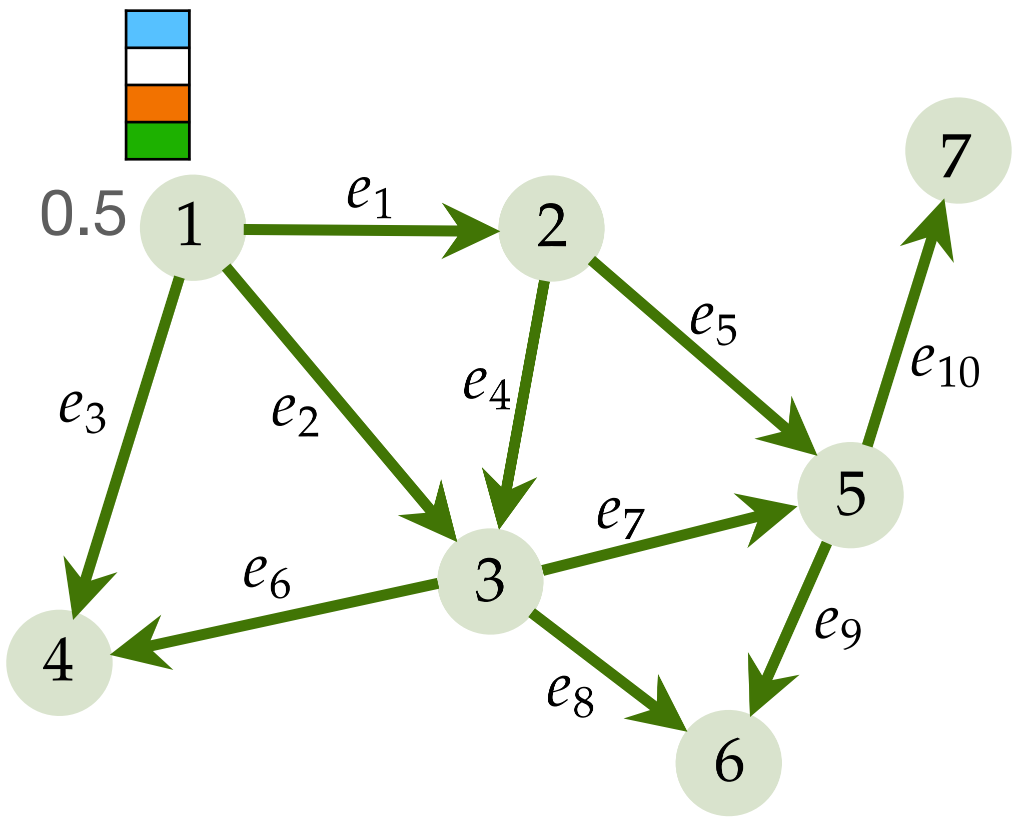
Functions on simplices

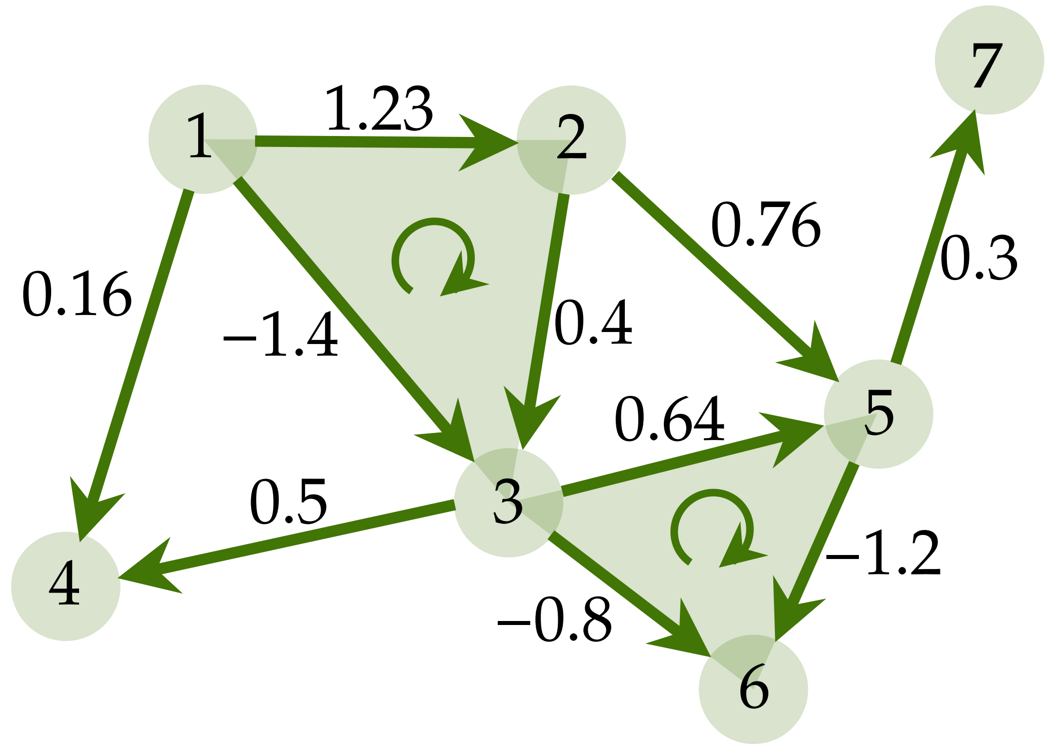
Signals on nodes, edges, triangles, …



Node function

$f_0 : V \to \mathbb{R}$

$\mathbf{f}_0 = (f_0(1), \ldots, f_0(N_0))^\top$



Edge function

$f_1 : E \to \mathbb{R}$

$\mathbf{f}_1 = (f_1(e_1), \ldots, f_1(e_{N_1}))^\top$

- Alternating property
- Magnitude and sign

- Flow-type data (natural)
  - Physical world: traffic flow, water flow, information flow…
  - Forex: exchange rates
  - Game theory (Candogan et al. 2011)
  - Ranking data (Jiang et al. 2011)
  - Edge-based vector field discretisation (computer graphics)
  - …

