Hodge-compositional Edge GPs

Curl-free, div-free GPs

$\mathbf{f}_G \sim \mathrm{GP}(\mathbf{0}, \mathbf{K}_G)$
$\mathbf{f}_H \sim \mathrm{GP}(\mathbf{0}, \mathbf{K}_H)$
$\mathbf{f}_C \sim \mathrm{GP}(\mathbf{0}, \mathbf{K}_C)$

- Gradient kernel $\mathbf{K}_G = \mathbf{U}_G \Psi_G(\mathbf{\Lambda}_G)\mathbf{U}_G^\top$; Curl kernel $\mathbf{K}_C = \mathbf{U}_C \Psi_C(\mathbf{\Lambda}_C)\mathbf{U}_C^\top$
- Matérn family: $\Psi_\square(\mathbf{\Lambda}_\square) = \sigma_\square^2 \left(\frac{2\nu_\square}{\kappa_\square^2}\mathbf{I} + \mathbf{\Lambda}_\square \right)^{-\nu_\square}, \quad \square = \mathrm{H}, \mathrm{G}, \mathrm{C}$
- Also as solutions of SDEs, e.g.,

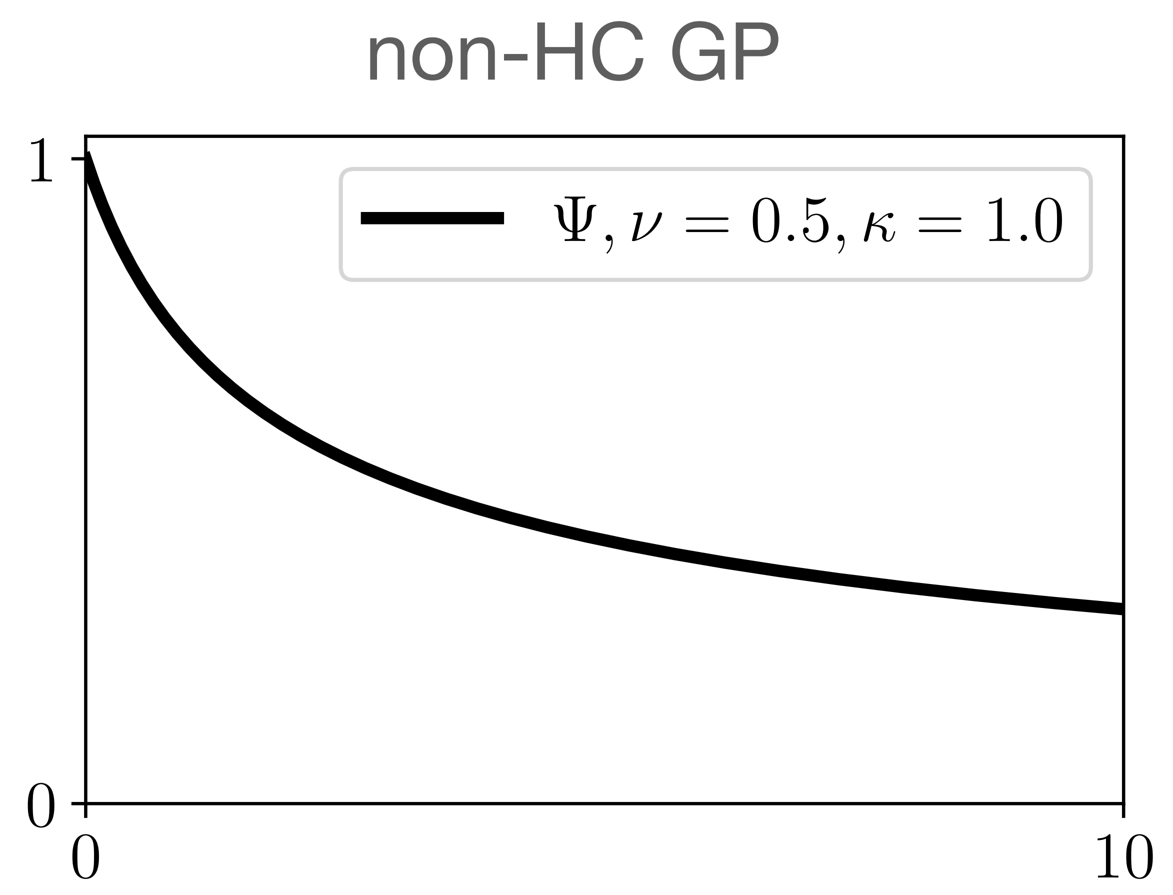
$\Phi_C(\mathbf{L}_{1,\mathrm{u}})\,\mathbf{f}_1 = \mathbf{w}_C$, with curl noise $\mathbf{w}_C \sim N(0, \sigma_C^2 \mathrm{U}_C \mathbf{U}_C^\top)$ and

