Filter Design

- Data-driven: given a training set $\mathcal{T}$ of input-output pairs

$$\min_{\boldsymbol{\alpha},\boldsymbol{\beta}} \frac{1}{|\mathcal{T}|} \sum \|\mathbf{Hf} - \mathbf{y}\|_2^2 + \gamma r(\boldsymbol{\alpha}, \boldsymbol{\beta})$$

- Spectral filter design
  - Least-Squares
  - Chebyshev polynomials

$$\begin{cases} \tilde{H}_{\mathrm{H}}(\lambda) = \alpha_0 + \beta_0 \approx g_0, & \text{for } \lambda \in \mathcal{Q}_{\mathrm{H}}, \\ \tilde{H}_{\mathrm{G}}(\lambda) = \sum_{k=0}^{K_d} \alpha_k \lambda^k \approx g_{\mathrm{G}}(\lambda), & \text{for } \lambda \in \mathcal{Q}_{\mathrm{G}}, \\ \tilde{H}_{\mathrm{C}}(\lambda) = \sum_{k=0}^{K_u} \beta_k \lambda^k \approx g_{\mathrm{C}}(\lambda), & \text{for } \lambda \in \mathcal{Q}_{\mathrm{C}} \end{cases}$$

Applications

Gradient projection op.
$\mathbf{P}_\mathrm{G} = \mathbf{B}_1^\top(\mathbf{B}_1\mathbf{B}_1^\top)^\dagger\mathbf{B}_1 = \mathbf{U}_\mathrm{G}\mathbf{U}_\mathrm{G}^\top$

- Hodge component extractions
- Solving LS problem: $\mathbf{f}_\mathrm{G} = \mathbf{P}_\mathrm{G}\mathbf{f},\ \mathbf{f}_\mathrm{C} = \mathbf{P}_\mathrm{C}\mathbf{f},\ \mathbf{f}_\mathrm{H} = \mathbf{f} - \mathbf{f}_\mathrm{G} - \mathbf{f}_\mathrm{C}$
- Convolutional filter implementation: closed form on coefficients