



Adversary-resilient Inference and Machine Learning: From Distributed to Decentralized

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Statistical inference and machine learning algorithms have traditionally been developed for data available at a single location. Unlike this *centralized setting*, modern datasets are increasingly being distributed across multiple physical entities (sensors, devices, machines, data centers, etc.) for a multitude of reasons that range from storage, memory, and computational constraints to privacy concerns and engineering needs. This has necessitated the development of inference and learning algorithms capable of operating on non-collocated data. Such algorithms can be divided into two broad categories, namely, *distributed algorithms* and *decentralized algorithms*. Distributed algorithms correspond to the setup in which the data-bearing entities (henceforth referred to as “nodes”) only communicate with a single entity (referred to as master node, central server, parameter server, fusion center, etc.), which is tasked with generating the final result. Such distributed setups arise in the context of parallel computing, where the focus is computational speedups and/or overcoming memory/storage bottlenecks, and federated systems, where “raw” data collected by individual nodes cannot be shared with the master node due to either communication constraints (e.g., sensor networks) or privacy concerns (e.g., smartphone data). Decentralized algorithms, on the other hand, correspond to the setup that lacks a central server; instead, individual nodes in this setup communicate among themselves over a network (often ad hoc) to reach a common solution (i.e., achieve *consensus*) at all nodes. Such decentralized setups arise either out of the need to eliminate single points of failure in distributed setups or due to practical constraints, as in the internet of things and autonomous systems. We refer the reader to Figure 1 for examples of distributed and decentralized setups.



Is it distributed or is it decentralized? Inference and learning from non-collocated data have been studied for decades in computer science, control, signal processing, and statistics. Both among and within these disciplines, however, there is no consensus on use of the terms “distributed” and “decentralized.” Though many works share the definitions provided in here, there are numerous authors who use these two terms interchangeably, while there are some other authors who reverse these definitions.

While the last few decades have witnessed a huge body of work devoted to inference and learning in distributed and decentralized setups, much of this work assumes a non-adversarial setting in which individual nodes—apart from occasional statistical failures—operate as intended within the algorithmic

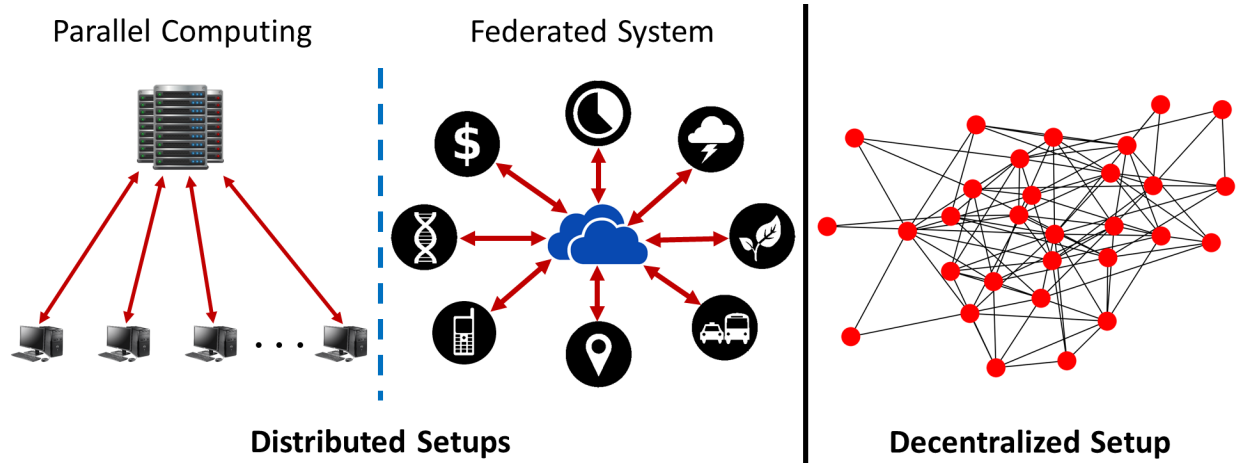


Fig. 1. Inference and machine learning algorithms involving non-located data are broadly divisible into the categories of (i) distributed algorithms and (ii) decentralized algorithms. The former category deals with distributed setups such as parallel computing and federated systems in which a single entity is tasked with generating the final result. The latter category deals with decentralized setups in which entities cooperate among themselves to produce the final result.

framework. In recent years, however, cybersecurity threats from malicious **non-state actors and rogue nations** have forced practitioners and researchers to rethink the robustness of distributed and decentralized algorithms against adversarial attacks. As a result, we now have a plethora of algorithmic approaches that guarantee robustness of distributed and/or decentralized inference and learning under different adversarial threat models; see, e.g., recent survey articles [1]–[3]. Driven in part by the world’s growing appetite for data-driven decision making, however, securing of distributed/decentralized frameworks for inference and learning against adversarial threats remains a rapidly evolving research area. In this article, we provide an overview of some of the most recent developments in this area under the threat model of *Byzantine attacks*. This threat model—which subsumes the case of malfunctioning nodes, referred to as *Byzantine faults/failures*—is one of the hardest to safeguard against since it allows for undetected takeover of nodes by the adversary. In particular, nodes affected under this threat model—termed *Byzantine nodes*—are assumed to have the potential to arbitrarily bias the outputs of the underlying algorithms by colluding among themselves, injecting false data and information into the distributed/decentralized system, etc.

The Byzantine threat model, since its introduction in 1982, has been extensively studied in the context of statistical inference from non-located data [1], [2]. However, its potential impact on more general decentralized consensus and distributed/decentralized machine learning problems has only been recently studied. It is against this backdrop that this article summarizes recent developments in Byzantine-resilient processing of non-located data, with a majority of the discussion focused on machine learning problems in distributed and decentralized settings.

Origination of the Byzantine threat model: The threat model of Byzantine attacks/faults/failures in its most general form was introduced and analyzed in [4] within the context of reliability of computer systems with potentially malfunctioning components. The overarching problem in [4] was abstracted as the *Byzantine Generals Problem*, in which several generals of the Byzantine army—some of whom are likely to be traitors—need to agree on attacking or retreating from an enemy city through exchange of messages via a messenger; see, e.g., Figure 2. The authors reported two key results in [4] for this problem. First, they established the impossibility of safeguarding against Byzantine nodes (traitor generals) when the number of uncompromised nodes (loyal generals) is not more than two-thirds of the total number of nodes. The threat from General 3 in Figure 2, therefore, cannot be neutralized by any algorithm since the number of loyal generals is only two in this scenario. Second, the authors proposed two algorithms for *complete* and *regular* graphs that provably counter the actions of Byzantine nodes in the case of binary (e.g., attack/retreat) decisions as long as the number of uncompromised nodes exceeds two-thirds of the total number of nodes.

