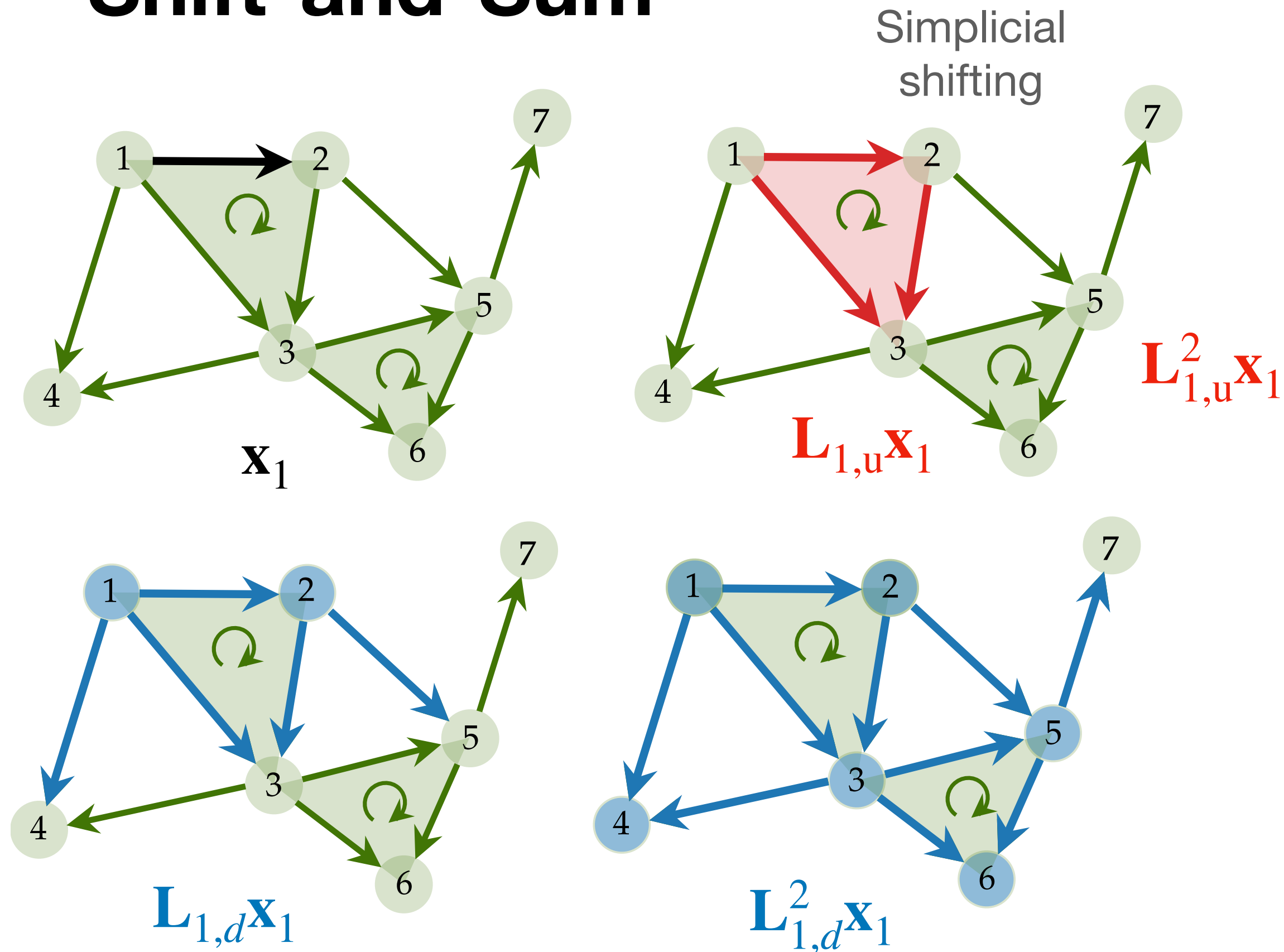


Edge Convolution

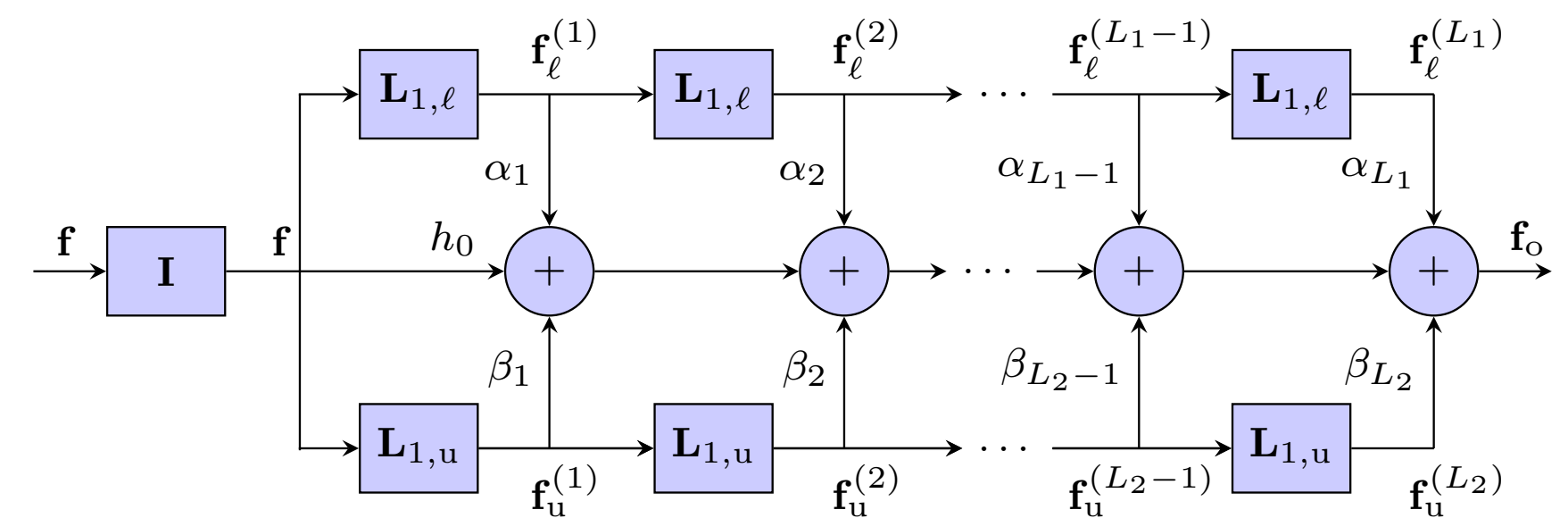
Shift-and-Sum



$$[\mathbf{L}_{1,d}\mathbf{f}]_i = \sum_{j \in \{\mathcal{N}_{1,i} \cup i\}} [\mathbf{L}_{1,d}]_{ij} [\mathbf{f}]_j$$

Simplicial locality

$$\mathbf{H} := \mathbf{H}(\mathbf{L}_d, \mathbf{L}_u; \alpha, \beta) = \sum_{k=0}^{K_d} \alpha_k \mathbf{L}_d^k + \sum_{k=0}^{K_u} \beta_k \mathbf{L}_u^k$$



- Efficient, distributed
- Expressive power (Cayley-Hamilton thm)
- Hodge-invariant operator

$$\mathbf{H}_1 \mathbf{x}_1 = \mathbf{H}_1|_{\text{im}(\mathbf{B}_1^T)} \mathbf{x}_{1,G} + \mathbf{H}_1|_{\text{im}(\mathbf{B}_2)} \mathbf{x}_{1,C} + \mathbf{H}_1|_{\text{ker}(\mathbf{L}_1)} \mathbf{x}_{1,H}$$

