

# Edge flow regularizations

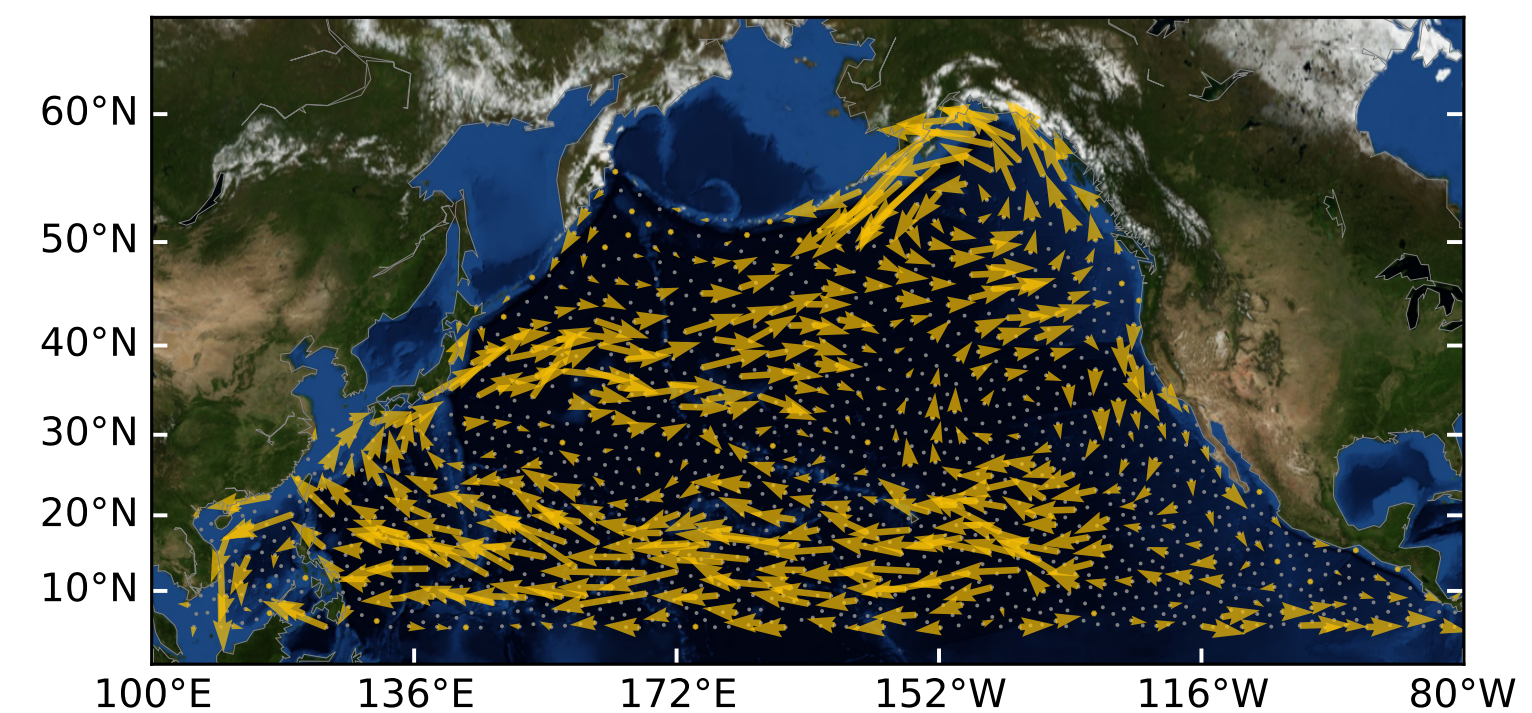
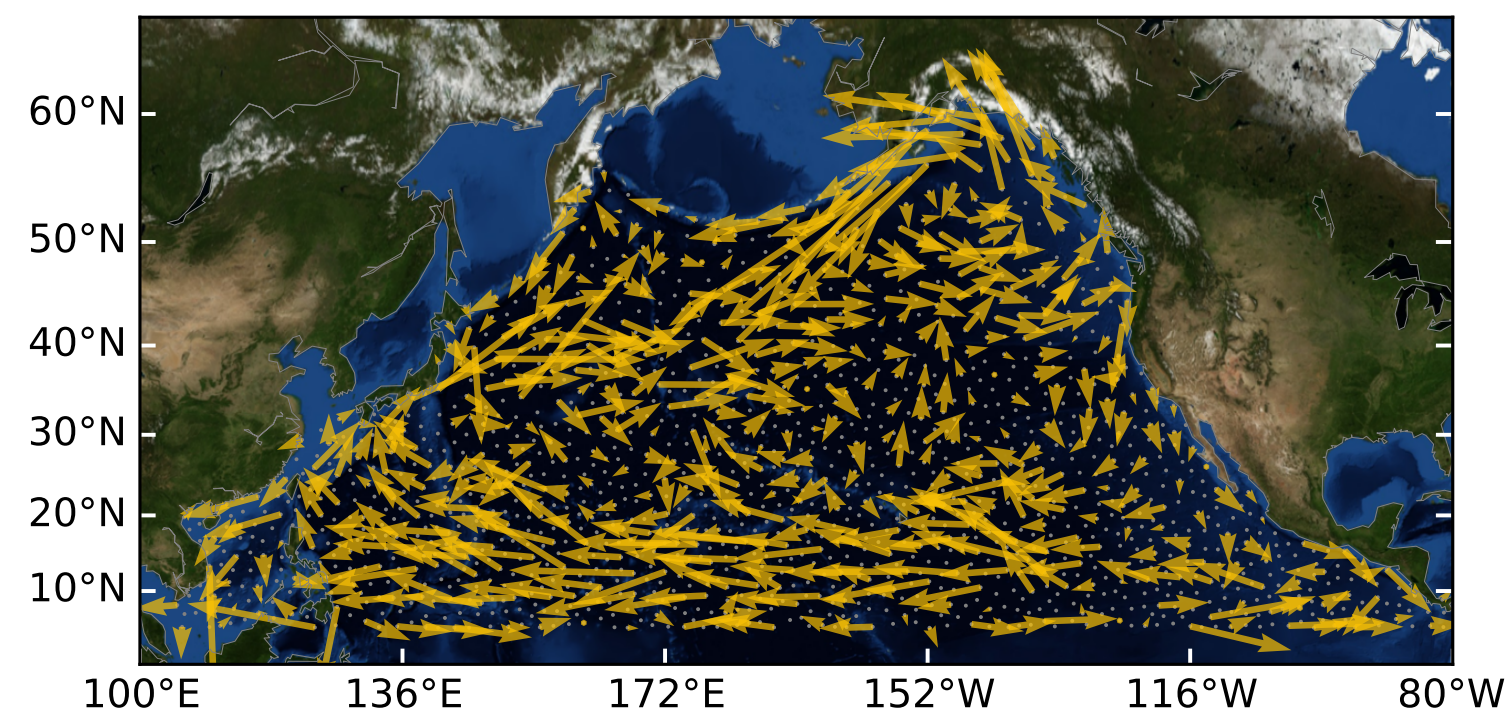
## $\ell_1, \ell_2$ regularization

