘的力量所吞噬了，所以请自己猜猜这道题到底考了什么。

2. [10 points] In the questions below,describe each sequence recursively.Include intial conditions and assume that the sequences begin with a1.

递推关系

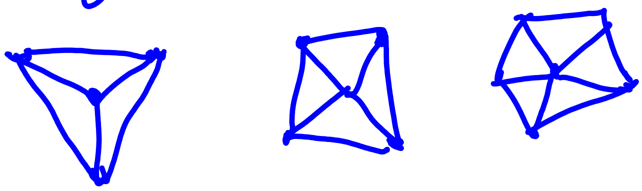
a) a_n=5^n

b) 1,101,10101,1010101 … …

c) a_n=the number of bit strings of length n with an even number of 0s.

d) a_n=the number of ways to go down an n-step staircase if you go down 1,2,or 3 step at a time.

6 8 5



3. [10 points]Suppose A={2,3,6,9,10,12,14,18,20} and R is the partial order relation balabalaba(原谅我，这个地方我实在看不清).

A where xRy means x is a divisor of y.

a) Draw the Hasse diagram for R.

b) Find all maximal elements.

c) Find all minimal elements.

d) Find lub({3,10})

e) Find glb({14,10})

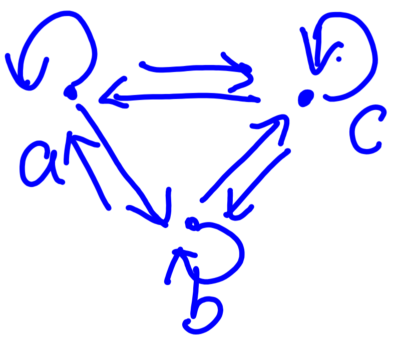
4. [10 points] In the questions below give an example or else prove that there are none.

a) A relation on {a,b,c} that is reflexive and transitive,but not antisymmetric. 不是反对称

b) A relation on {1,2} that is symmetric and transitive,but not reflexive.

对称 传递

c) A relation on {1,2,3} that is reflexive and transitive,but not symmetric.



轮图

5. [10 points] In the questions below fill in the blanks.

a) W_n has __2n__ edges and __n+1__ vertices.

b) The adjacency matrix for K_n has_________1s and _________0s.

Kn的邻接矩阵.

c) If G is a connected graph with 12 regions and 20 edges,then G has __10__vertices.

面

$r = e - v + 2$

$= 20 - v + 2 = 12$

$\Rightarrow v = 10$

d) The vertex-chromatic number for $K_{7,7}$=__2__.

e) If a regular graph G has 10 vertices and 45 edges,then each vertex of G has degree__9__.

正规图

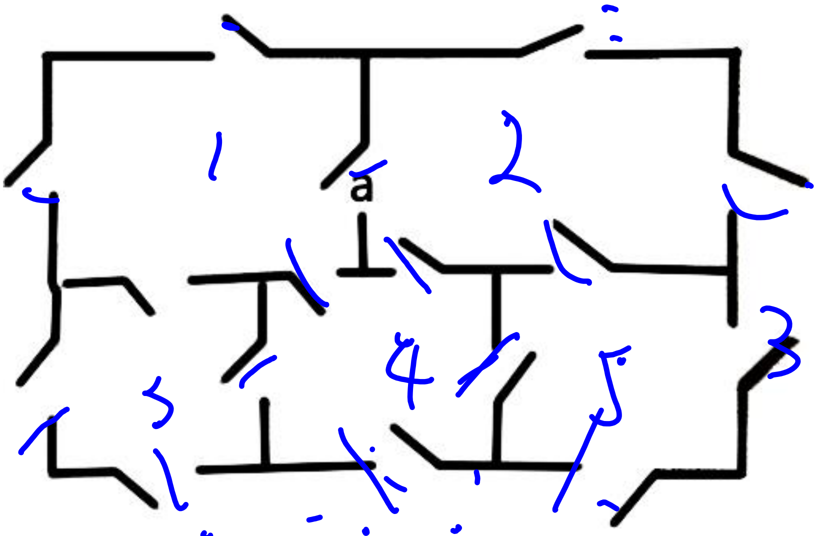
6. [10 points] An old puzzle presents a house with 5 rooms and 16 doors,as shown in the following figure.The problem is to figure out how to begin in a room or outside and take a walk that goes through each door exactly once.

b

a) Is such a walk possible?Explain.

b) How does your answer change if the door "a" adjoining the two large rooms is closed?

