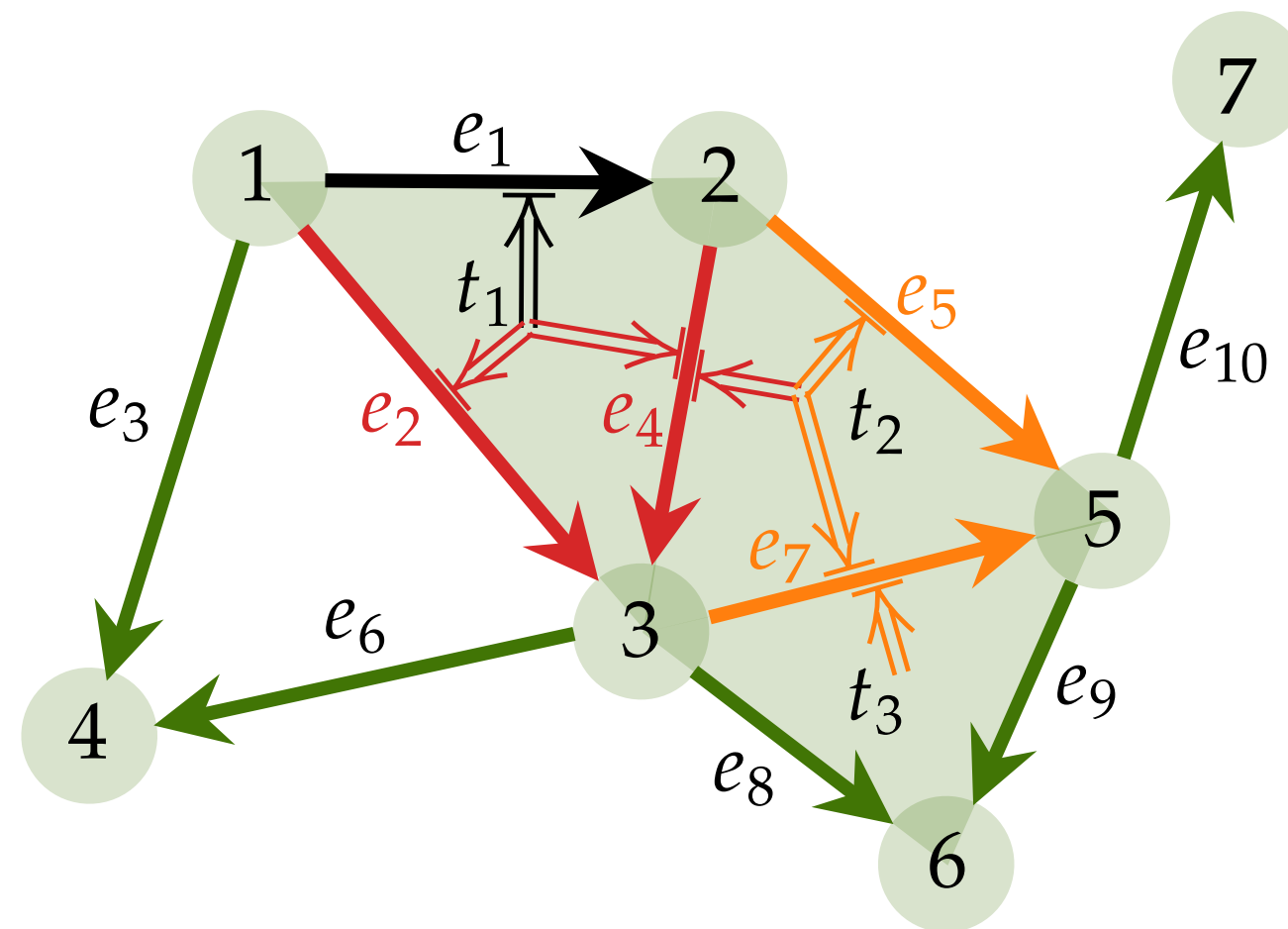
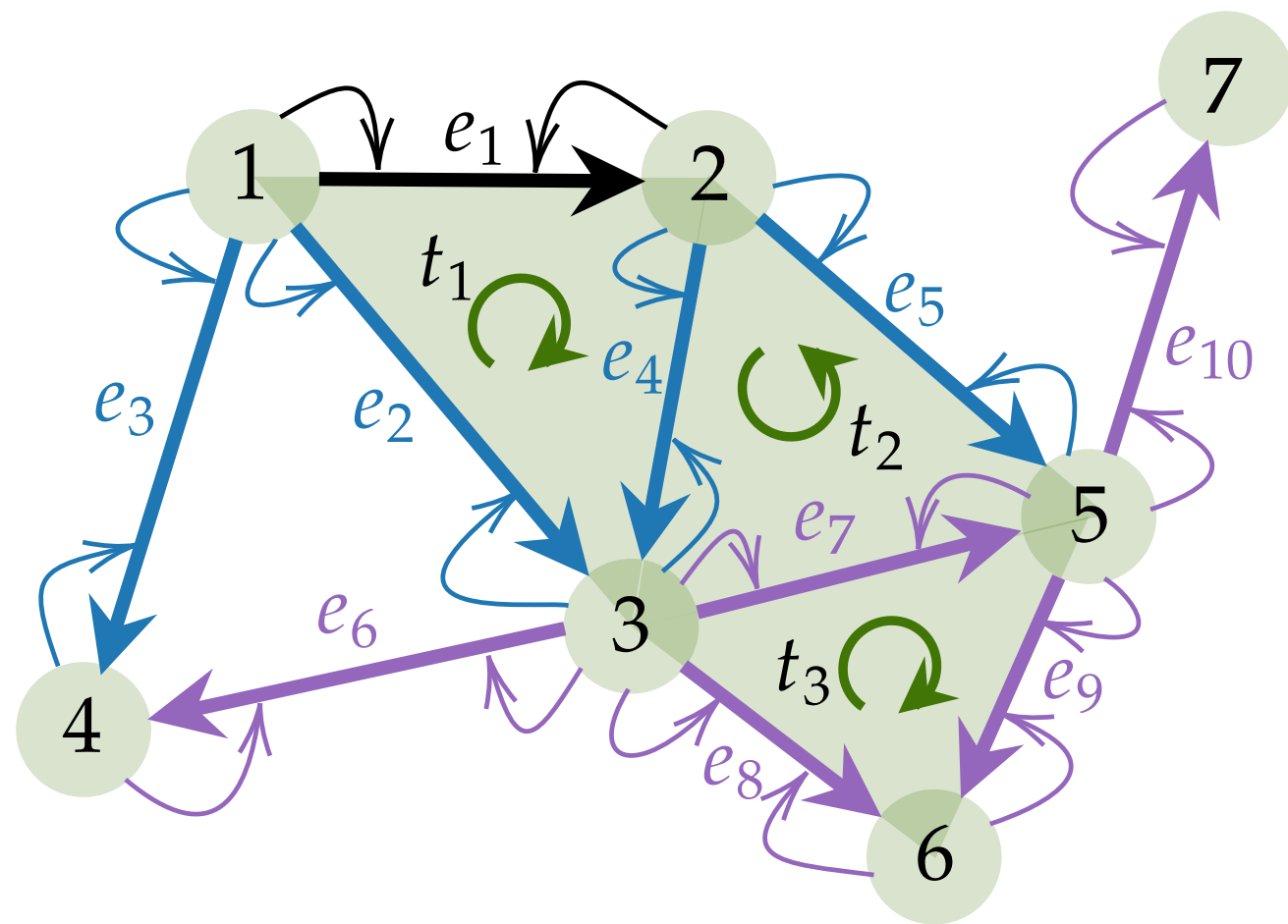


