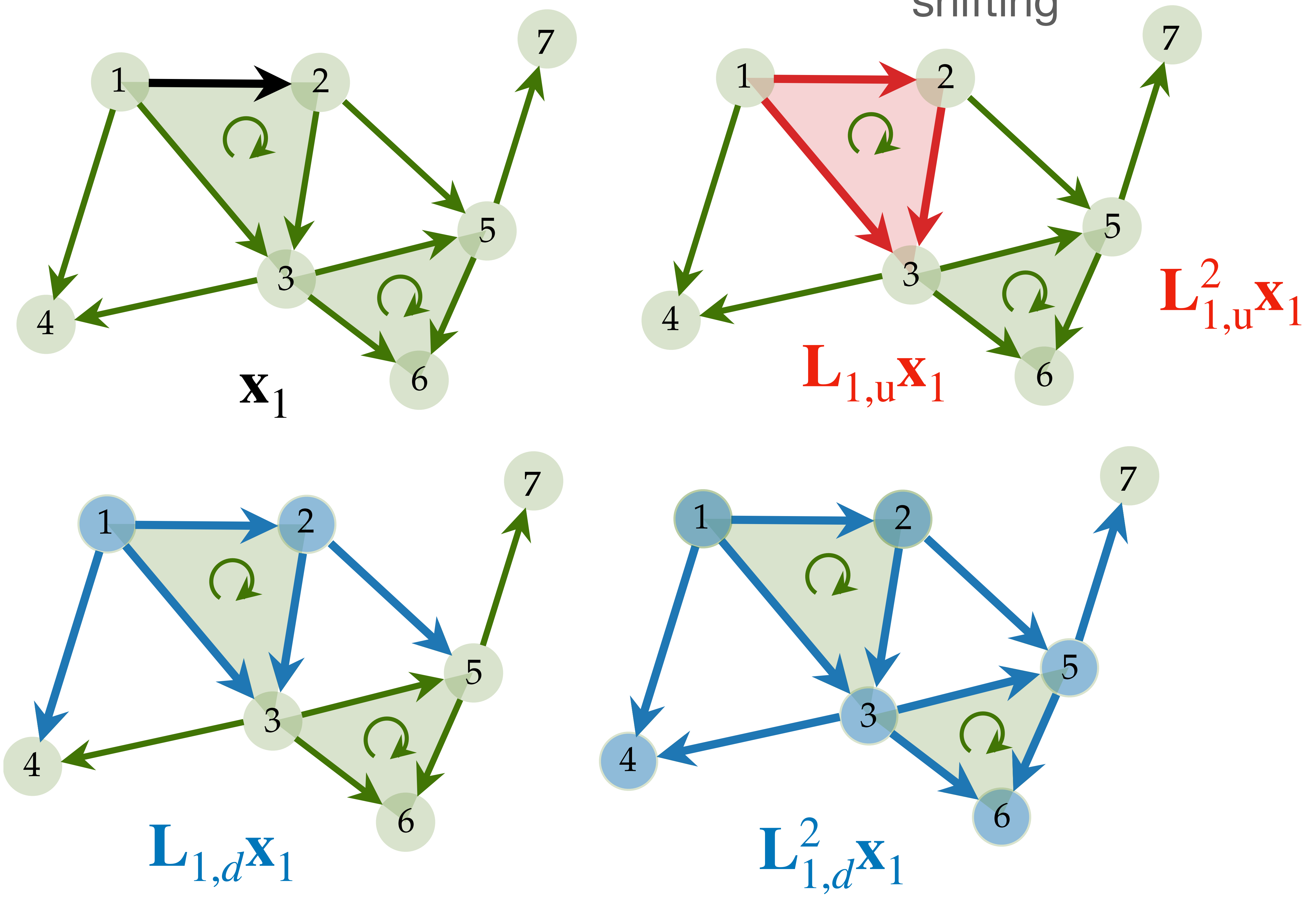
Edge Convolution

Shift-and-Sum

Simplicial shifting



$$[\mathbf{L}_{1,d}\mathbf{f}]_i = \sum_{j \in \{\mathcal{N}_{1,i} \cup i\}} [\mathbf{L}_{1,d}]_{ij}[\mathbf{f}]_j$$

Simplicial locality

Simplicial Convolutional Filter

$$\boxed{\mathbf{H} := \mathbf{H}(\mathbf{L}_\mathrm{d}, \mathbf{L}_\mathrm{u}; \boldsymbol{\alpha}, \boldsymbol{\beta}) = \sum_{k=0}^{K_\mathrm{d}} \alpha_k \mathbf{L}_\mathrm{d}^k + \sum_{k=0}^{K_\mathrm{u}} \beta_k \mathbf{L}_\mathrm{u}^k}$$

