

# Eigenspace of $L_1$ spans Hodge subspaces

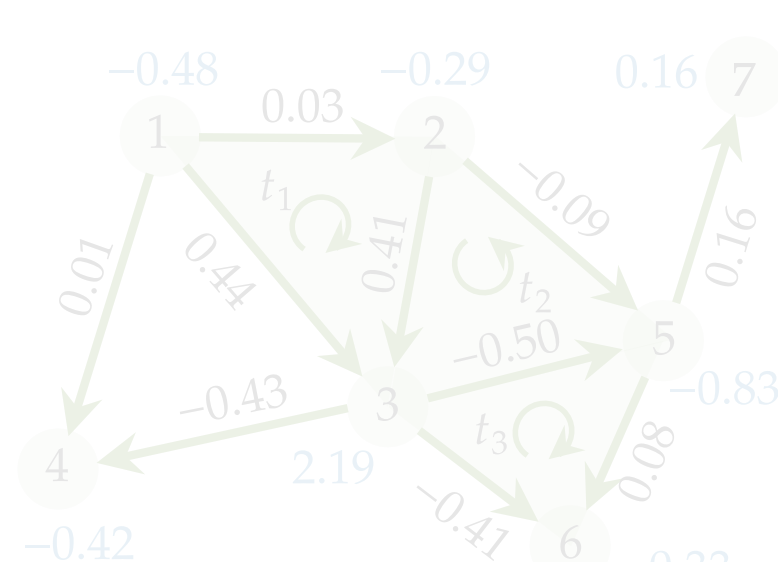
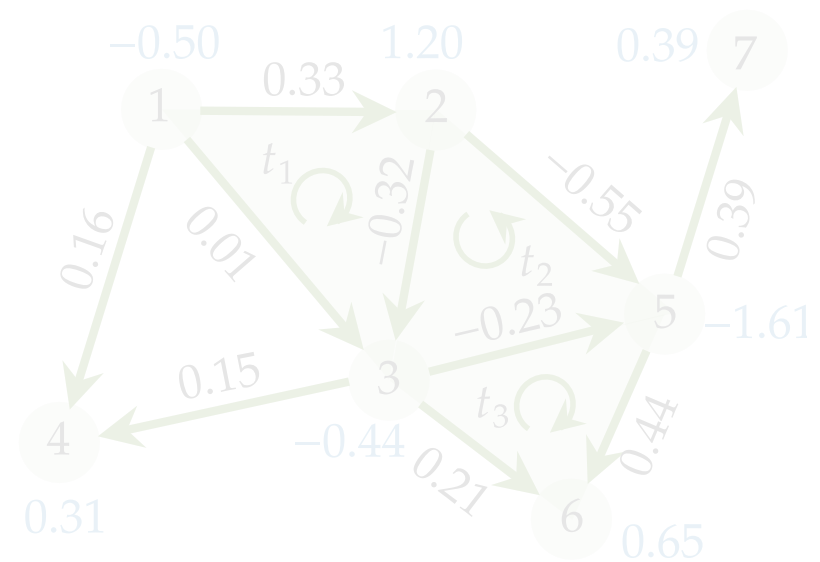
