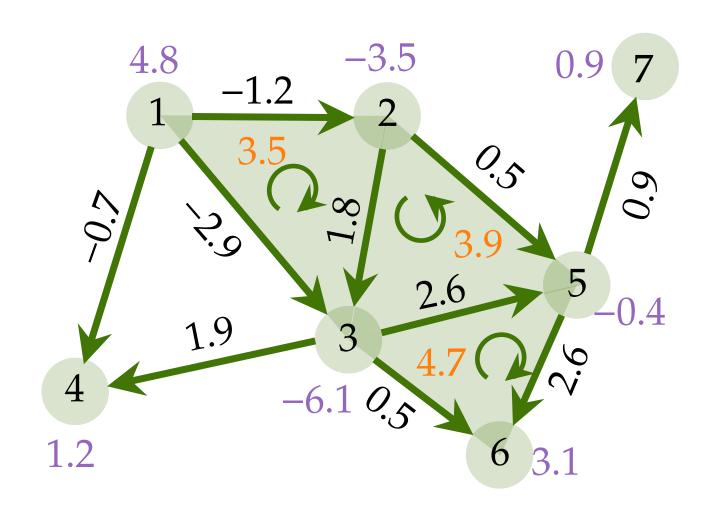
Incidence & Laplacians

1st and 2nd order Discrete Derivatives



Gradient of node signal:
$$\mathbf{B}_1^\mathsf{T}\mathbf{v}$$
 $[\mathbf{f}_G]_{[i,j]} = [\mathbf{v}]_j - [\mathbf{v}]_i$

Divergence of edge flows:
$$[\mathbf{B}_1\mathbf{f}]_{[i]} = \sum_{j < i} \mathbf{f}_{[j,i]} - \sum_{i < k} \mathbf{f}_{[i,k]}$$

Net-flow = in_flow - out_flow

Curl of edge flows:
$$[\mathbf{B}_{2}^{\mathsf{T}}\mathbf{f}]_{t} = \mathbf{f}_{[i,j]} + \mathbf{f}_{[j,k]} - \mathbf{f}_{[i,k]}$$
, for $t = [i,j,k]$

Net-circulation in triangles

$$[\mathbf{B}_1\mathbf{f}]_5 = 0.5 + 2.6 - (0.9 + 2.6) = -0.4$$

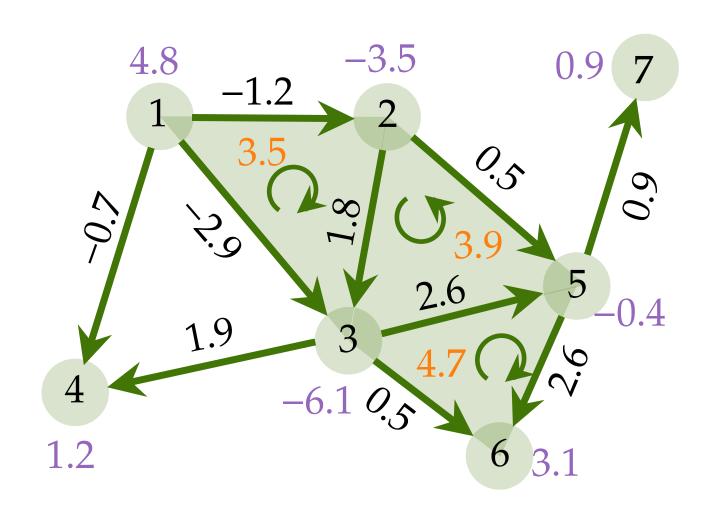
$$[\mathbf{B}_{2}^{\mathsf{T}}\mathbf{f}]_{[1,2,3]} = -1.2 + 1.8 - (-2.9) = 3.5$$

Laplacians = Grad Div + Curl* Curl* Hodge Laplacian:
$$\mathbf{L}_1 = \mathbf{B}_1^\mathsf{T} \mathbf{B}_1 + \mathbf{B}_2 \mathbf{B}_2^\mathsf{T}$$

$$\Delta_1 = \nabla(\nabla \cdot) + \nabla \times (\nabla \times)$$

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Hodge Laplacian:
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