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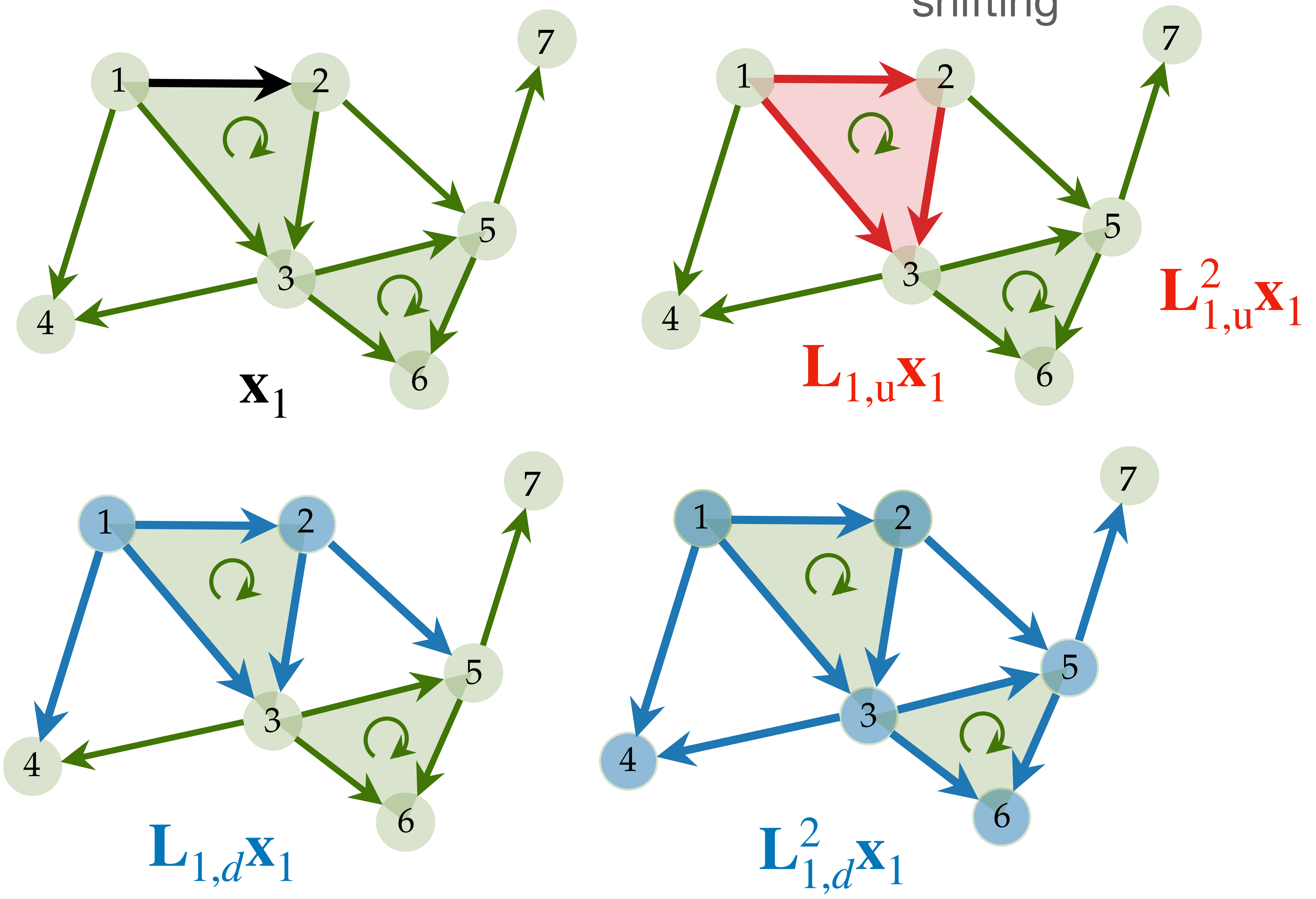

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- Efficient, distributed
- Expressive power (Cayley-Hamilton thm)
- Hodge-invariant operator

$$\mathbf{H}_1\mathbf{x}_1 = \mathbf{H}_1\,|_{\mathrm{im}(\mathbf{B}_1^\top)}\mathbf{x}_{1,\mathrm{G}} + \mathbf{H}_1\,|_{\mathrm{im}(\mathbf{B}_2)}\mathbf{x}_{1,\mathrm{C}} + \mathbf{H}_1\,|_{\ker(\mathbf{L}_1)}\mathbf{x}_{1,\mathrm{H}}$$

Hodge subspaces are invariant under $\mathbf{H}$