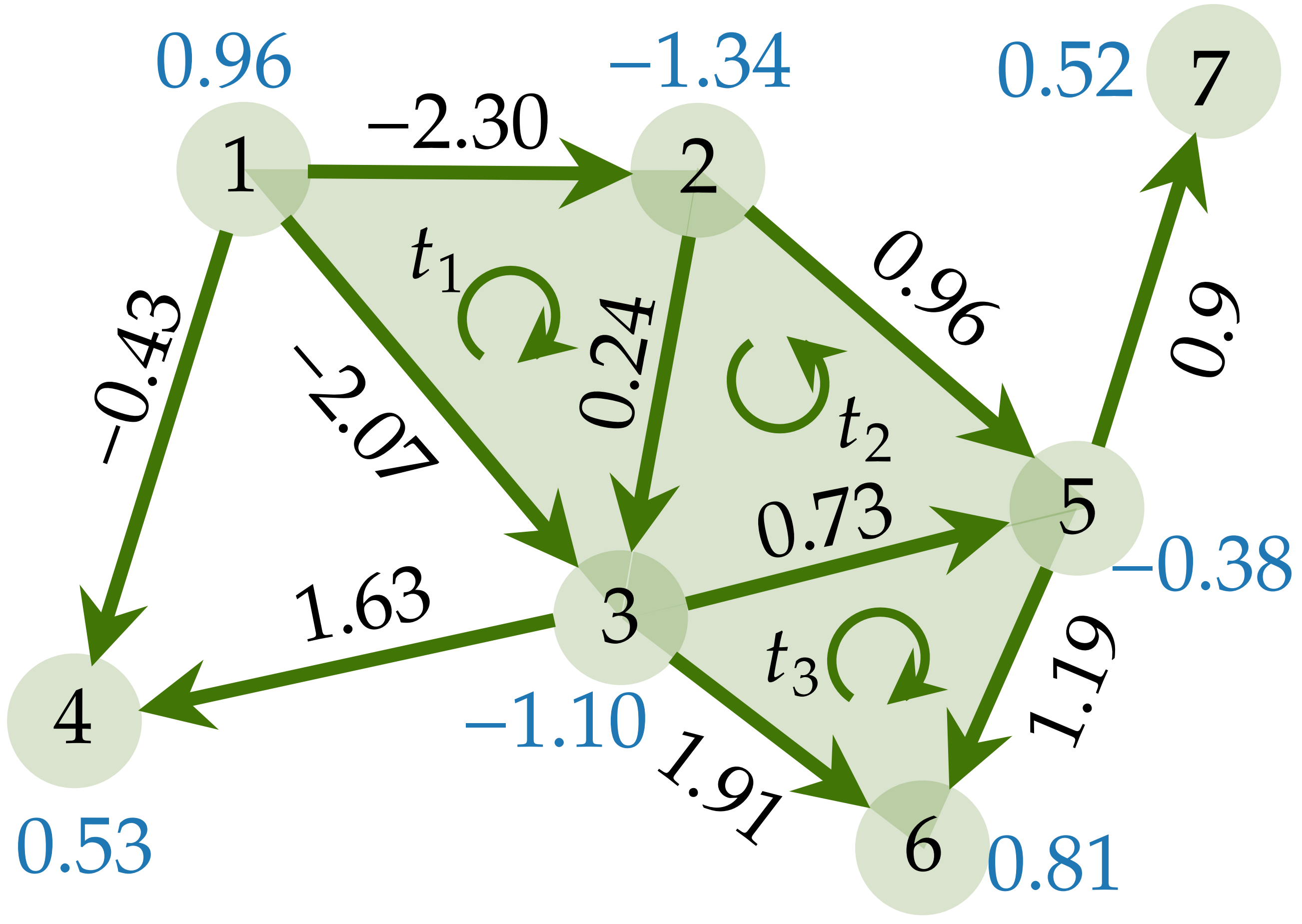
Incidence & Laplacians

1st and 2nd order Discrete Derivatives

- Node signal $\mathbf{v}$
- Edge flow $\mathbf{f}$



Gradient of node signal: $[\mathbf{f}_{\mathrm{G}}]_{[i,j]} = [\mathbf{B}_1^\top \mathbf{v}]_{[i,j]} = [\mathbf{v}]_j - [\mathbf{v}]_i$

$$[\mathbf{B}_1 \mathbf{f}]_{[i]} = \sum_{j<i} \mathbf{f}_{[j,i]} - \sum_{i<k} \mathbf{f}_{[i,k]}$$