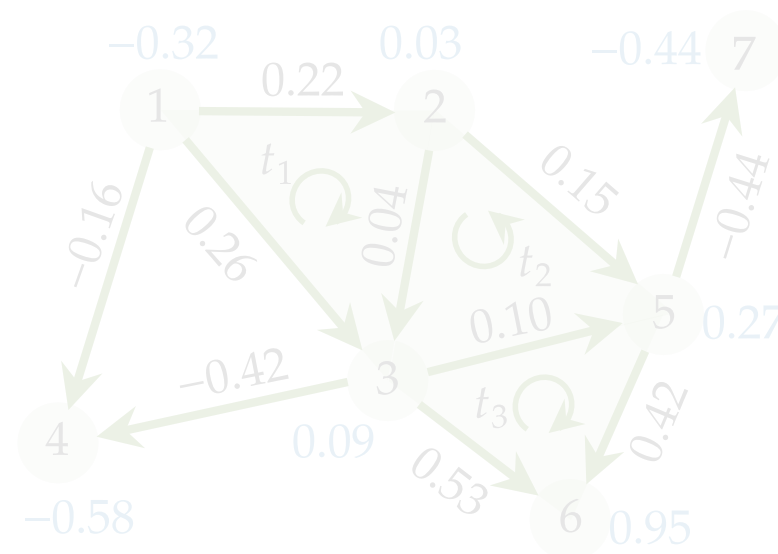
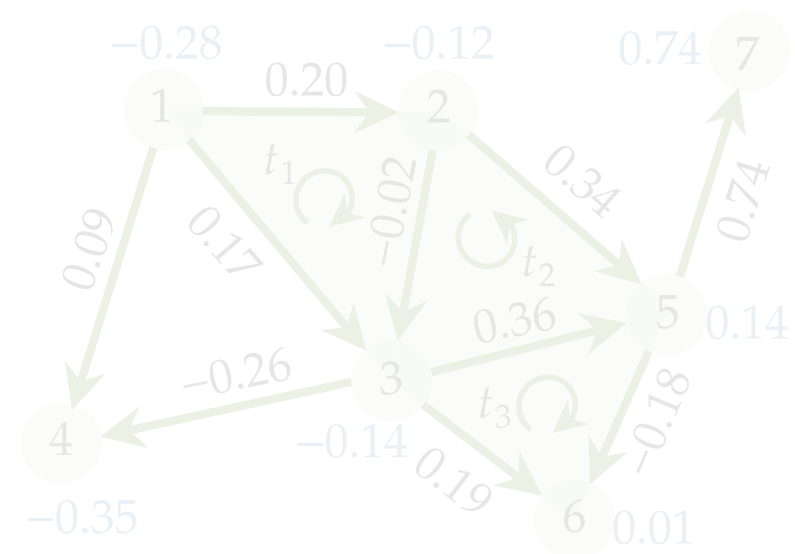
- Nonzero Eigenspace of **up Laplacian** spans the **curl** space

