Matérn Edge GPs

Derived from SDEs on the edge set

• $\mathbf{f}_1 \sim \mathrm{GP}(\mathbf{0}, \mathbf{K}_1)$

Matérn graph kernel

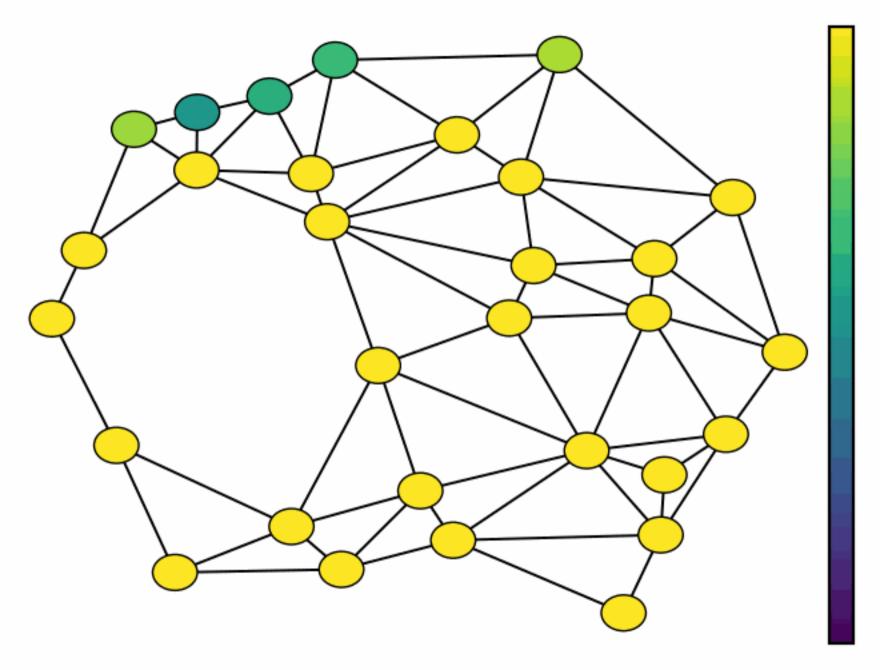
$$\Phi(\mathbf{L}_1)\mathbf{f}_1 = \mathbf{w}_1$$
, with

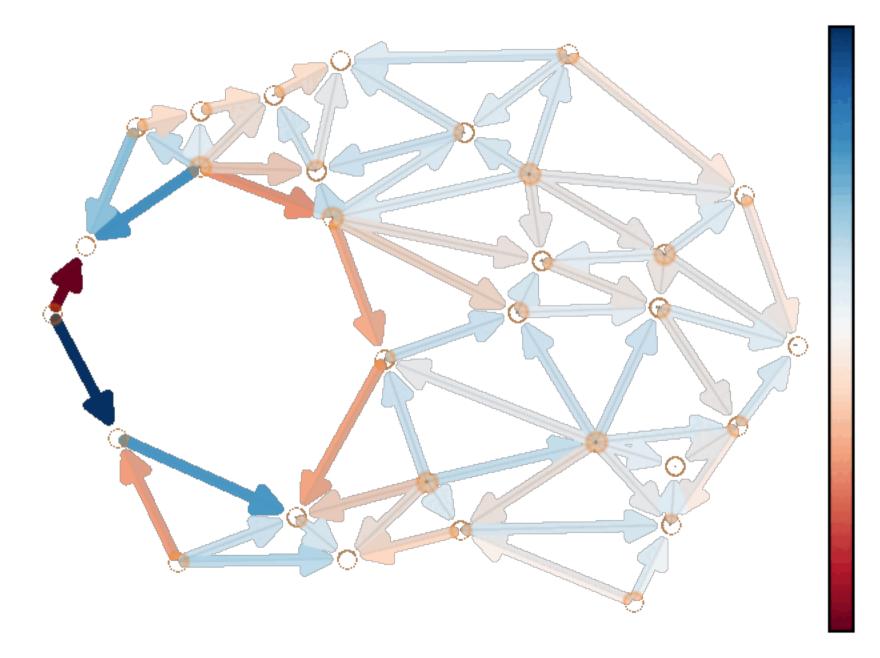
$$\Phi(\mathbf{L}_1) = \left(\frac{2\nu}{\kappa^2}\mathbf{I} + \mathbf{L}_1\right)^{\frac{\nu}{2}} \text{ and } \mathbf{w}_1 \sim N(\mathbf{0}, \sigma^2\mathbf{I})$$

