Overview

Convolution

- Simplicial Fourier transform, Frequency, Fourier basis
- Signal variations
- Convolutional filters
- Generalizes graph convolutions

Gaussian Process

- How to define GPs for different parts?
- Hodge-compositional idea
- Probabilistic methods

Hodge decomposition

Convolutional NNs

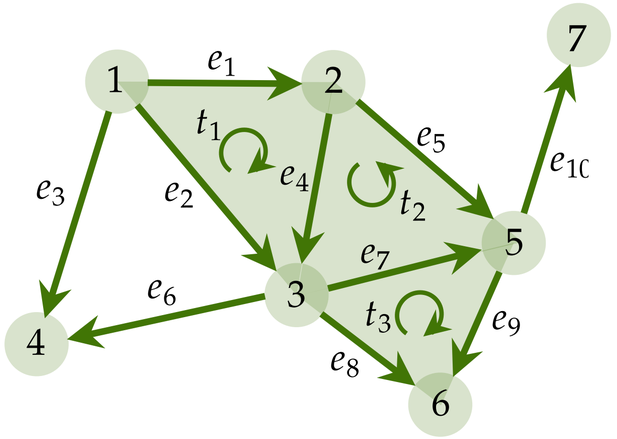
- Architecture (attention, message passing)
- Robustness
- Higher-order link predictions
- Generalizes GCNs, etc.

Generative learning

- (Dynamic) optimal transport (Schrödinger bridge) on SCs
- Gaussian bridge
- Generative models: diffusion, flow models on SCs

Repsentations of simplicial 2-complex

Incidences & Laplacians



Node-to-Edge

$$\mathbf{B}_1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} \begin{array}{c} \begin{array}{cccccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 & e_{10} \end{array} \\ \begin{pmatrix} -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array}, \quad \text{Edge-to-Faces} \quad \mathbf{B}_2 = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ e_7 \\ e_8 \\ e_9 \\ e_{10} \end{array} \begin{array}{c} \begin{array}{ccc} t_1 & t_2 & t_3 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \end{array}$$

Graph Laplacian: $\mathbf{L}_0 = \mathbf{B}_1\mathbf{B}_1^\top$

1-Hodge Laplacian: $\mathbf{L}_1 = \mathbf{B}_1^\top\mathbf{B}_1 + \mathbf{B}_2\mathbf{B}_2^\top := \mathbf{L}_{1,d} + \mathbf{L}_{1,u}$

Up

Down