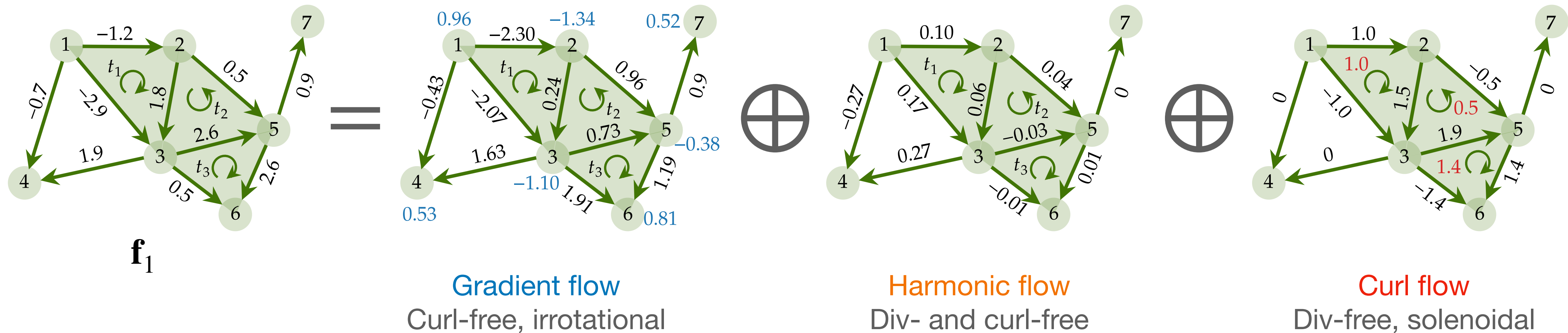


Hodge decomposition

Lovász et al. 2004; Lim et al. 2020

$$\mathbb{R}^{N_1} = \text{im}(\mathbf{B}_1^T) \oplus \text{ker}(\mathbf{L}_1) \oplus \text{im}(\mathbf{B}_2)$$

