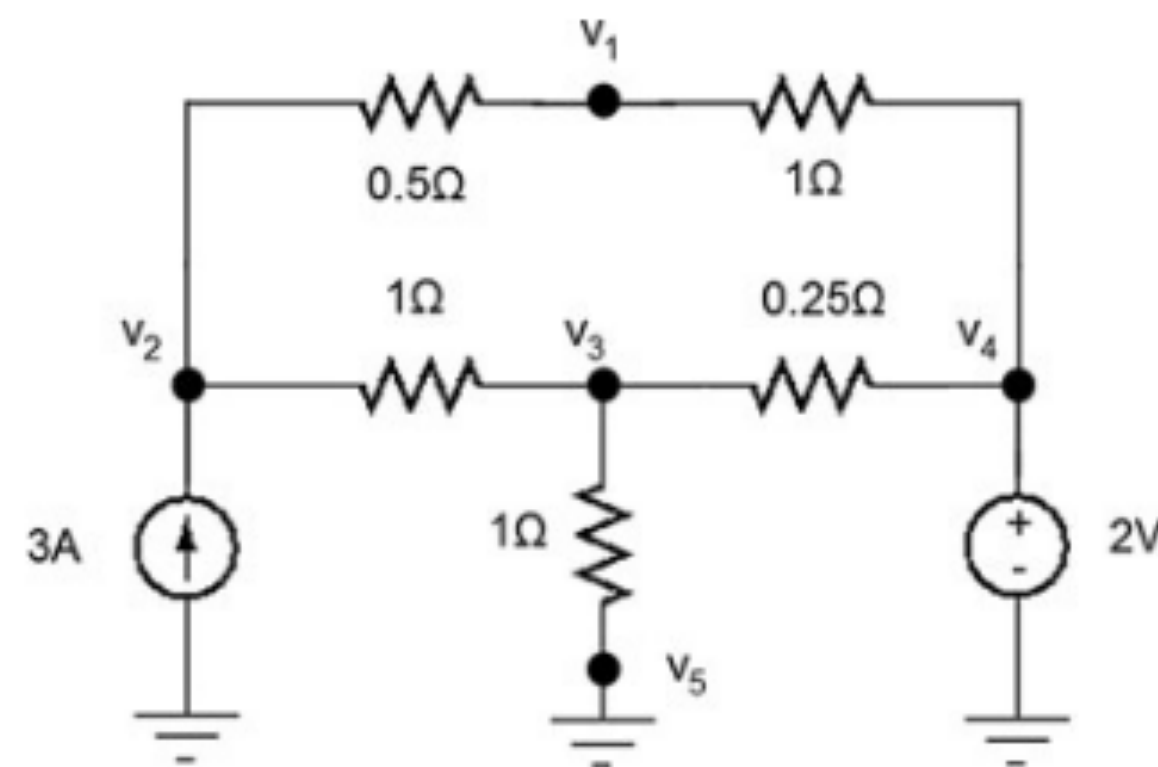
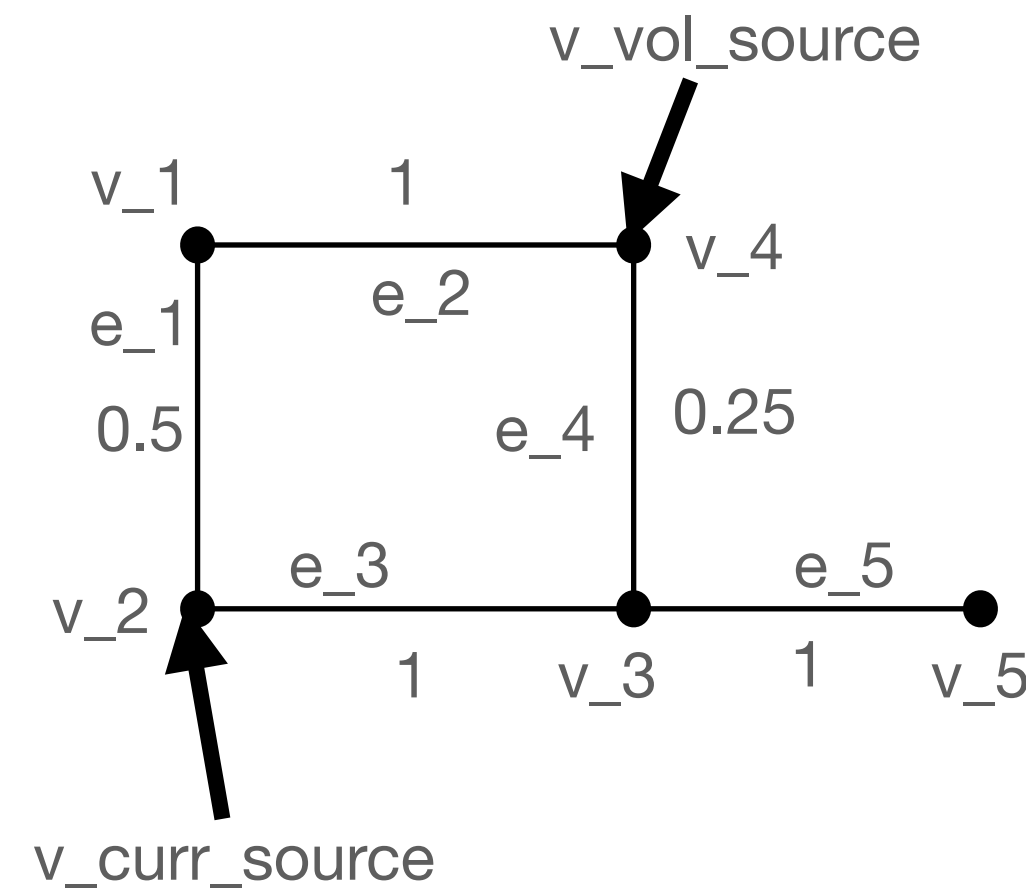


