Canceling Spiral/Chaotic Waves

In the case of Ventricular Fibrillation

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Abstract

The focus of this essay is to try to re-regulate the heart from a turbulence or spiral-wave state back to normal state during simulation. To achieve this, a normal state is generated, and then is turned into spiral/ turbulence states under different conditions. After these, suitable boundary conditions are explored and established in seek of bringing back the heart to normal state. The behavior of the heart is described by the Barkley Model.

Keywords

Barkley Model, spiral wave, chaotic wave, travelling wave, diffusion-reaction equation

Introduction

It is believed that spiral waves or worse, chaotic pattern waves can directly cause ventricular fibrillation, which is fatal in many cases and must be treated immediately. The conventional treatment is to rely on ECT or pacemakers to bring the regular electric behavior back. However these two methods are both expensive and render the patients painful. So a new way that's more convenient is desirable. A possible solution is to alternate/control the electric wave pattern of the heart. Barkley has viewed the heart as a system that can be described by the diffusion-reaction equation, given below:

$$\frac{\partial u}{\partial t} = -\frac{1}{\varepsilon}u\left(u-1\right)\left(u-\frac{v+b}{a}\right)+\nabla^2 u$$

$$\frac{\partial v}{\partial t} = f(u)-v$$

where

$f(u)=0$ if $0\le u<1/3$,

$f(u)=1-6.75u\left(u-1\right)^2$ if $1/3\le u<1$,

$f(u)=1$ if $u>1$

Here u and v describe the activator and the inhibitor variables, respectively, and ε denotes the excitability of the system. The higher ε is, the lower the excitability. In this case, u is the main dependent variable we focus on, as it represents the electric potential. Note that this is

NOT the general form of diffusion-reaction equation, but a heart-specific one. It will be shown that under the right circumstances, a series of uniform travelling wave can be generated. Then it is shown that such waves can totally run into disorder if a "suitable" ε is chosen. Last, attempts are made to cancel that disorder and bring back the normal travelling wave.

Tools Used

Instead of using MATLAB, free Python packages Numpy and Matplotlib were chosen to do the task, as they are lightweight and easy to use. The programing language is, of course, Python(3.6), and a demonstrating version of the code can be found in the appendix.

The Simulation

The numerical method of all of the simulation down below is Finite Difference Method (FDM). In addition, for empirical reason, a=0.84 and b=0.07 were chosen for all simulation. The simulation is done on a square with side length 150, which is divided into 150 nodes on every side, thus the $\mathrm{d}x$ and $\mathrm{d}y$ are both 1. To generate the travelling wave, the left side of the field is hold steady at 1, and no-flux boundary condition is applied. To satisfy the condition $(\mathrm{d}t)^2 < \frac{(\mathrm{d}x)^2 + (\mathrm{d}y)^2}{2}$, which ensures the simulation converges to "actual" values, $\mathrm{d}t = 0.04$ is chosen here. Also, ε is chosen to be 0.03, which stands for a somewhat high excitability, in this scenario. The results is shown in Figure.1

