

Problem Set 3

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Problem 1

Prove or disprove that the protocol BBA^* satisfies the consistency property as in Definition 1' (EX-POST consistency).

The protocol BBA^* is given in [Byzantine Agreement Made Trivial](#) page 7, and Definition 1' in [The Notion of Byzantine Agreement](#), page 2.

Problem 2

In class we saw that the protocol BA^* is an (n, t) -graded consensus protocol. Prove or disprove that it is an (n, t) -graded consensus protocol according to the following revised definition.

Definition (GC'). A protocol \mathcal{P} for a synchronous and complete network is an (n, t) -graded consensus protocol if, in every execution e of \mathcal{P} with n players, where each player i has a private initial value v_i' and at most t of the players are malicious, every honest player j halts outputting a value-grade pair (v_j, g_j) , where $g_j \in \{0, 1, 2\}$, so as to satisfy the following three conditions:

1. For all honest players i and j , $|g_i - g_j| \leq 1$.
2. For all honest players i and j , $g_i, g_j > 0 \implies v_i = v_j$.
3. If, for some value $v \in V$, $v_i' = v$ for all honest players i , then $v_i = v$ and $g_i = 2$ for all honest players i .

The classical definition is

Definition (GC). same as in the definition of [GC'](#), except property 3 is replaced with

- 3'. If, for some value $v \in V$, $v_1' = \dots = v_n' = v$, then $v_i = v$ and $g_i = 2$ for all honest players i .

The BA^* protocol:

Step 1. Each player i propagates v_i' .

Step 2. Each player i propagates the string x if and only if $\#_i^1(x) \geq 2t + 1$ (where $\#_i^s(x)$ is the number of players from which i has received the value x at step s).

Output Determination. Each player i outputs the pair (v_i, g_i) computed as follows:

- If, for some x , $\#_i^2(x) \geq 2t + 1$, then $v_i = x$ and $g_i = 2$.
- If, for some x , $\#_i^2(x) \geq t + 1$, then $v_i = x$ and $g_i = 1$.
- Else, $v_i = \perp$ and $g_i = 0$.

Problem 3

Let BA^{**} be the same as BA^* except that \perp is replaced with some value $v \in V$. Is BA^{**} an (n, t) -graded consensus protocol? If not, which property is not satisfied?