

## 0.1 Protocol $BA'$

(= Dolev-Strong')

**Nested Signatures** When a user  $X$  digitally signs the digital signature of another user  $Y$ ,  $SIG_Y(m)$ , the resulting string  $SIG_X(SIG_Y(m))$  is a *level-2 nested signature*, by signers  $X$  and  $Y$ , of value  $m$ . Generalizing, if  $X_k, \dots, X_1$  are distinct users, then  $s = SIG_{X_k}(\dots(SIG_{X_1}(m)\dots))$  is a *level- $k$  nested signature*, by signers  $X_1, \dots, X_k$ , of value  $m$ . Due to the retrievability property of digital signatures, anyone receiving  $s$  easily computes  $m$ . To highlight the first signer of  $s$ , we may refer to  $s$  as a  $(k, X_1)$ -signature of  $m$ .

**A Classical Protocol in Our Notation.** Except for the notation adopted and for the fact that messages are propagated in our network model, rather than sent to all players in a synchronous network,  $BA'$  coincides with the mentioned BA protocol of Dolev and Strong. Accordingly, the protocol is a slightly more specialized BA protocol, but perfectly adequate in our Algorand application. Namely, it makes use of digital signatures, and the initial value  $v_X$  of a player  $X$  is not an arbitrary value, but the value he has actually received from a distinguished player, *the leader*,  $\ell$ . More precisely, prior to the start of the protocol,  $\ell$  is supposed to have propagated  $SIG_\ell(v)$ .

Accordingly, “Step 0” is not technically part of  $BA'$ . (In our Algorand application, if  $BA'$  is indeed the chosen BA protocol, Step 0 has already been executed by the round- $r$  leader, when protocol  $ROUND(r)$  calls the execution of the chosen BA protocol.)

**Remark** As for protocol  $ROUND(r)$ , in describing  $BA'$  we omit to include, in each message  $m$ , a full specification of the “context” in which  $m$  is sent.<sup>1</sup>

For each player  $X$ , the set  $S_X$  is initially (reset to be) empty.

### Protocol $BA'$

COMMUNICATION STEP 0. For some value  $v$ , leader  $\ell$  propagates  $SIG_\ell(v)$ .

COMMUNICATION STEP  $i$  (for  $i = 1$  to  $t + 1$ ). For each player  $X$ :

- For each value  $x \notin S_X$  for which, in Step  $i - 1$ , he receives a  $(i, \ell)$ -signature of  $x$ , by signers other than himself,  $X$  arbitrarily chooses one such signature,  $s$ , and propagates  $SIG_X(s)$ .
- For each  $(i, \ell)$ -signature of a value  $x$  received in Step  $i - 1$ ,  $S_X := S_X \cup \{x\}$ .

COMPUTATION STEP  $t + 2$ . For each player  $X$ :

- If  $S_X$  contains exactly one value,  $S_X = \{x\}$ , then  $X$  sets  $OUT_X = x$ .
- Else,  $X$  sets  $OUT_X = \perp$ .