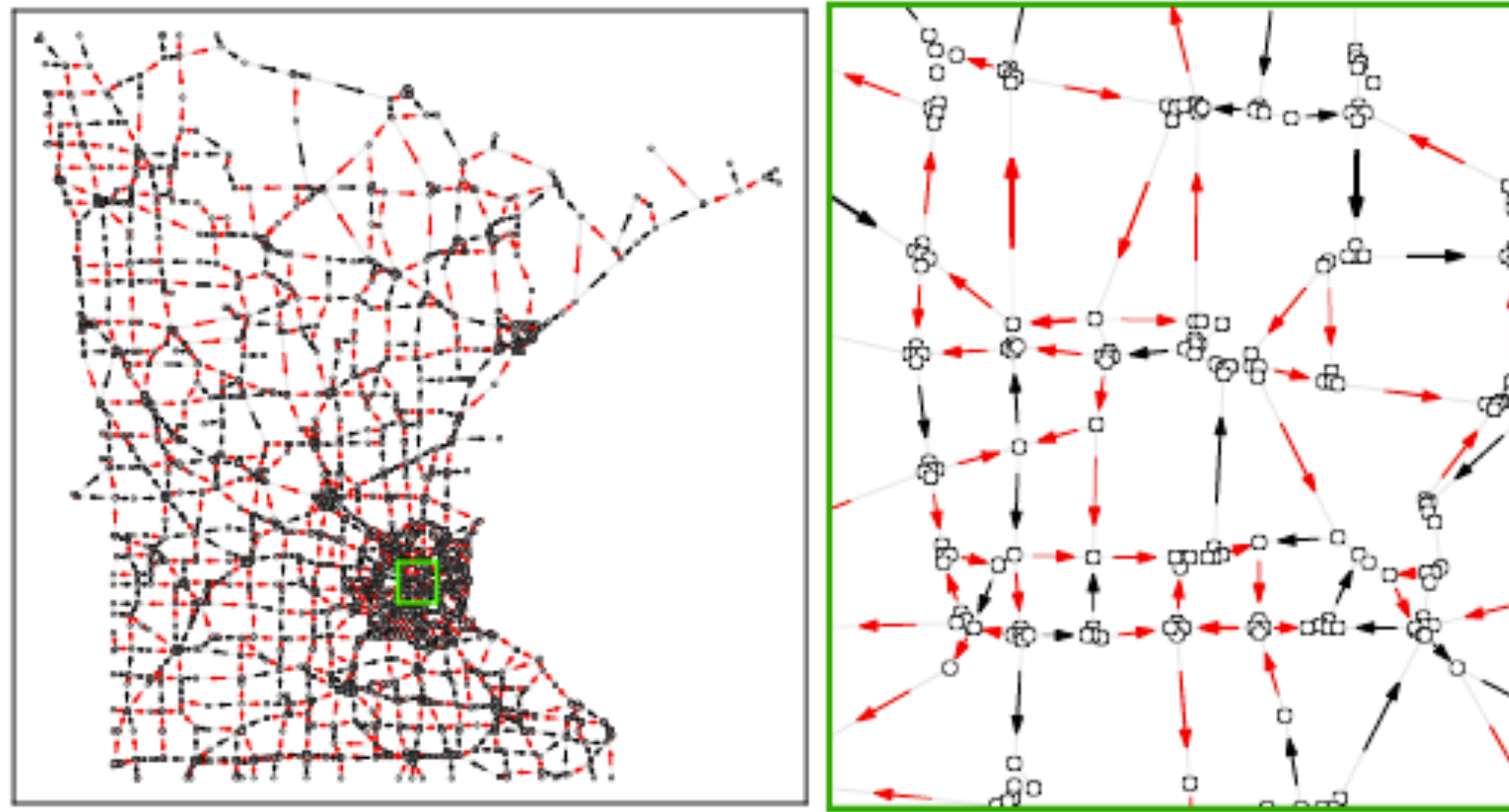
