

TSB-based learning model

- Under the recent SB-based learning framework [Vargas 2021, De Bortoli 2021, Chen 2022]

- Learnable models $(Z_t(\theta), \hat{Z}_t(\hat{\theta}))$ for optimal policies (Z_t, \hat{Z}_t)

- NNs, graph/simplicial NNs

- Trainable objective relating the TSBP objective and the models

$$\mathcal{L}_{TSB}(x_0) = \mathbb{E} [\log \nu_1(X_1)] - \int_0^1 \mathbb{E} \left[\frac{1}{2} \|Z_t\|^2 + \frac{1}{2} \|\hat{Z}_t\|^2 + \nabla \cdot (g_t \hat{Z}_t - f_t) + \hat{Z}_t^\top Z_t \right] dt$$

- Particular choices of models give topological variants

- diffusion models using **score-matching** [Song et al. 2021]

$$Z_t = 0, \quad \hat{Z}_t = g_t \nabla \log p_{t|0}$$

- **Diffusion bridge models** based on Doob's h -transform for a particular final distri.

- Probability flow ODE: **flow-matching** [Lipman et al. 2022]

TSB-learning model

$$Z_t \approx Z_t(\theta) \quad l(x_0; \phi)$$

$$\tilde{Z}_t \approx \tilde{Z}_t(\phi) \quad l(x_1; \theta)$$

Learnable

Trainable

Topological signal generation

- TSB-based models vs. SB-based models
- Generative modeling

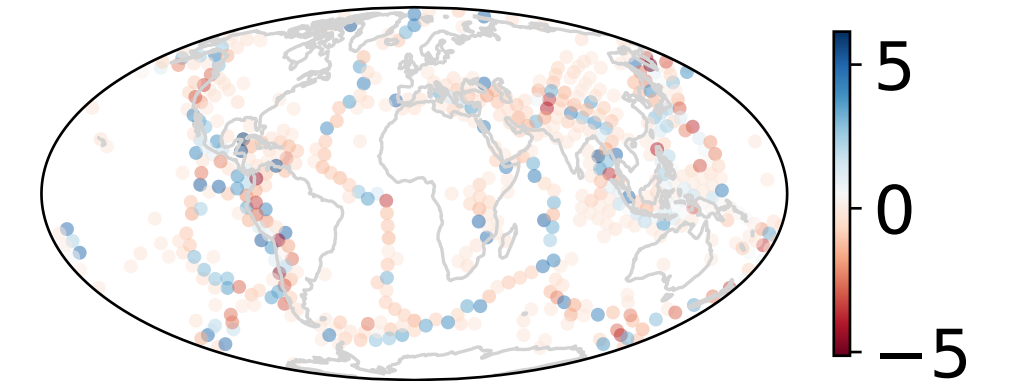
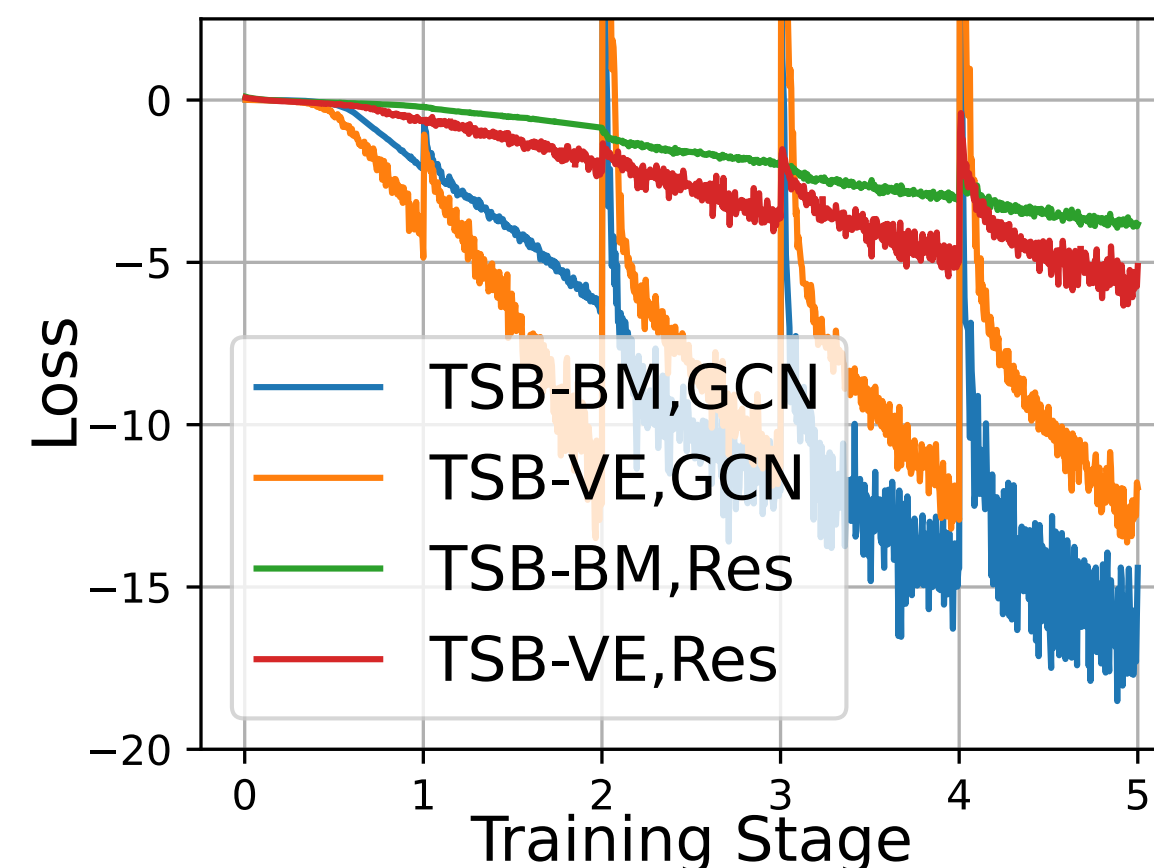


