Posterior distribution of Hodge components

$$
\begin{bmatrix} \boldsymbol{f}_H(\boldsymbol{x}) \\ \boldsymbol{f}_H(\boldsymbol{x}^*) \\ \boldsymbol{f}_G(\boldsymbol{x}) \\ \boldsymbol{f}_G(\boldsymbol{x}^*) \\ \boldsymbol{f}_C(\boldsymbol{x}) \\ \boldsymbol{f}_C(\boldsymbol{x}^*) \\ \boldsymbol{f}_1(\boldsymbol{x}) \\ \boldsymbol{f}_1(\boldsymbol{x}^*) \end{bmatrix} \sim \mathcal{N}\left(\mathbf{0}, \begin{bmatrix} \boldsymbol{K}_H & \boldsymbol{K}_H^* & & & & & \boldsymbol{K}_H & \boldsymbol{K}_H^* \\ \boldsymbol{K}_H^{*\top} & \boldsymbol{K}_H^{**} & & & & & \boldsymbol{K}_H^* & \boldsymbol{K}_H^{**} \\ & & \boldsymbol{K}_G & \boldsymbol{K}_G^* & & & \boldsymbol{K}_G & \boldsymbol{K}_G^* \\ & & \boldsymbol{K}_G^{*\top} & \boldsymbol{K}_G^{**} & & & \boldsymbol{K}_G^* & \boldsymbol{K}_G^{**} \\ & & & & \boldsymbol{K}_C & \boldsymbol{K}_C^* & \boldsymbol{K}_C & \boldsymbol{K}_C^* \\ & & & & \boldsymbol{K}_C^{*\top} & \boldsymbol{K}_C^{**} & \boldsymbol{K}_C^* & \boldsymbol{K}_C^{**} \\ \boldsymbol{K}_H & \boldsymbol{K}_H^{*\top} & \boldsymbol{K}_G & \boldsymbol{K}_G^{*\top} & \boldsymbol{K}_C & \boldsymbol{K}_C^{*\top} & \boldsymbol{K}_1 & \boldsymbol{K}_1^* \\ \boldsymbol{K}_H^{*\top} & \boldsymbol{K}_H^{**} & \boldsymbol{K}_G^{*\top} & \boldsymbol{K}_G^{**} & \boldsymbol{K}_C^{*\top} & \boldsymbol{K}_C^{**} & \boldsymbol{K}_1^{*\top} & \boldsymbol{K}_1^{**} \end{bmatrix}\right) \tag{B.26}
$$

where we represent the kernel matrices by $\boldsymbol{K}_1 = k_1(\boldsymbol{x}, \boldsymbol{x}), \boldsymbol{K}_1^* = k_1(\boldsymbol{x}, \boldsymbol{x}^*)$ and $\boldsymbol{K}_1^{**} = k_1(\boldsymbol{x}^*, \boldsymbol{x}^*)$, and likewise for the other kernel matrices. Given this joint distribution, we can obtain the posterior distributions of the three Hodge components as follows

$$
\boldsymbol{f}_H(\boldsymbol{x}^*)|\boldsymbol{f}_1(\boldsymbol{x}) \sim \mathcal{N}\left(\boldsymbol{K}_H^{*\top}\boldsymbol{K}_1^{-1}\boldsymbol{f}_1(\boldsymbol{x}), \boldsymbol{K}_H^{**} - \boldsymbol{K}_H^{*\top}\boldsymbol{K}_1^{-1}\boldsymbol{K}_H^*\right) \tag{B.27a}
$$

$$
\boldsymbol{f}_G(\boldsymbol{x}^*)|\boldsymbol{f}_1(\boldsymbol{x}) \sim \mathcal{N}\left(\boldsymbol{K}_G^{*\top}\boldsymbol{K}_1^{-1}\boldsymbol{f}_1(\boldsymbol{x}), \boldsymbol{K}_G^{**} - \boldsymbol{K}_G^{*\top}\boldsymbol{K}_1^{-1}\boldsymbol{K}_G^*\right) \tag{B.27b}
$$

$$
\boldsymbol{f}_C(\boldsymbol{x}^*)|\boldsymbol{f}_1(\boldsymbol{x}) \sim \mathcal{N}\left(\boldsymbol{K}_C^{*\top}\boldsymbol{K}_1^{-1}\boldsymbol{f}_1(\boldsymbol{x}), \boldsymbol{K}_C^{**} - \boldsymbol{K}_C^{*\top}\boldsymbol{K}_1^{-1}\boldsymbol{K}_C^*\right) \tag{B.27c}
$$

From these posterior distributions, we can directly obtain the means and the uncertainties of the Hodge components of the predicted edge function.