TSB-based learning model

- Under the recent SB-based learning framework [Vargas 2021, De Bortoli 2021, Chen 2022]
- Learnable models $(Z_t(\theta), \hat{Z}_t(\hat{\theta})$ for optimal policies $(Z_t, \hat{Z}_t)$
  - NNs, graph/simplicial NNs
- Trainable objective relating the TSBP objective and the models

$$\mathcal{L}_{TSB}(x_0) = \mathbb{E}\left[\log \nu_1(X_1)\right] - \int_0^1 \mathbb{E}\left[\frac{1}{2}\|Z_t\|^2 + \frac{1}{2}\|\hat{Z}_t\|^2 + \nabla \cdot (g_t \hat{Z}_t - f_t) + \hat{Z}_t^\top Z_t\right] dt$$

- Particular choices of models give topological variants
- diffusion models using score-matching [Song et al. 2021]

$$Z_t = 0, \quad \hat{Z}_t = g_t \nabla \log p_{t|0}$$

- Diffusion bridge models based on Doob's h-transform for a particular final distri.
- Probability flow ODE: flow-matching [Lipman et al. 2022]

TSB-learning model

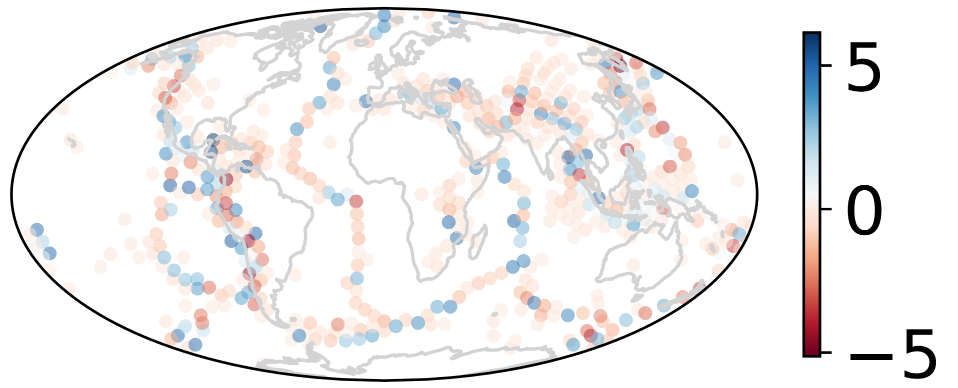
$$Z_t \approx Z_t(\theta) \qquad l(x_0; \phi)$$

