Self-Organizing Criticality Behavior in Human Brain

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Abstract

The focus of this essay is to explore the behavior of a neural system which models the human brain. We shall see a phenomenon called self-organized criticality. We change parameters such as threshold for the input to excite a certain brain area, the possibility for a brain area to excite itself without input, to see how the system will behave. By using Monte-Carlo method to simulate the system, we expect to see three different "criticality" states: 1. Subcritical, 2. Critical, and 3.Supercritical. After these we shall see explore how the correlation reacts as threshold varies, and we expect to spot an Ising Model-like relation.

Keywords

Self-Organizing Criticality, Monte-Carlo, Correlation Length

Introduction

It is somewhat hard to answer the question:

Why the fundamental laws of physics are so simple, yet the whole world is so complicated.

There's no "mathematical" answer to such question until a PRL paper published in 1987 by shed some light on this. That paper held a perspective that a complex system may cause avalanches in itself with no input from the outside world, and the possibility for an

$$P(S) = S^{-\alpha}$$

avalanche to occur is in exponential relation (decreasing) to scale of such avalanche. That is: Where S is the scale of avalanches and α is a constant related to a specific system. Different systems have different α, but they all obey this power law, even though they seem to be totally unrelated (such as neural system in human brain, forest fire, height of skyscrapers, literal texts, etc.).

In this essay we focus on human brain, and such "brain" is divided into 90 parts, with each of them representing an independent node. Each node is connected to all others by "synapses", and the connection strength can be depicted by a positive real number, all of which forms a 90 by 90 matrix, called the connectome data. In this research, connectome data from DTI is used.

Tools Used

Instead of using MATLAB, free Python packages Numpy, Matplotlib and Scipy.io (to load DTI connectome data) are chosen to do the task, as they are lightweight and easy to use. The programing language is, of course, Python(3.6), and a demonstrating version of the code can be found in the appendix.

The Simulation and Statistics

For the following simulation, r_1 represents the possibility for a certain brain area to excite without input, and r_2 represents the possibility for a brain area to be delayed to recover from refractory state to quiescent state. Also note that we multiplied the DTI data by 0.0005 before using it, only to find such a value seems to be “proper”.

We first set $r_1 = 0.001$, $r_2 = 0.1$ and let the threshold T vary, in order to find the subcritical state of the system. After finding a proper value of T, we decrease it in seek of the critical and supercritical states, the latter of which has a special property that in a certain range of scale of avalanches, the larger the scale, the higher the possibility for it to occur. Such property disobeys the power law and hence is abnormal.
The results are shown in Figure 1.

