Vulnerability of Information Transport on Temporal Networks to Link Removal

Li Zou and Huijuan Wang, *Member, IEEE*

***Abstract*—Communications networks such as vehicle and social contact networks are temporal networks, where autos/individuals are connected only when they are close to each other. These networks facilitate the propagation of information. Traffic between any two nodes demanded at any time is routed along the fastest time-respecting path. In this work, we investigate the vulnerability of information transport on temporal networks to link removal. The objective is to understand the removal of which types of links deteriorate the efficiency/speed of information transport the most. Identifying such critical links will enable better intervention to facilitate/prohibit the spread of (mis)information. To identify critical links, we propose link-removal strategies based on transport efficiency between two end nodes of each link, properties of links in the aggregated network, and in routing paths respectively. Each strategy ranks links according to their corresponding properties and removes links with the highest measures. Strategies are evaluated via the relative change in transport efficiency after link removal in real-world networks. We find that the path-based strategy performs the best: links appear more often and occur early in the fastest time-respecting paths tend to be critical. Via comprehensive analysis, we explain this strategy's out-performance and its dependency on network properties.**

***Index Terms*—Vulnerability, information transport, temporal network, link removal.**

I. Introduction

Complex systems can be represented as networks, where nodes represent the components of a system and links denote the interaction or relation between the components [1]. The interactions are, in many cases, not continuously active. For example, two autos or individuals in a communications network (such as an opportunistic mobile network, vehicle network, and social contact network) are connected (or have a contact) at a discrete time step if they are close to each other at that time step. Temporal networks [2], [3], [4], [5] could represent these systems more realistically with time-varying network topology. These communications networks facilitate the propagation of information where a piece of information is transmitted from one node (auto or individual) to another node through their contacts [6], [7]. In this work, we are going to explore the vulnerability of information transport on a temporal network to link removal. The objective is to understand the removal of which types of links deteriorate the efficiency of information diffusion the most. Identifying such critical links via their properties e.g., in the network will also allow us to better protect the network. Here, links are node pairs that have at least one contact in a temporal network. When a link is removed, all contacts along the link, i.e., between its two end nodes are removed.

Progress has been made in understanding the vulnerability of a temporal network, i.e., how the efficiency of information transport is influenced when the underlying network is subject to node removal. Scellato et al. [8], [9] and Lu et al. [10] investigated networks with which properties, are more robust against random node removal. The vulnerability of temporal networks subject to intentional attacks has also been investigated [11], [12], [13], [14]. Intentional attacks remove nodes based on nodal properties like nodal degree. Trajanovski et al. [11] explored the vulnerability of both real-world networks and networks generated by synthetic models, such as Erdős-Rényi and Markov temporal network models. The Erdős-Rényi temporal model, for example, generates a sequence of Erdős-Rényi static random graphs as a temporal network. They found in real-world networks, where nodes are heterogeneous in their properties (like node degree), information transport is affected more significantly by intentional attacks than by random node removal. In synthetic networks, where nodes are homogeneous in their network properties, random node removal, and intentional attacks affect similarly the efficiency of information transport on temporal networks. The change of network properties, such as the average degree [12] and the network reachability (the average number of nodes can be reached through time-respecting paths starting from any node at any time) [13], after node removal have also been studied.

Existing works have only considered basic attack methods that remove nodes based on simple nodal centrality metrics. However, how to identify the set of nodes whose removal decreases transport efficiency the most remains an open question. Beyond, the vulnerability of information transport on temporal networks to the removal of links, the fundamental building block of temporal networks, has not been explored either. Hence, in this work, we aim to understand the removal of which types

Received 21 December 2023; revised 24 March 2025; accepted 24 March 2025. Date of publication 27 March 2025; date of current version 27 June 2025. This work was supported in part by the Netherlands Organisation for Scientific Research NWO (TOP under Grant 612.001.802, Project FORT-PORT under Project KICH1.VE03.21.008) and in part by NExTWORKx, a collaboration between TU Delft and KPN on future telecommunication networks and the China Scholarship Council. Recommended for acceptance by Dr. Dusit Niyato. *(Corresponding authors: Li Zou; Huijuan Wang.)*

The authors are with the Faculty of Electrical Engineering, Mathematics and Computer Science, Delft University of Technology, 2628 CD Delft, The Netherlands (e-mail: L.Zou@tudelft.nl; H.Wang@tudelft.nl).

Digital Object Identifier 10.1109/TNSE.2025.3555409

of links deteriorate the efficiency of information diffusion the most.

Information transport demand (i, j, t_0) from any source node i to any target node j starting at any time t_0 is assumed to be routed along the fastest time-respecting path $p(i, j, t_0)$ on the underlying temporal network G. Time-respecting paths are sequences of links that can be traversed in a temporal network under the constraint that the next link to be traversed is activated (connected) at some point after the current one [15]. The duration of a time-respecting path is the length of time it takes to traverse that path. Specifically, a time-respecting path from node i starting at time t_0 to node j in a temporal network is a succession of contacts $(i, n_1, t_1), (n_1, n_2, t_2), \ldots, (n_k, j, t_k)$ that follow the time order $t_0 < t_1 < t_2 < \cdots < t_k$, and its duration is $t_k - t_0$. The fastest time-respecting path $p(i, j, t_0)$ for a traffic demand (i, j, t_0) is the time-respecting path with the minimum duration. Its duration $\tau(i, j, t_0)$ is called the temporal distance from the source node i to the target node j starting at t_0. The transport efficiency $\sigma(i, j, t_0)$ of traffic demand (i, j, t_0) is defined as the reciprocal temporal distance, i.e., $\sigma(i, j, t_0) = \frac{1}{\tau(i,j,t_0)}$. It measures how fast information can be transported from source node i to target node j starting at t_0. The efficiency of information transport on a temporal network G is defined as $\sigma(G) = \frac{1}{N(N-1)T} \sum_{i,j,t_0} \frac{1}{\tau(i,j,t_0)}$, the average transport efficiency over all possible traffic demands, where N is the number of nodes and T is the duration of the observation window $[1, T]$ of the temporal network [8]. Note that if the information cannot reach node j from node i starting at t_0, $\tau(i, j, t_0)$ is set as infinity. Similarly, the transport efficiency between a node pair is the average efficiency for all traffic demands between the node pair $\sigma(i, j) = \frac{1}{2T} \sum_{t_0} (\frac{1}{\tau(i,j,t_0)} + \frac{1}{\tau(j,i,t_0)})$.

In order to identify the links whose removal decreases the transport efficiency of the network the most, we propose link-removal strategies based on transport efficiency between two end nodes of each link, and based on the properties of each link in the time-aggregated network and in the fastest time-respecting paths, respectively. The rationale for using these three types of properties to design our link-removal strategies is as follows: (a) The efficiency of information transport on a temporal network is the average transport efficiency over all node pairs. Hence, removing links whose end nodes have the highest transport efficiency is likely to reduce network performance. (b) Link properties in the aggregated network have been proven effective in a variety of tasks, such as preventing information spread from a single seed node and minimizing the average prevalence of diffusion across the network [16]. (c) Links that do not contribute to information propagation—i.e., those not appearing in any fastest time-respecting path—are likely less critical.

Each strategy ranks links according to a property of links and then removes a given number of links with the highest rankings. The performance of each strategy is evaluated via the relative change in transport efficiency of the network after link removal. Our work reveals that our proposed strategies are more effective in identifying critical links for information transport than random link removal in seven physical contact networks and six virtual networks. Moreover, one strategy based on time-respecting paths tends to be the most effective. We observe that removing links, that appear frequently and are active earlier in the fastest time-respecting paths, deteriorates the efficiency of information transport the most.