## Simplicial Fourier transform

Frequency — eigenvalues

Fourier basis — eigenvectors

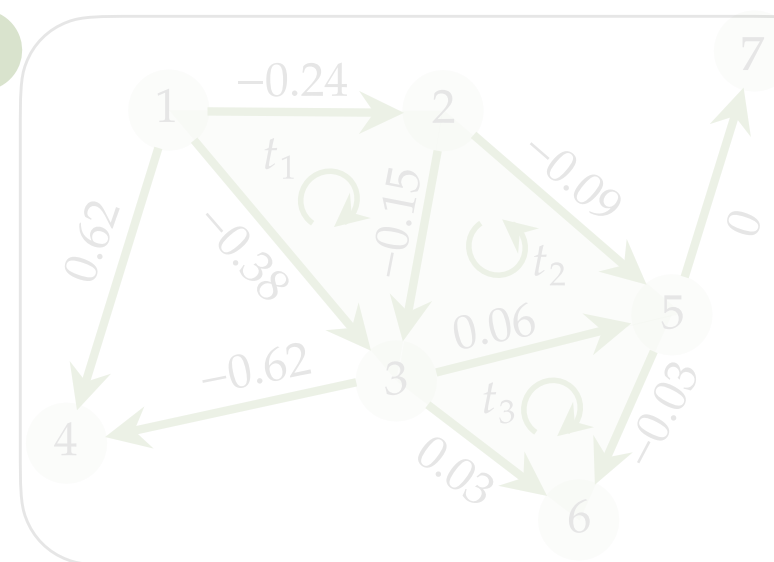
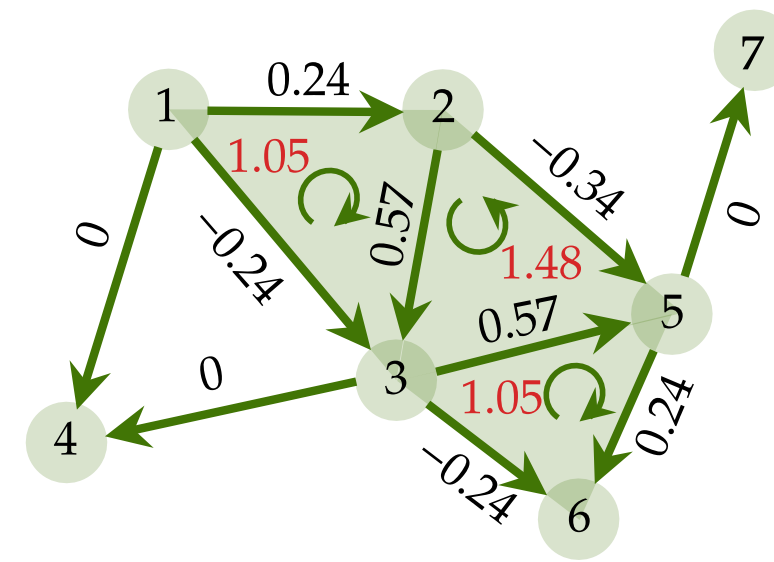
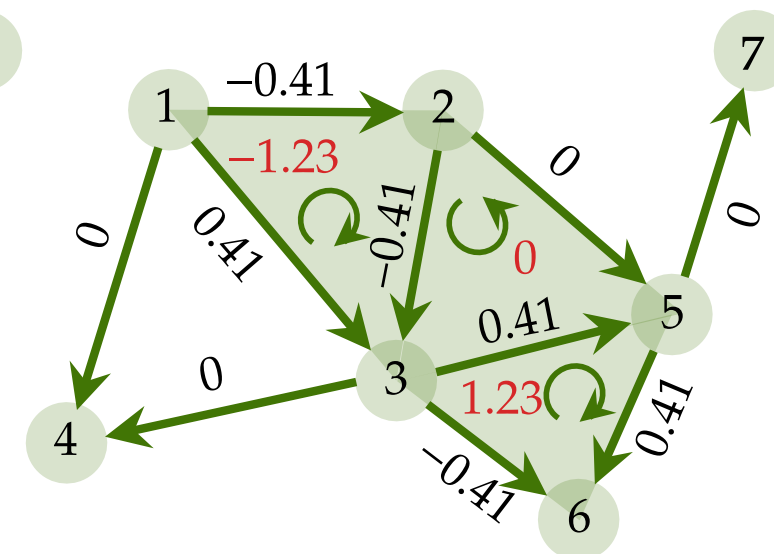
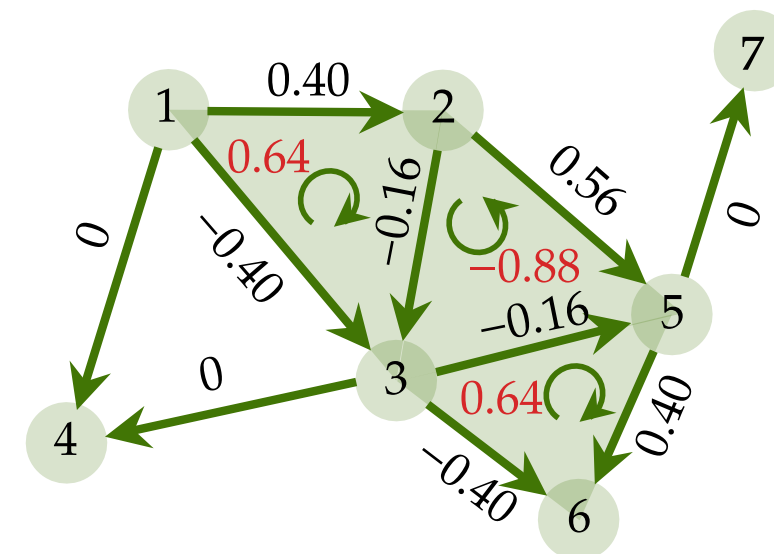
$$\lambda_G = \|\mathbf{B}_1 \mathbf{u}_G\|_2^2$$



