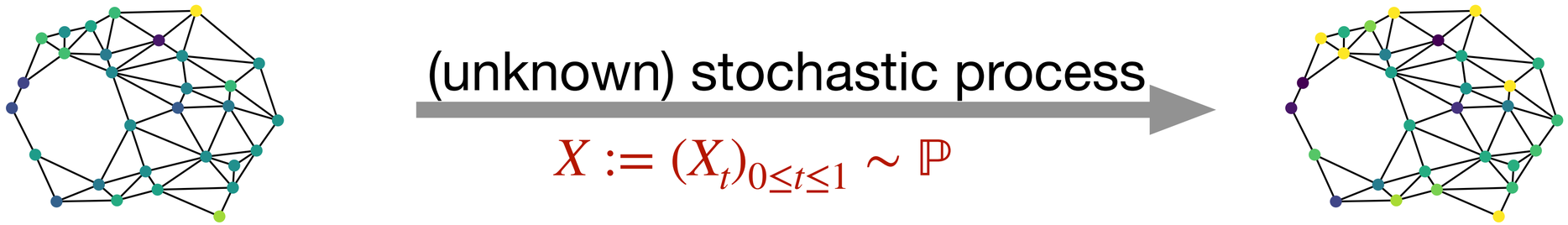
Topological signal distribution matching

- In a topological domain, e.g., a graph, signals on the node set.

- Given (empirical) signal distributions, $X_0 \sim \rho_0$ at $t = 0$ and $X_1 \sim \rho_1$ at $t = 1$



- Assume some *prior (reference)* process $Y \sim \mathbb{Q}_{\mathcal{T}}$ —— topology-aware

Topological Schrödinger Bridge Problem

$$\min D_{KL}(\mathbb{P} \| \mathbb{Q}_{\mathcal{T}}) \ s.t. \ \mathbb{P}_0 = \rho_0, \mathbb{P}_1 = \rho_1$$

Schrödinger's bridge problem

- Cloud of n independent Brownian particles
- Empirical distributions $\rho_0(x)$ and $\rho_1(y)$ at $t = 0$ and $t = 1$

$$\rho_0 \xrightarrow{\text{Wiener}} \rho_1$$

- Particles have been transported in an unlikely way
- Of the many possible (unlikely) ways, which one is the most likely? [Lénoard 2014]

$$\min D_{KL}(\mathbb{P}\|\mathbb{W}) \;\; s.t. \; \mathbb{P}_0 = \rho_0, \mathbb{P}_1 = \rho_1$$

- A dynamics formulation of entropic-regularized optimal transport [Vallani 2009]

$$\min_{\pi \in \Pi(\rho_0, \rho_1)} \int_{\mathbb{R}^n \times \mathbb{R}^n} \frac{1}{2} \|x_0 - x_1\|^2 \, d\pi(x_0, x_1) + \sigma^2 D_{KL}(\pi \| \rho_0 \otimes \rho_1)$$