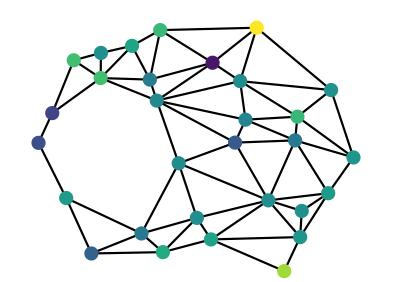
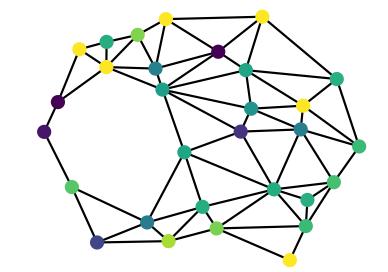
Topological signal distribution matching

- In a topological domain, e.g., a graph, signals on the node set.
- Given (empirical) signal distributions, $X_0 \sim \rho_0$ at t=0 and $X_1 \sim \rho_1$ at t=1



(unknown) stochastic process $X := (X_t)_{0 \le t \le 1} \sim \mathbb{P}$

$$X := (X_t)_{0 < t < 1} \sim \mathbb{P}$$



• Assume some *prior* (reference) process $Y \sim \mathbb{Q}_{\mathscr{T}} - -$ topology-aware

Topological Schrödinger Bridge Problem

$$\min D_{KL}(\mathbb{P}||\mathbb{Q}_{\mathcal{T}}) \ s.t. \mathbb{P}_0 = \rho_0, \mathbb{P}_1 = \rho_1$$

Schrödinger's bridge problem

- Cloud of n independent Brownian particles
- Empirical distributions $\rho_0(x)$ and $\rho_1(y)$ at t=0 and t=1



- Particles have been transported in an unlikely way
- Of the many possible (unlikely) ways, which one is the most likely? [Lénoard 2014]

$$\min D_{KL}(\mathbb{P}||\mathbb{W}) \ s.t. \mathbb{P}_0 = \rho_0, \mathbb{P}_1 = \rho_1$$

A dynamics formulation of entropic-regularized optimal transport [Vallani 2009]

$$\min_{\pi \in \Pi(\rho_0, \rho_1)} \int_{\mathbb{R}^n \times \mathbb{R}^n} \frac{1}{2} ||x_0 - x_1||^2 d\pi(x_0, x_1) + \sigma^2 D_{KL}(\pi || \rho_0 \otimes \rho_1)$$