Incidence & Laplacians

1st and 2nd order Discrete Derivatives

- Node signal v
- Edge flow **f**

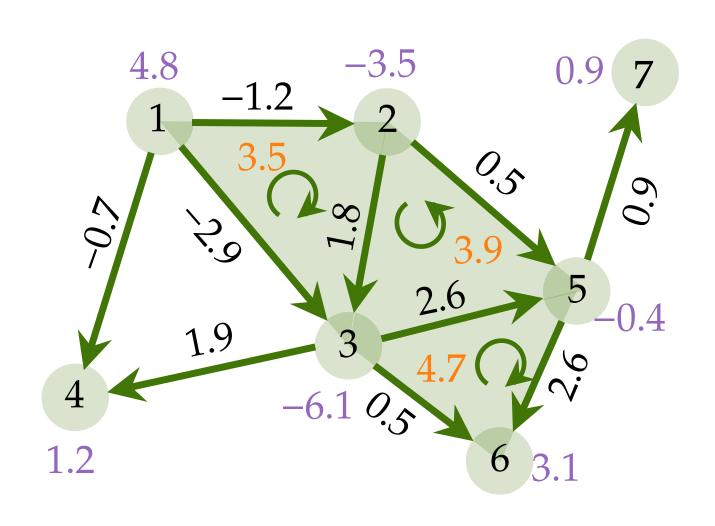
Gradient of node signal:
$$[\mathbf{f}_G]_{[i,j]} = [\mathbf{B}_1^\mathsf{T} \mathbf{v}]_{[i,j]} = [\mathbf{v}]_j - [\mathbf{v}]_i$$

$$[\mathbf{B}_1\mathbf{f}]_{[i]} = \sum_{j < i} \mathbf{f}_{[j,i]} - \sum_{i < k} \mathbf{f}_{[i,k]}$$

$$[\mathbf{B}_{1}^{\mathsf{T}}\mathbf{v}]_{[1,2]} = -1.34 - 0.96 = -2.30$$

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$$[\mathbf{f}_{\mathbf{G}}]_{[i,j]} = [\mathbf{v}]_j - [\mathbf{v}]_i$$

Divergence of edge flows:
$$[\mathbf{B}_1\mathbf{f}]_{[i]} = \sum_{j < i} \mathbf{f}_{[j,i]} - \sum_{i < k} \mathbf{f}_{[i,k]}$$

Net-flow = in_flow - out_flow

Net-circulation in triangles

$$[\mathbf{B}_1\mathbf{f}]_5 = 0.5 + 2.6 - (0.9 + 2.6) = -0.4$$

$$[\mathbf{B}_{2}^{\mathsf{T}}\mathbf{f}]_{[1,2,3]} = -1.2 + 1.8 - (-2.9) = 3.5$$

_aplacians = Grad Div + Curl* Curl