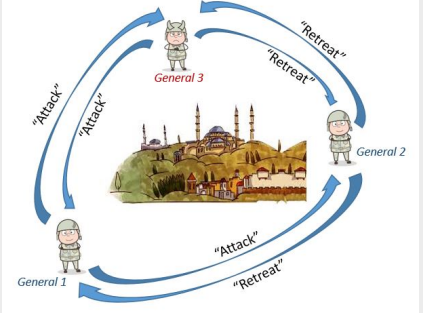


Fig. 2. A Byzantine army led by three generals, one of whom (General 3) is a traitor, surrounding an enemy city. The loyal generals are trying to reach a consensus on the plan of action against the enemy, while the traitor is trying to mislead them.

I. ADVERSARY-RESILIENT DISTRIBUTED PROCESSING OF DATA

In the classic setting of distributed systems tasked with processing of non-located data, there is one central server—referred to as master node, parameter server, fusion center, etc.—that coordinates with M data-bearing nodes (sensors, smartphones, worker nodes, etc.) for final decision making. While all nodes in this setting communicate with the server, they usually cannot communicate with each other (see “Distributed Setups” in Figure 1). The Byzantine threat model in this distributed setting assumes at most b nodes in the network have been compromised. This parameter b , which typically corresponds to a crude upper bound on the exact number of Byzantine nodes, often plays an important role in both the analysis and the performance of Byzantine-resilient algorithms. We summarize some of these algorithms and their theoretical guarantees in the following for statistical inference and machine learning problems.

A. Distributed Statistical Inference

Statistical inference leverages data samples in order to draw conclusions about the underlying probability distribution(s) generating the data. While statistical inference can take many forms, we limit our discussion in this article to Byzantine-resilient distributed detection and distributed estimation.

Distributed Detection: Distributed detection under both the *Neyman–Pearson* (NP) and the *Bayesian* frameworks has a rich history. Given two hypotheses \mathcal{H}_0 and \mathcal{H}_1 , a typical distributed detection algorithm first involves each node taking a local decision in favor of either \mathcal{H}_0 or \mathcal{H}_1 based on its own data samples. The nodes then send their decisions to the central server, which applies a fusion rule to the local decisions in order to reach the final (global) decision. Unfortunately, distributed detection algorithms designed without consideration of potential Byzantine failures break down in the presence of Byzantine nodes.

Despite the brittleness of traditional distributed detection techniques, investigation of Byzantine-resilient distributed detection only took off in the last decade. The survey article [1] provides an overview of many of the resulting methods and asymptotically characterizes the respective *critical fraction* α^* of Byzantine nodes, defined as $\alpha^* := \frac{b}{M}$, at (or beyond) which the central server cannot do better than random guessing for its final decision. An important insight from the earliest works on Byzantine-resilient distributed detection is that (asymptotically) α^* can be $\frac{1}{2}$ or higher [1]; in contrast, recall that $\alpha^* = \frac{1}{3}$ for the original Byzantine Generals Problem [4].

Since the appearance of [1] in 2013, a few other works on Byzantine-resilient distributed detection have appeared in the literature. It is shown in [5] that distributed detection can be more resilient to Byzantine failures in the case of a general Q -ary hypothesis testing problem, with the critical fraction given by $\alpha^* = \frac{Q-1}{Q}$. Byzantine-resilient distributed binary hypothesis testing is investigated for the first time under the Bayesian framework in [6], with the critical fraction also given by $\alpha^* = \frac{1}{2}$ in this case. Finally, it is established in [7] that—under certain conditions—it is possible to have $\alpha^* = 1$ in the case of Bayesian distributed binary detection as long as each node is allowed to replicate its message to the central server through one other node in the network. Strictly speaking, however, this coordination among pairs of nodes leads to a network topology that differs from the distributed setups of prior works and the convention used in this article. We conclude by noting that a tree network in which the central server sits at the root of the tree (Depth 0) and nodes in the network route their messages to the server through their parent nodes is another quasi-distributed architecture that does not fall under the convention used in this article. Byzantine resilience of such networks in distributed detection tasks is investigated in [8], with the critical fraction of Byzantine nodes defined in terms of the number of Byzantine nodes b_1 and the total number of nodes M_1 at Depth 1 of the tree. We also refer the reader to Table I for a summary of all the results that have been discussed in this article in relation to Byzantine-resilient distributed detection.

Distributed Estimation: Byzantine-resilient distributed estimation has received significant attention lately in the context of state estimation in cyberphysical systems. Much of the developments in this regard have been limited to linear models, with a typical observation model at any given time at node

TABLE I
SUMMARY OF RECENT RESULTS CONCERNING BYZANTINE-RESILIENT DISTRIBUTED DETECTION

Reference	Hypothesis Test	Detection Setup	Critical Fraction α^*	Network
Nadendla et al. [5]	Q -ary	Neyman–Pearson	$\alpha^* = \frac{Q-1}{Q}$	Star topology
Kailkhura et al. [6]	Binary	Bayesian	$\alpha^* = 0.5$	Star topology
Hashlamoun et al. [7]	Binary	Bayesian	Can have $\alpha^* = 1$	Quasi-star topology
Kailkhura et al. [8]	Binary	Neyman–Pearson	$b_1 = \lceil \frac{M_1}{2} \rceil$	Tree topology

j expressed as $y_j = \mathbf{h}_j^T \mathbf{w} + \eta_j$, $j = 1, \dots, M$, where $\mathbf{h}_j \in \mathbb{R}^d$ denotes a known vector, $\mathbf{w} \in \mathbb{R}^d$ is the unknown state vector that needs to be estimated at the central server, and η_j denotes the observation noise at node j . Traditionally, distributed estimation has involved the nodes transmitting y_j 's (or their quantized versions) to the central server and the server estimating \mathbf{w} using some variant of the following least-squares formulation [9] (or a maximum likelihood one in the case of quantized transmissions):

$$\hat{\mathbf{w}} = \arg \min_{\tilde{\mathbf{w}}} \|\mathbf{y} - \mathbf{H}\tilde{\mathbf{w}}\|_2, \quad (1)$$

where the matrix $\mathbf{H} \in \mathbb{R}^{M \times d}$ has \mathbf{h}_j 's as its rows. The quality of such solutions are assessed in terms of gap of their *mean squared error* (MSE), $\mathbb{E}[\|\hat{\mathbf{w}} - \mathbf{w}\|_2^2]$, from either the *minimum mean squared error* (MMSE) or the *Cramer–Rao lower bound* (CRLB) for unbiased estimators.

