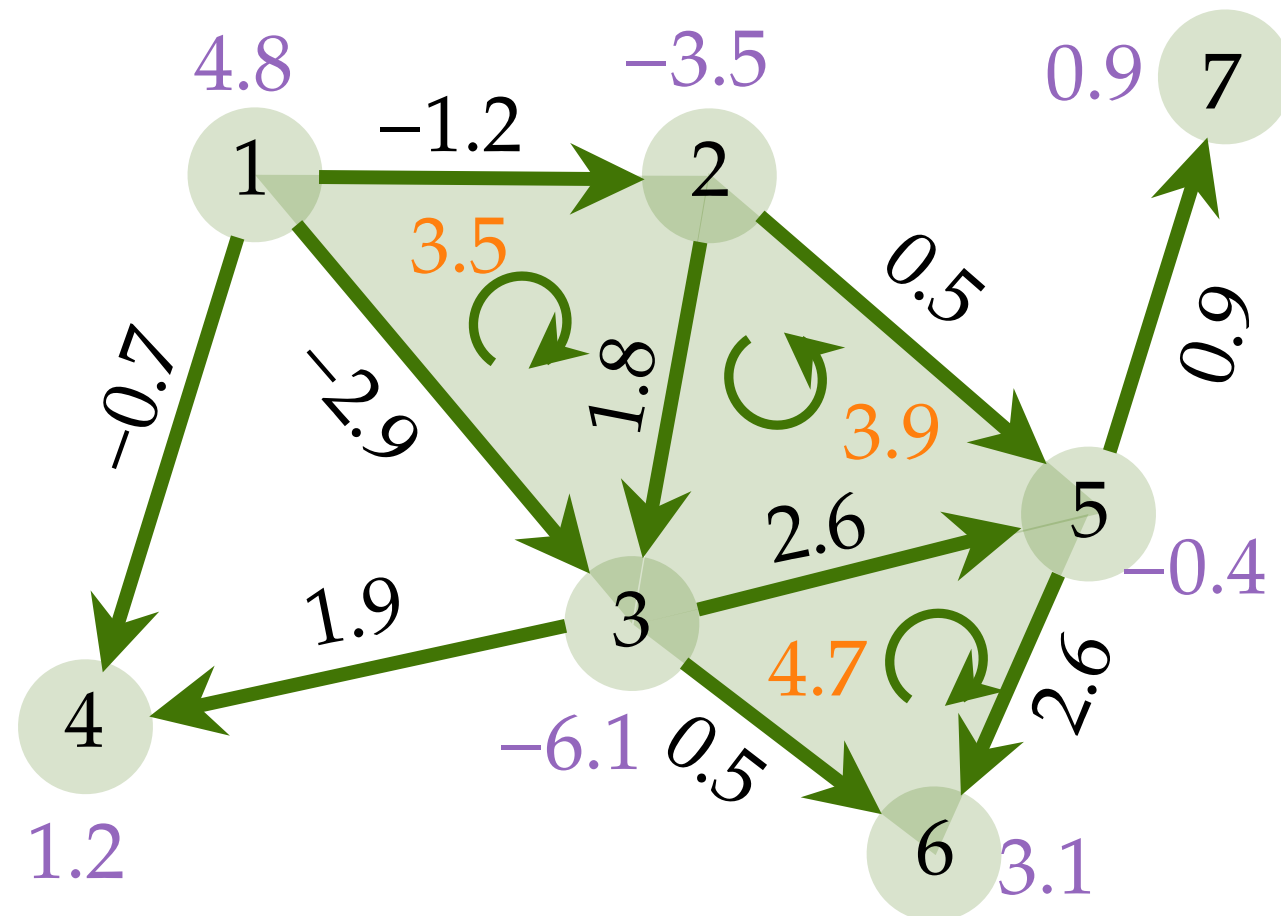


# Incidence & Laplacians

## 1st and 2nd order Discrete Derivatives



Gradient of node signal:  $\mathbf{B}_1^\top \mathbf{v}$        $[\mathbf{f}_G]_{[i,j]} = [\mathbf{v}]_j - [\mathbf{v}]_i$

Divergence of edge flows:  $[\mathbf{B}_1 \mathbf{f}]_i = \sum_{j < i} \mathbf{f}_{[j,i]} - \sum_{i < k} \mathbf{f}_{[i,k]}$

Net-flow = in\_flow - out\_flow

**Curl** of edge flows:  $[\mathbf{B}_2^\top \mathbf{f}]_t = \mathbf{f}_{[i,j]} + \mathbf{f}_{[j,k]} - \mathbf{f}_{[i,k]}$ , for  $t = [i, j, k]$