The rest of the paper is organized as follows. We will introduce the representation of temporal networks in Section II. Real-world temporal networks to be used to evaluate link-removal strategies are introduced in Section III. Their basic network properties are analyzed, which also inspires the design of link-removal methods. Key link-removal strategies are proposed in Section IV. Finally, our proposed link-removal strategies will be evaluated and interpreted in Section V.

II. Temporal Network

A temporal network measured at discrete times can be represented as a sequence of network snapshots $G = \{G_1, G_2, \ldots, G_T\}$, where T is the duration of the observation window $[1, T]$ and $G_t = (V; E_t)$ is the snapshot at time step t with V and E_t being the set of nodes and contacts, respectively [17]. If two nodes, j and k, have a contact at time step t, $(j, k) \in E_t$. Here, we assume all snapshots share the same set of nodes V and have the same duration.

The corresponding time aggregated network G^w contains the same set of nodes V and the set of links $E = \cup_{t=1}^{T} E_t$. That is, a pair of nodes is connected with a link in the aggregated network if at least one contact occurs between them in the temporal network. The weight of each link in the aggregated network is the total number of contacts occurring along the link within $[1, T]$. The total number of nodes and links are $N = |V|$ and $M = |E|$, respectively. In this paper, links refer to links in the aggregated network.

III. Empirical Data Sets

To evaluate the strategies that identify critical links for the transport of information on temporal networks, we consider 7 empirical physical contact networks measured at various contexts (Hospital [18], Workplace [19], PrimarySchool [20], HighSchool [21], LH10 [19], SFHH [22] and Hypertext2009 [23]) and 6 virtual contact networks (SMS [24], Call [24], Eu1 [25], Rad [26], DNC [27] and CollegeMsg [28]). All these networks can be collected on the SociaPatterns website.[1] To make the simulation more computationally feasible, we have selected a sample of the CollegeMsg network. Specifically, we focus on the contact sequence from the first 30 days of the network, while the original network spans a duration of 193 days. The basic properties of these data sets are given in Table I. The time steps at which there is no contact in the whole network have been deleted in order to consider the steps that are relevant for the transport of information and to avoid the periods that have no contact due to technical errors in measurements. In addition, we consider only the nodes that belong to the largest connected component in the aggregated network and the contacts among these nodes.

[1] SociaPatterns website: http://www.sociopatterns.org/datasets/

TABLE I
BASIC PROPERTIES OF EACH DATA SET: THE NUMBER OF NODES ($N = |V|$), THE NUMBER OF LINKS (M) IN THE GIANT COMPONENT OF THE AGGREGATED NETWORK, THE LENGTH OF THE OBSERVATION TIME WINDOW (T), THE TIME DURATION OF EACH SNAPSHOT (δ SEC), THE TYPE OF CONTACTS AND THE LOCATION WHERE THE DATA IS COLLECTED

Network	N	M	T	δ	Type	Location
Hospital	75	1139	9453	20	Physical	hospital
Hypertext2009	113	2196	5246	20	Physical	conference
Workplace	92	755	7104	20	Physical	office
LH10	73	1381	12605	20	Physical	hospital
HighSchool	327	5818	7375	20	Physical	school
PrimarySchool	242	8317	3100	20	Physical	school
SFHH	403	9565	3509	20	Physical	conference
Eu1	101	745	10354	1	Virtual	email
Rad	167	3250	57791	1	Virtual	email
DNC	1833	4366	18190	1	Virtual	email
SMS	457	628	21898	1	Virtual	phone
Call	347	477	2671	1	Virtual	phone
CollegeMsg	1089	5908	22101	1	Virtual	phone

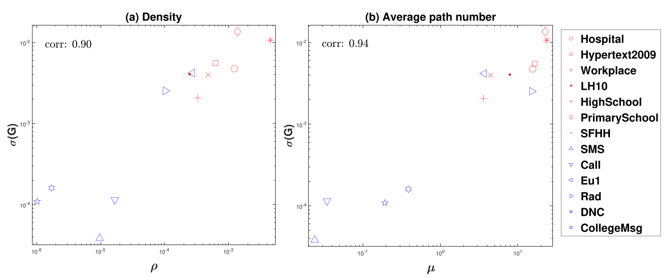

Fig. 1. The relation between the efficiency $\sigma(G)$ of information transport and (a) the density ρ of the temporal network and (b) the average number of paths μ between a node pair in the aggregated network, respectively. The results of physical and virtual networks are marked in red and blue, respectively. The Spearman correlation coefficient between the efficiency $\sigma(G)$ of information transport and the density ρ of the temporal network (the average number of paths μ between a node pair in the aggregated network) is also shown in the upper-left corner of the left (right) figure.

Fig. 1 illustrates the relation between a network property of a temporal network and the efficiency of transport on the temporal network. We consider two properties of a temporal network: density which is defined as the average number of contacts per link per time step and the average number of paths between a node pair in the aggregated network. Fig. 1 shows that the transport efficiency of a network is strongly and positively correlated with the density of the temporal network and the average number of paths of the network, respectively. This implies that the information tends to be transported faster in temporal networks that have a higher density, as well as in networks that have a larger number of paths between each node pair in their aggregated networks. Hence, the total number of contacts of a link and the number of paths that traverse a link will be used to design link-removal strategies in the following section.

IV. LINK-REMOVAL STRATEGIES

To understand the removal of which kind of links deteriorates the efficiency of information diffusion the most, in this section, we propose systematic link-removal strategies. These strategies are based on the transport efficiency between the two end nodes of each link and the properties of each link in the aggregated network and in the fastest time-respecting paths. Each strategy ranks links according to a property of links and then we remove a given number of links with the highest rankings.

A. Transport Efficiency Between the Two End Nodes of a Link

- **The transport efficiency (TE)** between the two end nodes of a link (i, j) has been defined as the average reciprocal temporal distance of traffic demands between node i and node j starting at all possible time, that is $\sigma(i, j) = \frac{1}{2T}\sum_{t_0}(\frac{1}{\tau(i,j,t_0)} + \frac{1}{\tau(j,i,t_0)})$. It measures how fast information can be transported between these two nodes. The transport efficiency of a temporal network is the average transport efficiency over all node pairs. Removing links with a larger contribution to the transport efficiency of a temporal network may reduce effectively the efficiency.

B. Aggregated Network-Based Properties

- **The number of contacts (NC)** of a link is the number of contacts occurring along the link within $[1, T]$. It is also the weight of the link in the aggregated network. In Fig. 1(a), we observed that temporal networks with more contacts per link per time step are more efficient for information transport. This motivates us to remove links with a large number of contacts to reduce the transport efficiency of a network.
- **Resistance Distance (RD)** of a link (i, j) is defined as the effective resistance $r(i, j)$ between node i and node j in the aggregated network G^w times the weight $w(i, j)$ of the link in G^w. The effective resistance $r(i, j)$ in G^w is the effect resistance between i and j in the electrical network that is constructed from G^w by replacing each link with a resistor whose resistance is $\frac{1}{w(i,j)}$ [29], [30]. Mathematically, the effective resistance $r(i, j)$ of link (i, j) can be calculated as:

$$r(i, j) = (\mathbf{e_i} - \mathbf{e_j})^T L^+(\mathbf{e_i} - \mathbf{e_j}). \tag{1}$$

where $\mathbf{e_i}$ is the column vector with all elements being 0 except that the i^{th} element is 1 and L^+ is the pseudoinverse of the Laplacian matrix L of the aggregated network G^w. The Laplacian matrix L of G^w is an $N \times N$ matrix defined by

$$L(i, j) = \begin{cases} \sum_{k=1}^{N} w(i, k), & \text{if } i = j; \\ -w(i, j), & \text{otherwise.} \end{cases} \tag{2}$$

The effective resistance $r(i, j)$ between i and j tends to be higher if there are fewer paths, and these paths tend to be longer between node i and node j in the aggregated network G^w. If node i and node j have many contacts and also few alternate paths in the aggregated network, their resistance distance tends to be high. In this case, the direct link between i and j is likely the only path between this node pair in the aggregated network. Removing such links with high resistance distances might significantly slow down the transport of information between these two nodes. Hence, we consider resistance distance as a link-removing strategy.