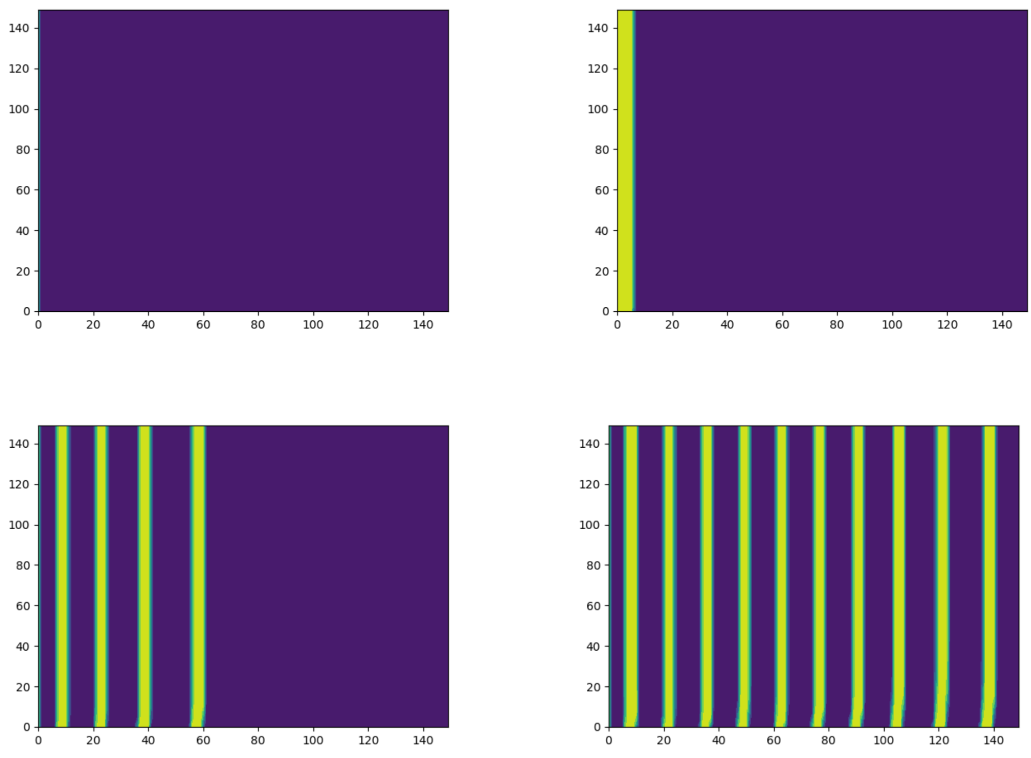

Figure.1 The four graphs has shown when the system has been iterated 0, 50, 400, 900 steps.

Then one wave in the middle of the field is manually truncated into a half and the field is set to 0 elsewhere, as shown in Figure2. This acts as a new initial value for the system. Also, the boundary value condition, $\partial\Omega = 1$ on the left side is deleted. This means that some part of the heart has some difficulty to conduct the wave(the truncated part), and we iterate the system to see how the “tissue” would react. Remember in this case the excitability is unchanged, thus $\varepsilon = 0.03$. The results are shown in Figure 2.