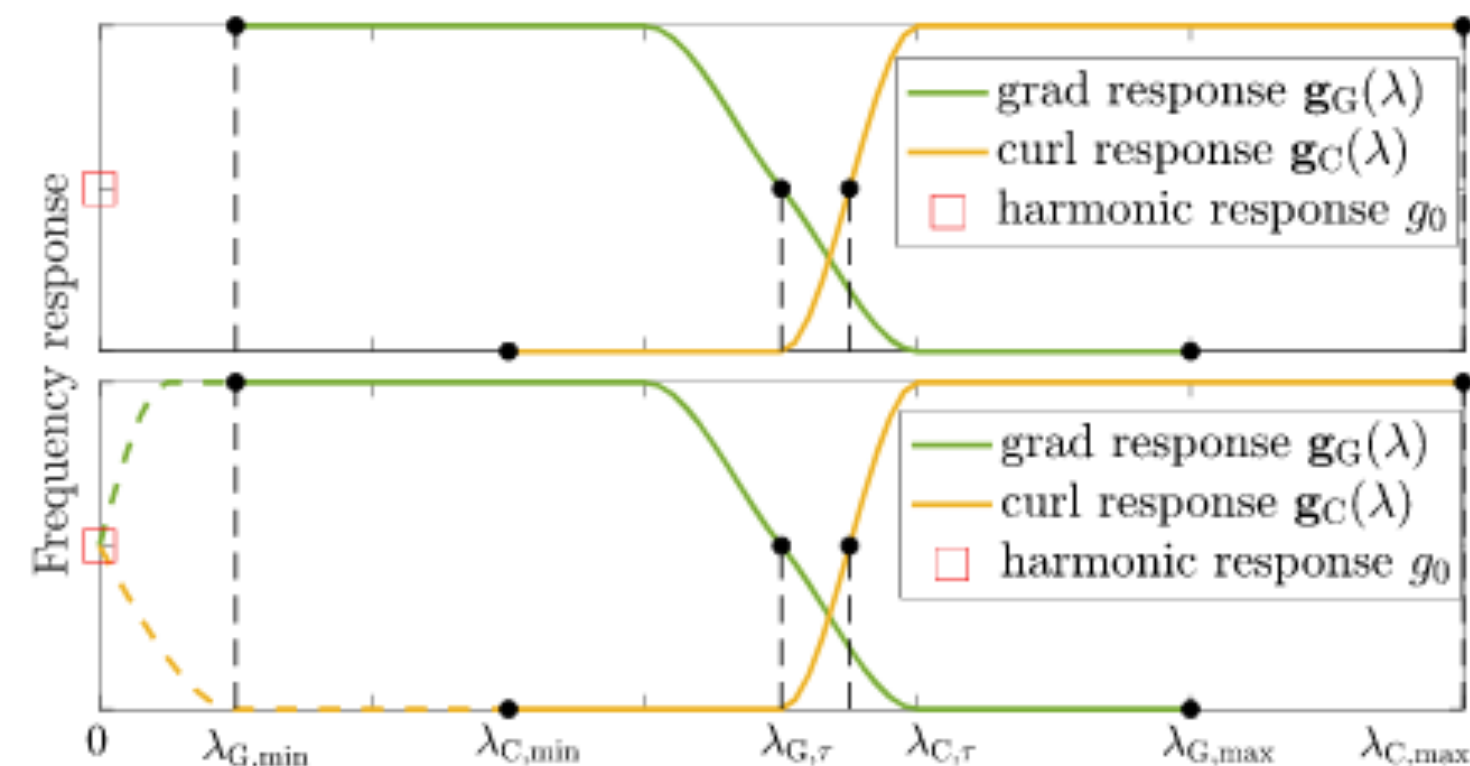
Hodge subspaces are invariant under \mathbf{H}