- Tikhonov regularization

$$\|\mathbf{y} - \mathbf{f}\|_2^2 + \gamma \cdot \|\text{div/curl}\|_2^2$$

- Trend filtering

$$\|\mathbf{y} - \mathbf{f}\|_2^2 + \gamma \cdot \|\text{div/curl}\|_1$$



# Tabular results

Table 1: Forex rates inference results.

| Method     | RMSE            |                 | NLPD             |                  |
|------------|-----------------|-----------------|------------------|------------------|
|            | Diffusion       | Matérn          | Diffusion        | Matérn           |
| Euclidean  | $2.17 \pm 0.13$ | $2.19 \pm 0.12$ | $2.12 \pm 0.07$  | $2.20 \pm 0.18$  |
| Line-Graph | $2.43 \pm 0.07$ | $2.46 \pm 0.07$ | $2.28 \pm 0.04$  | $2.32 \pm 0.03$  |
| Non-HC     | $2.48 \pm 0.07$ | $2.47 \pm 0.08$ | $2.36 \pm 0.07$  | $2.34 \pm 0.04$  |
| HC         | $0.08 \pm 0.12$ | $0.06 \pm 0.12$ | $-3.52 \pm 0.02$ | $-3.52 \pm 0.02$ |

Table 3: WSN inference results.

| Method            | Node Heads      |                  | Edge Flowrates  |                  |
|-------------------|-----------------|------------------|-----------------|------------------|
|                   | RMSE            | NLPD             | RMSE            | NLPD             |
| Diffusion, non-HC | $0.16 \pm 0.05$ | $0.72 \pm 2.06$  | $0.32 \pm 0.05$ | $0.97 \pm 1.80$  |
| Matérn, non-HC    | $0.16 \pm 0.04$ | $0.71 \pm 2.39$  | $0.26 \pm 0.05$ | $0.10 \pm 0.13$  |
| Diffusion, HC     | $0.15 \pm 0.04$ | $-0.47 \pm 0.14$ | $0.22 \pm 0.03$ | $-0.20 \pm 0.13$ |
| Matérn, HC        | $0.15 \pm 0.04$ | $-0.25 \pm 0.48$ | $0.23 \pm 0.03$ | $-0.45 \pm 0.49$ |

Table C.1: Ocean current inference results.

| Method     | RMSE            |                 |                 | NLPD            |                 |                 |
|------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|            | Diffusion       | Matérn          | Hodge Laplacian | Diffusion       | Matérn          | Hodge Laplacian |
| Euclidean  | $1.00 \pm 0.01$ | $1.00 \pm 0.00$ | —               | $1.42 \pm 0.01$ | $1.42 \pm 0.10$ | —               |
| Line-Graph | $0.99 \pm 0.00$ | $0.99 \pm 0.00$ | —               | $1.41 \pm 0.00$ | $1.41 \pm 0.00$ | —               |
| Non-HC     | $0.35 \pm 0.00$ | $0.35 \pm 0.00$ | $0.35 \pm 0.00$ | $0.33 \pm 0.00$ | $0.36 \pm 0.03$ | $0.33 \pm 0.01$ |
| HC         | $0.34 \pm 0.00$ | $0.35 \pm 0.00$ | $0.35 \pm 0.00$ | $0.33 \pm 0.01$ | $0.37 \pm 0.04$ | $0.33 \pm 0.01$ |