Distributed estimation and Byzantine failures: We illustrate the impact of Byzantine nodes on distributed estimation through a simple example of two-dimensional parameter estimation, in which the linear observation model corresponds to a line in (x, y) plane, using $M = 8$ nodes. The observations at each node in this example correspond to $y_j = \mathbf{h}_j^T \mathbf{w} + \eta_j$ with $\mathbf{h}_j^T := [x_j \ 1]$ and the two-dimensional vector \mathbf{w} describing the slope and y -intercept of the line. It can be seen from Figure 3 that Byzantine failures of just two nodes returns a least-squares solution that results in a model $\mathbf{h}_j^T \hat{\mathbf{w}}$ that significantly differs from the true model $\mathbf{h}_j^T \mathbf{w}$.

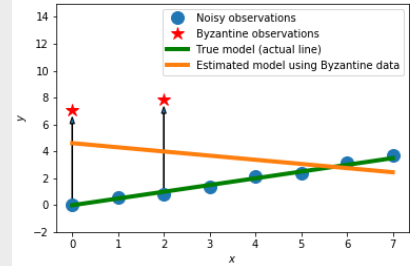


Fig. 3. The impact of Byzantine failures on traditional least-squares estimation.

In the presence of Byzantine nodes, in which case the y_j 's corresponding to the b Byzantine nodes can be arbitrarily different from $\mathbf{h}_j^T \mathbf{w}$, solutions of distributed estimation techniques based on (1) can be pushed far from the optimal (in terms of MMSE/CRLB); see, e.g., Figure 3. In this setting, one can again characterize the critical fraction α^* of Byzantine nodes at (or beyond) which the server can do no better than relying on prior knowledge about \mathbf{w} . It is, for instance, established in [5] that—under certain assumptions—it is possible to have $\alpha^* = \frac{Q-1}{Q}$ when nodes utilize Q -ary quantization in order

to transmit their data to the server. The survey articles [1]–[3] provide additional discussion of works on Byzantine-resilient distributed estimation, many of which are based on the idea of either detection of Byzantine attacks and/or identification of individual Byzantine nodes as part of their mitigation strategy. Finally, while much of the focus in Byzantine-resilient distributed estimation has been on linear models, there are works like [10] that focus on nonlinear models. Since there is a significant overlap between statistical estimation and machine learning problems, we do not indulge in discussion of such works; instead, we move towards our review of recent results on Byzantine-resilient distributed machine learning.

B. Distributed Machine Learning

A typical challenge in machine learning is to statistically minimize a function f , referred to as *loss function* or *risk function*, with respect to a candidate *model* $\mathbf{w} \in \mathbb{R}^d$ that describes the data, i.e.,

$$\min_{\mathbf{w}} \mathbb{E}_{\mathbf{z} \sim \mathcal{P}} [f(\mathbf{w}, \mathbf{z})], \quad (2)$$

where \mathbf{z} denotes data living in some Hilbert space that is drawn from an *unknown* probability distribution \mathcal{P} . To solve (2) without knowledge of \mathcal{P} , distributed machine learning focuses on minimizing an empirical variant of $\mathbb{E}_{\mathbf{z} \sim \mathcal{P}} [f(\mathbf{w}, \mathbf{z})]$ using samples of \mathbf{z} —termed *training data*—distributed across different nodes. The resulting objective, aptly termed distributed *empirical risk minimization* (ERM), can be expressed as $\min_{\mathbf{w}} \frac{1}{M} \sum_{j=1}^M f(\mathbf{w}, \mathbf{Z}_j)$, with \mathbf{Z}_j denoting the local training data at node j that comprises multiple samples drawn from \mathcal{P} and \mathbf{w} referred to as the *global* optimization variable. The model/variable \mathbf{w} in distributed machine learning is stored at the central server, which iteratively updates it based on messages received from individual nodes and subsequently sends the updated \mathbf{w} to all nodes. Each node, in turns, performs some computation according to its local data and the received \mathbf{w} , and sends a message back to the server.

Distributed Stochastic Gradient Descent: Similar to distributed statistical inference, Byzantine failures can lead to breakdowns of distributed machine learning methods. Motivated in part by the widespread adoption of deep neural networks and variance-reduction techniques like mini batching by the practitioners, we mainly focus here on robustification of synchronous distributed *stochastic gradient descent* (SGD) against Byzantine attacks. A typical distributed SGD algorithm proceeds iteratively. In each iteration, nodes compute the **gradient** of the loss function on their local data with respect to the current model and send their gradients to the server. The server, in turn, takes the average of all the **gradients** and updates the global variable according to the averaged gradient. In a faultless environment, distributed SGD—with proper choices of step size and batch size—has a linear convergence rate for strongly convex functions. However, a single Byzantine node can force the algorithm to converge to any model of its choice using a simple strategy. Suppose, for instance, the summation of gradients of faultless nodes is \mathbf{g}



and the Byzantine node wishes the server to operate on an alternate gradient \mathbf{g}' . The Byzantine node can accomplish this by sending $M\mathbf{g}' - \mathbf{g}$ as its gradient to the server, thereby controlling the update step at the server. Of course, as simple as this strategy is, it is not an optimal one for the Byzantine node; indeed, a large M will result in a large gradient, which is likely to make the malicious message detectable. We refer the reader to [11] for more sophisticated strategies that can be employed by Byzantine nodes.



Several algorithms have been put forth recently to safeguard distributed SGD against Byzantine failures. The central idea in all these approaches that imparts Byzantine resilience to distributed SGD involves the use of a *screening* procedure at the server while it aggregates the local gradients. Using an appropriate screening rule, the screened aggregated gradient can be shown to be close to the true average gradient, thereby enabling the server to approximately solve the distributed ERM problem. Algorithm 1 provides a general framework for Byzantine-resilient SGD that is based on this idea of screening. While some algorithms alter this framework a little bit, the general idea remains the same. Note that if one were to remove the screening procedure in Step 10 of the algorithm, it becomes vanilla distributed SGD.

Algorithm 1 A General Framework for Byzantine-resilient Distributed SGD

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1: for  $t = 1, 2, \dots$  do
2:   Server:
3:     Send the global optimization variable to all nodes
4:   Node:
5:     Receive the global optimization variable from the server
6:     Calculate gradient with respect to the local training set
7:     Send the local gradient to the server
8:   Server:
9:     Receive local gradients from all nodes
10:    Screen the received gradients for Byzantine resilience and aggregate them
11:    Update the global variable by taking a gradient step using the aggregated gradient
12: end for
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The State of the Art: We now discuss the various screening (and aggregation) procedures adopted in different algorithms. The algorithm introduced in [12], termed *robust distributed gradient descent*, uses two screening methods: *coordinate-wise median* and *coordinate-wise trimmed mean*. In coordinate-wise median, the server aggregates the local gradients by taking the median in each dimension of the gradients. Coordinate-wise trimmed mean, on the other hand, involves the server eliminating the smallest and the largest b values in each dimension of the gradients and coordinate-wise averaging the remaining values

for aggregation. The GeoMed algorithm [13], in contrast, uses the *geometric median* of local gradients as the screening and aggregation rule. The Krum [14] algorithm, on the other hand, finds the local gradient that has the smallest distance to its $M - b - 2$ closest gradients and uses this gradient for the update step. There is also a variant of the Krum algorithm, termed *Multi-Krum* [14], which finds $m \in \{1, \dots, M\}$ local gradients using the Krum principle and uses an average of these gradients for update.