- **Betweenness (BET)** of a link counts the number of shortest paths between all node pairs that traverse the link in the aggregated network [31]. To compute the shortest path of each node pair, the distance of each link is defined as $\frac{1}{w(i,j)}$, i.e., inversely proportional to its weight of the link in the aggregated network. The Betweenness strategy has been found effective in prohibiting the spreading of information starting from one seed node to minimize the average prevalence [16]. This motivates us to explore its performance in deteriorating the efficiency of information transport.

C. Path-Based Properties

We design link-removal strategies based on the properties of links in the fastest time-respecting paths, since links that do not contribute to the propagation of information, thus not appear in any path are likely less relevant to identify the critical link for information transport. Path-based or spreading trajectory-based properties are a special type of centrality metrics and have rarely been explored. Mainly the betweenness of a link in static networks has been studied. Recently, we have shown the effectiveness of using the frequency of a link appearing in the spreading trajectories/trees of an epidemic starting from each seed node respectively to identify the links to block in order to suppress the spreading [16], [32]. Hence, we design two path-based properties as a start and explore their performance in identifying the critical links for information transport.

- **The number of paths (NP)** of a link counts the number of fastest time-respecting paths between all node pairs starting from all possible time that traverse the link. Removing links that participate more in routing paths affects the information transport of more traffic demands and therefore may effectively reduce the efficiency of information transport $\sigma(G)$ in the network.
- **Generalized Number of Paths (GNP)**. The number of paths only counts how many times a link appears in the fastest time-respecting paths, and ignores the temporal information of the link, i.e., when the link is active (appearing) in each of the fastest time-respecting paths. Hence, we propose to consider the generalized number of paths of a link. The generalized number of paths for any link l is defined as the sum of the reciprocal relative active time of the link in each fastest time-respecting path. The relative active time $t_l(p(i,j,t_0))$ of the link l in the fastest time-respecting path $p(i,j,t_0)$ is the time when the link is active in the path minus the starting time t_0 of the path (traffic demand). If the path does not traverse link l, $t_l(p(i,j,t_0))$ is infinity. Thus, the generalized number of paths of a link l is $g(l) = \sum_{i,j,t_0} \frac{1}{t_l(p(i,j,t_0))}$. Removing links that participate early in many routing paths may increase the temporal distance much for many traffic demands and thus may reduce effectively the efficiency of information transport $\sigma(G)$ on the network.

Each strategy ranks links according to a link property and then removes a given number of links with the highest ranking. If two properties lead to the same ranking of links, the same set of links will be removed and their corresponding link-removal strategies perform the same in deteriorating the efficiency of information transport. We investigate the Spearman rank correlation between two link properties associated with two link-removal strategies respectively. This correlation provides insight into how similar or dissimilar the rankings assigned by different strategies are. In Fig. 2, we observe a positive correlation between the rankings assigned by most pairs of link-removal strategies. In particular, the link ranking assigned by the NC strategy exhibits a strong correlation with those of other strategies. On the other hand, the link rankings of the BET strategy are less correlated with other strategies. This correlation analysis will be used to explain the different performance of link-removal strategies in deteriorating the efficiency of information transport in the next section.

V. Results

In this section, we will evaluate and interpret the performance of our proposed link-removal strategies.

A. Evaluation

Given a temporal network G, each link-removal strategy ranks the links in the aggregated network G^w according to a property of each link. A fraction f, where $0 \le f \le 1$, of links with the highest ranks are selected/removed. After removing all the contacts associated with the selected links, the resultant temporal network is denoted as G_f. The vulnerability of information transport on the temporal network G to this removal of links, or equivalently the effectiveness of the strategy in deteriorating information transport, is evaluated by the relative change of transport efficiency $R_D(f) = \frac{\sigma(G_f)}{\sigma(G)}$ and $0 \le R_D(f) \le 1$. A more effective link-removal strategy will lead to a smaller $R_D(f)$.

Fig. 3 shows the relative change of transport efficiency $R_D(f)$ as a function of the fraction f of removed links in 13 real-world temporal networks. We find all our proposed strategies can reduce transport efficiency more significantly than random link removal in all networks. Moreover, for most link-removal strategies, removing just e.g., 10% of the links can result in a drop of over 50% in transport efficiency. This implies that the removal of a small percentage of links can lead to a large impact on the transport efficiency of temporal networks. This motivates us to find such critical links.

We also observe two link-removal strategies that rank links similarly (dissimilarly) (see Fig. 2) exhibit a small (large) discrepancy in their $R_D(f)$ curve in Fig. 3. For instance, the $R_D(f)$ curve corresponding to strategy BET is significantly higher than the other strategies at each f, whereas the correlation between BET and any other strategy in ranking links is relatively low.

We further calculate the area under each $R_D(f)$ curve, denoted as S, for each link-removal strategy in each temporal network. This variable implies the average effect of a link-removal method over all possible fraction f of links removed. The result is presented in Table II. A smaller S of a link-removal

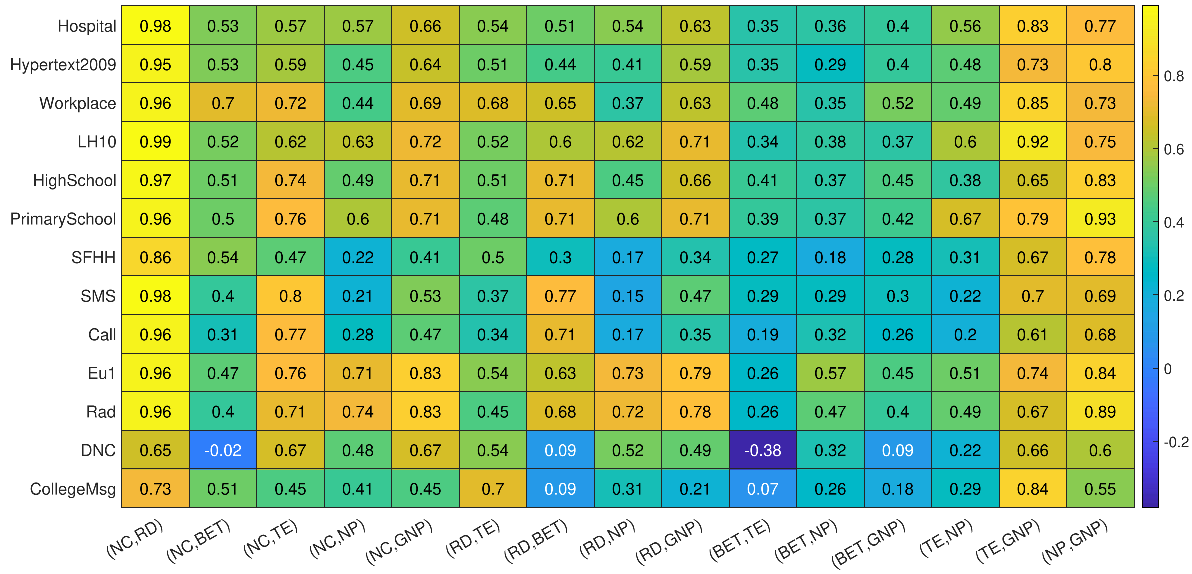

Fig. 2. Spearman rank correlation of two link properties corresponding to two link-removal strategies respectively in 13 temporal networks.

