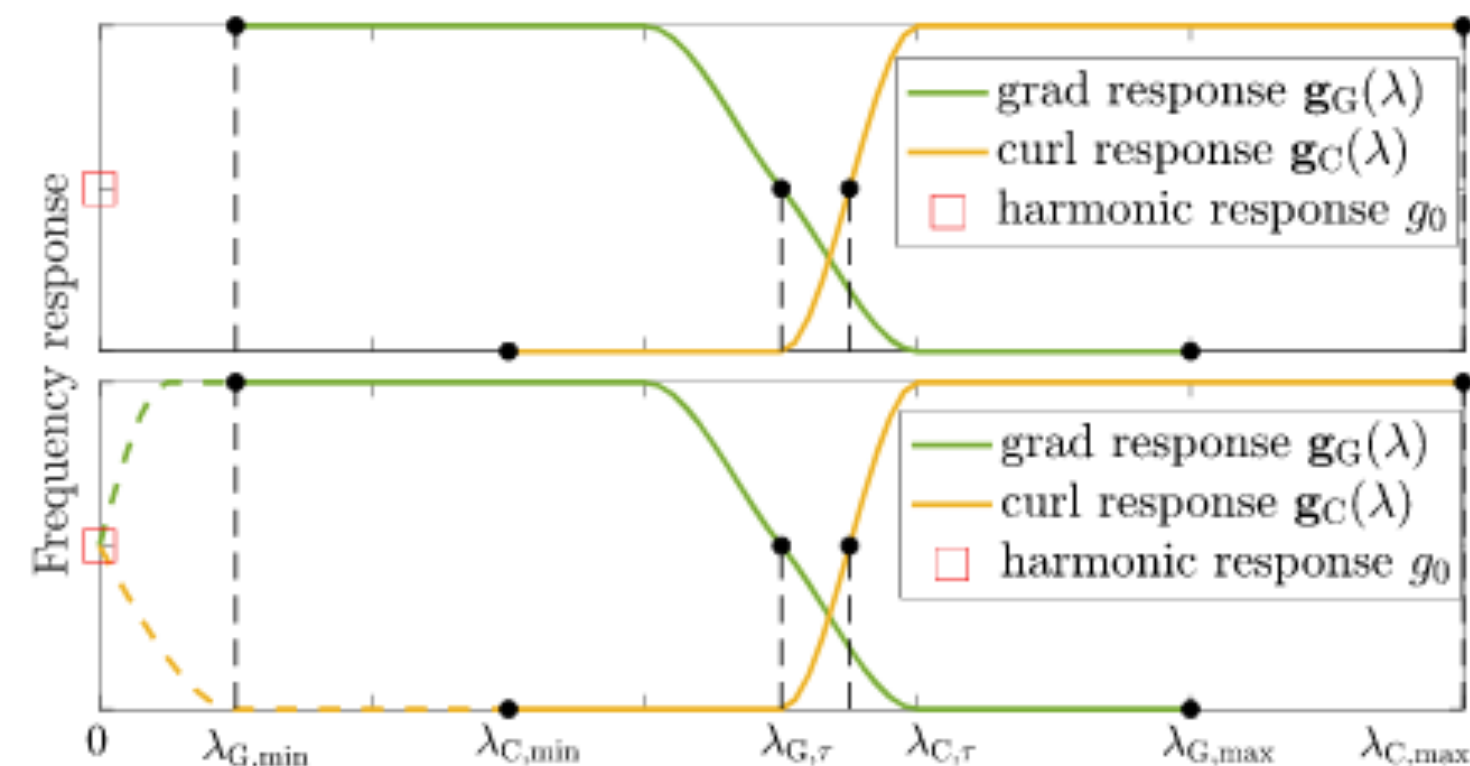


Edge Convolutions on SCs

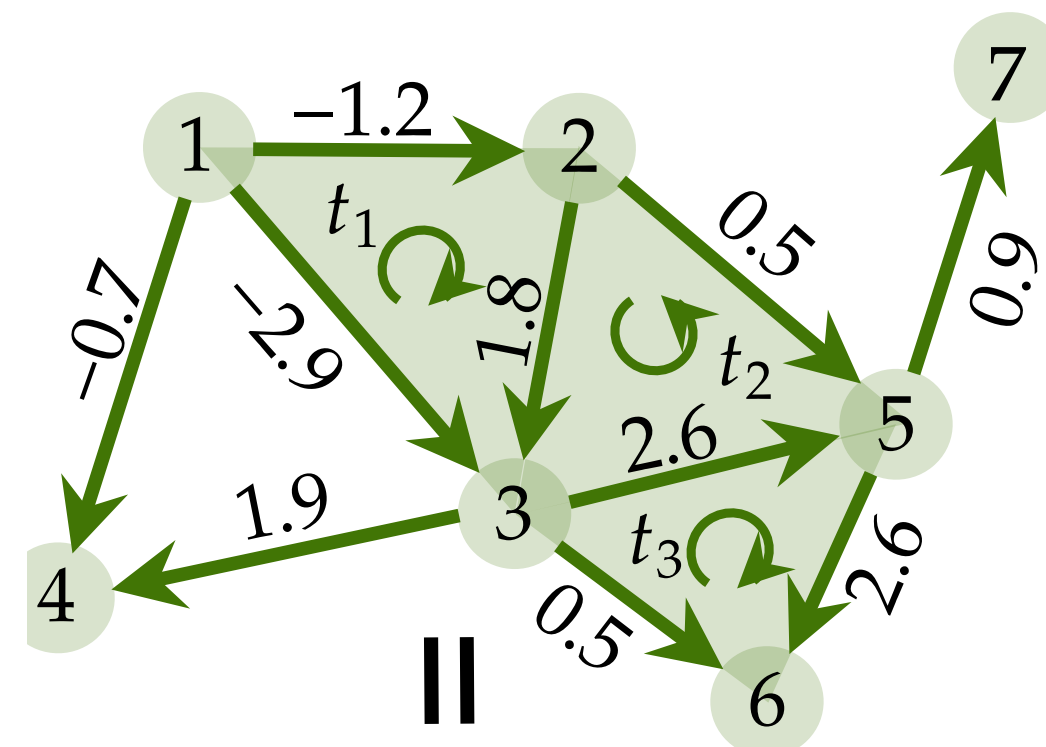
Pointwise Multiplication at frequencies

Spectral

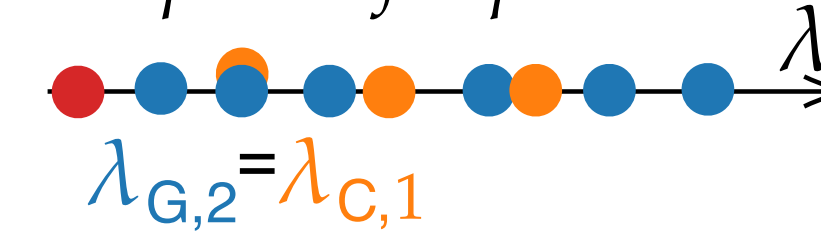
$$\begin{cases} \tilde{H}_H(\lambda) = \alpha_0 + \beta_0, & \text{for } \lambda \in \mathcal{Q}_H, \\ \tilde{H}_G(\lambda) = \sum_{k=0}^{K_d} \alpha_k \lambda^k, & \text{for } \lambda \in \mathcal{Q}_G, \\ \tilde{H}_C(\lambda) = \sum_{k=0}^{K_u} \beta_k \lambda^k, & \text{for } \lambda \in \mathcal{Q}_C \end{cases}$$



Why two sets of coefficients instead of one set?