The Bulyan algorithm [15], [16] is a two-stage algorithm. In the first stage, it recursively uses vector median methods such as Geometric median and Krum to select $M - 2b$ local gradients. In the second stage, it carries out a coordinate-wise operation on the $M - 2b$ selected gradients in which $M - 4b$ values in each coordinate are retained (and then averaged for aggregation) by eliminating $2b$ values that are farthest from the coordinate-wise median. Zeno/Zeno++ [17], [18] are related algorithms that require an oracle at the server, which can generate an estimate of the true gradient in each iteration, for screening/aggregation purposes. In particular, the screening procedure involves calculating a score for each local gradient based on its difference from the oracle gradient, with the surviving gradients being the ones that are the most similar to the oracle gradient. In order to reduce the communications overhead of Byzantine-resilient distributed SGD, [19] discusses the signSGD algorithm, which only uses the element-wise sign of the gradient—rather than the gradient itself—for the update step. In particular, it is established in [19] that signSGD with an element-wise majority vote on the signs of the local gradients as a screening/aggregation rule is a Byzantine-resilient learning method. Finally, the **RSA** algorithm of [20] robustifies distributed SGD by making individual nodes store and update local versions of the global optimization variable \mathbf{w} , which are then aggregated at the server in each iteration in a **Byzantine-resilient**.

We next summarize some key aspects of these algorithms in Table II in terms of the number of nodes M , the number of Byzantine nodes b , and the number of independent and identically distributed (i.i.d.) samples per node N . The last column in the table provides conditions on M and b necessary for well-posedness of different algorithms. The convergence rates correspond to the gap between the learned model after t iterations and the minimizer of the statistical risk (cf. (2)) in the limit of large N . The parameter c denotes a constant that may change from one algorithm to the other, the $\mathcal{O}(\cdot)$ scaling hides dependence on problem parameters such as the dimension d , N/A signifies an algorithm lacks a particular result, and — means guarantees for an algorithm are not directly comparable to other algorithms. While the convergence/learning rates in the table are for strongly convex and smooth loss functions, some of the works require further assumptions and/or also provide results under a relaxed set of assumptions on the loss function and training data. We refer the reader to the referenced works for further details.

TABLE II
SUMMARY OF RECENT RESULTS CONCERNING BYZANTINE-RESILIENT DISTRIBUTED MACHINE LEARNING

Algorithm	Convergence Rate	Learning Rate	Condition on (M, b)
Coordinate-wise Median [12]	$\mathcal{O}(c^t)$	$\mathcal{O}\left(\frac{b}{M\sqrt{N}} + \frac{1}{\sqrt{MN}} + \frac{1}{N}\right)$	$M \geq 2b + 1$
Coordinate-wise Trimmed Mean [12]	$\mathcal{O}(c^t)$	$\mathcal{O}\left(\frac{b}{M\sqrt{N}} + \frac{1}{\sqrt{MN}}\right)$	$M \geq 2b + 1$
GeoMed [13]	$\mathcal{O}(c^t)$	$\mathcal{O}\left(\frac{\sqrt{b}}{\sqrt{MN}}\right)$	$M \geq 2b + 1$
Krum [14]	N/A	N/A	$M \geq 2b + 3$
Multi-Krum [14]	N/A	N/A	$M \geq 2b + m + 2$
Bulyan [15]	N/A	N/A	$M \geq 4b + 3$
Zeno/Zeno++ [17], [18]	$\mathcal{O}(c^t) + \mathcal{O}(1)$	N/A	$M \geq b + 1$
signSGD [19]	—	N/A	$M \geq 2b + 1$
RSA [20]	$\mathcal{O}\left(\frac{1}{t}\right) + \mathcal{O}(1)$	N/A	$M \geq b + 1$

Screening and aggregation in two dimensions: We illustrate the robustness of different screening/aggregation methods against Byzantine attacks through a simple two-dimensional example. Consider a total of seven two-dimensional points (g_1, g_2) , out of which six represent correct data (black discs in Figure 4) and one of which has been falsified by a Byzantine node (red disc in Figure 4).

It can be seen from Figure 4, which is displayed with non-uniform g_1 - and g_2 -axes to capture the effect of the Byzantine node, that the ordinary average operation is highly susceptible to the falsified data point. On the other hand, screening and aggregation rules such as coordinate-wise median, coordinate-wise trimmed mean, Krum and Bulyan (using Krum in Figure 4) all produce final results that stay close to the true average of the faultless data points.

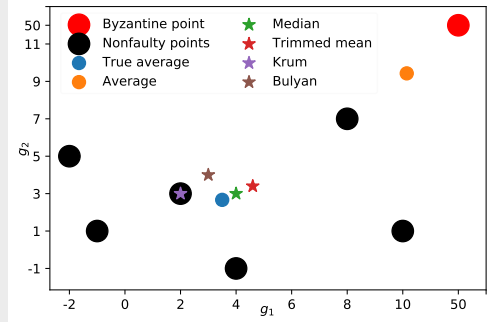


Fig. 4. An illustration of the effects of data falsified by Byzantine nodes on screening methods.

Numerical Experiments: We now numerically compare the performance of some of the algorithms listed in Table II. The learning task corresponds to classification of the MNIST dataset using a convolutional neural network with two convolution layers followed by two fully connected layers. Each of the two convolution layers are followed by max pooling and ReLU activation, while the output layer uses softmax activation. The distributed setup corresponds to one server and 20 nodes, where each node is provided with 2,000 images uniformly sampled from the MNIST training set. We run two rounds of experiments for each algorithm, where each round is repeated 10 times—with each trial corresponding to 10,000 algorithmic iterations—and the results on the MNIST test set are averaged over these 10 trials.