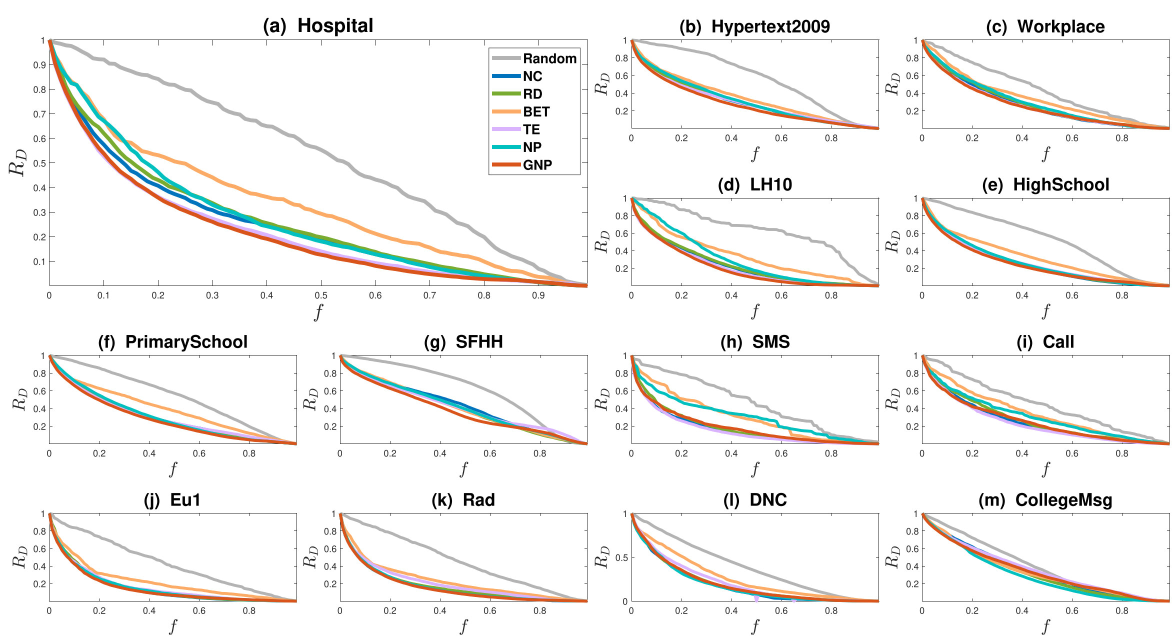

Fig. 3. The relative change of transport efficiency $R_D(f)$ as a function of the fraction f of links removed according to random removal and 6 link-removal strategies (NC, RD, BET, TE, NP, and GNP) respectively in each temporal network. The results of random removal are averaged over 1000 simulations for each temporal network.

strategy suggests that the strategy better identifies critical links for information transport. GNP strategy performs the best in most networks.

Fig. 1 shows that our networks are divided into two categories. The first category of networks (virtual network SMS, Call, DNC, and CollegeMsg) have evidently lower transport efficiency, lower network density, and a smaller number of paths between a node pair in aggregated networks than the second category (all the other networks: all physical contact networks and two virtual networks). We find that our GNP strategy works the best in networks of the second category. In the four networks of the first category, the performance of GNP is close to the best, whereas TE is the most effective in SMS and Call, and NP performs the best in DNC and CollegeMsg. In practice, usually, a small percentage of links are removed or attacked. The same has been observed when link-removal strategies are evaluated by the area under the $R_D(f)$ curve where $f \in [0, 0.5]$, i.e., the average effectiveness of a strategy when a percentage of links, between 0 and 50% are removed.

In general, strategies TE, NP, and GNP tend to outperform the aggregated network-based strategies NC, RD, and BET. The strategies TE, NP, and GNP select links to remove based on

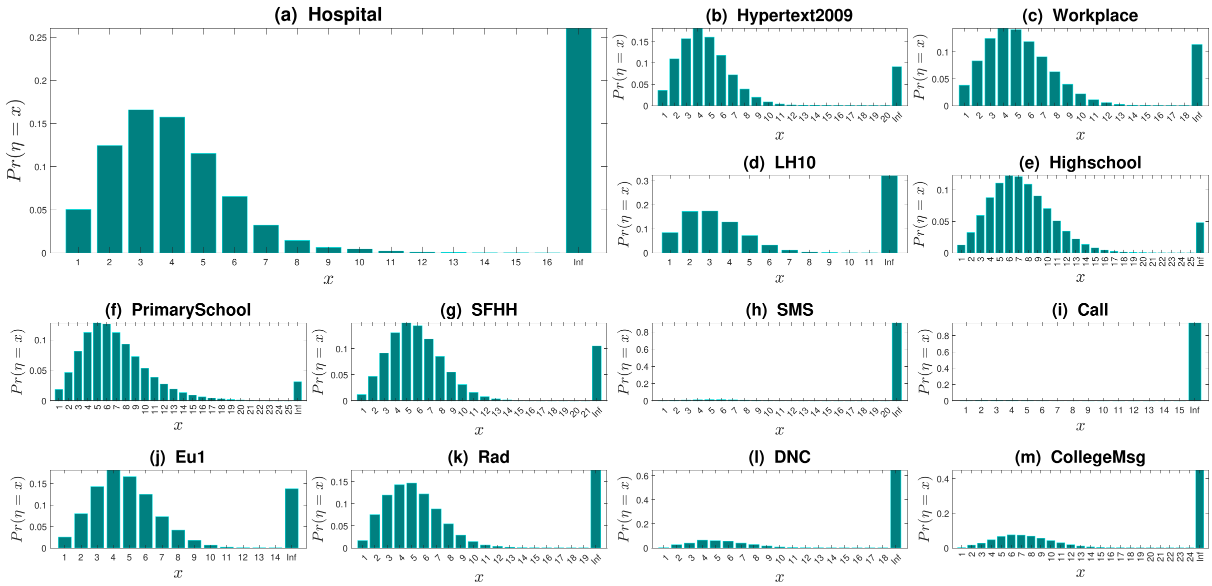

Fig. 4. The distribution of the hopcount η of the fastest time-respecting path to transport a random traffic demand from a random source node to a random target node starting at a random time on each temporal network.

