

Posterior distribution of Hodge components

$$\begin{bmatrix} f_H(\mathbf{x}) \\ f_H(\mathbf{x}^*) \\ f_G(\mathbf{x}) \\ f_G(\mathbf{x}^*) \\ f_C(\mathbf{x}) \\ f_C(\mathbf{x}^*) \\ f_1(\mathbf{x}) \\ f_1(\mathbf{x}^*) \end{bmatrix} \sim \mathcal{N} \left(\mathbf{0}, \begin{bmatrix} K_H & K_H^* & & & & & & \\ K_H^{*\top} & K_H^{**} & & & & & & \\ & & K_G & K_G^* & & & & \\ & & K_G^{*\top} & K_G^{**} & & & & \\ & & & & K_C & K_C^* & & \\ & & & & K_C^{*\top} & K_C^{**} & & \\ & & & & K_C^{*\top} & K_C^{**} & K_1 & K_1^* \\ K_H & K_H^{*\top} & K_G & K_G^{*\top} & K_C & K_C^{*\top} & K_1^{*\top} & K_1^* \\ K_H^{*\top} & K_H^{**} & K_G^{*\top} & K_G^{**} & K_C^{*\top} & K_C^{**} & K_1^* & K_1^{**} \end{bmatrix} \right) \quad (\text{B.26})$$

where we represent the kernel matrices by $K_1 = k_1(\mathbf{x}, \mathbf{x})$, $K_1^* = k_1(\mathbf{x}, \mathbf{x}^*)$ and $K_1^{**} = k_1(\mathbf{x}^*, \mathbf{x}^*)$, and likewise for the other kernel matrices. Given this joint distribution, we can obtain the posterior distributions of the three Hodge components as follows

$$f_H(\mathbf{x}^*) | f_1(\mathbf{x}) \sim \mathcal{N} \left(K_H^{*\top} K_1^{-1} f_1(\mathbf{x}), K_H^{**} - K_H^{*\top} K_1^{-1} K_H^* \right) \quad (\text{B.27a})$$

$$f_G(\mathbf{x}^*) | f_1(\mathbf{x}) \sim \mathcal{N} \left(K_G^{*\top} K_1^{-1} f_1(\mathbf{x}), K_G^{**} - K_G^{*\top} K_1^{-1} K_G^* \right) \quad (\text{B.27b})$$

$$f_C(\mathbf{x}^*) | f_1(\mathbf{x}) \sim \mathcal{N} \left(K_C^{*\top} K_1^{-1} f_1(\mathbf{x}), K_C^{**} - K_C^{*\top} K_1^{-1} K_C^* \right) \quad (\text{B.27c})$$

From these posterior distributions, we can directly obtain the means and the uncertainties of the Hodge components of the predicted edge function.