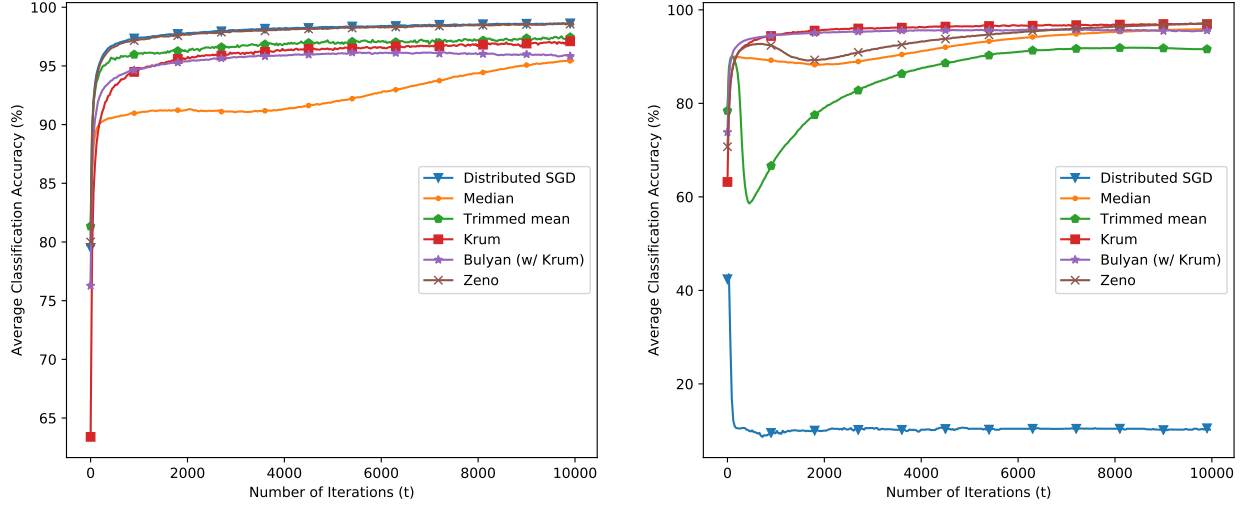


Fig. 5. Comparison of different distributed learning methods (with median and trimmed mean being coordinate wise) based on stochastic gradient descent. The left panel corresponds to the faultless setting, while the right panel corresponds to the case of four Byzantine nodes in the network. In both cases, the parameter b is set equal to 4 in all Byzantine-resilient algorithms.

In the first round, none of the nodes are taken to be Byzantine nodes. In the second round, four nodes are randomly selected as Byzantine nodes, with each Byzantine node sending a random vector, whose elements uniformly take values between 1 and 10, to the server in each iteration. While this strategy of Byzantine nodes may seem simple, it is an aggressive one that is hard to safeguard against in general [11]. We limit ourselves to four Byzantine nodes in order to provide a fairer comparison between different algorithms, since we observed a sharp drop in the performance of some methods for larger number of Byzantine nodes. In both rounds of experiments, the algorithms operate under the assumption of $b = 4$.

The results of the experiments are shown in Figure 5. It can be seen from this figure that while the Byzantine-resilient methods obtain good accuracy in both cases, the vanilla distributed SGD falls part under Byzantine attacks. The experimental results also highlight some trade-offs, which should help the practitioners select the algorithms that best fit their needs. In the faultless setting, for instance, all methods except Zeno fall short of distributed SGD’s performance. And while Zeno can match the accuracy of distributed SGD, it requires an oracle that can provide it with some knowledge of the true gradient. In the presence of Byzantine nodes, Bulyan seems to be the most stable. However, since Bulyan screens four times the number of Byzantine nodes in each iteration, this stability comes at the expense of a limited number of Byzantine nodes that it can handle.

II. ADVERSARY-RESILIENT DECENTRALIZED PROCESSING OF DATA

Unlike the distributed setup, a decentralized network lacks a central server that is connected to every node (see “Decentralized Setup” in Figure 1). Instead, all nodes in a decentralized network maintain local

copies of the decision variable/machine learning model and reach (approximate) consensus on a common solution through periodic exchange of messages with a subset of other nodes, termed *neighbors*. Since nodes lack access to the entire set of local variables, ensuring robustness against Byzantine failures is generally harder in decentralized networks. In particular, unlike simple characterizations of the feasible and/or necessary relationships between the number of nodes M and the number of Byzantine nodes b in Table I and Table II, the necessary and/or sufficient conditions for Byzantine-resilient decentralized algorithms are stated in terms of topology of the underlying network graph. In this article, we mostly limit our discussion to synchronous algorithms on decentralized networks whose topology does not change with time (i.e., *static graphs*). We begin by engaging in discussion of an algorithmic process, aptly termed *consensus*, that often forms the basis of algorithms for decentralized inference and machine learning.

A. Decentralized Consensus

Many decentralized inference and learning algorithms require a subprocess that ensures (approximate) agreement, i.e., consensus, among all nodes. In the context of Byzantine-resilient decentralized processing, therefore, it is instructive to understand the fundamentals of Byzantine-resilient consensus. Our discussion in this regard will be limited to linear strategies for consensus, which form the basis for Byzantine-resilient decision consensus, averaging consensus, convex consensus, etc. [21], [22].

Suppose each node j in the decentralized network has a local variable \mathbf{w}_j^0 and it is desired to reach consensus on the average of these variables at each node through in-network message passing, i.e., $\mathbf{w}_j \approx \frac{1}{M} \sum_{i=1}^M \mathbf{w}_i^0$ for every node j . Consensus algorithms usually proceed by having each node iteratively take a weighted average of their neighbors' variables. Mathematically, therefore, a consensus algorithm is associated with an $M \times M$ *weight matrix* whose (j, i) -th entry corresponds to the weight assigned by node j to the variable it receives from node i . In the absence of Byzantine nodes, a well-known result says that a doubly stochastic weight matrix whose smallest nonzero entry is lower bounded guarantees convergence of all \mathbf{w}_j 's to the average $\frac{1}{M} \sum_{i=1}^M \mathbf{w}_i^0$. Suppose, however, that node k in the network is Byzantine and it transmits $\mathbf{w}_k = \mathbf{w}'$ to its neighbors in each iteration. This simple "lazy" Byzantine strategy is enough for all nodes to converge to \mathbf{w}' . And if there were another Byzantine node k' in the network that always transmitted the same $\mathbf{w}_{k'} \neq \mathbf{w}'$, the network will not reach consensus at all.

The main idea behind ensuring robustness of consensus to Byzantine nodes is to screen potential outliers at each node before the weighting step in each iteration. Since the averaging process in one dimension is independent of other dimensions, much of the discussion in Byzantine-resilient consensus has focused on scalar-valued problems. Some variant of the trimmed mean, in which a node removes the largest b and the smallest b values received from its neighbors (including itself), is in particular a common

screening method used in scalar consensus. Clearly, each node has to have enough neighbors (e.g., more than $2b$ for trimmed mean) for screening-based consensus to be well defined. However, since the network must not become disconnected after screening, the exact constraint on network topology for different algorithms tends to be more involved; see [21], [22] for further discussion of different screening methods and the corresponding topology constraints. Note that decentralized inference and learning algorithms utilizing similar screening ideas for Byzantine resilience tend to have similar topology constraints.

