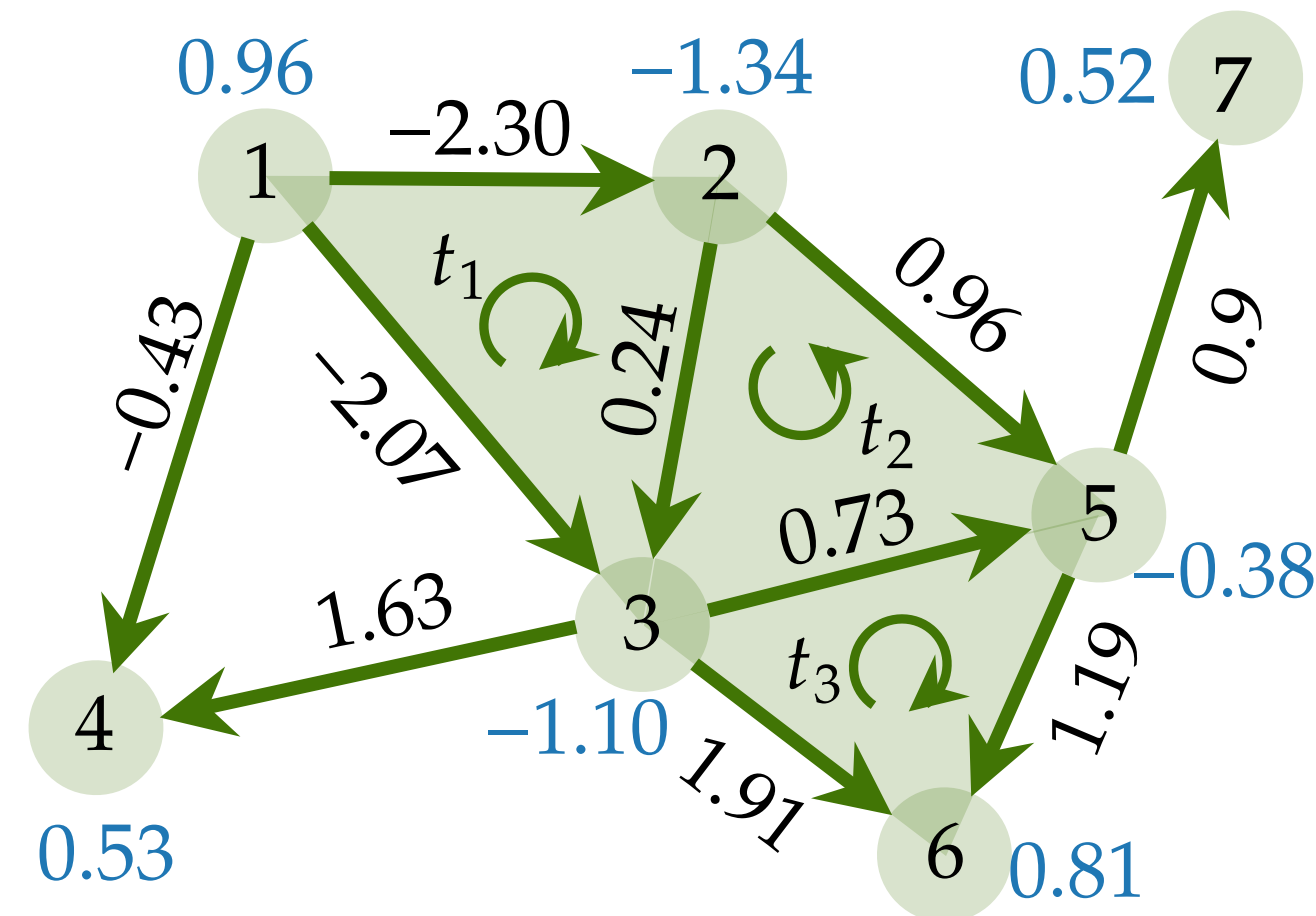# Incidence & Laplacians
1st and 2nd order Discrete Derivatives