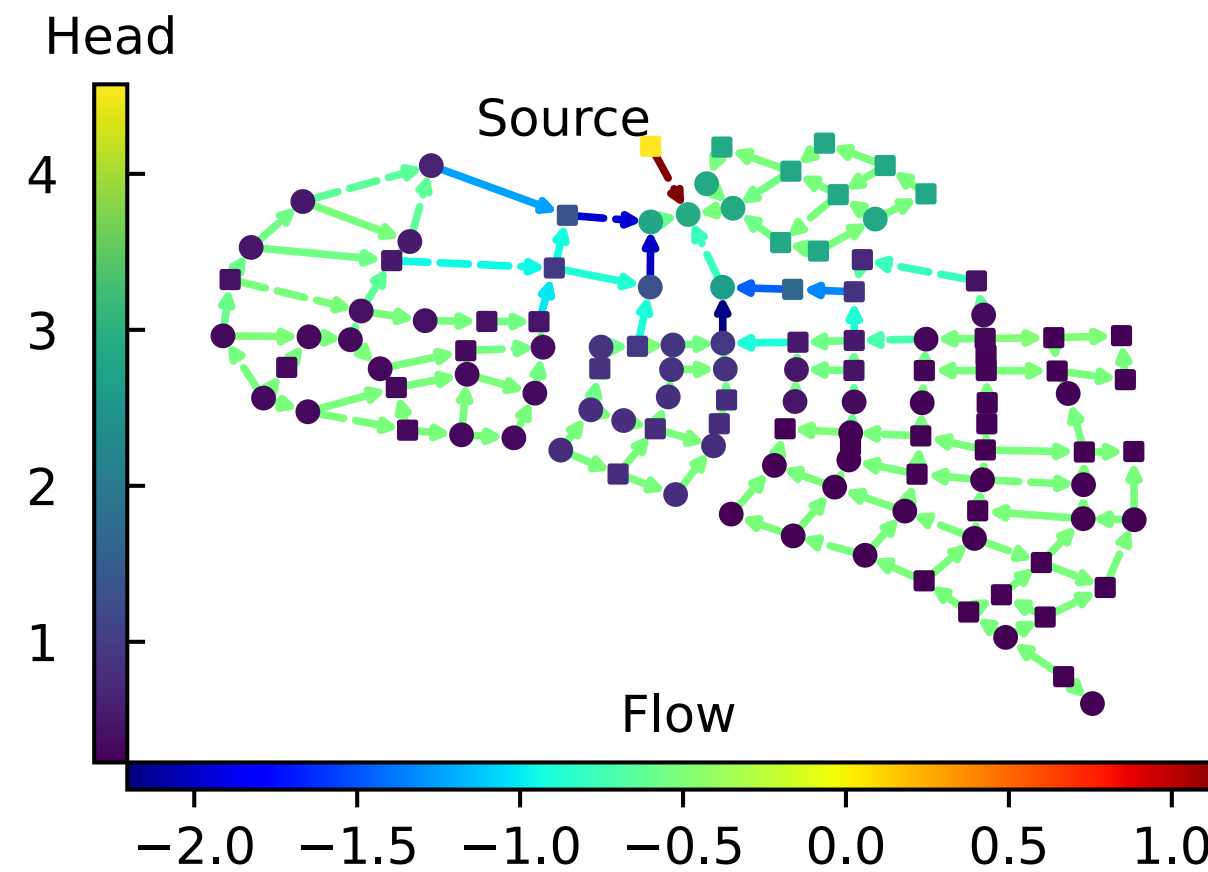