In terms of the performance of Byzantine-resilient consensus, even the best screening/aggregation method cannot guarantee removal of all falsified data. However, equipped with a reasonable strategy, a Byzantine-resilient scalar consensus algorithm can guarantee two things in each iteration: (i) the retained values are between the smallest and the largest scalars at all nonfaulty nodes; and (ii) the difference between the largest and the smallest nonfaulty values decreases. Together, these two conditions can ensure that the nonfaulty nodes converge to a common value. However, since a node can also retain falsified data and/or eliminate nonfaulty one, consensus to the true average is no longer possible. Instead, guarantees can only be given to an approximate average such as a convex combination of initial values. This in particular is what makes Byzantine-resilient decentralized inference and learning so challenging.

B. Decentralized Statistical Inference

Similar to the case of distributed inference, we limit ourselves in here to decentralized detection and estimation. In both problems, decentralized consensus makes an integral part of the developed algorithms.

Decentralized Detection: Similar to distributed detection, a typical setup in decentralized detection involves nodes in the network making observations under one of two (or more) hypotheses. Unlike distributed detection, however, nodes must collaborate among themselves to reach consensus in favor of one of the hypotheses. Vulnerability of the vanilla consensus framework to Byzantine nodes, therefore, also makes decentralized detection highly susceptible to Byzantine attacks.

While several application scenarios call for Byzantine-resilient decentralized detection, it has received less attention compared to its distributed counterpart. Some of the most relevant works in this regard include [23]–[25], all of which focus on scalar-valued problems. In the spirit of Byzantine-resilient scalar consensus, an adaptive threshold-based screening method is utilized in [23] to mitigate the impact of Byzantine nodes on the final decision. Similarly, a robust variant of distributed *alternating direction method of multipliers* (ADMM) is proposed in [24] that uses trimmed mean to eliminate $2b$ scalars at each node in every iteration. But other than the trivial topology constraint imposed by the trimmed mean (i.e., in-degree of nodes being greater than $2b$), this work lacks guarantees. In contrast, [25] proposes and analyzes a robust detection scheme that involves identification of Byzantine nodes and use of a weighted

average consensus algorithm. However, this work focuses on a particular variant of Byzantine attacks and does not characterize topology constraints as a function of the number of Byzantine nodes.

Decentralized Estimation: Decentralized estimation, in which nodes use local observations and messages from neighbors to reach consensus on the estimate of an unknown parameter $\mathbf{w} \in \mathbb{R}^d$, is often also studied under the linear model. Similar to decentralized detection, Byzantine-resilient decentralized estimation is a relatively recent topic of interest. Among relevant works, [26] focuses on a specialized estimation problem in which node j need only estimate the j -th entry of \mathbf{w} and there are three types of nodes in the network: *reliable* nodes, which are special nodes having perfect knowledge of the entry of \mathbf{w} associated with them, ordinary (normal) nodes, and Byzantine nodes. Under this setup and the assumption of a time-varying directed graph, [26] proposes and analyzes a trimmed mean-type (scalar-valued) screening procedure for Byzantine resilience. The works [27], [28] study vector-valued dynamic estimation under the noiseless observation model of $\mathbf{y}_j[n] = \mathbf{w}$, $n = 1, \dots$, for nonfaulty nodes. While the threat model in [27], [28] is a specialized variant of the Byzantine model, it allows the indices of the nodes under attack to change from one time instance to the next one. Both [27], [28] use similar consensus+innovations algorithms coupled with time-varying gains to achieve resilience. The main difference in these works is that [28] can achieve linear convergence, as opposed to sublinear convergence for [27], but it can only tolerate less than 30% of the nodes being under attack, in contrast to 50% for [27]. Finally, [29] accomplishes Byzantine-resilient decentralized dynamic estimation under the general linear model $\mathbf{y}_j[n] = \mathbf{H}_j \mathbf{w} + \boldsymbol{\eta}_j[n]$ by focusing on explicit detection of adversaries. We conclude by noting that the review article [2] provides more detailed overview of [26], [27], [29].

C. Decentralized Machine Learning

Decentralized machine learning algorithms, which can be considered a combination of consensus and distributed learning frameworks, approximately solve (2) by minimizing a *global* loss function on the non-located data in a decentralized manner while reaching an agreement among all nodes i.e.,

$$\min_{\{\mathbf{w}_1, \dots, \mathbf{w}_M\}} \frac{1}{M} \sum_{j=1}^M f(\mathbf{w}_j, \mathbf{Z}_j) \quad \text{subject to} \quad \mathbf{w}_i = \mathbf{w}_j \quad \forall i, j, \quad (3)$$

where $\mathbf{w}_j \in \mathbb{R}^d$ denotes the model learned at node j in the network. We refer to (3) as the *decentralized* ERM problem, which approximately solves the statistical risk minimization problem (2).

We focus here on synchronous gradient descent-based methods for solving (3). The classic *decentralized gradient descent* (DGD) method [30], for instance, involves each node j exchanging its current local iterate

in every iteration t with all nodes in its neighborhood \mathcal{N}_j and then updating the local iterate using a consensus-type weighted averaging step and a local gradient descent step, i.e.,

$$\mathbf{w}_j^{t+1} = \underbrace{\alpha_{jj}\mathbf{w}_j^t + \sum_{i \in \mathcal{N}_j} \alpha_{ji}\mathbf{w}_i^t}_{\text{consensus}} \underbrace{- \rho(t)\mathbf{g}_j(\mathbf{w}_j^t)}_{\text{local gradient descent}}, \quad (4)$$

where $\{\alpha_{ji}\}$ is the collection of averaging weights, $\rho(t)$ denotes the step size, and $\mathbf{g}_j(\mathbf{w}) := \nabla_{\mathbf{w}} f(\mathbf{w}, \mathbf{Z}_j)$ denotes the local gradient. Similar to the case of “Decentralized Consensus,” however, a simple lazy Byzantine strategy at one or more nodes will lead to breakdown of DGD-based learning methods.

