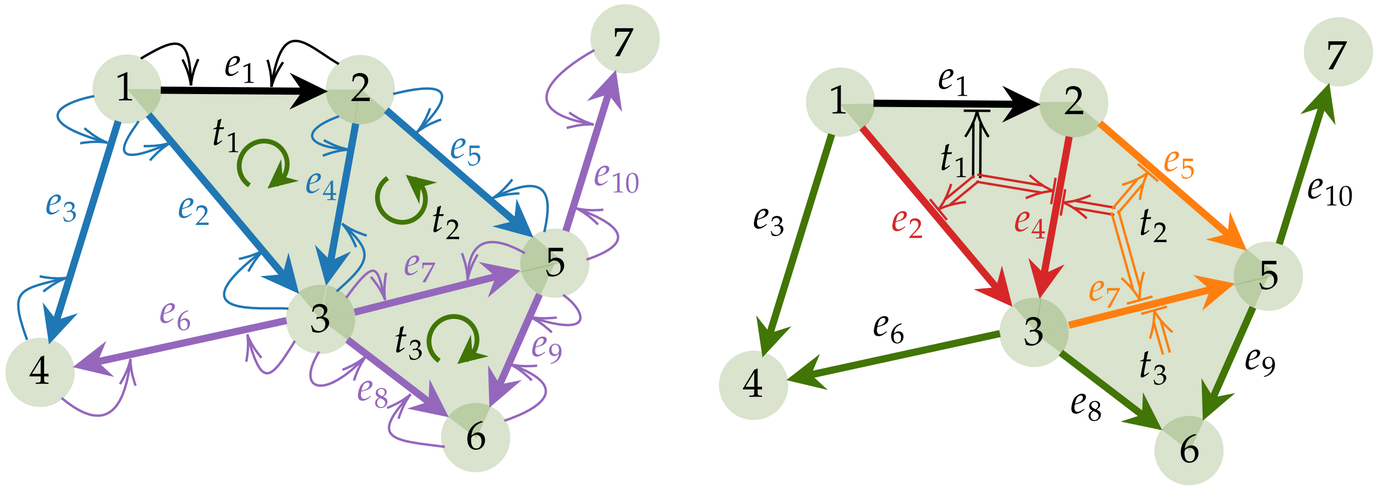
Simplicial complex CNNs

Node-edge-triangle interactions

- $\text{SCCNN}_k^l : \{x_{k-1}^{l-1}, x_k^{l-1}, x_{k+1}^{l-1}\} \to x_k^l$, with simplicial order k and layer l

$$\mathbf{x}_k^l = \sigma(\mathbf{H}_{k,\mathrm{d}}^l \mathbf{x}_{k,\mathrm{d}}^{l-1} + \mathbf{H}_k^l \mathbf{x}_k^{l-1} + \mathbf{H}_{k,\mathrm{u}}^l \mathbf{x}_{k,\mathrm{u}}^{l-1})$$



$$\mathbf{x}_0^l = \sigma(\mathbf{H}_0^l \mathbf{x}_0^{l-1} + \mathbf{H}_{0,\mathrm{u}}^l \mathbf{B}_1 \mathbf{x}_1^{l-1})$$
$$\mathbf{x}_1^l = \sigma(\mathbf{H}_{1,\mathrm{d}}^l \mathbf{B}_1^\top \mathbf{x}_0^{l-1} + \mathbf{H}_1^l \mathbf{x}_1^{l-1} + \mathbf{H}_{1,\mathrm{u}}^l \mathbf{B}_2 \mathbf{x}_2^{l-1})$$
$$\mathbf{x}_2^l = \sigma(\mathbf{H}_{2,\mathrm{d}}^l \mathbf{B}_2^\top \mathbf{x}_1^{l-1} + \mathbf{H}_2^l \mathbf{x}_2^{l-1})$$

Convolution based (Ebli et al. 2020; Roddenberry et al. 2021; Yang et al. 2022, 2023)
Message passing (Bodnar et al. 2021)

Properties: locality, symmetry
Dirichlet energy perspective
Hodge-invariant
Stability to weights perturbations

Simplex prediction

Generalization of link prediction

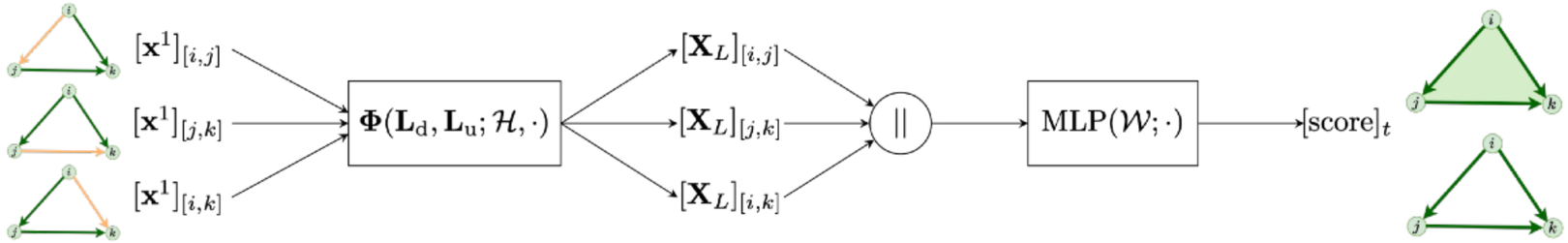


Table 2: Simplex prediction (AUC, ↑) .

Methods	2-simplex	3-simplex
Mean (Benson et al., 2018)	62.8±2.7	63.6±1.6
MLP	68.5±1.6	69.0±2.2
GNN (Defferrard et al., 2016)	93.9±1.0	96.6±0.5
SNN (Ebli et al., 2020)	92.0±1.8	95.1±1.2
PSNN (Roddenberry et al., 2021)	95.6±1.3	98.1±0.5
SCNN (Yang et al., 2022a)	96.5±1.5	98.3±0.4
Bunch (Bunch et al., 2020)	98.3±0.5	98.5±0.5
MPSN (Bodnar et al., 2021b)	98.1±0.5	99.2±0.3
SCCNN	**98.7±0.5**	**99.4±0.3**