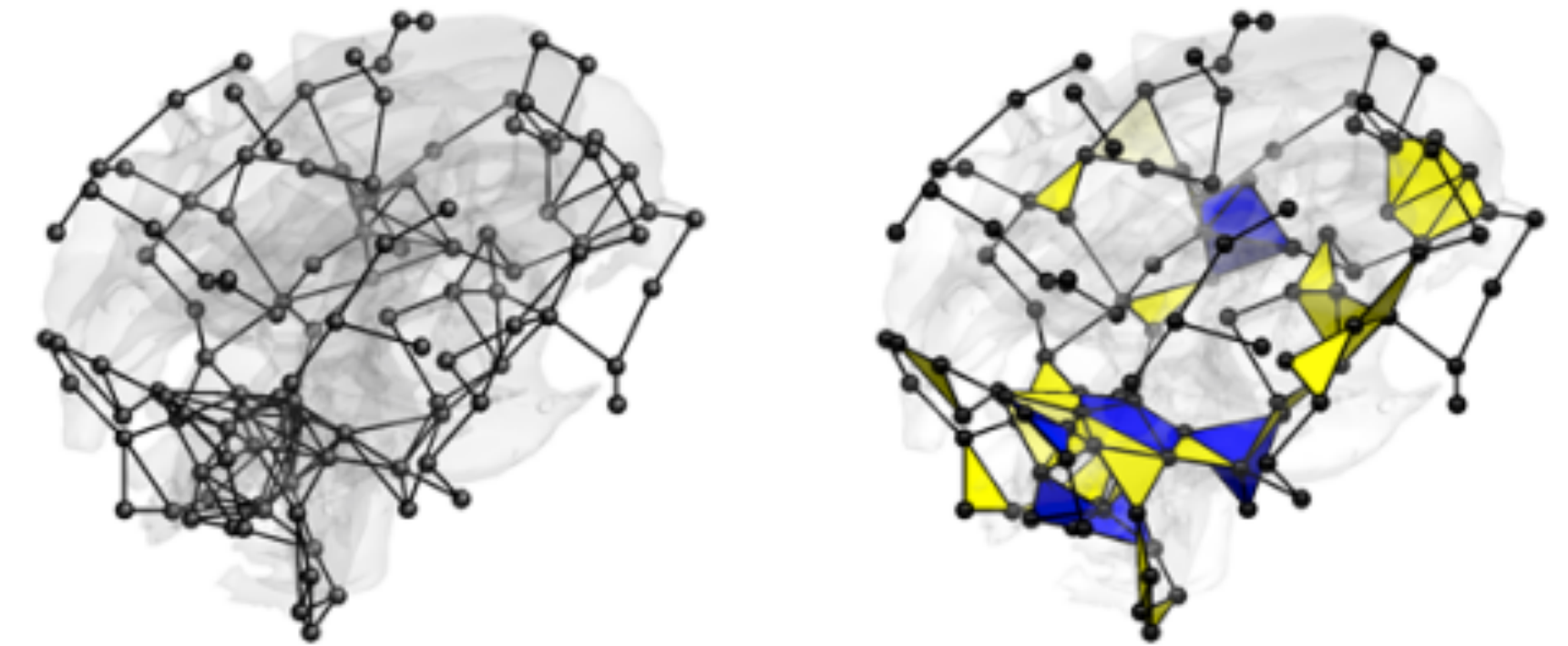
Simplicial complexes and Data in real world



