Optimal topological SB

- Schrödinger system characterizes the optimality
- Disintegration of measures: gives us static TSBP (OT formulation)

$$\min D_{KL}(\mathbb{P}_{01} \| \mathbb{Q}_{\mathcal{T}01}) \ s.t. \mathbb{P}_0 = \rho_0, \mathbb{P}_1 = \rho_1$$

  - An E-OT with transport cost: $\left\| y_1 - \Psi_1 y_0 - \xi_1 \right\|^2_{K_{11}^{-1}}$
  - Lagrange multipliers: gives us a topological Schrödinger system —— iterative proportional fitting (cont. Sinkhorn alg.) for half-bridge prob

- Stochastic control: tells us how optimal TSB follows a forward-backward SDE system
  - Forward: $dX_t = dY_t + Z_t dt,\ X_0 \sim \rho_0$
  - Backward: $dX_t = dY_t - \tilde{Z}_t dt,\ X_1 \sim \rho_1$

- Nonlinear Feynman-Kac formula: gives us a likelihood

TSB-learning model

Learnable	Trainable
$Z_t \approx Z_t(\theta)$	$l(x_0; \phi)$
$\tilde{Z}_t \approx \tilde{Z}_t(\phi)$	$l(x_1; \theta)$

Diffusion on nodes and edges