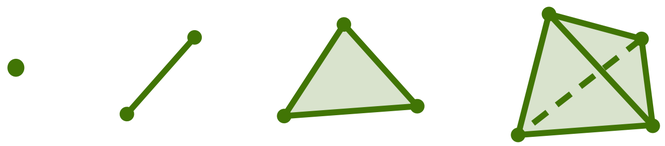
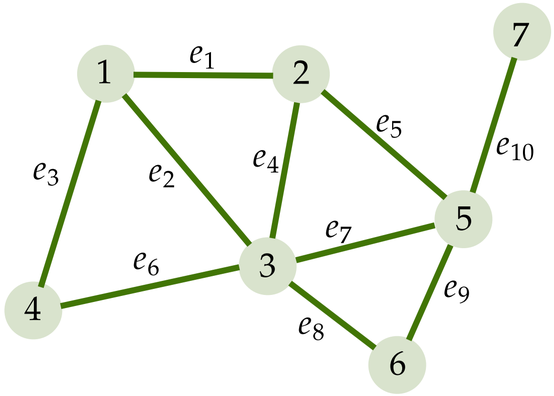
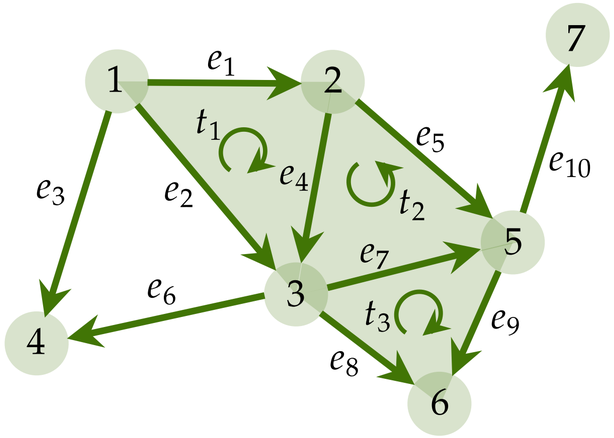
Graphs vs Simplicial 2-Complexes



0-, 1-, 2-, 3-simplices

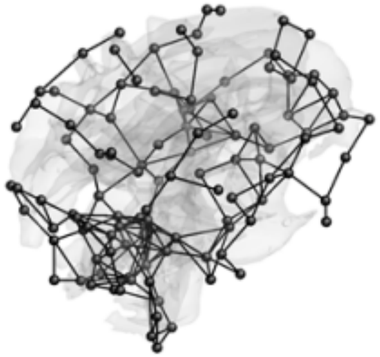


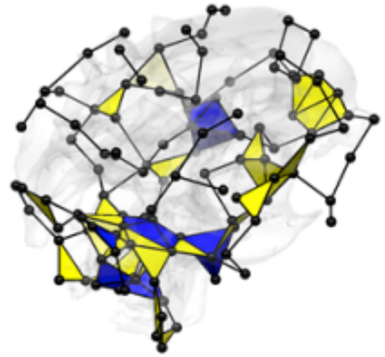
Graph
Simplicial 1-complex
$G = (V, E)$



Simplicial 2-complex
$\mathrm{SC}_2 = (V, E, T)$

- Oriented simplices (equivalence class of permutations)



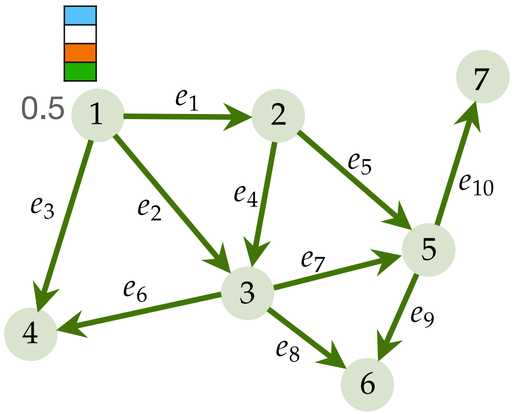


Neuroscience (Anand et al. 2023):
1. Firing of neurons
2. Activation of multiple brain regions

- Network analysis
- Topological data analysis
- Topological signal processing
- Topological deep learning

Functions on simplices

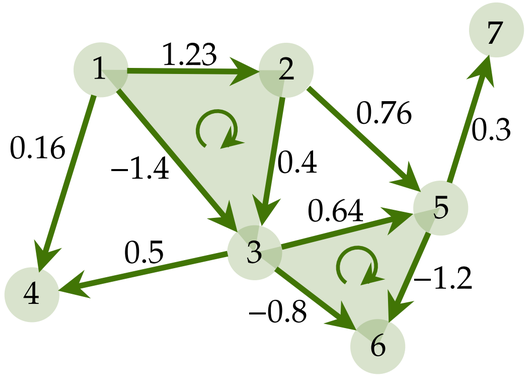
Signals on nodes, edges, triangles, …



Node function

$$f_0 : V \to \mathbb{R}$$

$$\mathbf{f}_0 = (f_0(1), \ldots, f_0(N_0))^\top$$



Edge function

$$f_1 : E \to \mathbb{R}$$

$$\mathbf{f}_1 = (f_1(e_1), \ldots, f_1(e_{N_1}))^\top$$

- Alternating property
- Magnitude and sign

- Flow-type data (natural)
  - Physical world: traffic flow, water flow, information flow…
  - Forex: exchange rates
  - Game theory (Candogan et al. 2011)
  - Ranking data (Jiang et al. 2011)
  - Edge-based vector field discretisation (computer graphics)
  - …

Triangle function

$$f_2 : T \to \mathbb{R}$$

0-, 1-, 2-cochains in topology