$$\lambda_C = \|\mathbf{B}_2^T \mathbf{u}_C\|_2^2$$

Curl eigenvector

Fourier basis reflecting **rotational** properties



$$\text{EVD: } \mathbf{L}_1 = \mathbf{U}_1 \mathbf{\Lambda}_1 \mathbf{U}_1^T$$

$$\mathbf{U}_1 = [\mathbf{U}_H \ \mathbf{U}_G \ \mathbf{U}_C]$$

$$\text{span}(\mathbf{U}_C) = \text{im}(\mathbf{B}_2)$$

# Eigenspace of $L_1$ spans Hodge subspaces

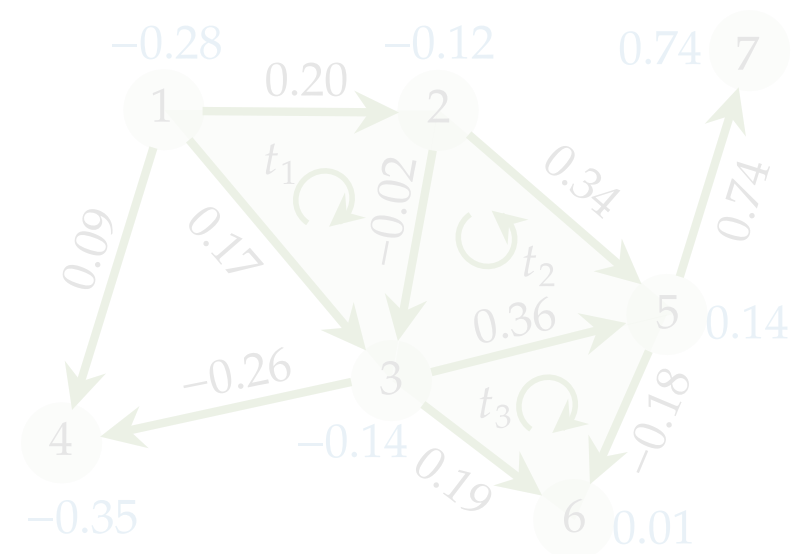
- **Kernel** of Laplacian spans the **harmonic** space

## Simplicial Fourier transform

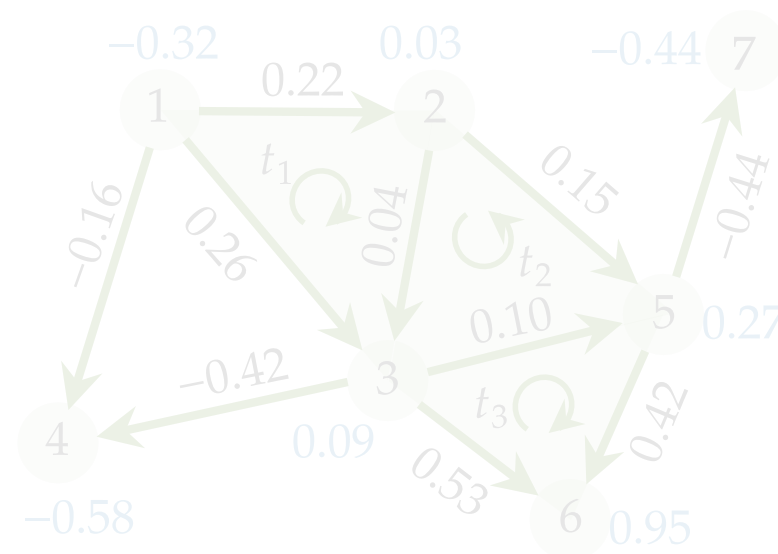
Frequency — eigenvalues

Fourier basis — eigenvectors

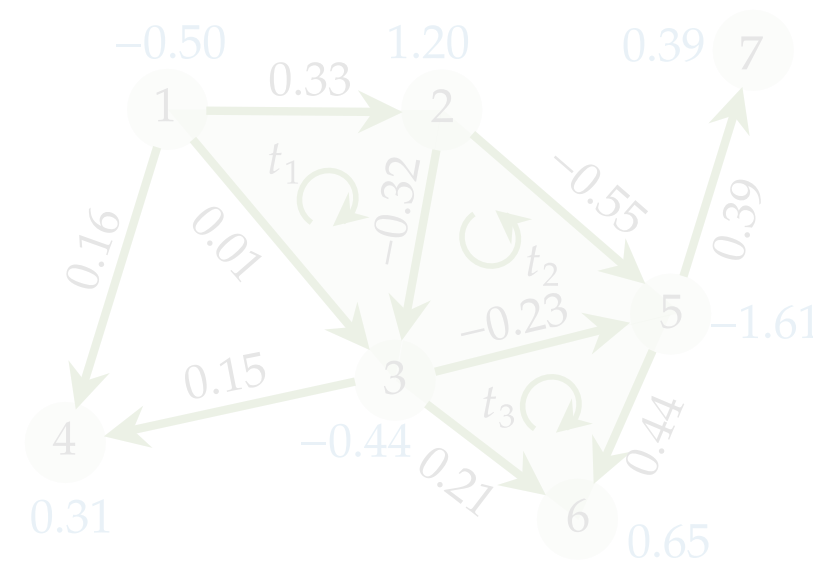
$$\lambda_G = \|\mathbf{B}_1 \mathbf{u}_G\|_2^2$$



