

Tabular results

Table 1: Forex rates inference results.

Method	RMSE		NLPD	
	Diffusion	Matérn	Diffusion	Matérn
Euclidean	2.17 ± 0.13	2.19 ± 0.12	2.12 ± 0.07	2.20 ± 0.18
Line-Graph	2.43 ± 0.07	2.46 ± 0.07	2.28 ± 0.04	2.32 ± 0.03
Non-HC	2.48 ± 0.07	2.47 ± 0.08	2.36 ± 0.07	2.34 ± 0.04
HC	0.08 ± 0.12	0.06 ± 0.12	-3.52 ± 0.02	-3.52 ± 0.02

Table 3: WSN inference results.

Method	Node Heads		Edge Flowrates	
	RMSE	NLPD	RMSE	NLPD
Diffusion, non-HC	0.16 ± 0.05	0.72 ± 2.06	0.32 ± 0.05	0.97 ± 1.80
Matérn, non-HC	0.16 ± 0.04	0.71 ± 2.39	0.26 ± 0.05	0.10 ± 0.13
Diffusion, HC	0.15 ± 0.04	-0.47 ± 0.14	0.22 ± 0.03	-0.20 ± 0.13
Matérn, HC	0.15 ± 0.04	-0.25 ± 0.48	0.23 ± 0.03	-0.45 ± 0.49

Table C.1: Ocean current inference results.

Method	RMSE			NLPD		
	Diffusion	Matérn	Hodge Laplacian	Diffusion	Matérn	Hodge Laplacian
Euclidean	1.00 ± 0.01	1.00 ± 0.00	—	1.42 ± 0.01	1.42 ± 0.10	—
Line-Graph	0.99 ± 0.00	0.99 ± 0.00	—	1.41 ± 0.00	1.41 ± 0.00	—
Non-HC	0.35 ± 0.00	0.35 ± 0.00	0.35 ± 0.00	0.33 ± 0.00	0.36 ± 0.03	0.33 ± 0.01
HC	0.34 ± 0.00	0.35 ± 0.00	0.35 ± 0.00	0.33 ± 0.01	0.37 ± 0.04	0.33 ± 0.01

Sampling gradient and curl edge GPs

Proof. We focus on the case of gradient GPs. First, we can decompose the gradient kernel in terms of $U_1 = [U_H \ U_G \ U_C]$ as

$$K_G = U_1 \begin{pmatrix} \mathbf{0} & & \\ & \Psi_G(\Lambda_G) & \\ & & \mathbf{0} \end{pmatrix} U_1^\top. \quad (\text{B.9})$$

From a vector $\mathbf{v} = (v_1, \dots, v_{N_1})^\top$ of variables following independent normal distribution, we can draw a random sample of gradient function as

$$\mathbf{f}_G = U_1 \text{diag}([\mathbf{0}, \Psi_G^{\frac{1}{2}}(\Lambda_G), \mathbf{0}]) \mathbf{v} \quad (\text{B.10})$$

where $\text{diag}([\mathbf{a}, \mathbf{b}, \mathbf{c}])$ is the diagonal matrix with $(\mathbf{a}, \mathbf{b}, \mathbf{c})^\top$ on its diagonal.

Therefore, their curls are

$$\text{curl } \mathbf{f}_G = \mathbf{B}_2^\top U_1 \text{diag}([\mathbf{0}, \Psi_G^{\frac{1}{2}}(\Lambda_G), \mathbf{0}]) = \mathbf{B}_2^\top U_G \Psi_G^{\frac{1}{2}}(\Lambda_G) = \mathbf{0}. \quad (\text{B.11})$$

Likewise, we can show the samples of a curl GP are div-free.