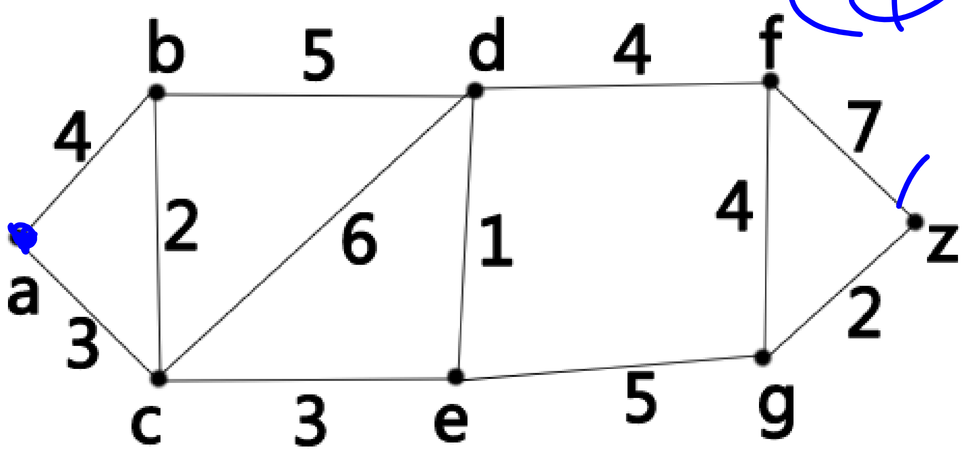
$P_G = x(x-1)(x-2)\cdots(x-6)$



7.[10 points] Use Dijkstra's Algorithm to find the shortest path length between the vertices a and z in this weighted graph.(Please give the process!)

4 (a,b)

3 (a,c)



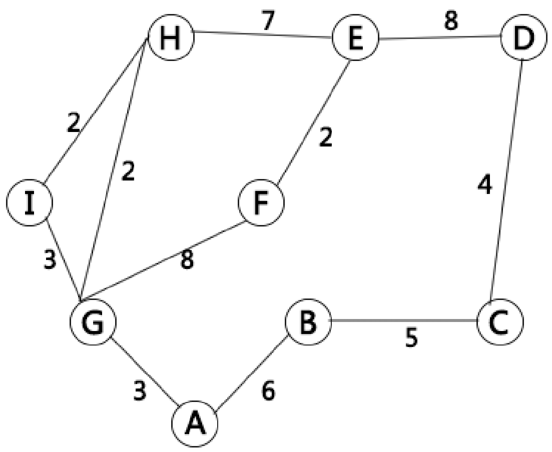
(a,c)

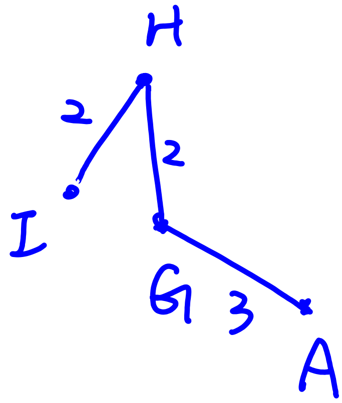
8.[10 points] Find a minimal spanning tree for the relation given by the graph .

a) Use Prim's algorithm,star from node H.(Write down the detail process)

b) Use Kruskal’ s algorithm.List the edges in the order in which they are chosen.

a)





9. [10 points] Let(S,*) be the semigroup whose operation table is given below.Let R be the equivalence relation on S defined by the partition{{x,y},{z,w}}.Show that R is a congruence relation on (S,*),and construct the operation table for quotient semigroup(S/R, ⊙).

*	x	y	z	w
x	x	y	z	w
y	y	z	w	z
z	z	z	z	z
w	w	w	w	w

aRa'

bRb'

$(a*b)R(a'*b')$

(yRy)

(xRy)

$(y*x)R(y*y)$

y

x