

Edge flow processing, learning

- perform Hodge decomposition
- signal smoothing, filtering
- learning from edge flows

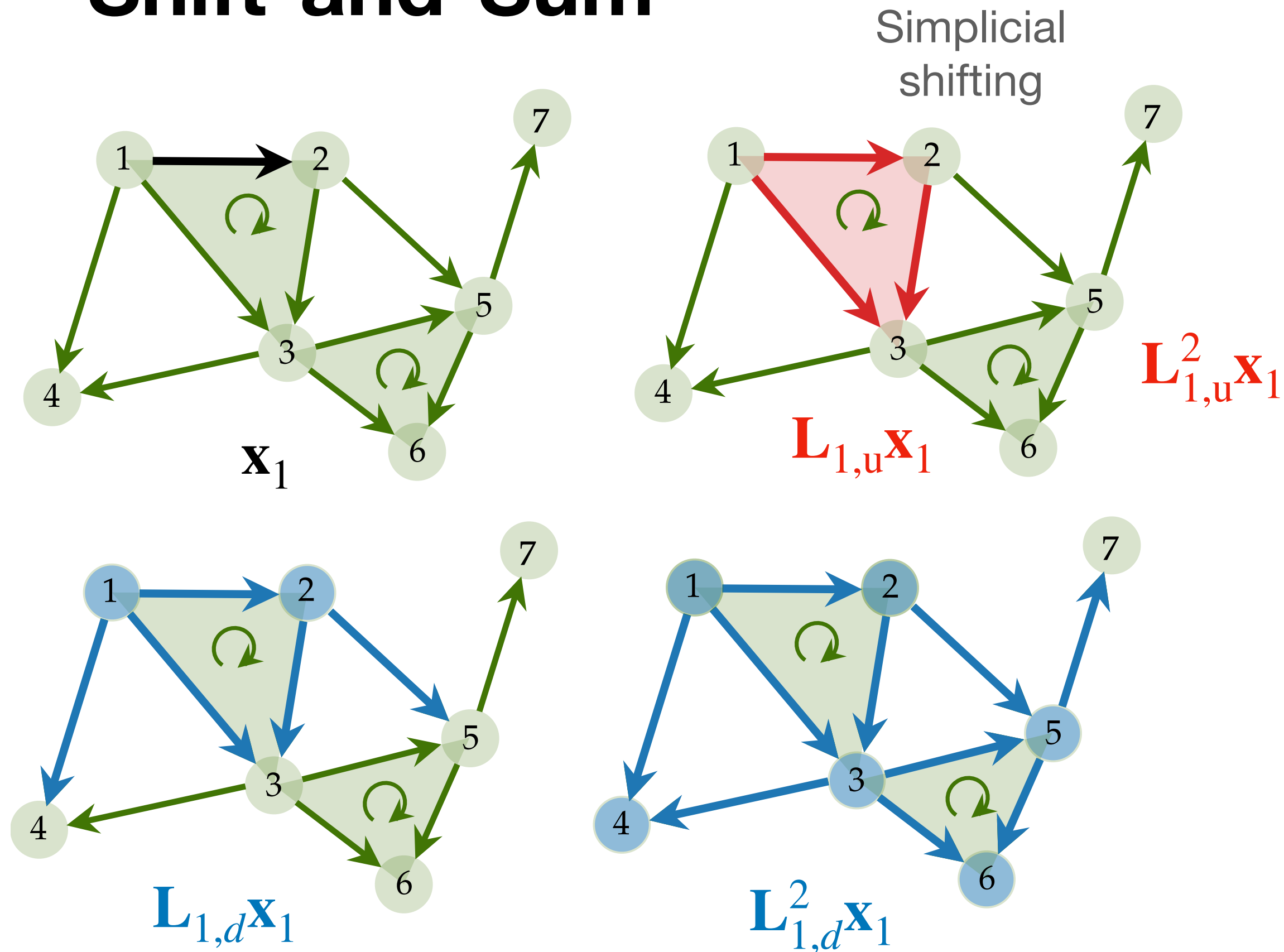
Simplicial Convolutional Filter

Simplicial CNN

Gaussian processes

Edge Convolution

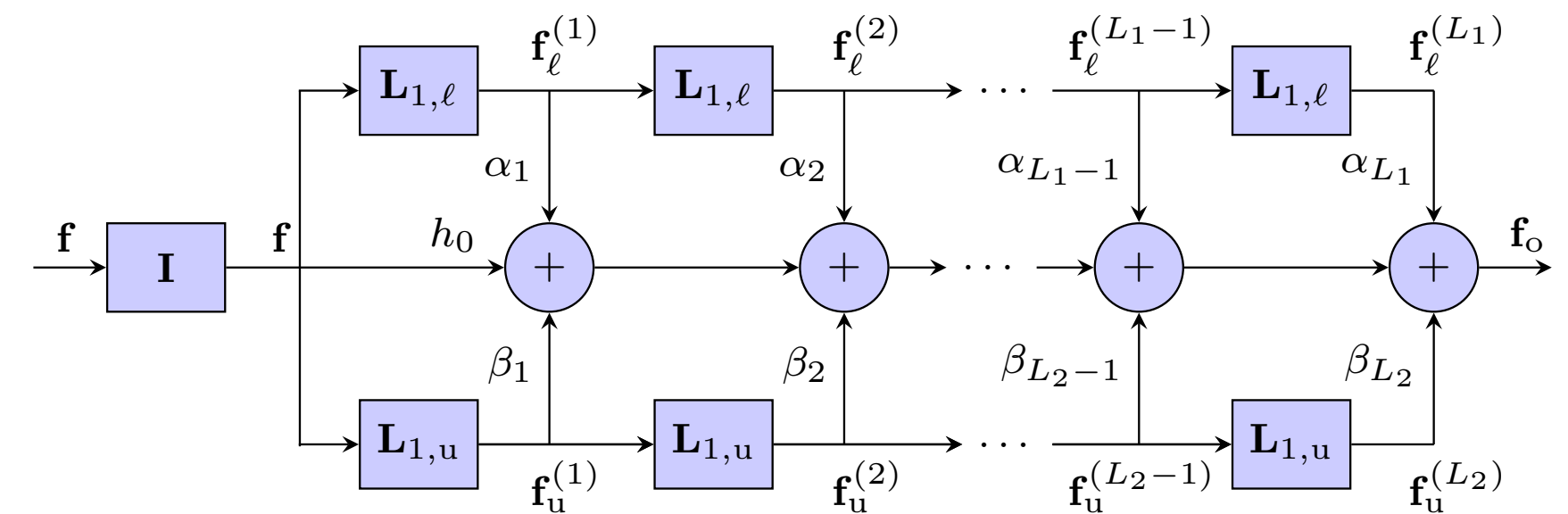
Shift-and-Sum



$$[\mathbf{L}_{1,d}\mathbf{f}]_i = \sum_{j \in \{\mathcal{N}_{1,i} \cup i\}} [\mathbf{L}_{1,d}]_{ij} [\mathbf{f}]_j$$

Simplicial locality

$$\mathbf{H} := \mathbf{H}(\mathbf{L}_d, \mathbf{L}_u; \alpha, \beta) = \sum_{k=0}^{K_d} \alpha_k \mathbf{L}_d^k + \sum_{k=0}^{K_u} \beta_k \mathbf{L}_u^k$$



- Efficient, distributed
- Expressive power (Cayley-Hamilton thm)
- Hodge-invariant operator

$$\mathbf{H}_1 \mathbf{x}_1 = \mathbf{H}_1|_{\text{im}(\mathbf{B}_1^T)} \mathbf{x}_{1,G} + \mathbf{H}_1|_{\text{im}(\mathbf{B}_2)} \mathbf{x}_{1,C} + \mathbf{H}_1|_{\text{ker}(\mathbf{L}_1)} \mathbf{x}_{1,H}$$

Hodge subspaces are invariant under \mathbf{H}