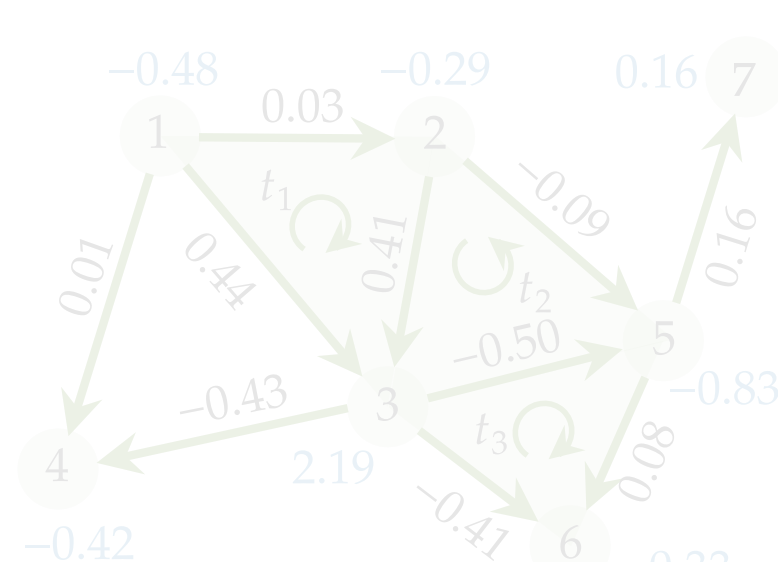
$$\lambda_{G,1} = 0.80$$



$$\lambda_{G,2} = 1.61$$

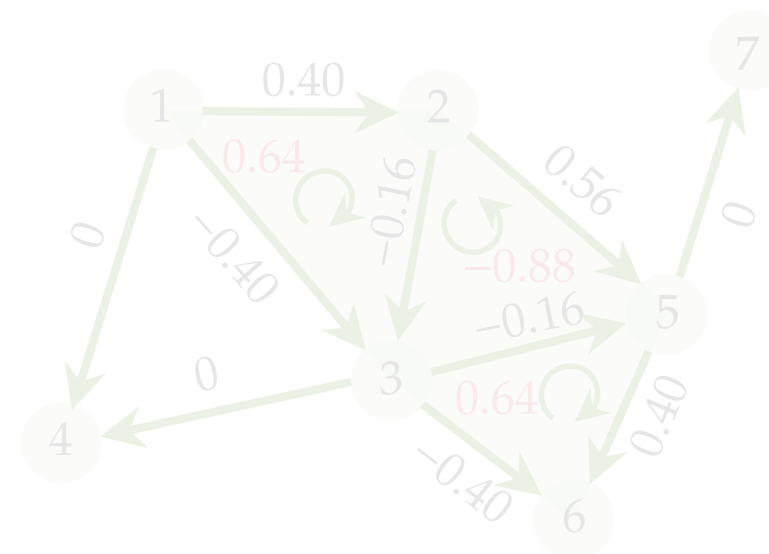


$$\lambda_{G,5} = 5.12$$



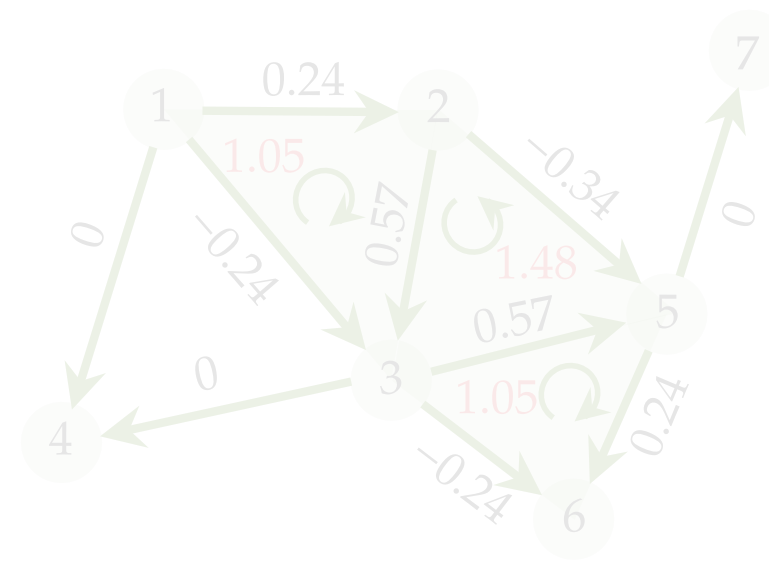
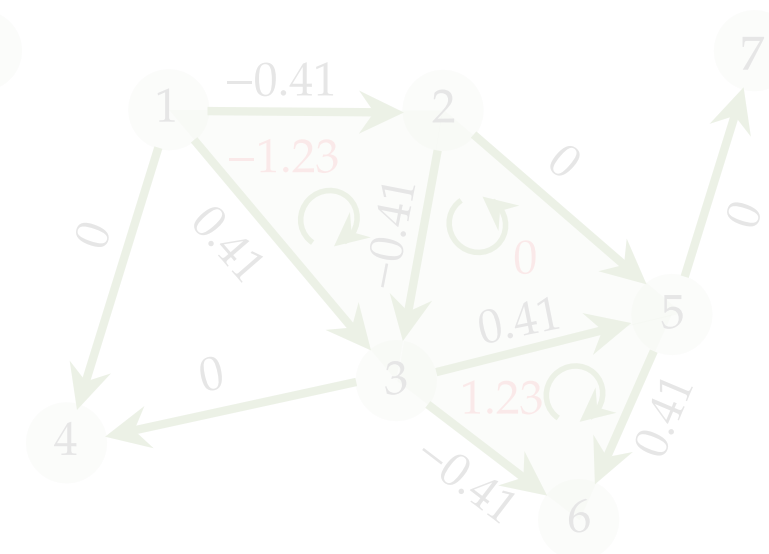
$$\lambda_{G,6} = 6.08$$

$$\lambda_C = \|\mathbf{B}_2^T \mathbf{u}_C\|_2^2$$

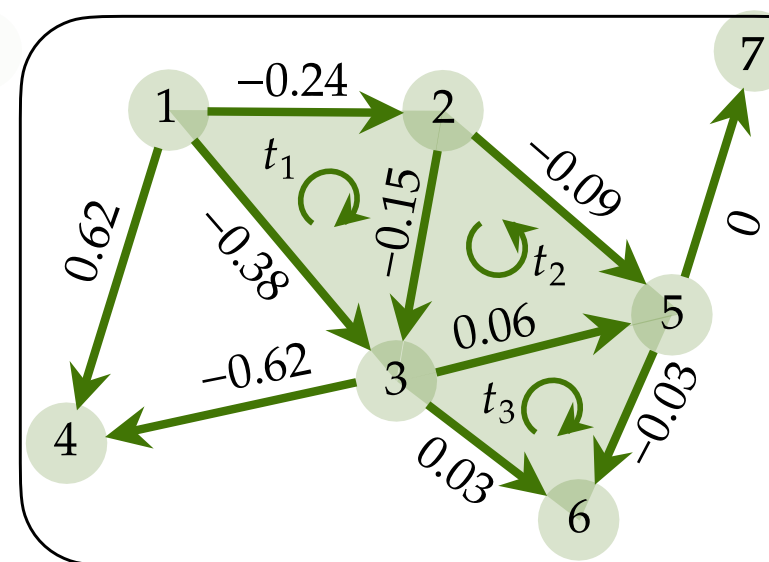


$$\lambda_{C,1} = 1.59$$

$$\|\mathbf{B}_2^T \mathbf{u}_H\|_2^2 = 0, \|\mathbf{B}_1 \mathbf{u}_H\|_2^2 = 0$$



$$\lambda_{C,3} = 4.41$$



$$\lambda_{H,1} = 0$$

$$\text{EVD: } \mathbf{L}_1 = \mathbf{U}_1 \mathbf{\Lambda}_1 \mathbf{U}_1^T$$

$$\mathbf{U}_1 = [\mathbf{U}_H \ \mathbf{U}_G \ \mathbf{U}_C]$$

$$\text{span}(\mathbf{U}_H) = \ker(\mathbf{L}_1)$$