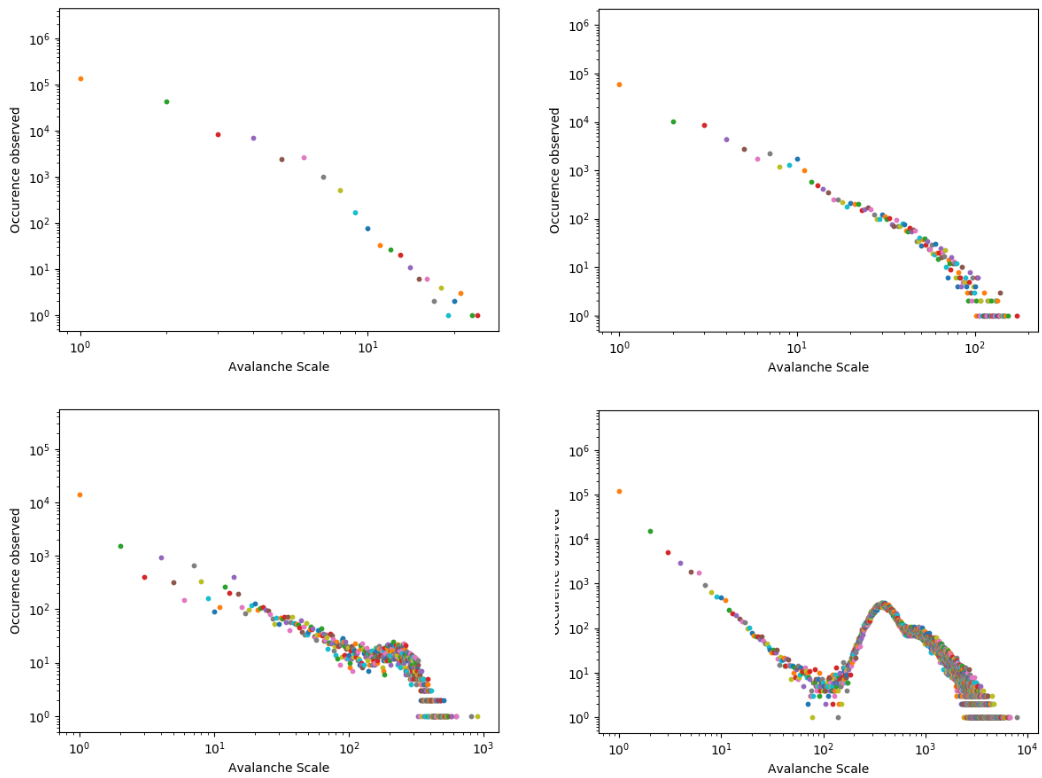

Figure 1. From top left to bottom right: T=0.055, 0.045, 0.035, 0.025.

As shown, when threshold is 0.055, the statistics presents a more noticeable curvature, thus the system is in subcritical state (Note that log scale are chosen for both x and y.) As T decreases, the system eventually transits to critical state and then supercritical state, as there is no clear boundary to differentiate these states. When the system is in the affinity to its critical state, the occurrence-scale shows a pretty decent “exponentially decreasing” relation. Notice there is already a small “bump” when T=0.035 and such bump swells significantly when T is further decreased down to 0.025. Remember that the absolute times of occurrence of avalanches is not important across different T, as our focus is the relative distribution of occurrence. (Actually the system is iterated different steps for different T.)

For comparison, we set $r_1 = 0.01$ and hold other parameters unaltered to see what difference it would make. The results are shown in Figure 2.