TABLE II
THE AVERAGE EFFECTIVENESS S OF EACH LINK-REMOVAL STRATEGY. FOR EACH TEMPORAL NETWORK, THE MOST EFFECTIVE LINK-REMOVAL STRATEGY IS HIGHLIGHTED IN BLUE COLOR

Data	Ran.	NC	RD	BET	TE	NP	GNP
Hospital	52.63	24.74	26.05	33.10	21.84	26.58	21.27
Hypertext2009	56.88	29.90	30.29	34.47	29.78	31.06	27.28
Workplace	43.78	27.18	28.11	33.95	26.08	29.42	25.89
LH10	62.75	23.06	23.69	33.99	20.53	29.36	19.76
HighSchool	52.43	24.51	25.13	31.98	25.35	25.76	23.76
PrimarySchool	53.87	30.42	30.15	38.90	30.21	30.54	27.96
SFHH	60.93	42.22	39.99	40.90	41.51	41.45	37.33
SMS	46.17	16.28	18.21	30.26	15.39	30.69	18.58
Call	44.55	24.79	26.98	34.14	23.01	30.88	25.25
Eu1	42.99	14.77	15.68	22.44	16.57	15.90	14.72
Rad	45.72	16.52	17.61	23.12	20.13	16.40	15.84
DNC	38.92	18.44	19.34	26.63	21.13	18.31	19.44
CollegeMsg	38.31	33.71	31.49	32.98	35.43	28.70	33.43

links' properties related to the fastest time-respecting paths. In contrast, strategies NC, RD, and BET choose links to remove based on their properties in the aggregated network without the temporal information of the network. This helps explain why aggregated network-based strategies NC, RD, and BET perform worse than TE, NP, and GNP. However, computing TE, NP, and GNP requires finding the fastest time-respecting paths, a process that is highly time-consuming. The computation of a fastest time-respecting path can be performed in $O(\Xi)$ time [33], where $\Xi = \sum_{t=1}^{T} |E_t|$ is the total number of contacts in the temporal network. Time-respecting path-based strategies TE, NP, and GNP have the same computational complexity $O(\Xi \cdot T \cdot N^2)$ and GNP outperforms the best among them. Among all aggregated network-based strategies, NC performs the best while has the lowest computational complexity $O(\Xi)$.

B. Explanation

Here, we perform further analysis to explain: (1) why the transport efficiency $\sigma(G)$ in networks of the first category is lower than that of the second category of networks; (2) why GNP works better than NP in most networks; and (3) why TE works the best in SMS and Call in identifying critical links for information transport.

1) Transport Efficiency in Relation to Network Properties: As traffic demand from any source node to any target node at any time is routed along the fastest time-respecting path, we will begin with examining the properties of these paths, specifically the distribution of their hopcounts. The hopcount η of a path is the number of links contained in the path.

The distribution of the hopcount η of the fastest time-respecting path to transport a random traffic demand from a random source node to a random target node starting at a random time on each temporal network is given in Fig. 4. This distribution is derived from the hopcounts of all fastest time-respecting paths to transport all traffic demands, between all possible node pairs starting at each possible time step. If there is not any feasible fastest time-respecting path for a given traffic demand, the corresponding hopcount is defined as infinity. The networks (virtual network SMS, Call, DNC, and CollegeMsg) in the first category have an evidently higher probability for the hopcount to be infinity than those in the second category, in line with their lower network density compared with networks of the second category. In other words, a higher percentage of traffic demands have no time-respecting paths in the first category of networks than that in the second category. This explains why the transport efficiency $\sigma(G)$ in networks of the first category is relatively lower.

2) Why GNP Works Better Than NP: Our GNP strategy performs better than NP in all networks except DNC and CollegeMsg. Here, we explain why GNP works better than NP.

The removal of any link l in the fastest time-respective path $p(i, j, t_0)$ that routes the traffic demand (i, j, t_0) will lead to a larger temporal distance $\tau(i, j, t_0)$ of the traffic demand. If

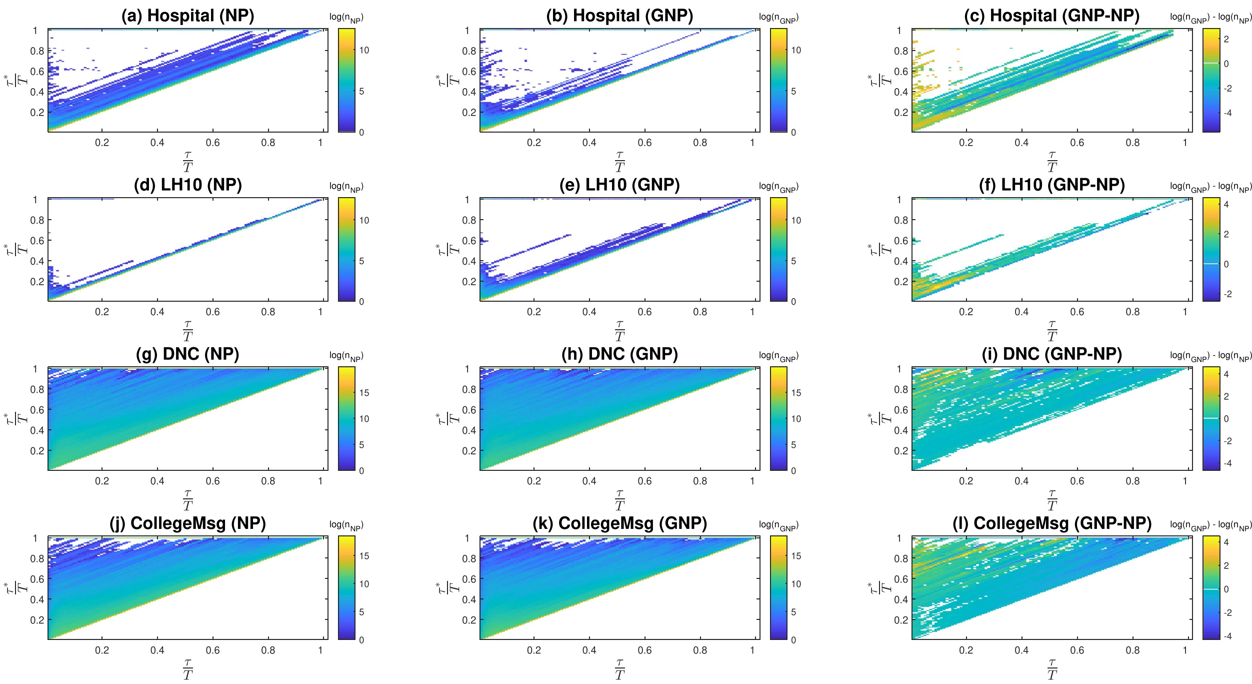

Fig. 5. Normalized temporal distance of traffic demands before (τ/T) and after (τ^*/T) link removal in Hospital, LH10, DNC, and CollegeMsg. (a) and (b) illustrates the number of traffic demands in logarithm that have a temporal distance τ in the original network Hospital and temporal distance τ^* after link removal according to NP and GNP strategy respectively. (c) shows the difference between (a) and (b). Same for other rows for other networks. A fraction $f = 10\%$ links are removed.

the relative time $t_l(p(i,j,t_0))$ that link l is active in the fastest time-respective path $p(i,j,t_0)$ is small, the temporal distance $\tau(i,j,t_0)$ tends to small, because $\tau(i,j,t_0) > t_l(p(i,j,t_0))$. The removal of such a link l with a small $t_l(p(i,j,t_0))$, tends to lead to a far smaller transport efficiency $\frac{1}{\tau^*(i,j,t_0)}$ than the efficiency $\frac{1}{\tau(i,j,t_0)}$ before link removal. Hence, removing links that appear in many fastest paths and appear relatively early in each path tends to effectively reduce the transport efficiency of the given network. This explanation is further supported by the following analysis.