Edge Convolutions on SCs

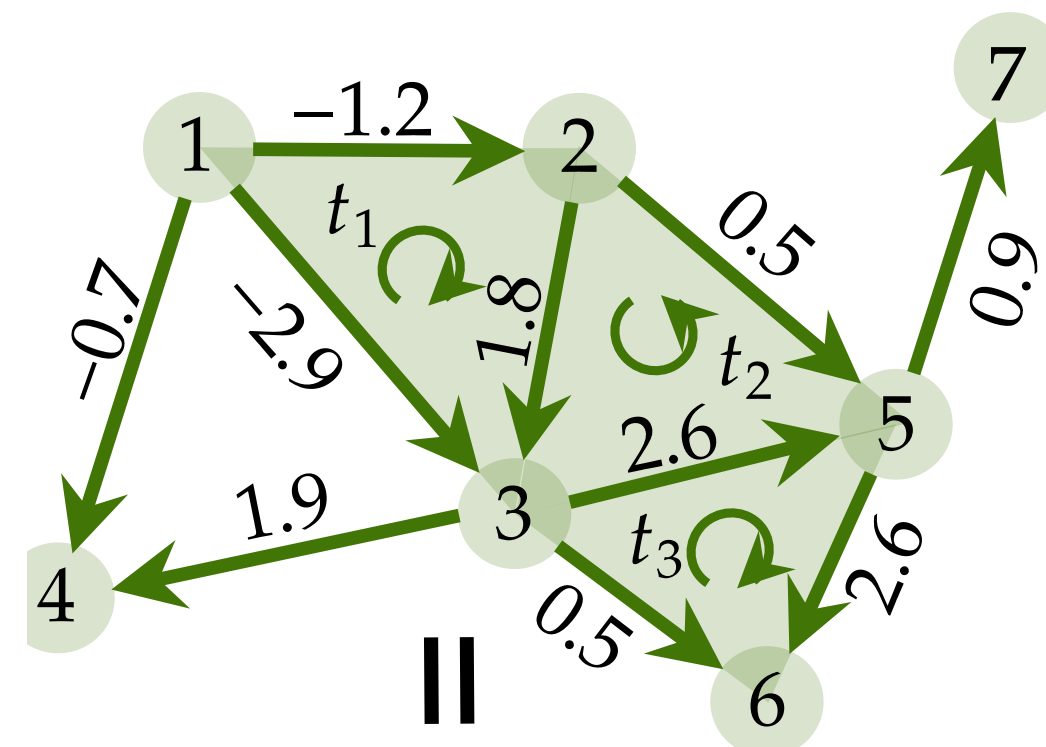
Pointwise Multiplication at frequencies

Spectral

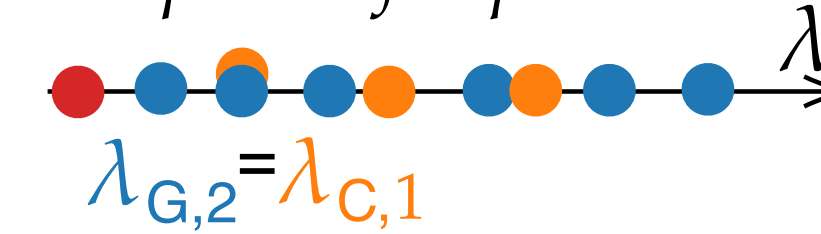
$$\begin{cases} \tilde{H}_H(\lambda) = \alpha_0 + \beta_0, & \text{for } \lambda \in \mathcal{Q}_H, \\ \tilde{H}_G(\lambda) = \sum_{k=0}^{K_d} \alpha_k \lambda^k, & \text{for } \lambda \in \mathcal{Q}_G, \\ \tilde{H}_C(\lambda) = \sum_{k=0}^{K_u} \beta_k \lambda^k, & \text{for } \lambda \in \mathcal{Q}_C \end{cases}$$



Why two sets of coefficients instead of one set?



simplicial frequencies



- blue dot: gradient freq. λ_G
- orange dot: curl freq. λ_C
- red dot: harmonic freq. λ_H

blue arrow: gradient flow

orange arrow: curlflow

red arrow: harmonic flow

$H_1|_{\text{im}(\mathbf{B}_1^T)} : \text{im}(\mathbf{B}_1^T) \rightarrow \text{im}(\mathbf{B}_1^T)$

$H_1|_{\text{im}(\mathbf{B}_2)} : \text{im}(\mathbf{B}_2) \rightarrow \text{im}(\mathbf{B}_2)$

$H_1|_{\text{ker}(\mathbf{L}_1)} : \text{ker}(\mathbf{L}_1) \rightarrow \text{ker}(\mathbf{L}_1)$

