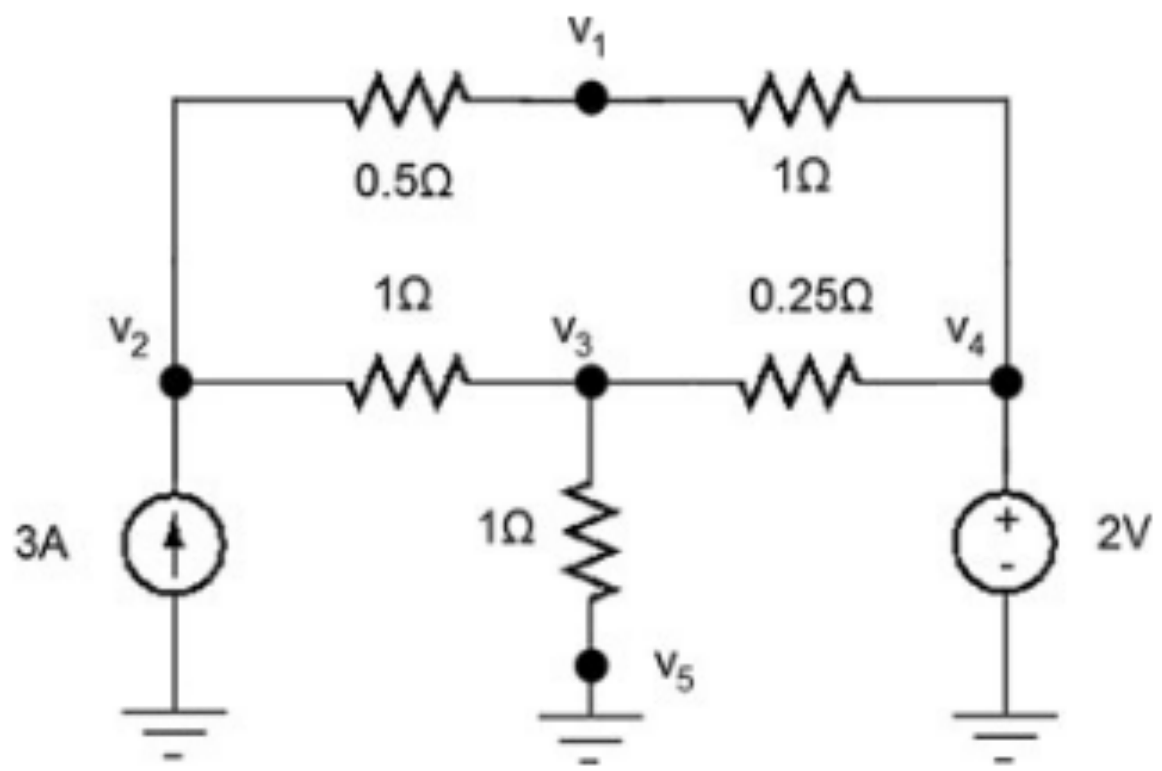
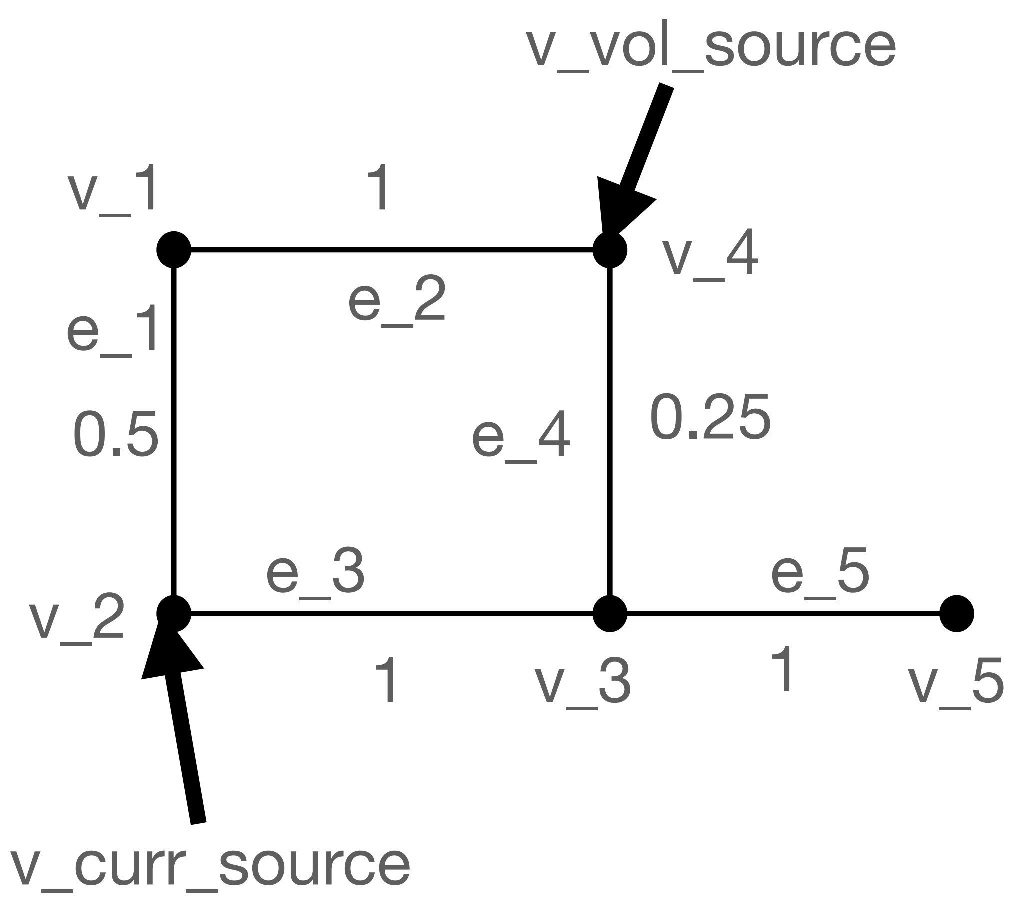
A Circuit toy example



