Schrödinger's bridge problem

- Cloud of n independent Brownian particles

- Empirical distributions $\rho_0(x)$ and $\rho_1(y)$ at $t = 0$ and $t = 1$

$$\rho_0 \xrightarrow{\text{Wiener process}} \rho_1$$

- Particles have been transported in an unlikely way

- Of the many possible (unlikely) ways, which one is the most likely? [Lénoard 2014]

$$\min D_{KL}(\mathbb{P} \| \mathbb{W}) \ s.t.\ \mathbb{P}_0 = \rho_0, \mathbb{P}_1 = \rho_1$$

- A dynamics formulation of entropic-regularized optimal transport [Vallani 2009]

$$\min_{\pi \in \Pi(\rho_0, \rho_1)} \int_{\mathbb{R}^n \times \mathbb{R}^n} \frac{1}{2} \|x_0 - x_1\|^2 \, d\pi(x_0, x_1) + \sigma^2 D_{KL}(\pi \| \rho_0 \otimes \rho_1)$$

Reference topological dynamics

- $Y \sim \mathbb{Q}_{\mathcal{T}}$: topology-aware stochastic dynamics, tractable

- Topological stochastic dynamics: $dY_t = f(t, Y_t; L)dt + g_t dW_t$

  - $f_t = H_t(L)Y_t + \alpha_t$ with $H_t(L)$ a topological convolution operator

- Topological stochastic heat diffusion: $dY_t = -cLY_t dt + g_t dW_t$