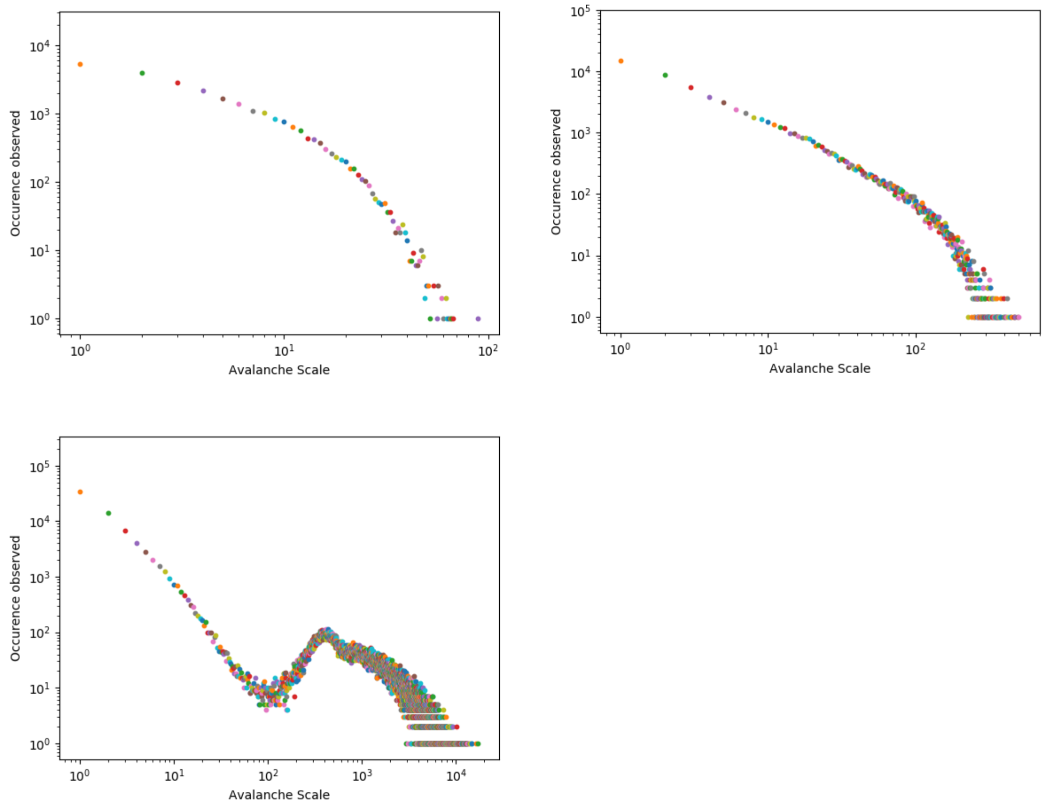

Figure 2. From top left to bottom left: T=0.055, 0.045, 0.025.

As we can see, the general behavior of the system is largely unchanged, with a slightly more pronounced curve appears when T=0.055.

The next step of our research is to find when the system has the largest correlation among the nodes (among different brain areas). The correlation we are interested here is the counterpart to the correlation distance/length in the famous Ising Model. Here, such value represents how possible for a different brain area to excite *together with* a specific brain area. To calculate this correlation, we first iterate the system for many steps (say, 10000), and then find the average value for each brain area, then we calculate the correlation for each step using its definition: $C_{ij} = \left(i - \langle i \rangle\right)\left(j - \langle j \rangle\right)$ for every step and then sum them up against steps. Finally we sum C_{ij} element-wise: $C_{total} = \sum_{i,j} C_{ij}$. Then C_{total} shows the tendency for the nodes in the system to correlate with other nodes. We calculate C_{total} for a range of T (from 0.02 to 0.06, with interval of 0.001). The total steps iterated for each T is held the same at 10000 to make C_{total} comparable. For two different $r_1 = 0.001$ and $r_1 = 0.01$, the results are shown in Figure 3.

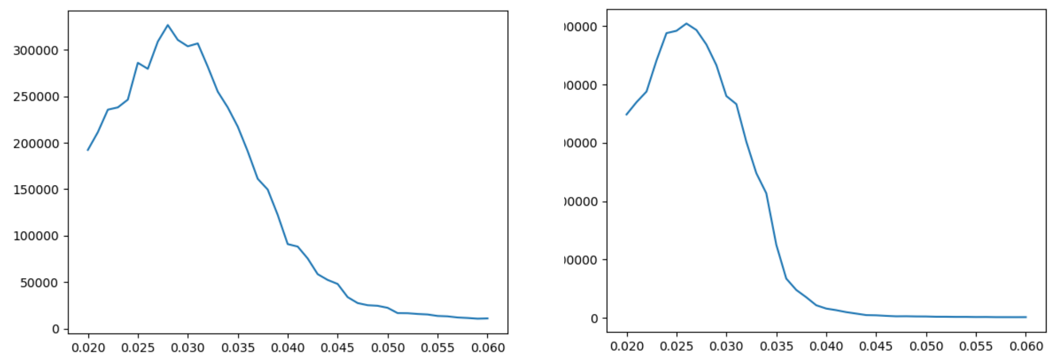

Figure 3. left: $r_1 = 0.01$, right: $r_1 = 0.001$

We can actually see, no matter what value r_1 is, the “real” critical T for our system lies between 0.025 and 0.03, at which the correlation is the highest. In theory, the correlation should be ∞ for a critical threshold. However as the whole simulation is run on a personal computer, the results can be somewhat inaccurate (10000 steps for each T is not that sufficient and causes some roughness in the graphs.), due to lack of computation capability.

Afterthoughts

We verified the power law of occurrence-avalanche scale in a complex system. Further study to such systems would yield many useful properties and insights of them, with which we may be able to control the “complexity” of those systems by changing certain parameters.

Reference

1. Bak, P., Tang, C. and Wiesenfeld, K. (1987). "Self-organized criticality: an explanation of 1/f noise". *Physical Review Letters*. **59** (4): 381–384.
2. Vladyslav A. Golyk , Self-organized criticality , Massachusetts Institute of Technology, Department of Physics, Cambridge
3. https://en.wikipedia.org/wiki/Self-organized_criticality