We compare temporal distance of traffic demands before and after link removal, using Hospital, LH10, DNC, and CollegeMsg as examples. The objective is to understand the temporal distance of which traffic demands are more likely to be influenced by link removal. Fig. 5(a) and (b) illustrates the number of traffic demands in logarithm that have a temporal distance τ in the original network Hospital and temporal distance τ^* after link removal according to NP and GNP strategy respectively. We find that strategy GNP tends to affect more traffic demands that have a low temporal distance before link removal, than NP. This comparison is clearer in Fig. 5(c) which shows the difference between Fig. 5(a) and (b). We observe that GNP (NP) affects more traffic demands with a small (medium and large) temporal distance τ in the original network to have a large temporal distance τ^* after link removal. This is reflected in the positive values on the left top corner in Fig. 5(c), as well as (f), but less evidently in (i) and (l). As a consequence, GNP effectively reduces the transport efficiency of traffic demands with a large transport efficiency in the original network. This is supported by the probability density function of the transport efficiency σ of traffic demands from any source node to any target node starting at any time in Fig. 6 before and after link removal (a). After removing links according to GNP (NP), the probability of having a large transport efficiency is evidently (slightly) lower than that in the original network. The same has been observed in most networks (not evident in DNC and CollegeMsg).

From Fig. 7, we can see that the percentage of traffic demands with a large efficiency is the smallest in DNC and CollegeMsg among all networks. In other words, transport efficiency of traffic demands varies less in DNC and CollegeMsg. The trend that, if the relative time $t_l(p(i,j,t_0))$ that link l is active in the fastest time-respective path $p(i,j,t_0)$ is small, the temporal distance $\tau(i,j,t_0)$ tends to small, is thus not evident in DNC and CollegeMsg. This might limit the effectiveness of GNP. On the other hand, DNC and CollegeMsg have the lowest density among all networks. Removing links that appear in many fastest paths could be possibly the most effective in reducing network efficiency. These two perspectives might explain why NP performs the best, slightly better than GNP in DNC and CollegeMsg.

3) Why TE Works the Best in SMS and Call: The removal of a link could decrease the transport efficiency of traffic demands between its two end nodes and also other node pairs if this link appears in the fastest time-respective paths of these node pairs. SMS and Call differ from all the other networks in the sense that the probability that there is not any feasible time-respective path for a traffic demand is the highest in these two networks, as shown in Fig. 4. Hence, once a link is removed from SMS or

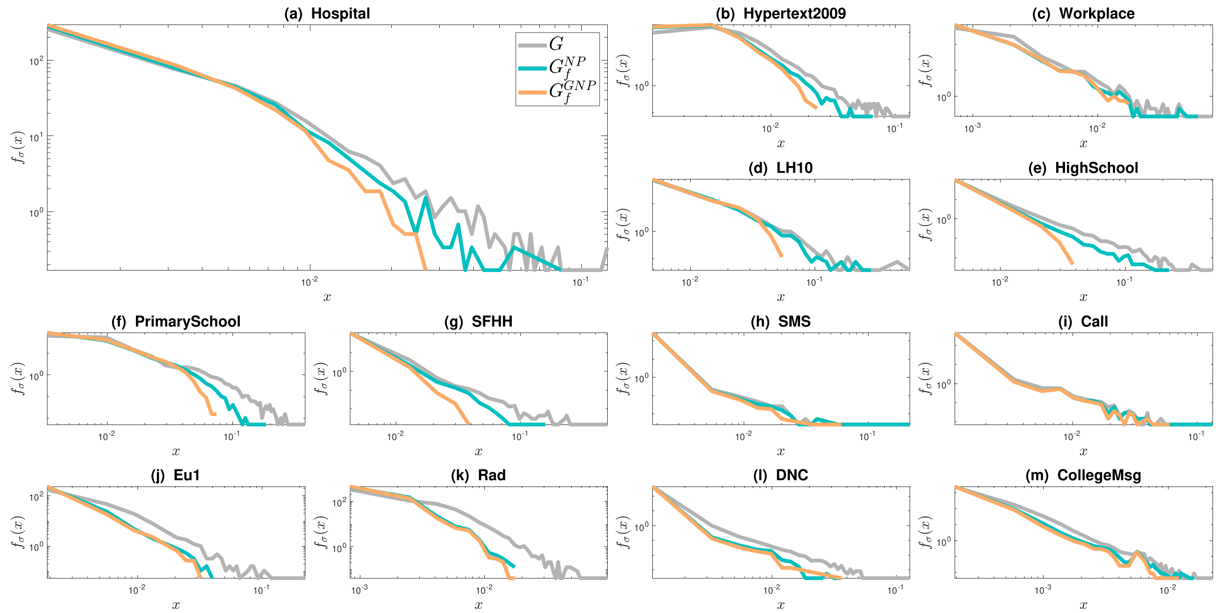

Fig. 6. The probability density function of transport efficiency σ of traffic demands from any source node to any target node starting at any time. After removing all the contacts associated with the selected $10\%M$ links based on NP (GNP) strategy, the resultant temporal network is denoted as G_f^{NP} (G_f^{GNP}). The probability density function in the networks G, G_f^{NP}, and G_f^{GNP} are denoted by grey, blue, and orange, respectively.

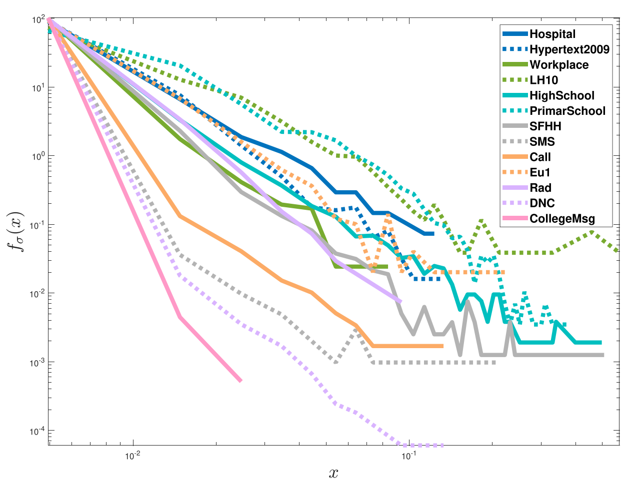

Fig. 7. The probability density function of transport efficiency σ of traffic demands from any source node to any target node starting at any time before link removal in each temporal network.

Call, the chance is high that there is no time-respective path for any traffic demand between its end nodes starting at any time. This is supported by Fig. 8, where we examine the transport efficiency of each node pair before and after removing $10\%M$ links based on GNP strategy. The node pairs are grouped into Type 1 (including the pair of end nodes of each removed link) and Type 2 node pairs (all the other node pairs). In SMS and Call, the efficiency of Type 1 node pairs is almost reduced to 0 after link removal. This confirms that after link removal there is hardly any feasible time-respective path between the end nodes of each removed link. In contrast, the transport efficiency of Type 2 node pairs hardly decreases in SMS and Call. This is because the removed links seldom appear in the fastest time-respective path of a traffic demand between a type 2 node pair, supported by the low probability for the hopcount of a random traffic demand to be larger than one but not infinity (see Fig. 4). Even if the removed link appears in the path between a type 2 node pair, the influence of the link removal on the efficiency of a type 2 node pair is small. This is because the efficiency of any node pair depends largely on the efficiency of the traffic demands between the node pair that have hopcount 1, as shown in Fig. 9(h) and (i).

In general, removing links in SMS and Call only reduces the efficiency of type 1 node pairs to zero without affecting evidently the efficiency of type 2 node pairs. This has been observed when different percentages of links are removed according to diverse link-removal methods. Hence, removing links whose end nodes have the the highest transport efficiency TE is the most effective in reducing the transport efficiency of the network in SMS and Call.