- Node signal $\mathbf{v}$
- Edge flows $\mathbf{f}$

Gradient of node signal: $[\mathbf{f}_G]_{[i,j]} = [\mathbf{B}_1^\top \mathbf{v}]_{[i,j]} = [\mathbf{v}]_j - [\mathbf{v}]_i$
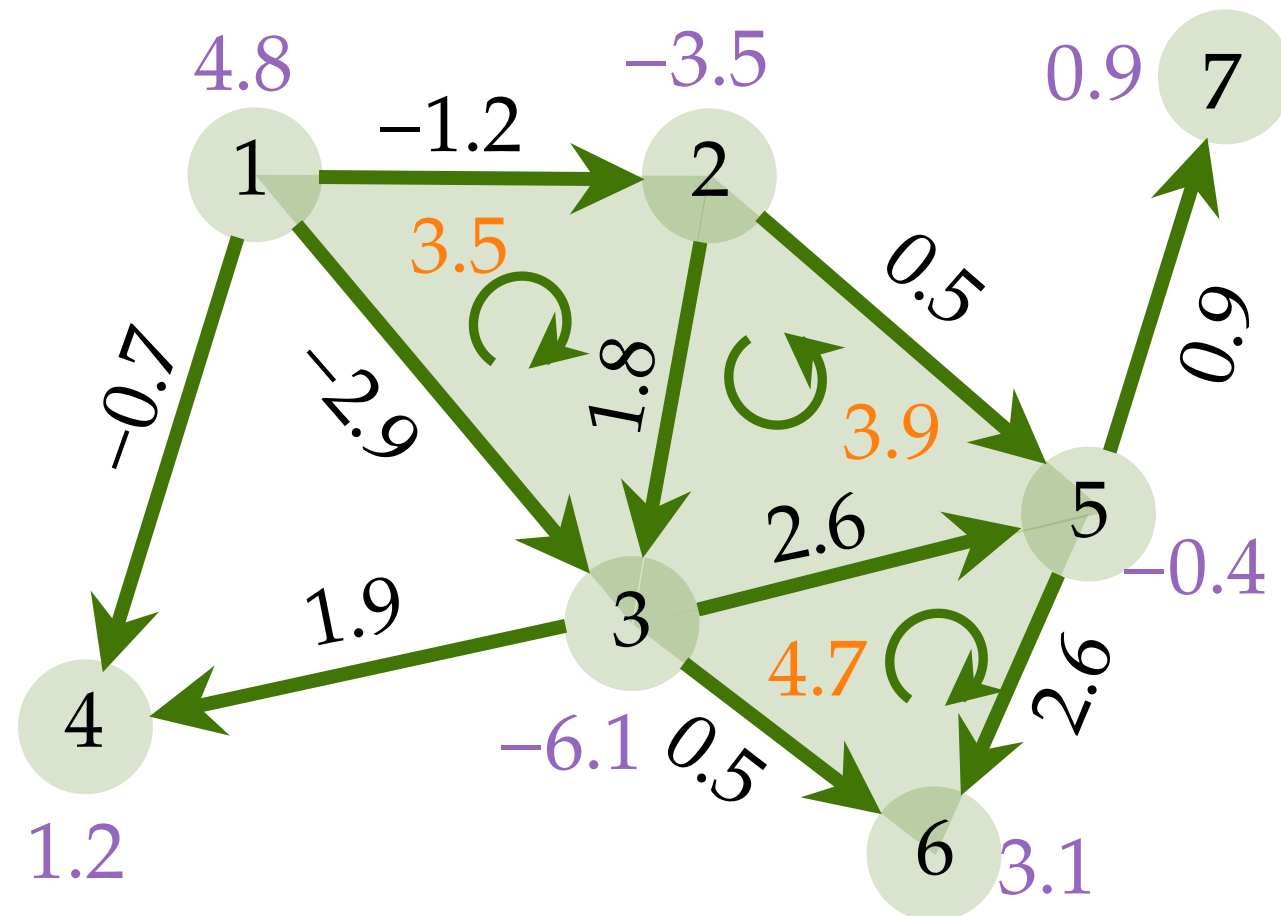
Divergence of edge flows: $[\mathbf{B}_1 \mathbf{f}]_{[i]} = \sum_{j<i} \mathbf{f}_{[j,i]} - \sum_{i<k} \mathbf{f}_{[i,k]}$

Curl of edge flows: $[\mathbf{B}_2^\top \mathbf{f}]_t = \mathbf{f}_{[i,j]} + \mathbf{f}_{[j,k]} - \mathbf{f}_{[i,k]}$, for $t = [i,j,k]$

$$[\mathbf{B}_1^\top \mathbf{v}]_{[1,2]} = -1.34 - 0.96 = -2.30$$

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$$\text{Gradient of node signal: } \mathbf{B}_1^\top \mathbf{v} \qquad [\mathbf{f}_G]_{[i,j]} = [\mathbf{v}]_j - [\mathbf{v}]_i$$

Divergence of edge flows:
$$[\mathbf{B}_1 \mathbf{f}]_{[i]} = \sum_{j<i} \mathbf{f}_{[j,i]} - \sum_{i<k} \mathbf{f}_{[i,k]}$$

Net-flow = in_flow - out_flow

$$\text{Curl of edge flows: } [\mathbf{B}_2^\top \mathbf{f}]_t = \mathbf{f}_{[i,j]} + \mathbf{f}_{[j,k]} - \mathbf{f}_{[i,k]}, \text{ for } t = [i,j,k]$$

Net-circulation in triangles

$$[\mathbf{B}_1 \mathbf{f}]_5 = 0.5 + 2.6 - (0.9 + 2.6) = -0.4$$

$$\text{Laplacians = Grad Div + Curl* Curl}$$

$$\text{Hodge Laplacian: } \mathbf{L}_1 = \mathbf{B}_1^\top \mathbf{B}_1 + \mathbf{B}_2 \mathbf{B}_2^\top$$

$$[\mathbf{B}_2^\top \mathbf{f}]_{[1,2,3]} = -1.2 + 1.8 - (-2.9) = 3.5$$

$$\Delta_1 = \nabla(\nabla \cdot) + \nabla \times (\nabla \times)$$