Table 1: Generative modeling results (NLL).

| | SB-BM | SB-VE | SB-VP | TSB-BM | TSB-VE | TSB-VP |
|--------------|-----------------|------------------|------------------|------------------|------------------|------------------|
| Seismic, Res | 2.78 ± 0.01 | 2.94 ± 0.03 | 2.28 ± 0.02 | 2.13 ± 0.01 | 2.22 ± 0.02 | 2.00 ± 0.02 |
| Seismic, GCN | 2.71 ± 0.03 | 2.73 ± 0.05 | 2.01 ± 0.03 | 1.82 ± 0.02 | 1.53 ± 0.03 | 1.51 ± 0.02 |
| Traffic, Res | 0.82 ± 0.00 | 0.77 ± 0.00 | 0.79 ± 0.00 | 0.40 ± 0.00 | 0.01 ± 0.00 | 0.02 ± 0.00 |
| Traffic, SNN | 0.18 ± 0.02 | -0.42 ± 0.01 | -0.09 ± 0.01 | -0.83 ± 0.05 | -1.26 ± 0.05 | -1.01 ± 0.03 |

- Effect of policy models
 - ResBlock vs. topological NNs



| Method | Seismic magnitudes | | Traffic flows | |
|--------|--------------------|-----------------|------------------|------------------|
| | W_1 | W_2 | W_1 | W_2 |
| SB-BM | 11.73 ± 0.05 | 8.29 ± 0.04 | 18.69 ± 0.02 | 13.36 ± 0.01 |
| SB-VE | 11.49 ± 0.04 | 8.13 ± 0.03 | 19.04 ± 0.02 | 13.61 ± 0.02 |
| SB-VP | 12.61 ± 0.06 | 8.92 ± 0.04 | 18.22 ± 0.03 | 13.02 ± 0.02 |
| TSB-BM | 9.01 ± 0.03 | 6.37 ± 0.03 | 10.57 ± 0.02 | 7.62 ± 0.01 |
| TSB-VE | 7.69 ± 0.04 | 5.44 ± 0.03 | 10.51 ± 0.02 | 7.58 ± 0.01 |
| TSB-VP | 8.40 ± 0.04 | 5.95 ± 0.03 | 9.92 ± 0.02 | 7.16 ± 0.01 |