VI. Conclusion

In this work, we investigate the vulnerability of information transport on temporal networks to link removal. This aims to understand the removal of which types of links deteriorate the efficiency of information transport the most. Link-removal strategies have been proposed systematically, based on the transport efficiency between the two end nodes of each link, properties of each link in the aggregated network, and in the fastest time-respecting paths.

We evaluate the effectiveness of these link-removal strategies in reducing the efficiency of information transport in seven physical contact networks and six virtual networks. Interestingly, we find that the strategy based on the property of each link in the fastest time-respecting paths performs the best in most networks. Links that appear in more paths and occur earlier in time in each path tends to be critical for information transport. We explain

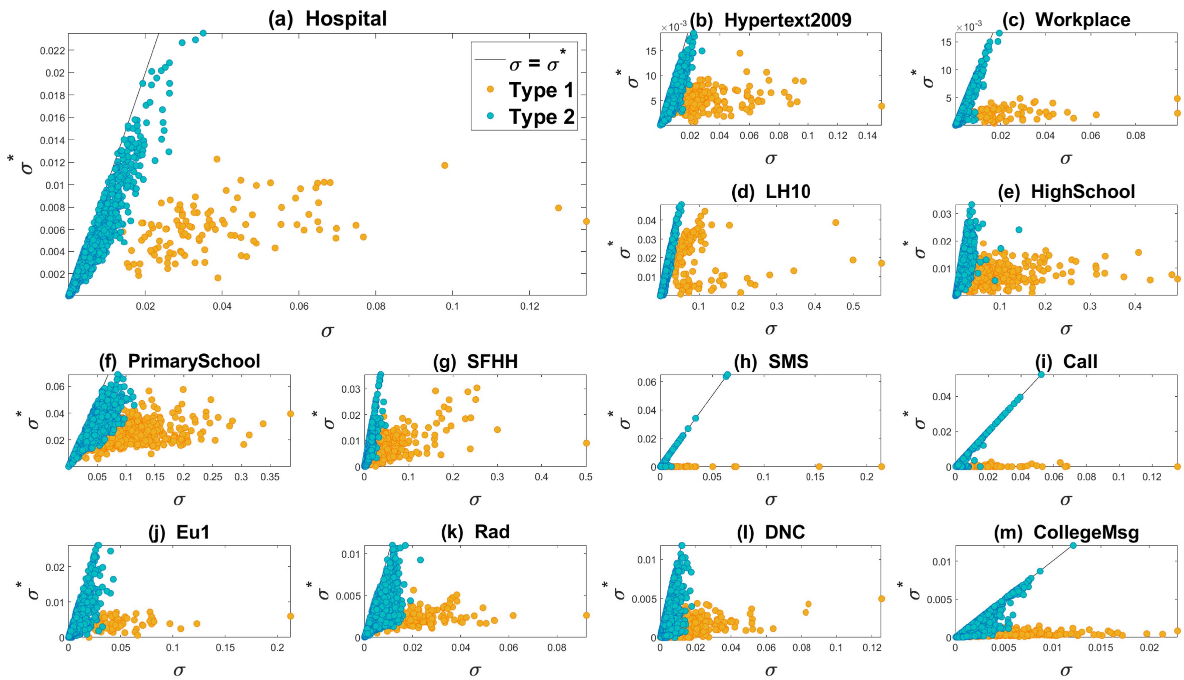

Fig. 8. The transport efficiency of each traffic demand before (σ) and after (σ^*) $10\%M$ links are removed based on GNP strategy in 13 temporal networks. Each point in each subgraph corresponds to a traffic demand. Traffic demands between two end nodes of a removed link are marked in orange otherwise in blue. The black dashed line represents $\sigma = \sigma^*$.

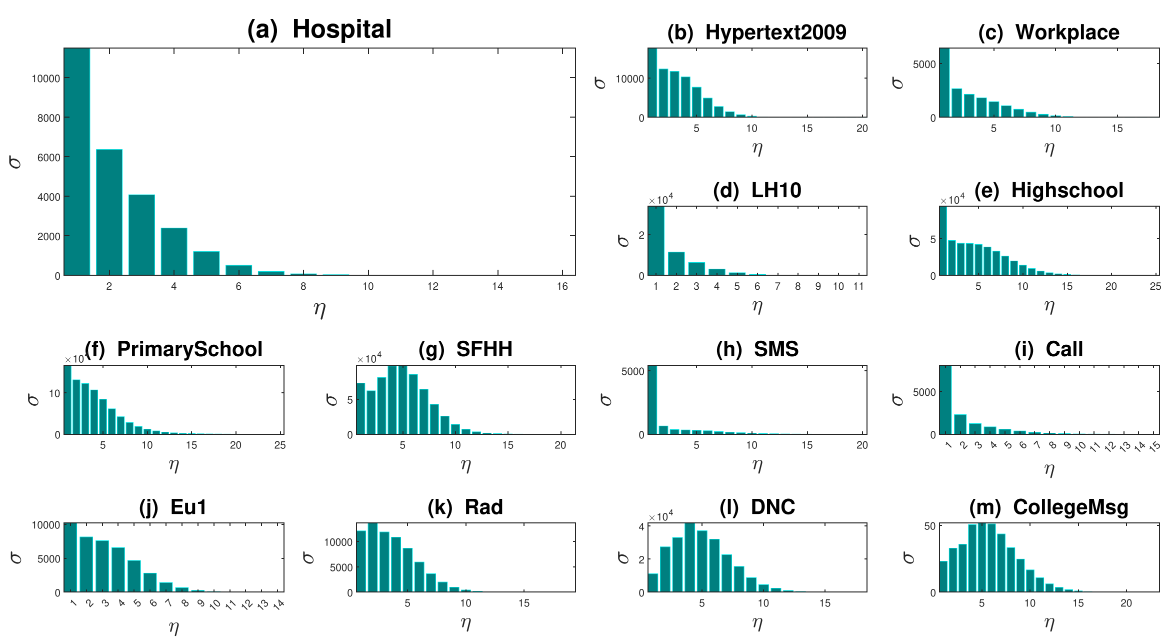

Fig. 9. The total transport efficiency $\sigma(\eta)$ of traffic demands that have hopcount η, as a function of η in each temporal network.

why and in which kind of networks this strategy performs the best via comprehensive analysis, such as the temporal network properties, and the influence of link removal on the transport efficiency of different types of traffic demands and of different kinds of node pairs.

Our work is a starting point to explore the vulnerability of dynamic processes on temporal networks to link removal. Properties of links in spreading paths can be further designed, e.g., by considering the properties of the paths that a link belongs to. Properties of links in spreading paths usually have a high computational complexity. It is interesting to explore their relation with other network properties of links, especially those with a low computational complexity. This may enable efficient identification of critical links.

ACKNOWLEDGMENT

The authors would also like to express their gratitude to Alberto Ceria for his discussion and to Mathieu Jung Muller for his helpful comments on the draft.