$$\mathbf{f}_1 = \mathbf{f}_G + \mathbf{f}_H + \mathbf{f}_C$$

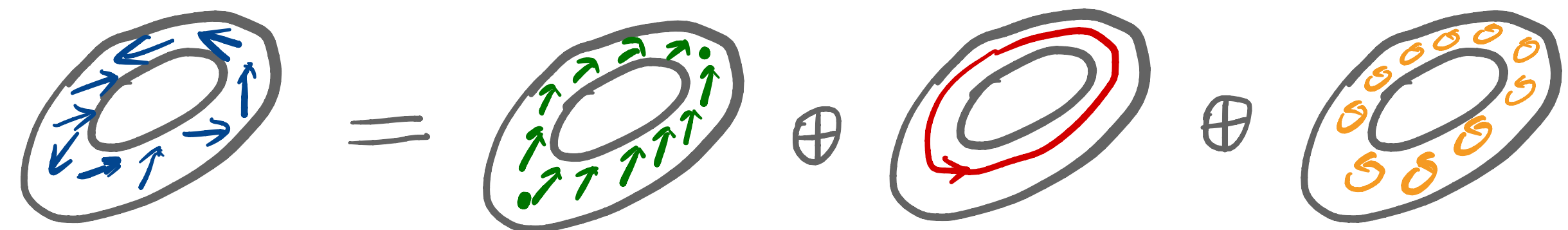


- This holds for any simplex order k

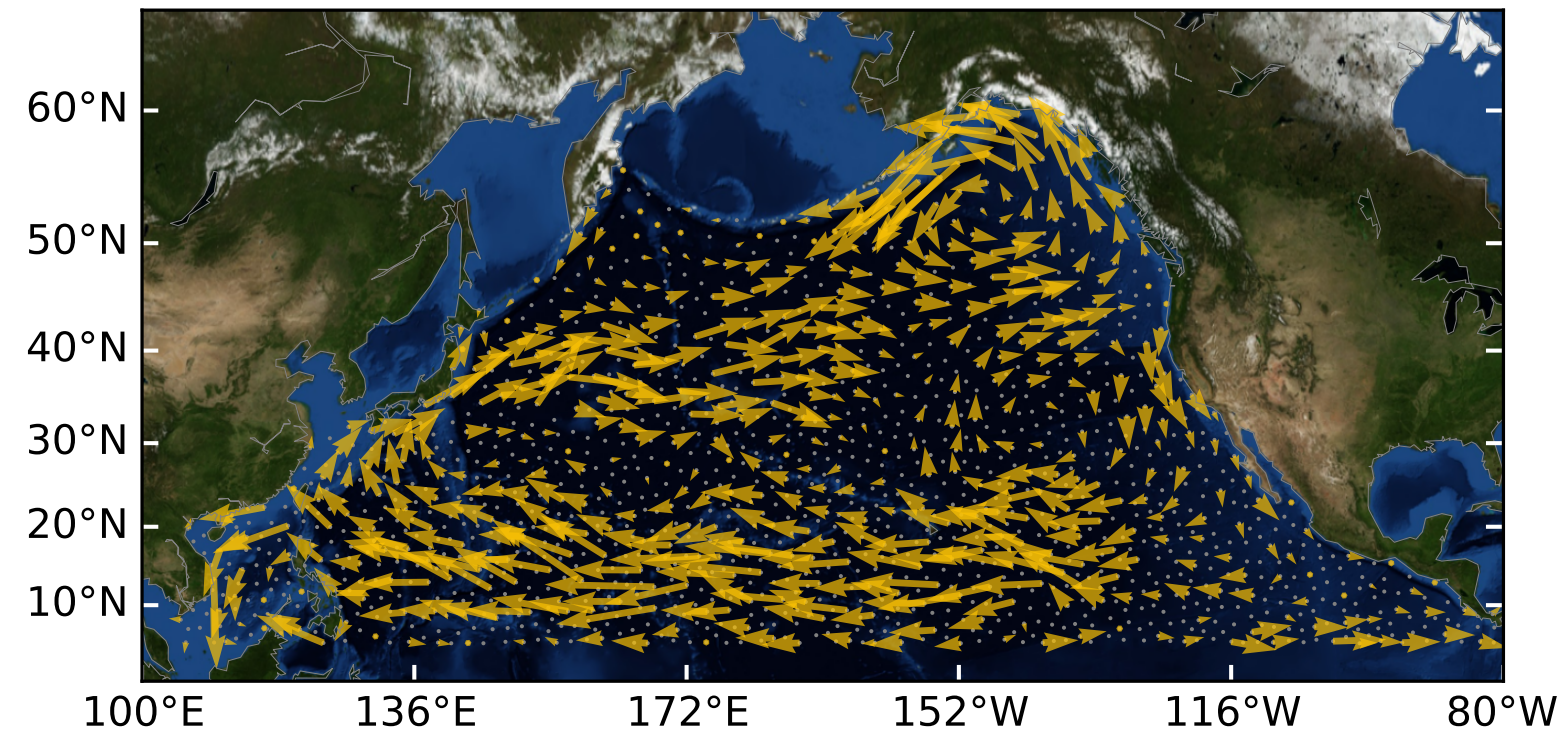
- What is the case for $k = 0$?