The solution gives edge GPs

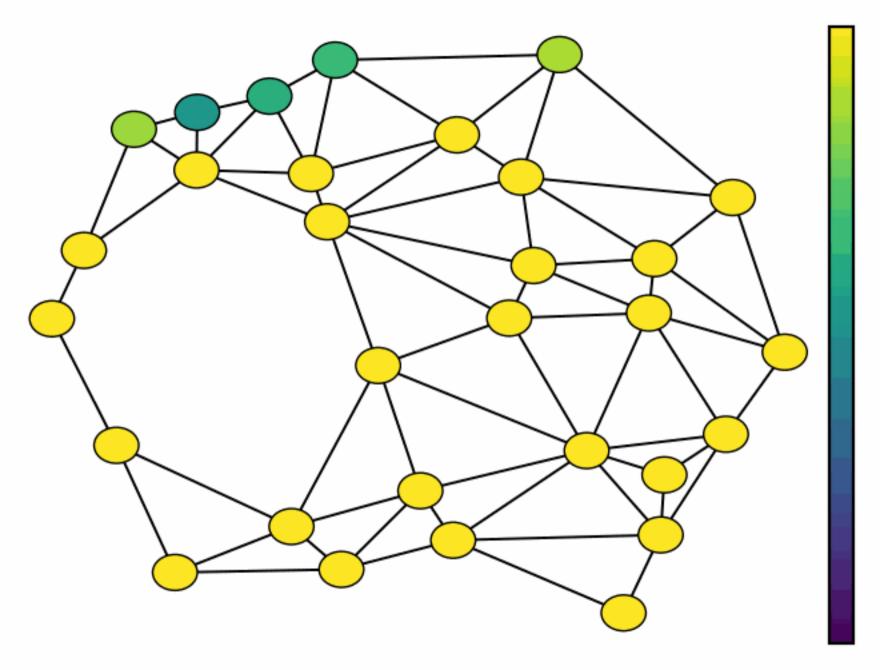
Matérn:
$$\mathbf{f}_1 \sim \mathrm{GP}\Big(0, \Big(\frac{2\nu}{\kappa^2}\mathbf{I} + \mathbf{L}_1\Big)^{-\nu}\Big)$$
Diffusion: $\mathbf{f}_1 \sim \mathrm{GP}\Big(0, e^{-\frac{\kappa^2}{2}\mathbf{L}_1}\Big)$

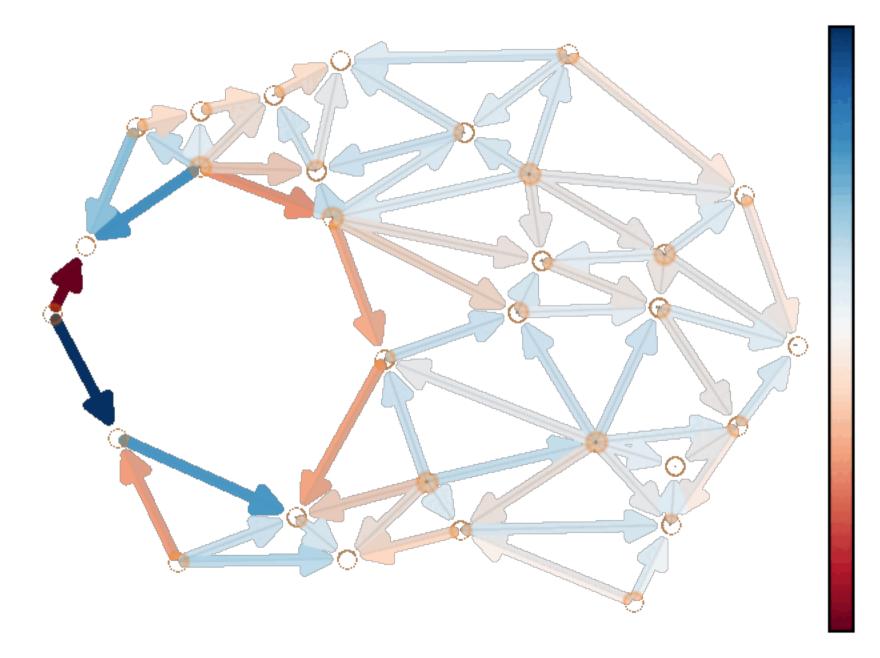
EVD: $\mathbf{L}_1 = \mathbf{U}_1 \mathbf{\Lambda}_1 \mathbf{U}_1^{\mathsf{T}}$