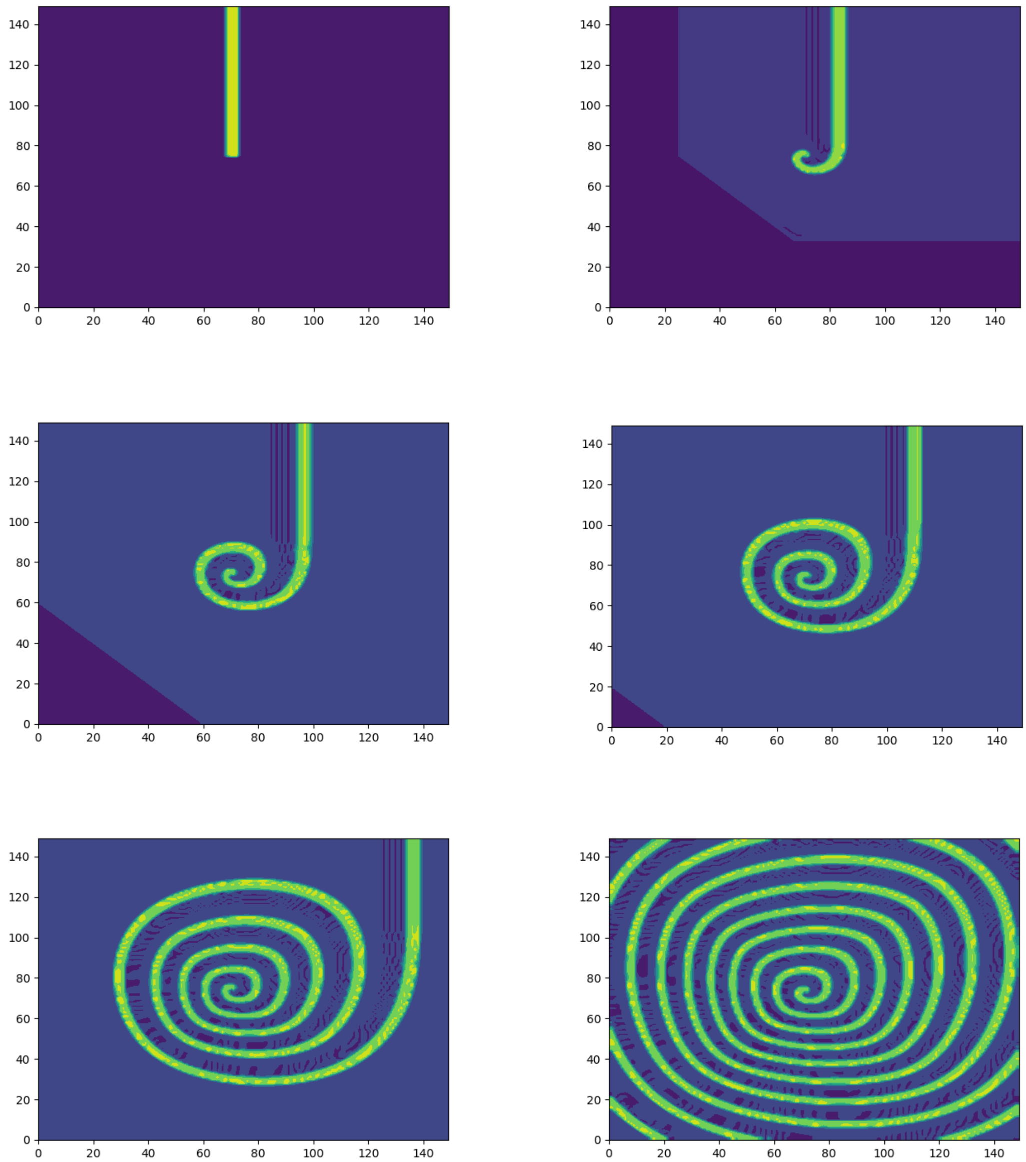

Figure 2. The first graph shows the initially truncated wave. Then the wave grows to a spiral wave from that point; the following graphs show this. The system has been iterated 0, 40, 80, 120, 200, 400 steps respectively.

We can see a spiral wave forms in the center and grows outwardly. Such wave is harmful to heart tissue and make it dysfunctional. In a worse case, on a unhealthy heart, the excitability of the heart is lower, so ε is higher. To simulate the scene, we set $\varepsilon = 0.09$ and run the simulation again. The results are shown in Figure3.