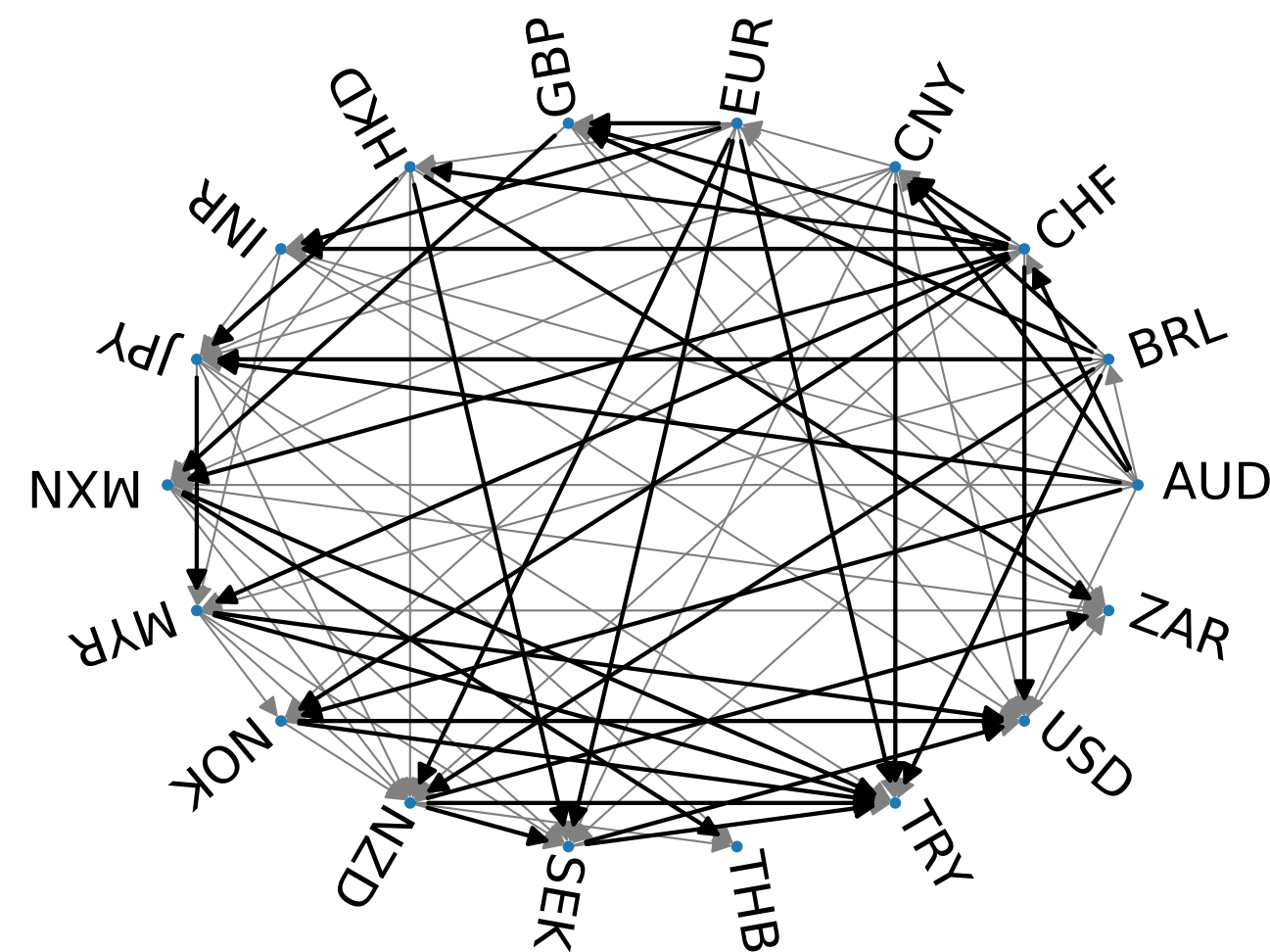
Traffic flows (Jia et al. 2019)