**Net-circulation in triangles**

$$[\mathbf{B}_1 \mathbf{f}]_5 = 0.5 + 2.6 - (0.9 + 2.6) = -0.4$$

$$[\mathbf{B}_2^\top \mathbf{f}]_{[1,2,3]} = -1.2 + 1.8 - (-2.9) = 3.5$$

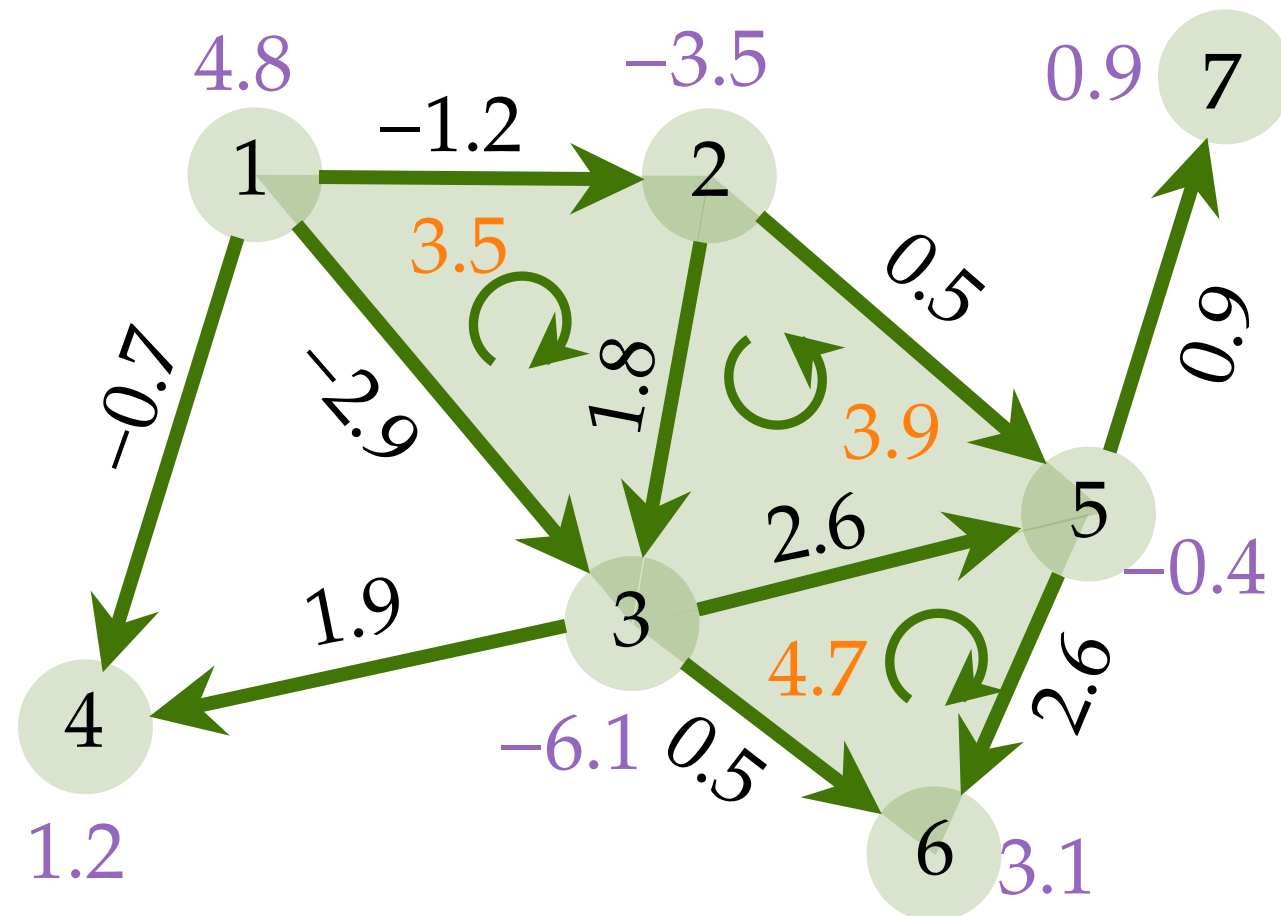
Laplacians = Grad Div + Curl\* Curl

$$\text{Hodge Laplacian: } \mathbf{L}_1 = \mathbf{B}_1^\top \mathbf{B}_1 + \mathbf{B}_2 \mathbf{B}_2^\top$$

$$\Delta_1 = \nabla(\nabla \cdot) + \nabla \times (\nabla \times)$$

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