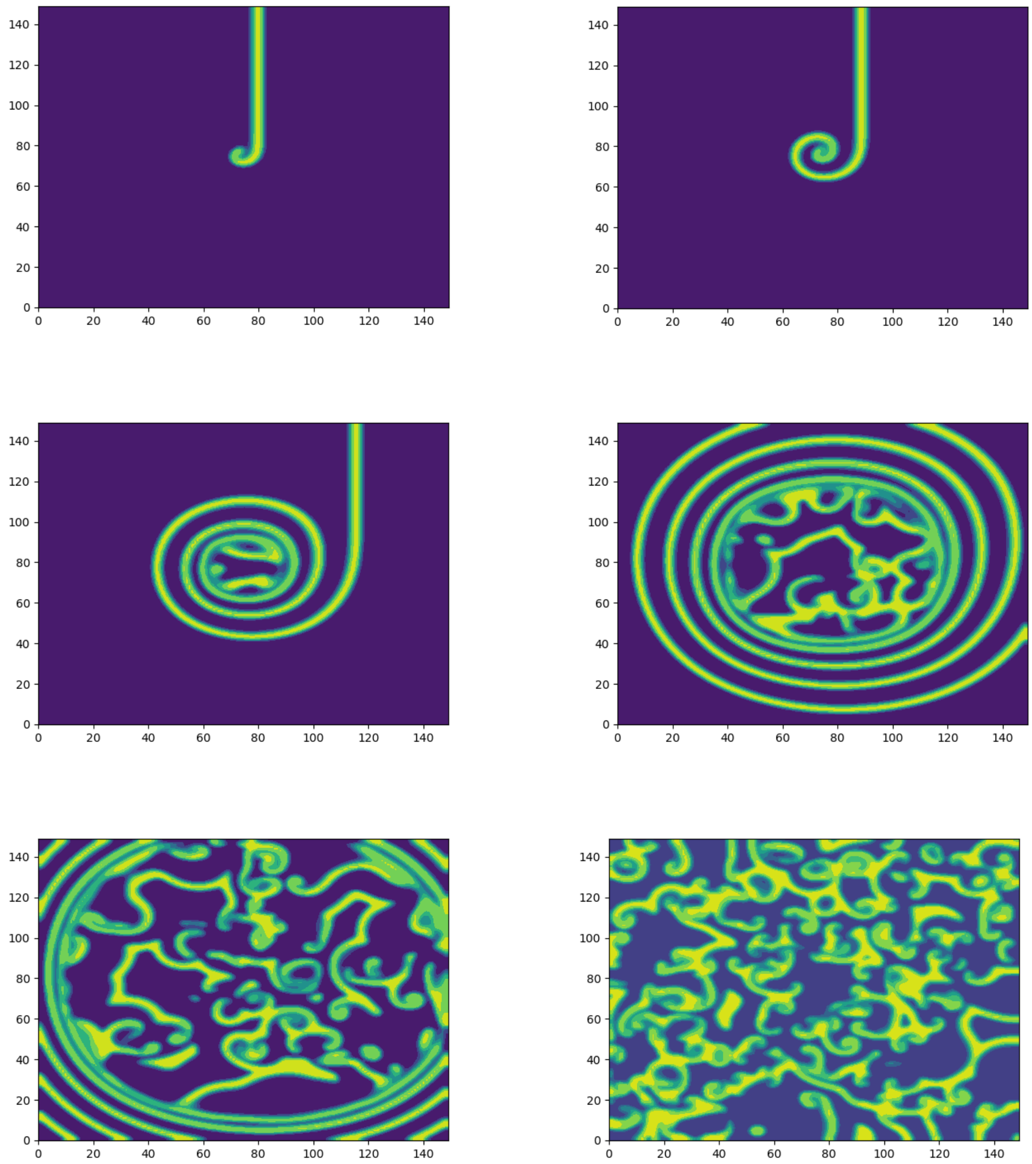

Figure 3. The first graph shows the initially truncated wave as in Figure 2. At first the system grows into a spiral wave with no problem, but before long it goes chaotic. The following graphs show this. The system has been iterated 40, 80, 200, 400, 640, 1000 steps, respectively.

In such a system of low excitability, we can see at first a spiral wave generates as the former scene, however before long the waves begin to meander into chaos from the center and spreads out. If we keep the system running for a long time, the whole field become fully chaotic.

The next part of our simulation, being the most valuable/meaningful part, is to try to use a special boundary value condition to “generate” a new series of travelling wave to “flush” away the spiral/turbulence waves.

We first try to eliminate the chaotic wave. In this case boundary value on the left side of the field is set to 1 again, in the hope of the generated wave would push through the field. The results are shown in Figure4.

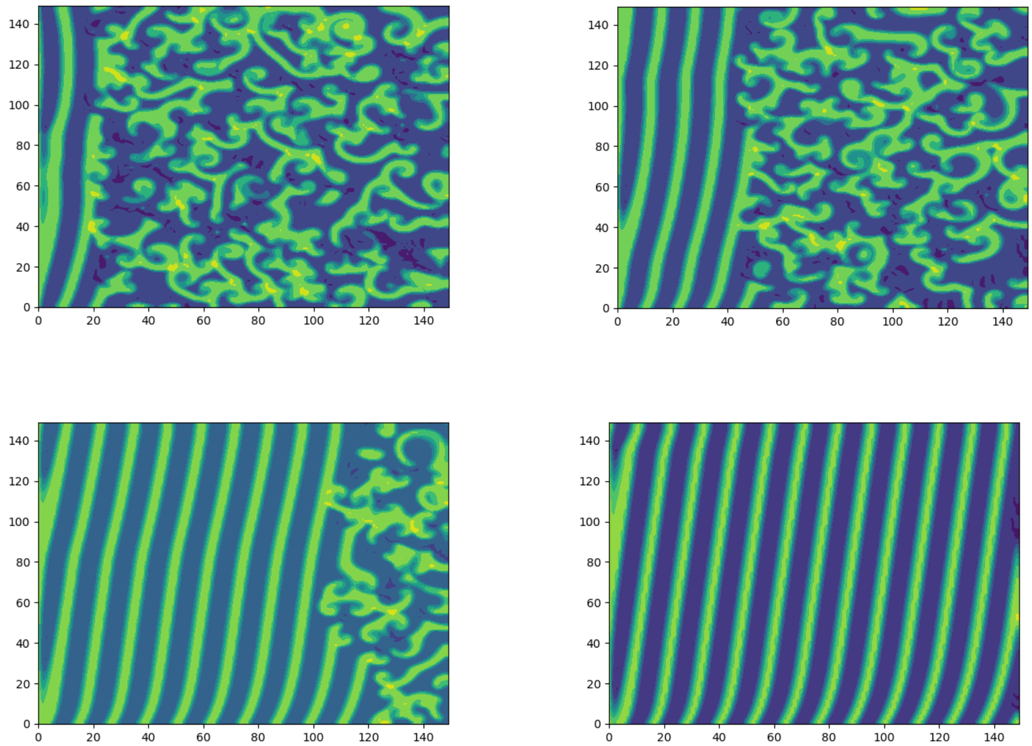

Figure 4. The system has been iterated 1200, 2700, 7200, 11000 steps, respectively.