# A Circuit toy example



(Grady et al. 2010)



$\mathbf{v} \in \mathbb{R}^{|\mathcal{N}|}$ : Electric potential on nodes

$$\mathbf{f}_{Vol} = \mathbf{B}_1^T \mathbf{v}: \text{(Kirchhoff's voltage law)}$$

$$\mathbf{f}_{currents} = \mathbf{G}^{-1} \mathbf{f}_{Vol}: \text{currents (Ohm's law)}$$

Diagonal resistance/conductance

$$\text{Kirchhoff's current law: } \mathbf{B}_1 \mathbf{f}_{currents} = \mathbf{0}$$

$$\text{Or } \mathbf{B}_1 \mathbf{f}_{currents} + \mathbf{v}_{curr source} = \mathbf{0}$$

$$\mathbf{B}_1 = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 \\ 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix} \quad \mathbf{v}_{vol} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ 2 \\ 0 \end{pmatrix}$$

$$\mathbf{B}_1 \mathbf{G}^{-1} \mathbf{B}_1^T \mathbf{v}_{vol} + \mathbf{v}_{curr source} = \mathbf{0}$$

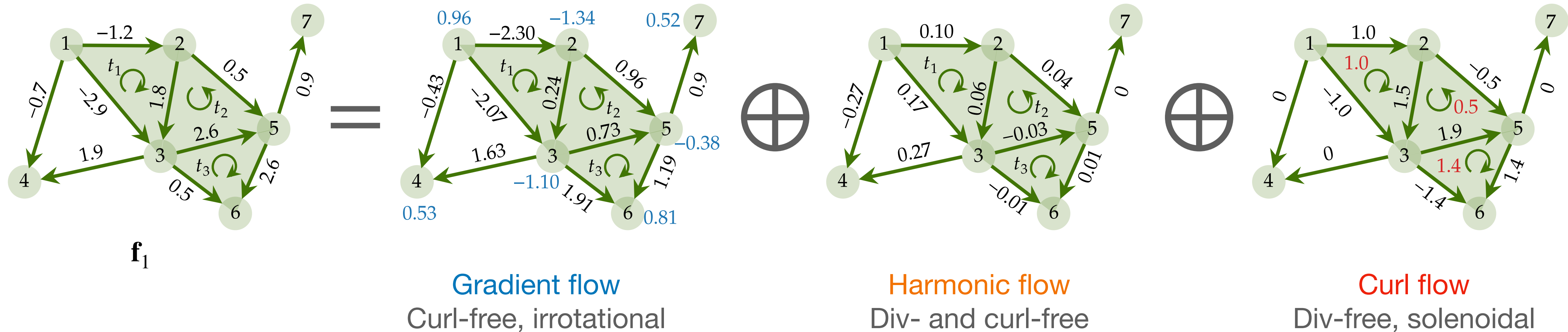
Resistance — Metrics?  
Power, energy?

# Hodge decomposition

Lovász et al. 2004; Lim et al. 2020

$$\mathbb{R}^{N_1} = \text{im}(\mathbf{B}_1^T) \oplus \text{ker}(\mathbf{L}_1) \oplus \text{im}(\mathbf{B}_2)$$

$$\mathbf{f}_1 = \mathbf{f}_G + \mathbf{f}_H + \mathbf{f}_C$$



- This holds for any simplex order  $k$
- What is the case for  $k = 0$ ?
- Characteristic decomposition