(Grady et al. 2010)



$\mathbf{v} \in \mathbb{R}^{|\mathcal{N}|}$: Electric potential on nodes

$\mathbf{f}_{Vol} = \mathbf{B}_1^\top \mathbf{v}$: (Kirchhoff's voltage law)

$\mathbf{f}_{currents} = \mathbf{G}^{-1}\mathbf{f}_{Vol}$: currents (Ohm's law)

Diagonal resistance/conductance

Kirchhoff's current law: $\mathbf{B}_1 \mathbf{f}_{currents} = \mathbf{0}$

Or $\mathbf{B}_1 \mathbf{f}_{currents} + \mathbf{v}_{curr\,source} = \mathbf{0}$

$$\mathbf{B}_1 = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 \\ 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix} \qquad \mathbf{v}_{vol} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ 2 \\ 0 \end{pmatrix}$$

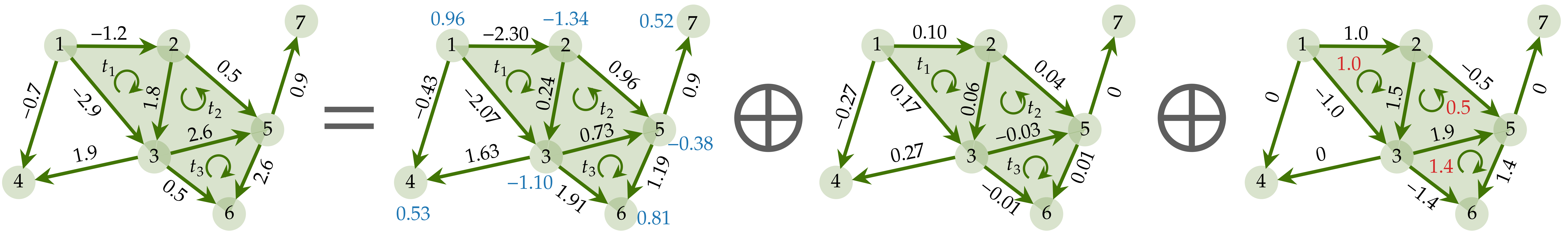
$$\mathbf{B}_1 \mathbf{G}^{-1} \mathbf{B}_1^\top \mathbf{v}_{vol} + \mathbf{v}_{curr\,source} = \mathbf{0}$$

Resistance — Metrics?
Power, energy?

Hodge decomposition

Lovász et al. 2004; Lim et al. 2020

$$\mathbb{R}^{N_1} = \mathrm{im}(\mathbf{B}_1^\top) \oplus \ker(\mathbf{L}_1) \oplus \mathrm{im}(\mathbf{B}_2)$$
$$\mathbf{f}_1 = \mathbf{f}_G + \mathbf{f}_H + \mathbf{f}_C$$



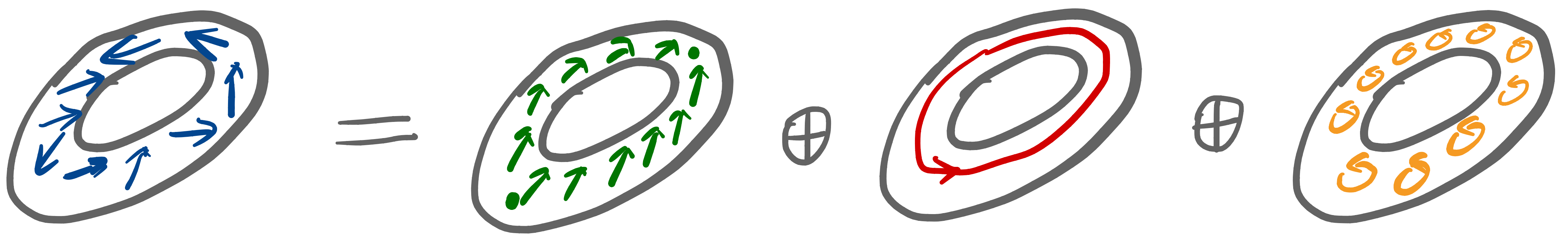
$\mathbf{f}_1$

Gradient flow
Curl-free, irrotational

Harmonic flow
Div- and curl-free

Curl flow
Div-free, solenoidal

- This holds for any simplex order k



- What is the case for $k = 0$?

$$\mathbb{R}^{N_0} = \mathrm{im}(\mathbf{B}_1) \oplus \ker(\mathbf{L}_0)$$

- Characteristic decomposition