We can see if given a period of time that is long enough, the travelling wave will finally push away the chaotic waves and restore the normal condition of the heart. However, it cannot be ignored that the time for the travelling wave to dominate the field is much longer (about 15-20 times) than fot the chaotic wave to form in the field. Also, the travelling waves go askew for no specific reason.

For the harder-to-eliminate spiral waves (due to the higher excitability), instead of holding a steady 1 on the left side for boundary condition, a varying, periodic function to time is chosen. The form of such a function is $\partial\Omega = \cos\left(2k\mathrm{d}t\right)$, with k being the specific iteration step number. Note that ε is chosen back to 0.03. The results are shown in Figure5.

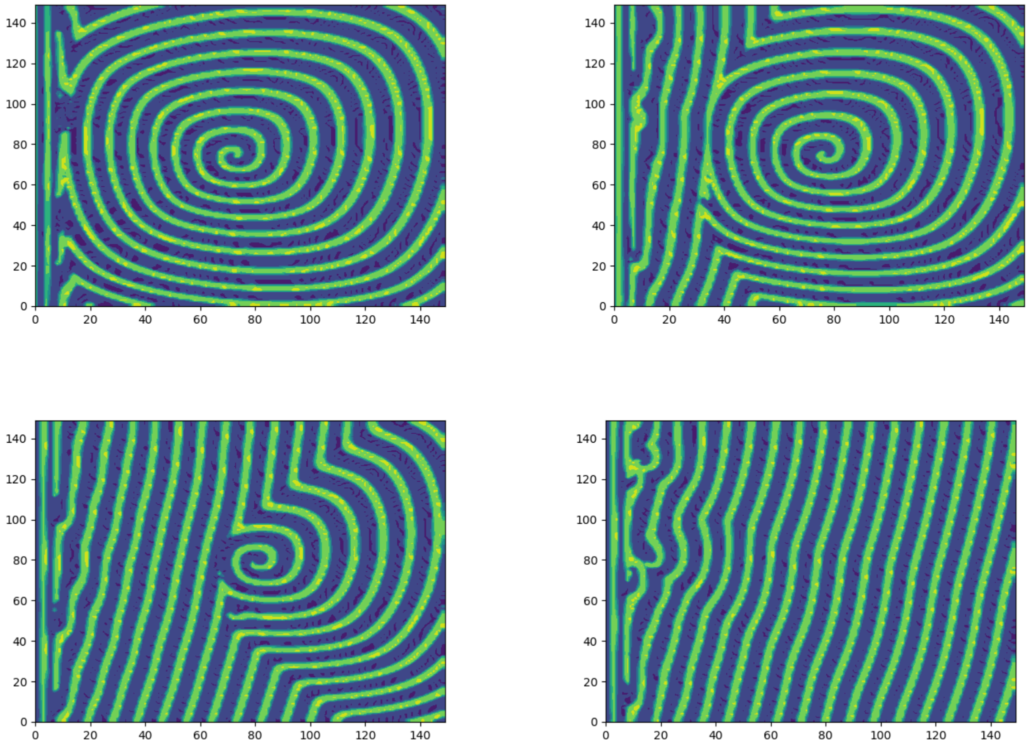

Figure 5. The system has been iterated 500, 3500, 8000, 20000 steps, respectively. Though consuming a much longer time, the travelling wave will finally dominates the field.

Afterthoughts

It can be seen that if proper boundary value conditions are provided, the “normal ” travelling wave will re-dominate the field, thus providing a possible cure for ventricular fibrillation. Such method has much more value and potential to be explored.

References

1. Spiral-wave dynamics in a simple model of excitable media: The transition from simple to compound rotation, ***Dwight Barkley, Princeton University, Physical Review A.***

2. A model for fast computer simulation of waves in excitable media, ***Dwight Barkley, Princeton University, Physica D***

3. *http://www.scholarpedia.org/article/Barkley_model*