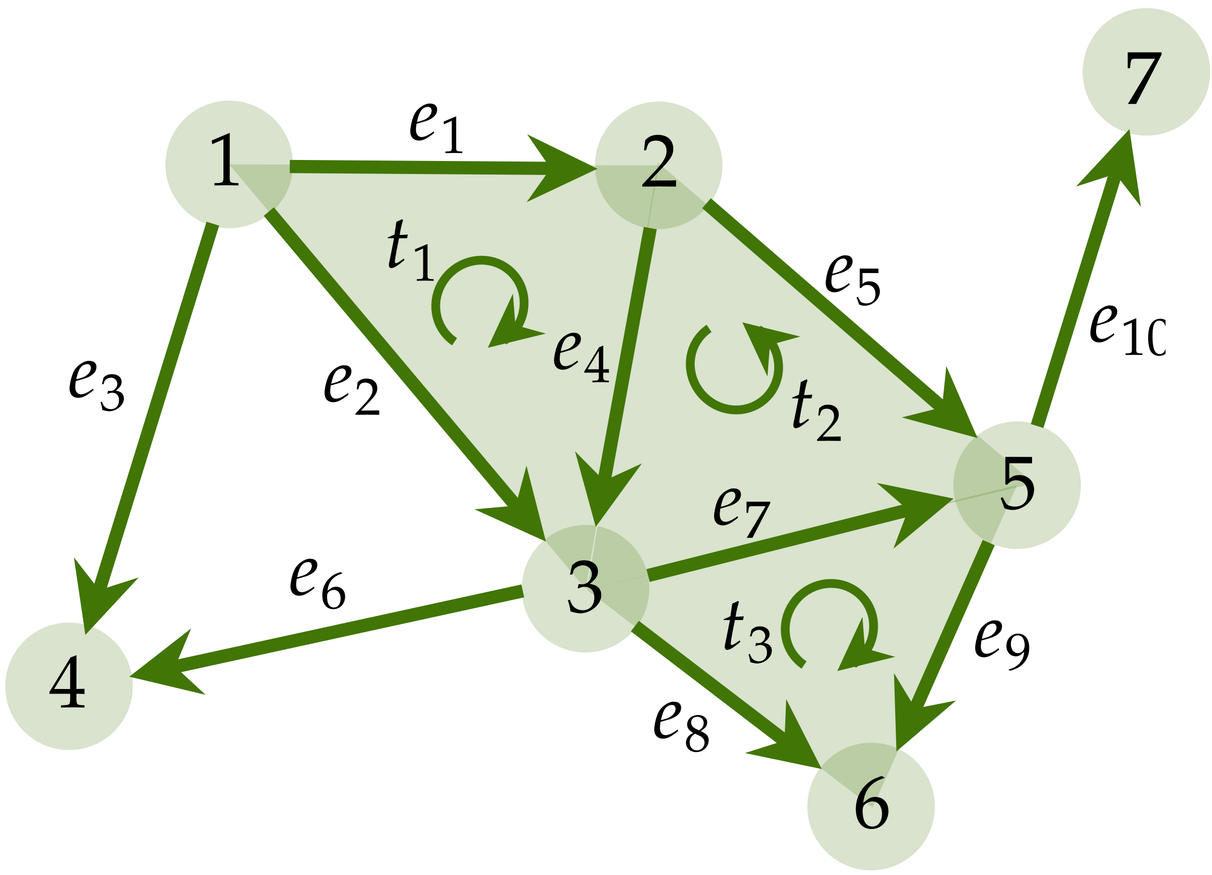
Triangle function

$f_2 : T \to \mathbb{R}$

0-, 1-, 2-cochains in topology

Algebraic reps. of simplicial 2-complex

Incidences & Laplacians



Node-to-Edge

Edge-to-Faces

$$\mathbf{B}_1 = \begin{array}{cc} & \begin{array}{cccccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 & e_{10} \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} & \begin{pmatrix} -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array}, \quad \mathbf{B}_2 = \begin{array}{cc} & \begin{array}{ccc} t_1 & t_2 & t_3 \end{array} \\ \begin{array}{c} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ e_7 \\ e_8 \\ e_9 \\ e_{10} \end{array} & \begin{pmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \end{array}$$

Graph Laplacian: $\mathbf{L}_0 = \mathbf{B}_1 \mathbf{B}_1^\top$

Up

1-Hodge Laplacian: $\mathbf{L}_1 = \mathbf{B}_1^\top \mathbf{B}_1 + \mathbf{B}_2 \mathbf{B}_2^\top := \mathbf{L}_{1,d} + \mathbf{L}_{1,u}$

Down