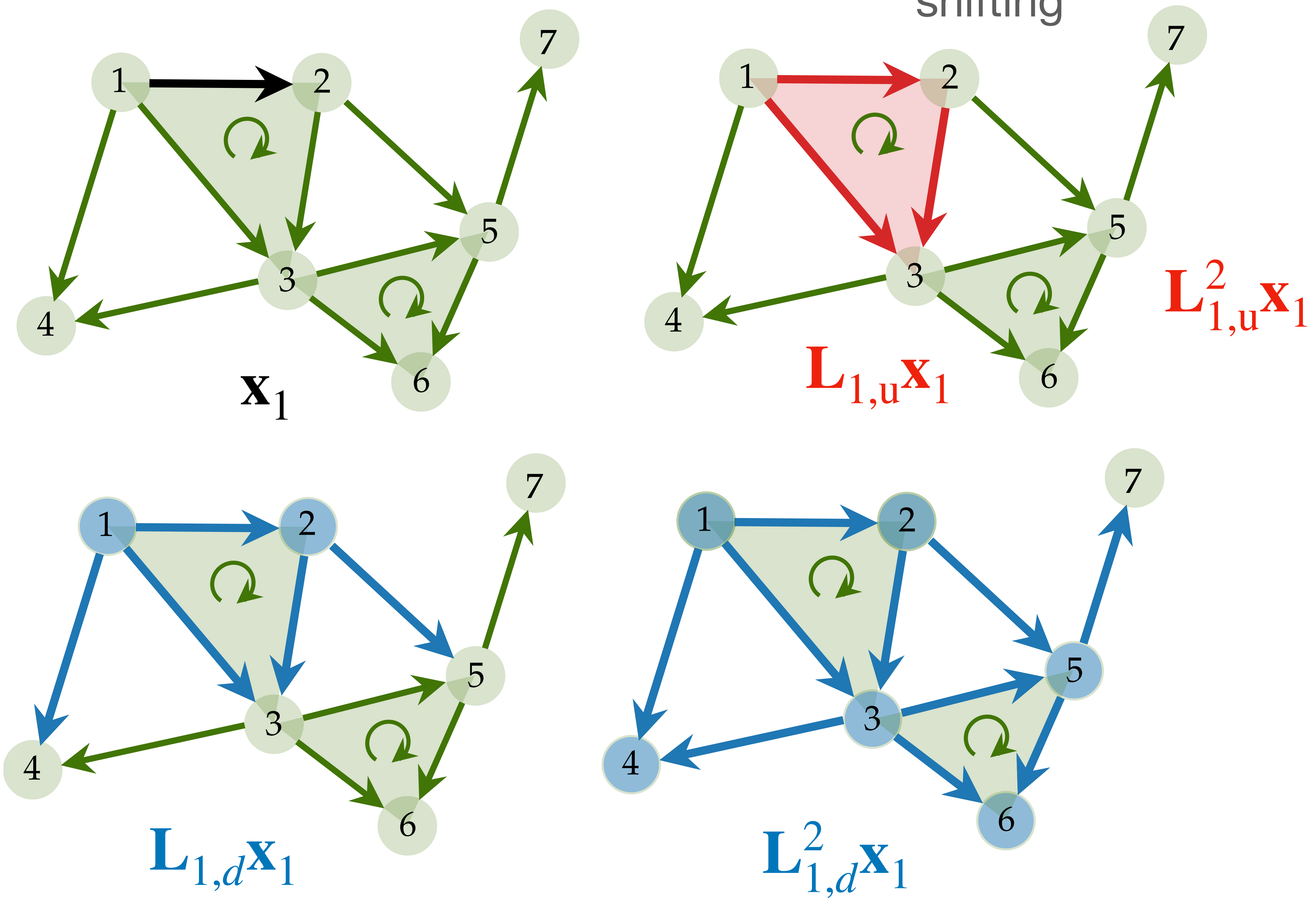
Edge Convolution

Shift-and-Sum

Spatial/Topological

Simplicial shifting

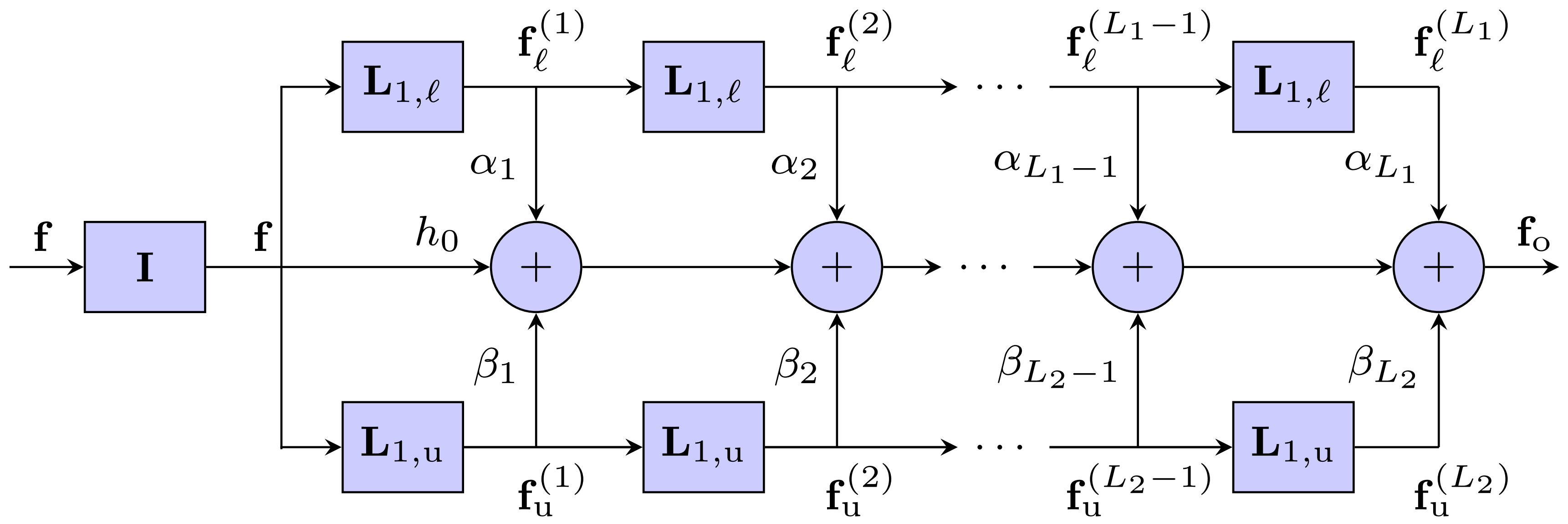


$$[\mathbf{L}_{1,d}\mathbf{f}]_i = \sum_{j \in \{\mathcal{N}_{1,i} \cup i\}} [\mathbf{L}_{1,d}]_{ij}[\mathbf{f}]_j$$

Simplicial locality

Convolutional filter

$$\boxed{\mathbf{H} := \mathbf{H}(\mathbf{L}_\mathrm{d}, \mathbf{L}_\mathrm{u}; \boldsymbol{\alpha}, \boldsymbol{\beta}) = \sum_{k=0}^{K_\mathrm{d}} \alpha_k \mathbf{L}_\mathrm{d}^k + \sum_{k=0}^{K_\mathrm{u}} \beta_k \mathbf{L}_\mathrm{u}^k}$$



- Efficient, distributed
- Expressive power (Cayley-Hamilton thm)
- Hodge-invariant operator

$$\mathbf{H}_1\mathbf{x}_1 = \mathbf{H}_1\,|_{\mathrm{im}(\mathbf{B}_1^\top)}\mathbf{x}_{1,\mathrm{G}} + \mathbf{H}_1\,|_{\mathrm{im}(\mathbf{B}_2)}\mathbf{x}_{1,\mathrm{C}} + \mathbf{H}_1\,|_{\ker(\mathbf{L}_1)}\mathbf{x}_{1,\mathrm{H}}$$