Diffusion on nodes vs on edges





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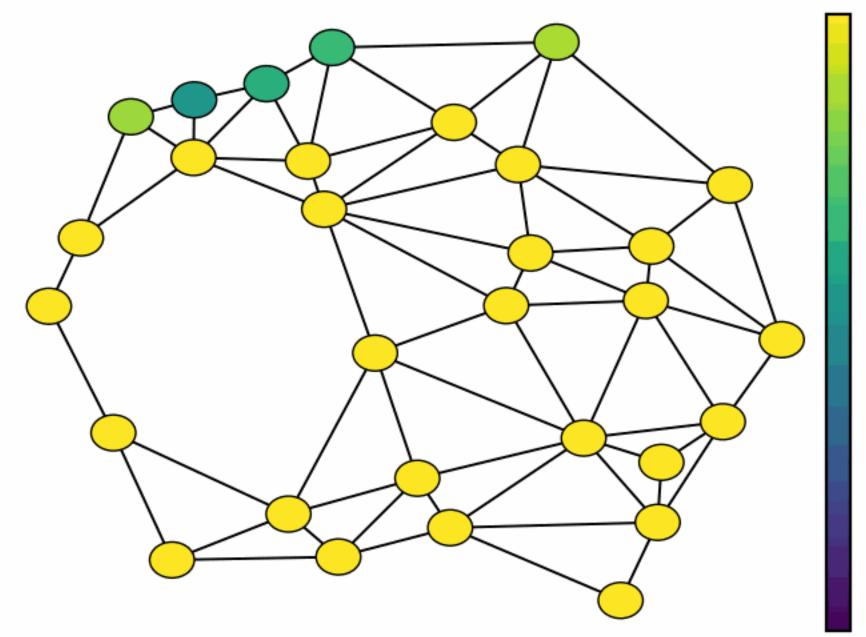
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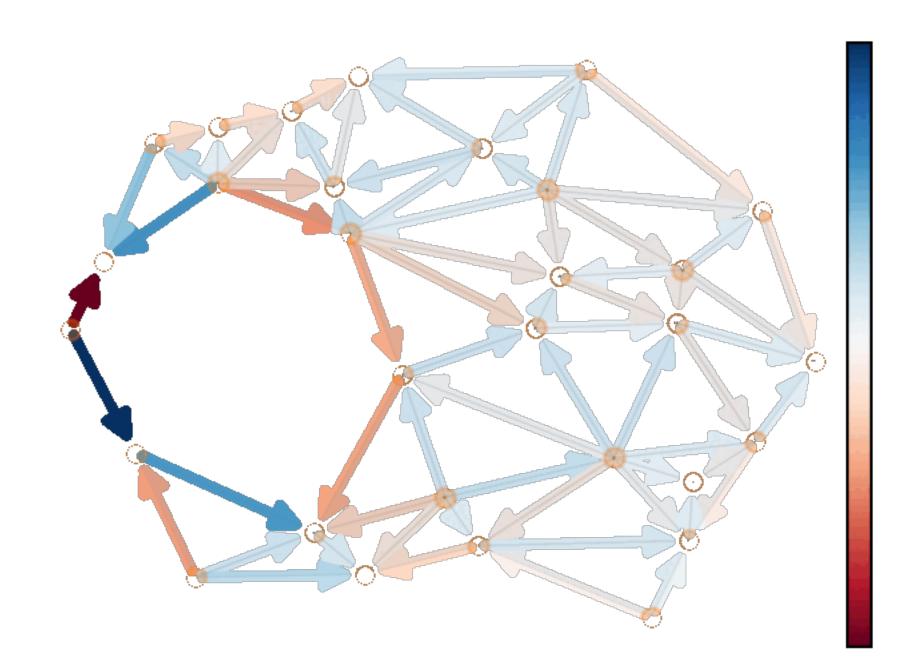
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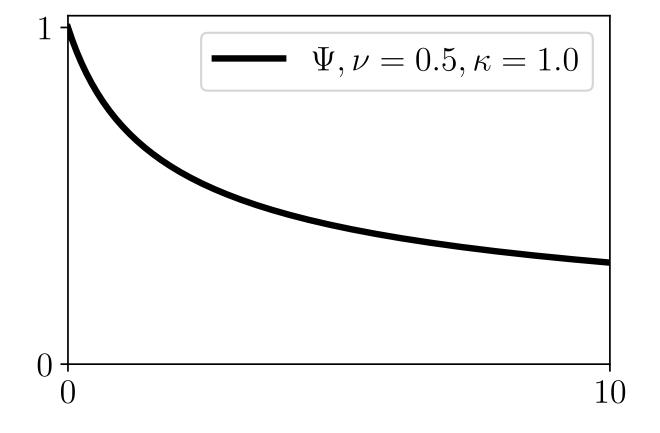
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- Low-pass in the eigen-spectrum

Smoothness

Node function — 0-form (scalar field) Edge function — 1-form (vector field)

> Divergence Curl