

# Filter Design

- Data-driven: given a training set  $\mathcal{T}$  of input-output pairs

$$\min_{\alpha, \beta} \frac{1}{|\mathcal{T}|} \sum \|\mathbf{H}\mathbf{f} - \mathbf{y}\|_2^2 + \gamma r(\alpha, \beta)$$

- Spectral filter design
  - Least-Squares
  - Chebyshev polynomials

$$\begin{cases} \tilde{H}_H(\lambda) = \alpha_0 + \beta_0 \approx g_0, & \text{for } \lambda \in \mathcal{Q}_H, \\ \tilde{H}_G(\lambda) = \sum_{k=0}^{K_d} \alpha_k \lambda^k \approx g_G(\lambda), & \text{for } \lambda \in \mathcal{Q}_G, \\ \tilde{H}_C(\lambda) = \sum_{k=0}^{K_u} \beta_k \lambda^k \approx g_C(\lambda), & \text{for } \lambda \in \mathcal{Q}_C \end{cases}$$

# Applications

- Hodge component extractions
- Solving LS problem:  $\mathbf{f}_G = \mathbf{P}_G \mathbf{f}$ ,  $\mathbf{f}_C = \mathbf{P}_C \mathbf{f}$ ,  $\mathbf{f}_H = \mathbf{f} - \mathbf{f}_G - \mathbf{f}_C$
- Convolutional filter implementation: closed form on coefficients

Gradient projection op.  
 $\mathbf{P}_G = \mathbf{B}_1^\top (\mathbf{B}_1 \mathbf{B}_1^\top)^\dagger \mathbf{B}_1 = \mathbf{U}_G \mathbf{U}_G^\top$