$$[\mathbf{B}_1^\top \mathbf{v}]_{[1,2]} = -1.34 - 0.96 = -2.30$$

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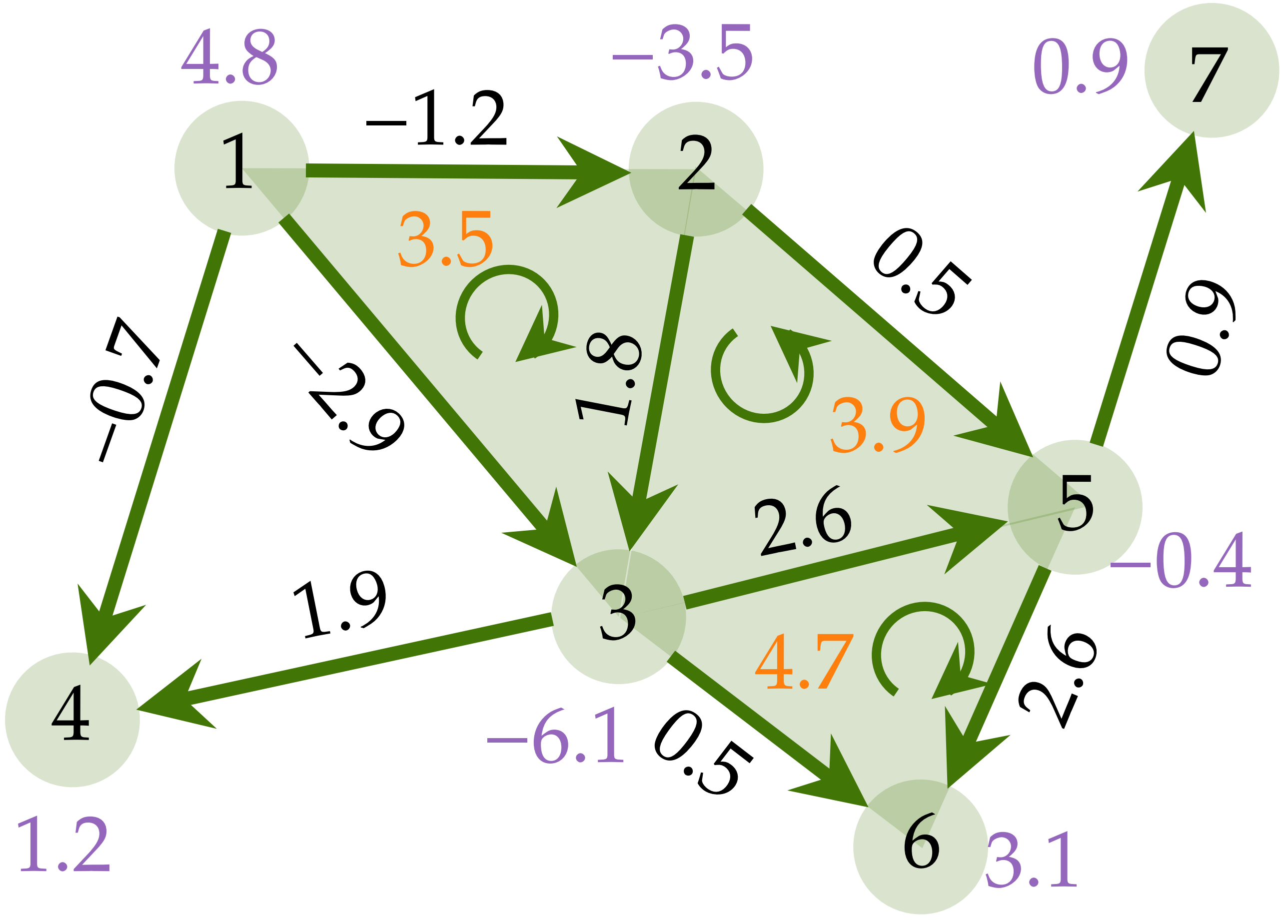

$$[\mathbf{f}_{\mathrm{G}}]_{[i,j]} = [\mathbf{v}]_j - [\mathbf{v}]_i$$

Divergence of edge flows: $[\mathbf{B}_1\mathbf{f}]_{[i]} = \sum_{j<i} \mathbf{f}_{[j,i]} - \sum_{i<k} \mathbf{f}_{[i,k]}$

Net-flow = in_flow - out_flow

Net-circulation in triangles

Laplacians = Grad Div + Curl* Curl

$$[\mathbf{B}_1\mathbf{f}]_5 = 0.5 + 2.6 - (0.9 + 2.6) = -0.4$$

$$[\mathbf{B}_2^{\top}\mathbf{f}]_{[1,2,3]} = -1.2 + 1.8 - (-2.9) = 3.5$$