REFERENCES

[1] M. Brede, "Networks—An introduction. Mark E.J. Newman. (2010, Oxford University Press.)," *Artif. Life*, vol. 18, pp. 241–242, 2012.
[2] P. Holme and J. Saramäki, "Temporal networks," *Phys. Rep.*, vol. 519, no. 3, pp. 97–125, 2012.
[3] A. Ceria, S. Havlin, A. Hanjalic, and H. Wang, "Topological–temporal properties of evolving networks," *J. Complex Netw.*, vol. 10, no. 5, 2022, Art. no. cnac041.
[4] L. Zou and H. Wang, "Memory based temporal network prediction," in *Proc. Complex Netw. Appl. XI*, 2023, pp. 661–673.
[5] N. Masuda and R. Lambiotte, *A Guide to Temporal Networks (Series Complexity Science)*, vol 4. Singapore: World Scientific, 2016.
[6] M. Karsai, N. Perra, and A. Vespignani, "Time varying networks and the weakness of strong ties," *Sci. Reports*, vol. 4, 2014, Art. no. 4001.
[7] J.-P. Eckmann, E. Moses, and D. Sergi, "Entropy of dialogues creates coherent structures in e-mail traffic," in *Proc. Nat. Acad. Sci.*, vol. 101, no. 40, pp. 14333–14337, 2004.
[8] S. Scellato, I. Leontiadis, C. Mascolo, P. Basu, and M. Zafer, "Understanding robustness of mobile networks through temporal network measures," in *Proc. 2011 IEEE INFOCOM*, 2011, pp. 1–5.
[9] S. Scellato, I. Leontiadis, C. Mascolo, P. Basu, and M. Zafer, "Evaluating temporal robustness of mobile networks," *IEEE Trans. Mobile Comput.*, vol. 12, no. 1, pp. 105–117, Jan. 2013.
[10] X. Lu, H. Wang, and Y. Deng, "Evaluating the robustness of temporal networks considering spatiality of connections," *Chaos, Solitons Fractals*, vol. 78, pp. 176–184, 2015.
[11] S. Trajanovski, S. Scellato, and I. Leontiadis, "Error and attack vulnerability of temporal networks," *Phys. Rev. E*, vol. 85, Jun. 2012, Art. no. 066105. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevE.85.066105
[12] N. Wang, N. Wu, L.-l. Dong, H.-k. Yan, and D. Wu, "A study of the temporal robustness of the growing global container-shipping network," *Sci. Reports*, vol. 6, 2016, Art. no. 34217.
[13] H. H. Sano and L. Berton, "Topology and robustness analysis of temporal air transport network," in *Proc. 35th Annu. ACM Symp. Appl. Comput.*, 2020, pp. 1881–1884.
[14] J. W. Matthew and M. Mirco, "Spatio-temporal networks: Reachability, centrality and robustness," *R. Soc. Open Sci.*, vol. 3, 2016, Art. no. 160196.
[15] R. K. Pan and J. Saramäki, "Path lengths, correlations, and centrality in temporal networks," *Phys. Rev. E*, vol. 84, Jul. 2011, Art. no. 016105.
[16] X.-X. Zhan, A. Hanjalic, and H. Wang, "Suppressing information diffusion via link blocking in temporal networks," in *Proc. Complex Netw. Appl., VIII*, 2020, pp. 448–458.
[17] L. Zou, X.-X. Zhan, J. Sun, A. Hanjalic, and H. Wang, "Temporal network prediction and interpretation," *IEEE Trans. Netw. Sci. Eng.*, vol. 9, no. 3, pp. 1215–1224, May/Jun., 2022.
[18] P. Vanhems et al., "Estimating potential infection transmission routes in hospital wards using wearable proximity sensors," *PLoS One*, vol. 8, no. 9, pp. 1–9, 2013. [Online]. Available: https://doi.org/10.1371/journal.pone.0073970
[19] M. G'enois and A. Barrat, "Can co-location be used as a proxy for face-to-face contacts," *EPJ Data Sci.*, vol. 7, no. 1, pp. 1–18, May 2018. [Online]. Available: https://doi.org/10.1140/epjds/s13688-018-0140-1
[20] J. Stehlé et al., "High-resolution measurements of face-to-face contact patterns in a primary school," *PLoS One*, vol. 6, no. 8, 2011, Art. no. e23176. [Online]. Available: http://dx.doi.org/10.1371/journal.pone.0023176
[21] R. Mastrandrea, J. Fournet, and A. Barrat, "Contact patterns in a high school: A comparison between data collected using wearable sensors, contact diaries and friendship surveys," *PLoS One*, vol. 10, no. 9, pp. 1–26, 2015. [Online]. Available: https://doi.org/10.1371/journal.pone.0136497
[22] R. A. Rossi and N. K. Ahmed, "The network data repository with interactive graph analytics and visualization," in *Proc. AAAI*, 2015. [Online]. Available: https://networkrepository.com
[23] L. Isella, J. Stehlé, A. Barrat, C. Cattuto, J.-F. Pinton, and W. V. den Broeck, "What's in a crowd? analysis of face-to-face behavioral networks," *J. Theor. Biol.*, vol. 271, pp. 166–180, 2011.
[24] P. Sapiezynski, A. Stopczynski, D. D. Lassen, and S. Lehmann, "Interaction data from the copenhagen networks study," vol. 6, pp. 1–10, 2019.
[25] A. Paranjape, A. R. Benson, and J. Leskovec, "Motifs in temporal networks," in *Proc. 10th ACM Int. Conf. Web Search Data Mining*, 2017, pp. 601–610. [Online]. Available: https://doi.org/10.1145/3018661.3018731
[26] R. Michalski, S. Palus, and P. Kazienko, "Matching organizational structure and social network extracted from email communication," in *Proc. Lecture Notes Business Inform. Process.*, 2011, vol. 87, pp. 197–206.
[27] J. Kunegis, "KONECT: The Koblenz network collection," in *Proc. 22nd Int. Conf. World Wide Web*, 2013, pp. 1343–1350.
[28] P. Panzarasa, T. Opsahl, and K. Carley, "Patterns and dynamics of users' behavior and interaction: Network analysis of an online community," *JASIST*, vol. 60, pp. 911–932, 2009.
[29] A. Mercier, S. Scarpin, and C. Moore, "Effective resistance against pandemics: Mobility network sparsification for high-fidelity epidemic simulations," *PLoS Comput. Biol.*, vol. 18, 2022, Art. no. e1010650.
[30] D. A. Spielman and N. Srivastava, "Graph sparsification by effective resistances," *SIAM J. Comput.*, vol. 40, no. 6, pp. 1913–1926, 2011.
[31] H. Wang, J. M. Hernandez, and P. Van Mieghem, "Betweenness centrality in a weighted network," *Phys. Rev. E*, vol. 77, Apr. 2008, Art. no. 046105. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevE.77.046105
[32] X.-X. Zhan, A. Hanjalic, and H. Wang, "Information diffusion backbones in temporal networks," *Sci. Reports*, vol. 9, 2019, Art. no. 6798.
[33] P. Holme, "Network reachability of real-world contact sequences," *Phys. Rev. E*, vol. 71, 2005, Art. no. 046119.

Li Zou received the Ph.D. degree in computer science from the Delft University of Technology, Delft, The Netherlands, in 2024. She is currently a Postdoctoral Researcher with the Department of Cognitive Robotics, Delft University of Technology, specializing in explainable machine learning for haptics. She is deeply passionate about leveraging algorithms, modeling, and data to create practical solutions for real-world challenges.

Huijuan Wang (Member, IEEE) received the M.Sc. and Ph.D. degrees in electrical engineering from the Delft University of Technology, Delft, The Netherlands. She is currently an Associate Professor with the Multimedia Computing Group, Department of Intelligent Systems, Delft University of Technology. She has been a Visiting Scientist with Boston University, Stanford University, and Princeton University. Her research interests include prediction, modeling, and control of dynamic processes (such as information, disease, failure propagation and social or financial contagion) on interdependent and time-evolving networks, combining data and network science approaches.