Scalar-valued Problems: Robustification of decentralized ERM-based learning in the presence of Byzantine attacks requires resilient variants of DGD. The works in [31], [32] focus on this for scalar-valued (i.e., $d = 1$) decentralized optimization. Similar to distributed learning and decentralized consensus, resilience in these methods is also achieved through the use of a screening-based aggregation procedure within the consensus step in (4). The works [31], [32] in particular focus on trimmed-mean screening: after node j receives $w_i \in \mathbb{R}$ from its neighbors, it removes the largest b and the smallest b w_i ’s and takes the average of the remaining scalars for consensus. This leads to the following update rule:

$$w_j^{t+1} = \frac{1}{|\mathcal{N}_{j*}^t|} \sum_{i \in \mathcal{N}_{j*}^t} w_i^t - \rho(t)g_j(w_j^t), \quad (5)$$

where \mathcal{N}_{j*}^t is the set of nodes (possibly including node j itself) that survive the screening at node j . This update can be shown to be Byzantine resilient in the sense that the w_j^t ’s converge to the minimizer of *some* convex combination of local losses $f(w, \mathbf{Z}_j)$. We also have a result from [31] that the minimum of the decentralized ERM in (3), even when restricted to the set of nonfaulty nodes, cannot be achieved in the presence of Byzantine nodes. Despite this negative result, it can be shown that any convex combination of local losses will converge in probability to the global statistical risk in the case of i.i.d. training samples. This can then be leveraged to establish the robustness of (5) for scalar-valued decentralized learning [33].

Vector-valued Problems: The algorithms in [31], [32] cannot be directly utilized in vector-valued problems (i.e., $\mathbf{w} \in \mathbb{R}^d$ for $d > 1$). On the one hand, unless a problem decouples over different coordinates of the optimization variable \mathbf{w} , minimizing the objective function along one coordinate *independent* of the other coordinates does not yield the right solution. On the other hand, since the trimmed-mean procedure of [31], [32] requires sorting of values received from one’s neighbors, it cannot be directly applied to members of an *unordered space* like \mathbb{R}^d . This limitation of [31], [32] is overcome in [33], which proposes an algorithm termed *Byzantine-resilient decentralized coordinate descent* (ByRDiE) for vector-valued decentralized learning in the presence of Byzantine nodes. ByRDiE, fundamentally being a coordinate descent method, cyclically updates one coordinate at a time in a decentralized manner. And

since each subproblem in coordinate descent becomes a scalar-valued problem, ByRDiE uses trimmed-mean screening in each inner (coordinate-wise) iteration for Byzantine resilience. The final update rule in each coordinate-wise iteration of ByRDiE takes a form similar to (5), except that the gradient term also depends on other coordinates of \mathbf{w} . For strictly convex and smooth loss functions, [33] guarantees (algorithmic and statistical) convergence of the iterates of ByRDiE to the statistical minimizer, with the convergence rate being sublinear and the learning rate being a function of different problem parameters. Despite these guarantees, the one-coordinate-at-a-time update of ByRDiE can be inefficient for high-dimensional (i.e., $d \gg 1$) problems. Indeed, since the coordinate-wise gradient update step depends on the updates of other coordinates, the iterates in ByRDiE cannot be updated in a coordinate-wise parallel fashion, leading to high network coordination and local computation overheads in decentralized learning.

To curtail the high overheads of ByRDiE, [34] presents another algorithm—termed *Byzantine-resilient decentralized gradient descent* (BRIDGE)—that is based on gradient descent and coordinate-wise trimmed mean. Similar to DGD, each node j in BRIDGE also exchanges its *entire* current iterate \mathbf{w}_j^t in every iteration t with all nodes in its neighborhood \mathcal{N}_j . The update step in BRIDGE, however, involves coordinate-wise screening/aggregation of \mathbf{w}_j^t 's using trimmed mean with parameter $2b$, which is followed by a local gradient descent step. Mathematically, the (parallel) update of the k -th coordinate is given by

$$\forall k \in \{1, \dots, d\} \text{ (in parallel), } w_j^{t+1}(k) = \frac{1}{|\mathcal{N}_{j^*}^{t,k}|} \sum_{i \in \mathcal{N}_{j^*}^{t,k}} w_i^t(k) - \rho(t) g_j^k(\mathbf{w}_j^t), \quad (6)$$

where $\mathcal{N}_{j^*}^{t,k}$ is the set of nodes whose k -th coordinates survive the screening at node j and $g_j^k(\mathbf{w})$ denotes the k -th coordinate of the local gradient $\mathbf{g}_j(\mathbf{w})$. In the case of strongly convex and smooth loss functions, theoretical guarantees for BRIDGE match those for ByRDiE.

Topology Constraints: We noted earlier (see “Decentralized Consensus”) that Byzantine resilience in decentralized setups depends on network topology. We now describe two related topology constraints for trimmed mean-based decentralized learning. The constraint in [31] requires that, after removing all b Byzantine nodes and *any* combination of remaining b (incoming) edges from each node, there is always a group of nodes—termed *source component*—of cardinality at least $(b+1)$ that has a directed path to every other node. In words, this constraint means every nonfaulty node, even after trimmed-mean screening, can always receive information—directly or indirectly—from the source component. The topology constraints for ByRDiE and BRIDGE are also based on this condition. The constraint in [32], on the other hand, is based on the idea that any two arbitrary partitions of the network must result in one of the partitions having at least one node with $(2b+1)$ neighbors outside the partition. This, in turn, guarantees that b Byzantine nodes cannot isolate any subset of nonfaulty nodes during trimmed-mean screening.

Beyond Trimmed-mean Screening: Unlike distributed learning, where several screening methods and aggregation rules have been proposed and analyzed for Byzantine resilience (Table II), utilization of screening methods in decentralized learning has been limited to (coordinate-wise) trimmed mean. The main reason for this is the need for iterate consensus in decentralized learning, which makes analysis of other screening/aggregation methods challenging. (It is also worth noting that our focus here is on iterate screening, as opposed to gradient screening in distributed learning.) In practice, it is possible to merge the ideas behind BRIDGE and screening methods such as coordinate-wise median, Krum, and Bulyan to develop other Byzantine-resilient decentralized learning methods. This is indeed the approach we have taken in the numerical experiments discussed in the following. Consensus, convergence analysis, and statistical learning rates of such methods, however, remain an open problem.