simplicial frequencies



- blue dot: gradient freq. λ_G
- orange dot: curl freq. λ_C
- red dot: harmonic freq. λ_H

blue arrow: gradient flow

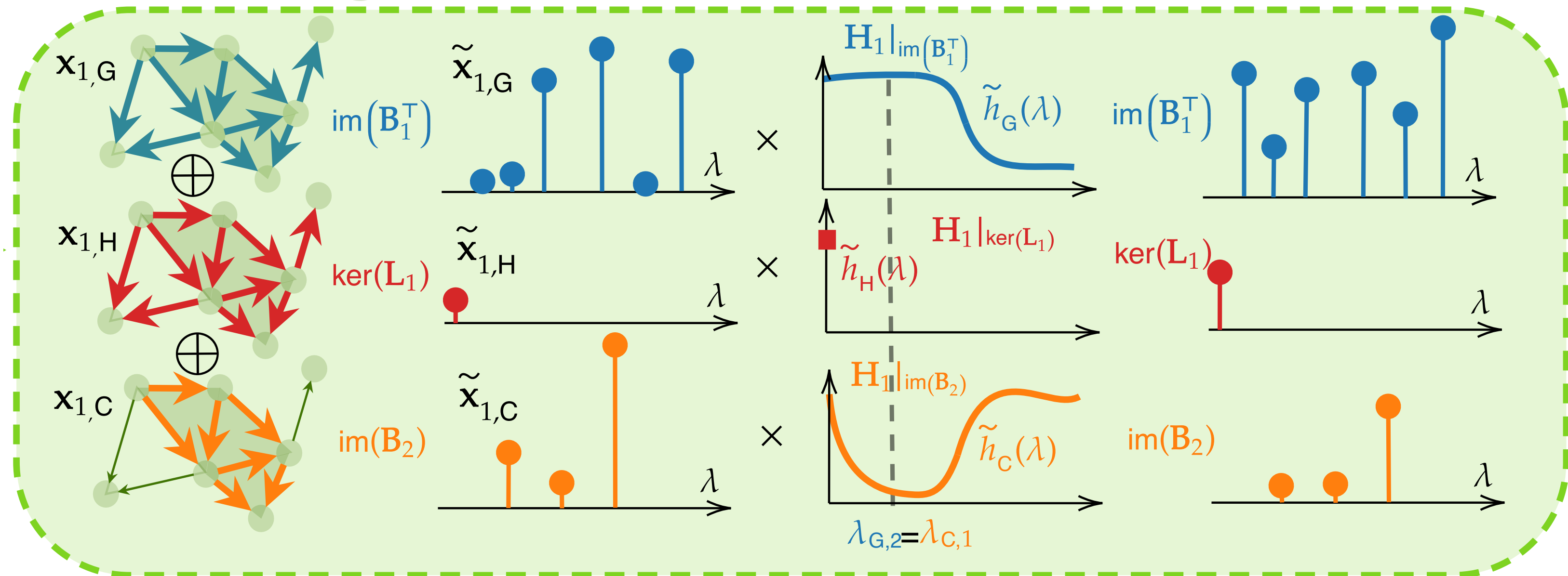
orange arrow: curlflow

red arrow: harmonic flow

$H_1|_{\text{im}(\mathbf{B}_1^T)} : \text{im}(\mathbf{B}_1^T) \rightarrow \text{im}(\mathbf{B}_1^T)$

$H_1|_{\text{im}(\mathbf{B}_2)} : \text{im}(\mathbf{B}_2) \rightarrow \text{im}(\mathbf{B}_2)$

$H_1|_{\ker(\mathbf{L}_1)} : \ker(\mathbf{L}_1) \rightarrow \ker(\mathbf{L}_1)$



Filter Design

- Data-driven: given a training set \mathcal{T} of input-output pairs

$$\min_{\alpha, \beta} \frac{1}{|\mathcal{T}|} \sum \|\mathbf{H}\mathbf{f} - \mathbf{y}\|_2^2 + \gamma r(\alpha, \beta)$$

- Spectral filter design
 - Least-Squares
 - Chebyshev polynomials

$$\begin{cases} \tilde{H}_H(\lambda) = \alpha_0 + \beta_0 \approx g_0, & \text{for } \lambda \in \mathcal{Q}_H, \\ \tilde{H}_G(\lambda) = \sum_{k=0}^{K_d} \alpha_k \lambda^k \approx g_G(\lambda), & \text{for } \lambda \in \mathcal{Q}_G, \\ \tilde{H}_C(\lambda) = \sum_{k=0}^{K_u} \beta_k \lambda^k \approx g_C(\lambda), & \text{for } \lambda \in \mathcal{Q}_C \end{cases}$$