

Hodge-compositional Edge GPs

Curl-free, div-free GPs

$$\begin{aligned} \mathbf{f}_G &\sim \text{GP}(\mathbf{0}, \mathbf{K}_G) \\ \mathbf{f}_H &\sim \text{GP}(\mathbf{0}, \mathbf{K}_H) \\ \mathbf{f}_C &\sim \text{GP}(\mathbf{0}, \mathbf{K}_C) \end{aligned}$$

- Gradient kernel $\mathbf{K}_G = \mathbf{U}_G \Psi_G(\Lambda_G) \mathbf{U}_G^\top$; Curl kernel $\mathbf{K}_C = \mathbf{U}_C \Psi_C(\Lambda_C) \mathbf{U}_C^\top$

- Matérn family: $\Psi_\square(\Lambda_\square) = \sigma_\square^2 \left(\frac{2\nu_\square}{\kappa_\square^2} \mathbf{I} + \Lambda_\square \right)^{-\nu_\square}$, $\square = H, G, C$

- Also as solutions of SDEs, e.g.,

$\Phi_C(\mathbf{L}_{1,u}) \mathbf{f}_1 = \mathbf{w}_C$, with curl noise $\mathbf{w}_C \sim N(0, \sigma_C^2 \mathbf{U}_C \mathbf{U}_C^\top)$ and

$$\Phi(\mathbf{L}_{1,u}) = \left(\frac{2\nu_C}{\kappa_C^2} \mathbf{I} + \mathbf{L}_{1,u} \right)^{\frac{\nu_C}{2}} \text{ or } \Phi(\mathbf{L}_{1,u}) = e^{\frac{\kappa_C^2}{4} \mathbf{L}_{1,u}}$$

Hodge-compositional Edge GPs

Composition of three GPs on the Hodge subspaces

- Kernel: $K_1 = K_G + K_H + K_C$
- Mutual independence hypothesis
- Separate learning of different components
- Automatic determination of Hodge components, instead of solving Hodge decomp.
- Edge Fourier Feature perspective

Alternative formulation

via node-edge-triangle interactions

- Derivatives of GPs are also GPs
- Induce edge GPs from node and triangle GPs

$$K_1 = K_H + B_1^\top K_0 B_1 + B_2 K_2 B_2^\top$$

- Induce node GPs from edge GPs