Numerical Experiments: We now compare the performance of DGD, ByRDIE, BRIDGE and three variants of BRIDGE that come about from incorporation of the screening principles of (coordinate-wise) median, Krum, and Bulyan (with Krum) within the BRIDGE framework. With the exception of DGD, all methods are initialized with parameter $b = 2$ in *all* experiments. The experimental setup corresponds to multiclass classification of MNSIT dataset using a linear one-layer neural network, which results in a loss function that satisfies the theoretical conditions for convergence of ByRDIE and BRIDGE. There are $M = 20$ nodes in the network, with an edge between two nodes decided by a random coin flip. Once a random topology is generated, we ensure each node has at least $4b + 1$ nodes in its neighborhood (a condition imposed due to Bulyan screening). The training data at each node corresponds to $N = 2000$ samples randomly selected from the MNIST dataset. The performance of each method is reported in terms of classification accuracy, averaged over $(M - b)$ nodes and a total of 10 independent Monte Carlo trials, as a function of the number of scalars broadcast per node. The final results, shown in Figure 6, correspond to two sets of experiments: (i) the faultless setting in which none of the nodes actually behaves maliciously; and (ii) the setting in which two of the 20 nodes are Byzantine, with each Byzantine node broadcasting every coordinate of the iterate as a uniform random variable between 0 and 10.

It can be seen from Figure 6 that, other than ByRDIE and Krum-based screening, all methods perform almost as well as DGD in the faultless case. In the presence of Byzantine nodes, however, DGD completely falls apart, whereas the performances of all screening methods remain comparable to the faultless setting. It is also worth comparing these results to those for Byzantine-resilient distributed learning (see Figure 5). While (coordinate-wise) median and trimmed mean appear to be the worst performers in Figure 5, Krum-based screening is the least effective in Figure 6. In both cases, however, Bulyan is quite effective, except that it has stringent topology requirements. Finally, the (averaged over nodes) curves in Figure 6 are less

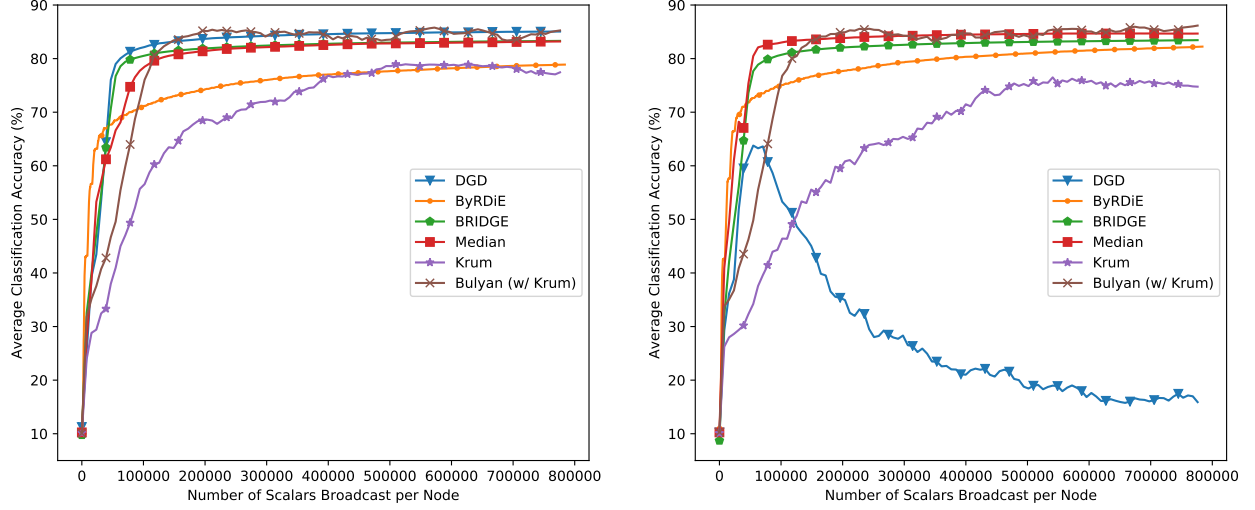


Fig. 6. Performance comparison of different decentralized learning methods in both faultless (left panel) and Byzantine settings (right panel). Byzantine-resilient algorithms in both settings operate under the assumption of $b = 2$. The algorithms entitled *Median*, *Krum*, and *Bulyan* are effectively BRIDGE combined with the screening procedures advocated in distributed learning.

smooth than in Figure 5. This is to be expected since consensus and learning in decentralized settings require diffusion of information over the network that, depending on network topology, can take a while.

III. SOME OPEN RESEARCH PROBLEMS

Despite recent advances, Byzantine-resilient inference and learning remains an active area of research with several open problems. Much of the focus in distributed inference has been on the somewhat restrictive model in which Byzantine nodes do not collude. Collaborative Byzantine attacks, on the other hand, can be much more potent than independent ones. A fundamental understanding of mechanisms for safeguarding against such attacks remains a relatively open problem in distributed inference. Byzantine-resilient distributed estimation under nonlinear models is another problem that has been relatively unexplored. In the case of Byzantine-resilient distributed learning, existing works have only scratched the surface. Convergence and/or learning rates of many of the proposed methods remain unknown (Table II). In addition, while SGD is a workhorse of machine learning, approaches such as accelerated first-order methods (e.g., accelerated gradient descent), first-order dual methods (e.g., ADMM), and second-order methods (e.g., Newton’s method) do play important roles in machine learning. However, resilience of distributed variants of such methods to Byzantine attacks has not been investigated in the literature.

The lack of a central server, the need for consensus, and an ad-hoc topology make it even more challenging to develop and analyze Byzantine-resilient methods for decentralized inference and learning. Much of the work in this regard is based on screening methods such as trimmed mean and median that

originated in the literature on Byzantine-resilient scalar-valued consensus. This has left open the question of how other screening methods, such as the ones explored within distributed learning, might handle Byzantine attacks—both in theory and in practice—in various decentralized problems. Unlike distributed learning, any such efforts will also have to characterize the interplay between the network topology and the effectiveness of the screening procedure. The fundamental tradeoffs between the robustness and the (faultless) performance of Byzantine-resilient methods also remain largely unknown for decentralized setups. Finally, existing works on decentralized learning only guarantee sublinear convergence for strictly/strongly convex and smooth functions. Whether this can be improved by taking advantage of faster distributed optimization frameworks or different screening methods also remains an open question.

IV. CONCLUSIONS

In this article, we have presented an overview of latest advances in Byzantine-resilient inference and learning. In the distributed setting, which is characterized by the presence of a central server that computes the final solution, we discussed recent results concerning resilience of distributed detection, estimation, and machine learning against Byzantine attacks. Within distributed machine learning, we focused on Byzantine-resilient variants of distributed stochastic gradient descent, whose performances were compared using numerical experiments. In the decentralized setting, which typically requires consensus due to lack of a central server, we first discussed the principles behind Byzantine-resilient consensus. This was followed by a discussion of latest results on decentralized detection, estimation, and learning in the presence of Byzantine nodes. We also compared and contrasted different Byzantine-resilient decentralized learning methods using numerical experiments, and discussed the similarities and difference between them and distributed learning methods. Byzantine-resilient inference and learning has a number of research challenges that remain unaddressed, some of which are also briefly discussed in the article.

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