# Convolutional Learning on SCs

## Node-edge-triangle interactions

- $\text{SCCNN}_k^l : \{x_{k-1}^{l-1}, x_k^{l-1}, x_{k+1}^{l-1}\} \rightarrow x_k^l$ , with simplicial order  $k$  and layer  $l$

$$\mathbf{x}_k^l = \sigma(\mathbf{H}_{k,d}^l \mathbf{x}_{k,d}^{l-1} + \mathbf{H}_k^l \mathbf{x}_k^{l-1} + \mathbf{H}_{k,u}^l \mathbf{x}_{k,u}^{l-1})$$



$$\begin{aligned} \mathbf{x}_0^l &= \sigma(\mathbf{H}_0^l \mathbf{x}_0^{l-1} + \mathbf{H}_{0,u}^l \mathbf{B}_1 \mathbf{x}_1^{l-1}) \\ \mathbf{x}_1^l &= \sigma(\mathbf{H}_{1,d}^l \mathbf{B}_1^\top \mathbf{x}_0^{l-1} + \mathbf{H}_1^l \mathbf{x}_1^{l-1} + \mathbf{H}_{1,u}^l \mathbf{B}_2 \mathbf{x}_2^{l-1}) \\ \mathbf{x}_2^l &= \sigma(\mathbf{H}_{2,d}^l \mathbf{B}_2^\top \mathbf{x}_1^{l-1} + \mathbf{H}_2^l \mathbf{x}_2^{l-1}) \end{aligned}$$

Properties: locality, symmetry  
Dirichlet energy perspective  
Hodge-invariant  
Stability to weights perturbations

Convolution based (Ebli et al. 2020; Roddenberry et al. 2021; Yang et al. 2022, 2023)  
Message passing (Bodnar et al. 2021)

# Simplex prediction

## Generalization of link prediction

Table 2: Simplex prediction (AUC,  $\uparrow$ ) .

Methods	2-simplex	3-simplex
Mean (Benson et al., 2018)	62.8 $\pm$ 2.7	63.6 $\pm$ 1.6
MLP	68.5 $\pm$ 1.6	69.0 $\pm$ 2.2
GNN (Defferrard et al., 2016)	93.9 $\pm$ 1.0	96.6 $\pm$ 0.5
SNN (Ebli et al., 2020)	92.0 $\pm$ 1.8	95.1 $\pm$ 1.2
PSNN (Roddenberry et al., 2021)	95.6 $\pm$ 1.3	98.1 $\pm$ 0.5
SCNN (Yang et al., 2022a)	96.5 $\pm$ 1.5	98.3 $\pm$ 0.4
Bunch (Bunch et al., 2020)	98.3 $\pm$ 0.5	98.5 $\pm$ 0.5
MPSN (Bodnar et al., 2021b)	98.1 $\pm$ 0.5	99.2 $\pm$ 0.3
<b>SCCNN</b>	<b>98.7<math>\pm</math>0.5</b>	<b>99.4<math>\pm</math>0.3</b>