Water flows (Yang et al. 2023)



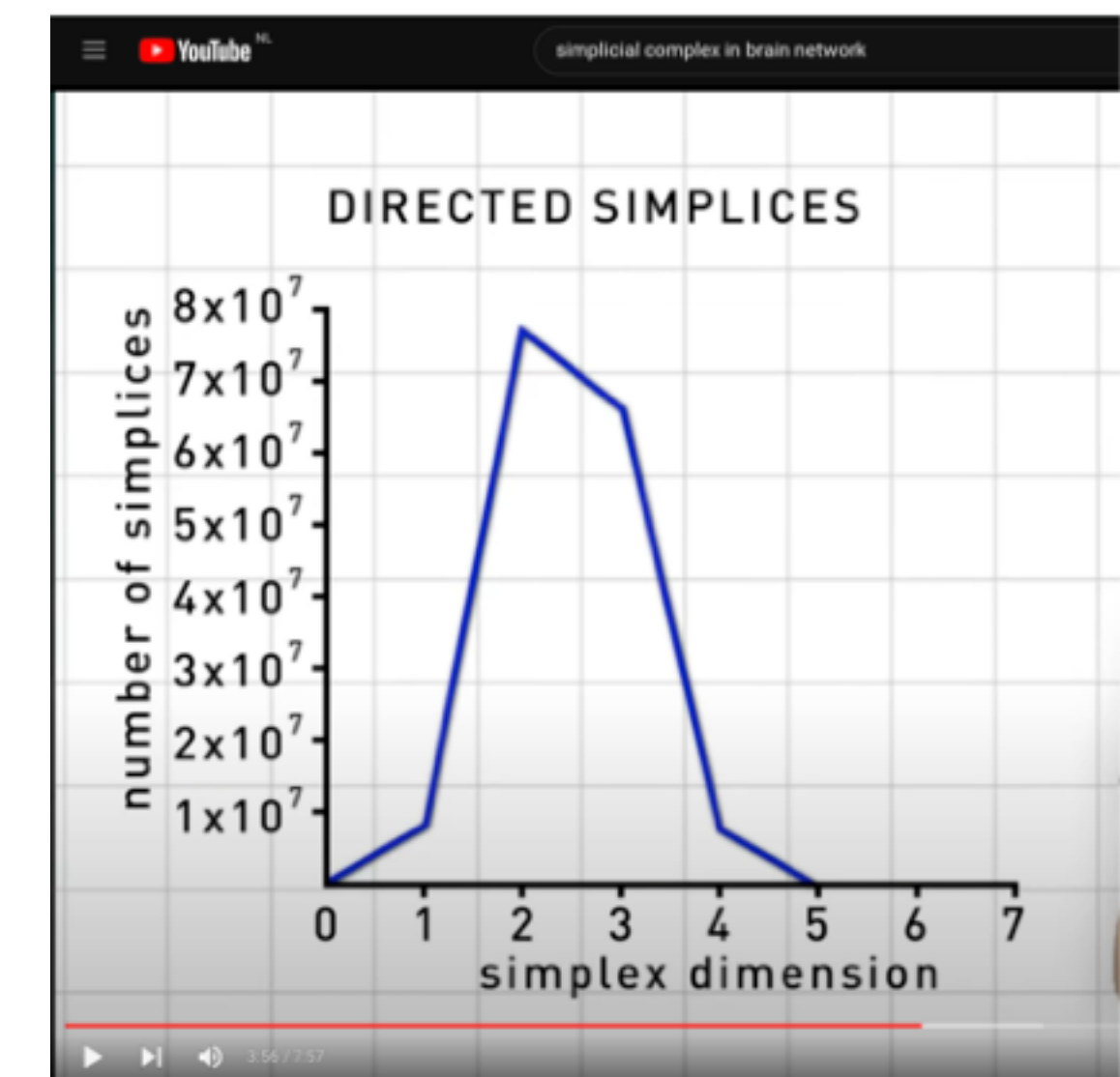
Neuroscience (Anand et al. 2023):
1. Firing of neurons
2. Activation of multiple brain regions



Foreign currency exchange (Jiang et al. 2011)

Others:

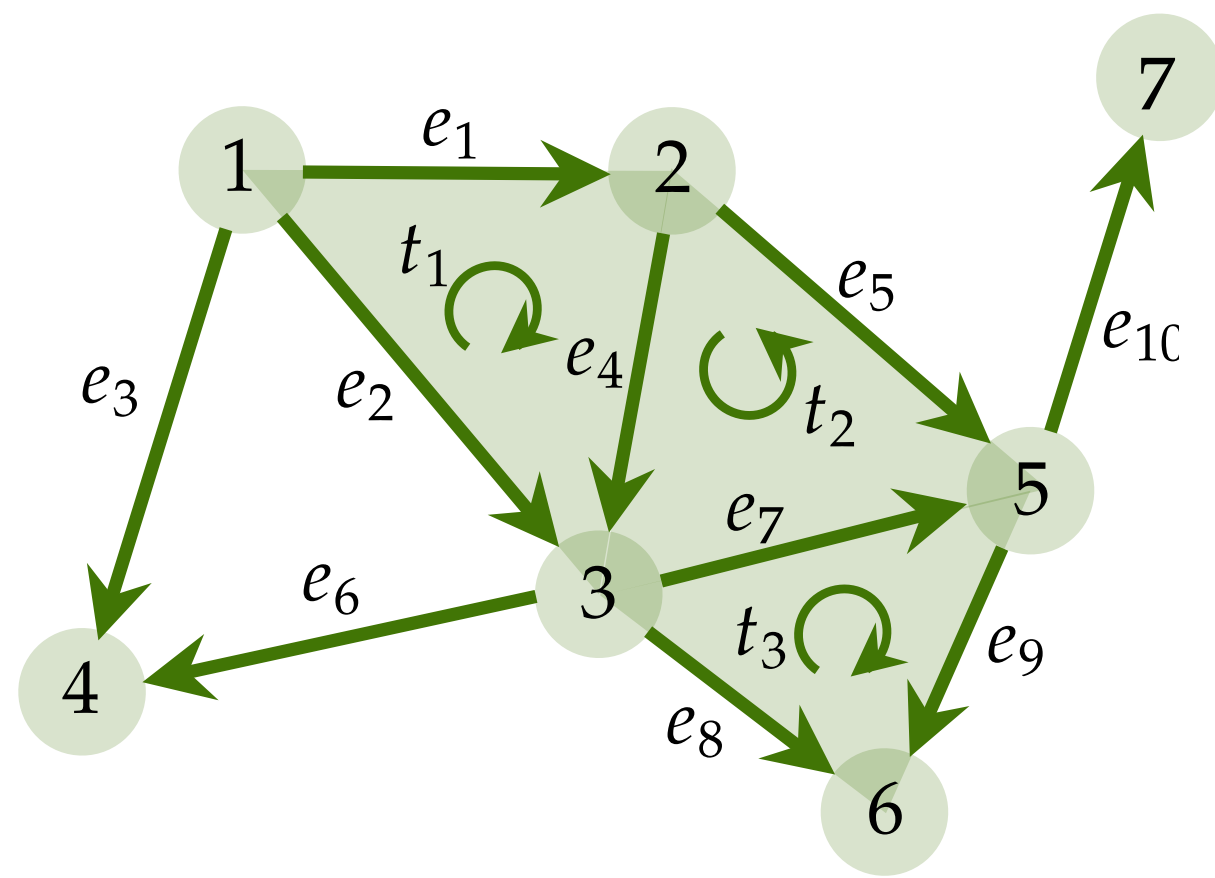
- Currents/Voltage in electric circuits/grid
- Game theory (Candogan et al. 2011)
- Ranking theory (Jiang et al. 2011)
- Information flows
- ...
- Discrete vector fields



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Algebraic reps. of simplicial 2-complex

Incidences & Laplacians



Node-to-Edge

$$\mathbf{B}_1 = \begin{matrix} & \begin{matrix} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 & e_{10} \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{matrix} & \begin{pmatrix} -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{matrix},$$

Edge-to-Faces

$$\mathbf{B}_2 = \begin{matrix} & \begin{matrix} t_1 & t_2 & t_3 \end{matrix} \\ \begin{matrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ e_7 \\ e_8 \\ e_9 \\ e_{10} \end{matrix} & \begin{pmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

Graph Laplacian: $\mathbf{L}_0 = \mathbf{B}_1 \mathbf{B}_1^\top$

1-Hodge Laplacian: $\mathbf{L}_1 = \underbrace{\mathbf{B}_1^\top \mathbf{B}_1}_{\text{Down}} + \underbrace{\mathbf{B}_2 \mathbf{B}_2^\top}_{\text{Up}} := \mathbf{L}_{1,d} + \mathbf{L}_{1,u}$