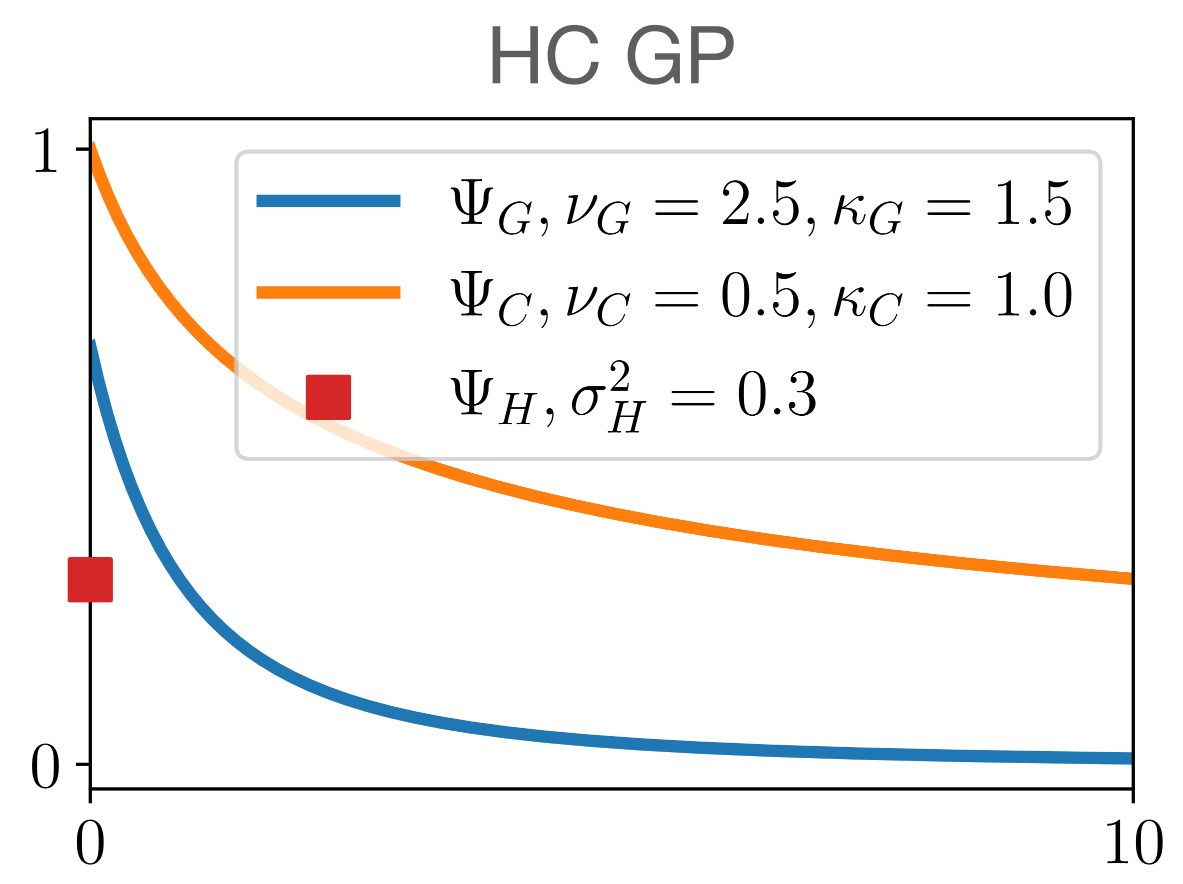
$$\Phi(\mathbf{L}_{1,u}) = \left(\frac{2\nu_C}{\kappa_C^2}\mathbf{I} + \mathbf{L}_{1,u} \right)^{\frac{\nu_C}{2}} \text{ or } \Phi(\mathbf{L}_{1,u}) = e^{\frac{\kappa_C^2}{4}\mathbf{L}_{1,u}}$$

Hodge-compositional Edge GPs

Composition of three GPs on the Hodge subspaces

- Kernel: $K_1 = K_G + K_H + K_C$
- Mutual independence hypothesis
- Separate learning of different components
- Automatic determination of Hodge components, instead of solving Hodge decomp.
- Edge Fourier Feature perspective





Alternative formulation

via node-edge-triangle interactions

- Derivatives of GPs are also GPs
- Induce edge GPs from node and triangle GPs

$$K_1 = K_H + B_1^\top K_0 B_1 + B_2 K_2 B_2^\top$$

- Induce node GPs from edge GPs