$$\tilde{Z}_t \approx \tilde{Z}_t(\phi) \qquad l(x_1; \theta)$$

Learnable **Trainable**

Topological signal generation

- TSB-based models vs. SB-based models

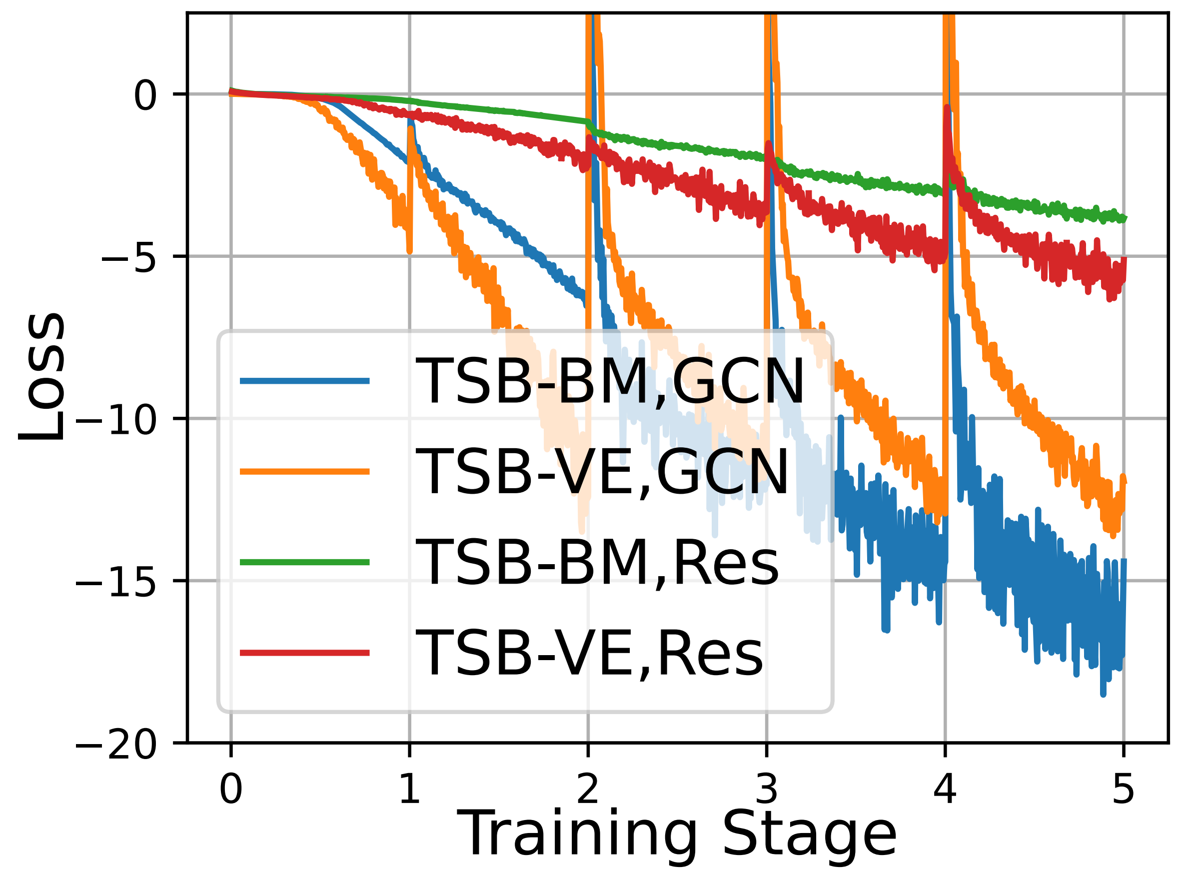


- Generative modeling

Table 1: Generative modeling results (NLL).

	SB-BM	SB-VE	SB-VP	TSB-BM	TSB-VE	TSB-VP
Seismic, Res	$2.78_{\pm 0.01}$	$2.94_{\pm 0.03}$	$2.28_{\pm 0.02}$	$2.13_{\pm 0.01}$	$2.22_{\pm 0.02}$	$2.00_{\pm 0.02}$
Seismic, GCN	$2.71_{\pm 0.03}$	$2.73_{\pm 0.05}$	$2.01_{\pm 0.03}$	$1.82_{\pm 0.02}$	$1.53_{\pm 0.03}$	$1.51_{\pm 0.02}$
Traffic, Res	$0.82_{\pm 0.00}$	$0.77_{\pm 0.00}$	$0.79_{\pm 0.00}$	$0.40_{\pm 0.00}$	$0.01_{\pm 0.00}$	$0.02_{\pm 0.00}$
Traffic, SNN	$0.18_{\pm 0.02}$	$-0.42_{\pm 0.01}$	$-0.09_{\pm 0.01}$	$-0.83_{\pm 0.05}$	$-1.26_{\pm 0.05}$	$-1.01_{\pm 0.03}$

- Effect of policy models
  - ResBlock vs. topological NNs



Method	Seismic magnitudes		Traffic flows	
	W_1	W_2	W_1	W_2
SB-BM	$11.73_{\pm 0.05}$	$8.29_{\pm 0.04}$	$18.69_{\pm 0.02}$	$13.36_{\pm 0.01}$
SB-VE	$11.49_{\pm 0.04}$	$8.13_{\pm 0.03}$	$19.04_{\pm 0.02}$	$13.61_{\pm 0.02}$
SB-VP	$12.61_{\pm 0.06}$	$8.92_{\pm 0.04}$	$18.22_{\pm 0.03}$	$13.02_{\pm 0.02}$
TSB-BM	$9.01_{\pm 0.03}$	$6.37_{\pm 0.03}$	$10.57_{\pm 0.02}$	$7.62_{\pm 0.01}$
TSB-VE	$7.69_{\pm 0.04}$	$5.44_{\pm 0.03}$	$10.51_{\pm 0.02}$	$7.58_{\pm 0.01}$
TSB-VP	$8.40_{\pm 0.04}$	$5.95_{\pm 0.03}$	$9.92_{\pm 0.02}$	$7.16_{\pm 0.01}$