$$\mathbb{R}^{N_0} = \text{im}(\mathbf{B}_1) \oplus \text{ker}(\mathbf{L}_0)$$

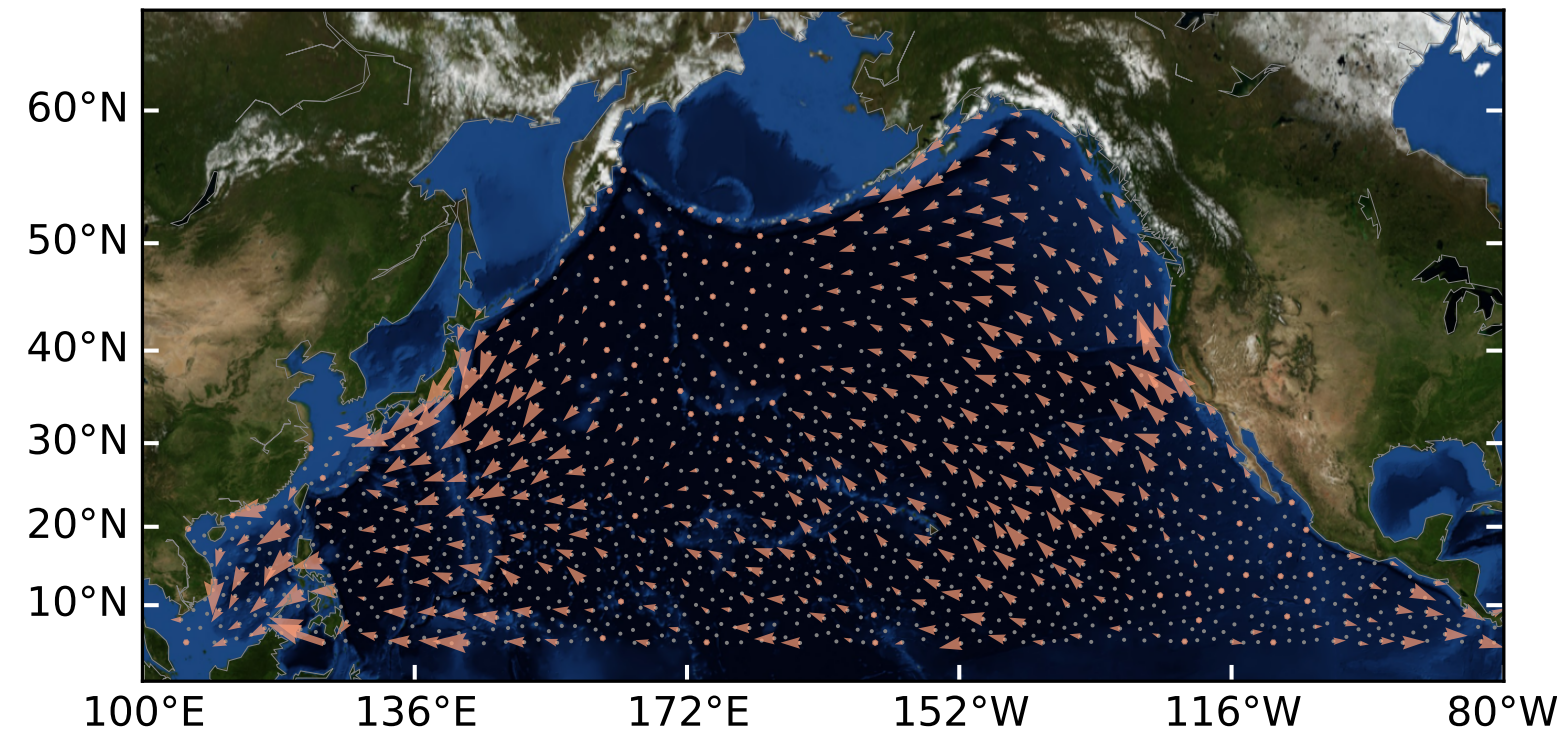
- Characteristic decomposition



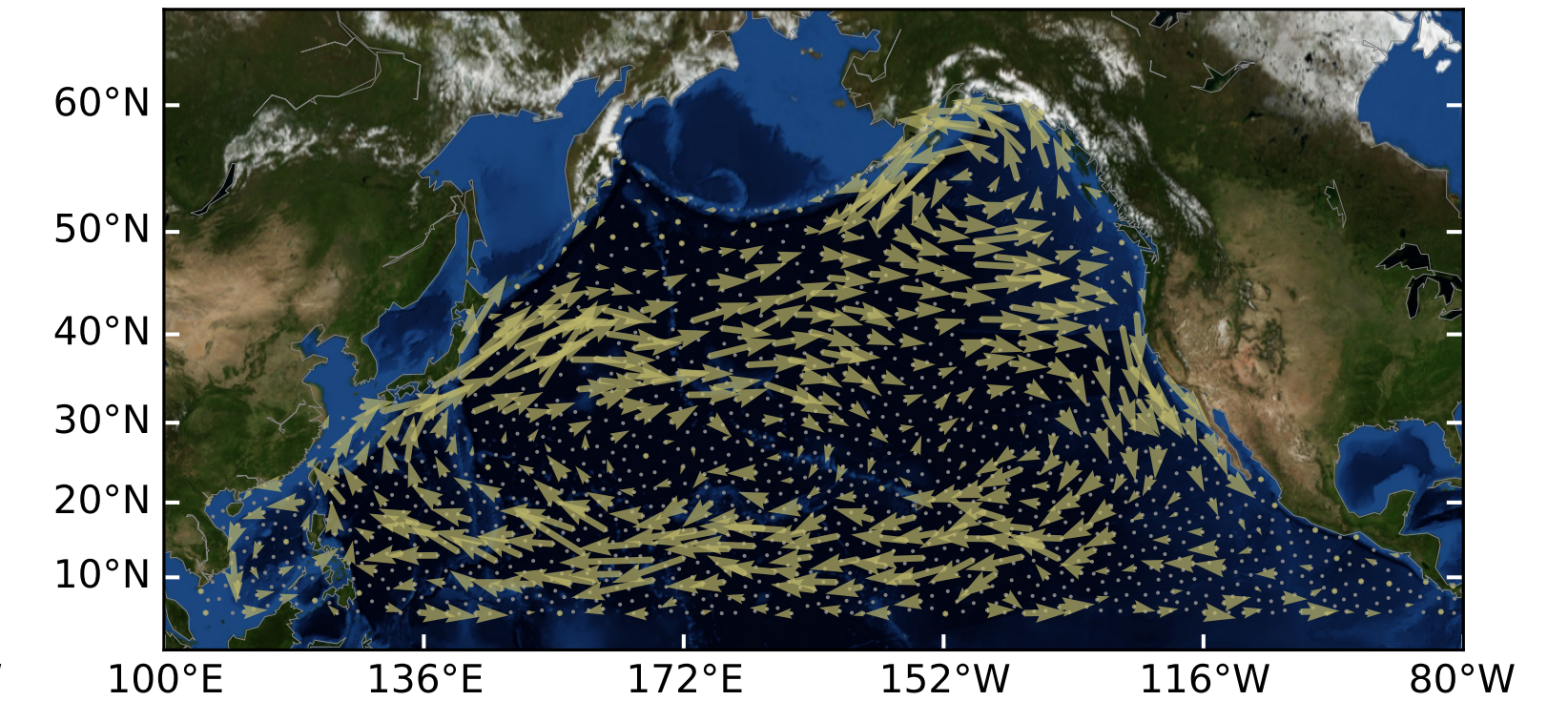
Applications of Hodge decomposition



Ocean currents

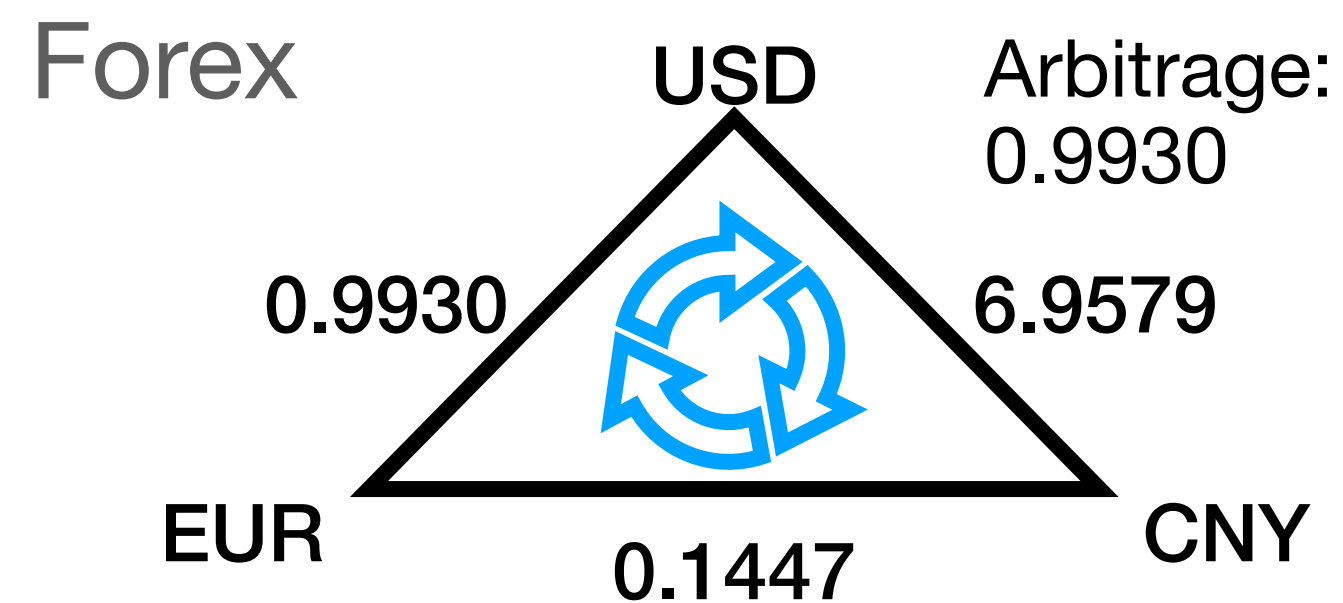


Gradient flow
Curl-free, irrotational



Curl flow
Div-free, solenoidal

Chen, Yu-Chia et al. (2021) "Helmholtzian Eigenmap."



$$r^{a/b} r^{b/c} = r^{a/c} \quad \text{Arbitrage-free}$$

$$f_{[a,b]} + f_{[b,c]} - f_{[a,c]} = 0 \quad \text{Curl-free}$$

- Water flows (div-free)
- Electrical currents, voltages

- Brain networks (Anand et al. 2022)
- Game theory (Candogan et al. 2011)
- Ranking problems (Jiang et al. 2011)
- Random walks (Strang et al. 2020)
- ...