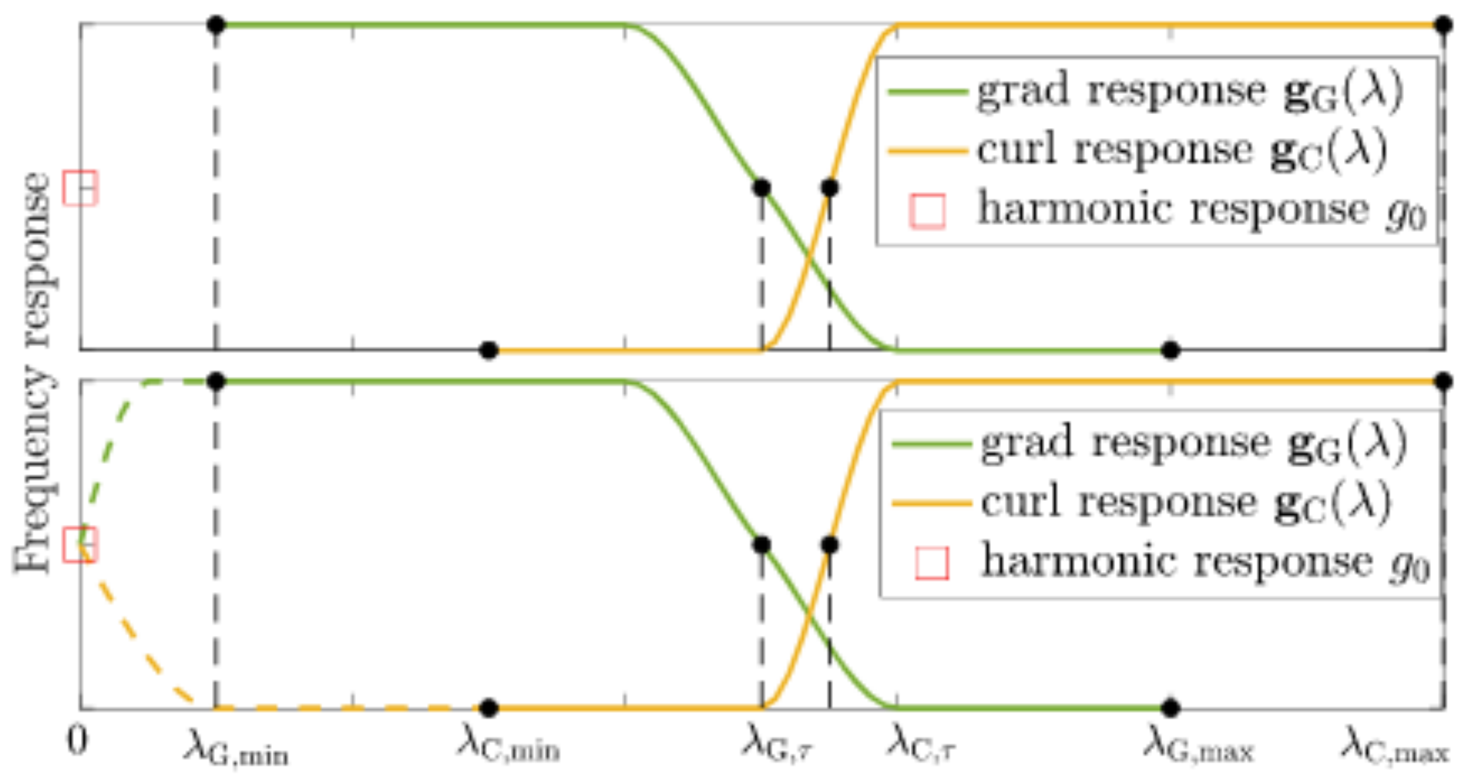
Hodge subspaces are invariant under $\mathbf{H}$

Edge Convolutions on SCs

Pointwise Multiplication at frequencies

Spectral

$$\begin{cases} \tilde{H}_{\mathrm{H}}(\lambda) = \alpha_0 + \beta_0, & \text{for } \lambda \in \mathcal{Q}_{\mathrm{H}}, \\ \tilde{H}_{\mathrm{G}}(\lambda) = \sum_{k=0}^{K_d} \alpha_k \lambda^k, & \text{for } \lambda \in \mathcal{Q}_{\mathrm{G}}, \\ \tilde{H}_{\mathrm{C}}(\lambda) = \sum_{k=0}^{K_u} \beta_k \lambda^k, & \text{for } \lambda \in \mathcal{Q}_{\mathrm{C}} \end{cases}$$



Why two sets of coefficients instead of one set?