Tabular results

Table 1: Forex rates inference results.

Method	RMSE		NLPD	
	Diffusion	Matérn	Diffusion	Matérn
Euclidean	2.17 ± 0.13	2.19 ± 0.12	2.12 ± 0.07	2.20 ± 0.18
Line-Graph	2.43 ± 0.07	2.46 ± 0.07	2.28 ± 0.04	2.32 ± 0.03
Non-HC	2.48 ± 0.07	2.47 ± 0.08	2.36 ± 0.07	2.34 ± 0.04
HC	0.08 ± 0.12	0.06 ± 0.12	-3.52 ± 0.02	-3.52 ± 0.02

Table 3: WSN inference results.

Method	Node Heads		Edge Flowrates	
	RMSE	NLPD	RMSE	NLPD
Diffusion, non-HC	0.16 ± 0.05	0.72 ± 2.06	0.32 ± 0.05	0.97 ± 1.80
Matérn, non-HC	0.16 ± 0.04	0.71 ± 2.39	0.26 ± 0.05	0.10 ± 0.13
Diffusion, HC	0.15 ± 0.04	-0.47 ± 0.14	0.22 ± 0.03	-0.20 ± 0.13
Matérn, HC	0.15 ± 0.04	-0.25 ± 0.48	0.23 ± 0.03	-0.45 ± 0.49

Table C.1: Ocean current inference results.

Method	RMSE			NLPD		
	Diffusion	Matérn	Hodge Laplacian	Diffusion	Matérn	Hodge Laplacian
Euclidean	1.00 ± 0.01	1.00 ± 0.00	—	1.42 ± 0.01	1.42 ± 0.10	—
Line-Graph	0.99 ± 0.00	0.99 ± 0.00	—	1.41 ± 0.00	1.41 ± 0.00	—
Non-HC	0.35 ± 0.00	0.35 ± 0.00	0.35 ± 0.00	0.33 ± 0.00	0.36 ± 0.03	0.33 ± 0.01
HC	0.34 ± 0.00	0.35 ± 0.00	0.35 ± 0.00	0.33 ± 0.01	0.37 ± 0.04	0.33 ± 0.01

Sampling gradient and curl edge GPs

Proof. We focus on the case of gradient GPs. First, we can decompose the gradient kernel in terms of $\boldsymbol{U}_1 = [\boldsymbol{U}_H \ \boldsymbol{U}_G \ \boldsymbol{U}_C]$ as

$$\boldsymbol{K}_G = \boldsymbol{U}_1 \begin{pmatrix} \mathbf{0} & & \\ & \Psi_G(\boldsymbol{\Lambda}_G) & \\ & & \mathbf{0} \end{pmatrix} \boldsymbol{U}_1^\top. \tag{B.9}$$

From a vector $\boldsymbol{v} = (v_1, \ldots, v_{N_1})^\top$ of variables following independent normal distribution, we can draw a random sample of gradient function as

$$\boldsymbol{f}_G = \boldsymbol{U}_1 \mathrm{diag}([\mathbf{0}, \Psi_G^{\frac{1}{2}}(\boldsymbol{\Lambda}_G), \mathbf{0}])\boldsymbol{v} \tag{B.10}$$

where $\mathrm{diag}([\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}])$ is the diagonal matrix with $(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c})^\top$ on its diagonal.

Therefore, their curls are

$$\mathrm{curl}\, \boldsymbol{f}_G = \boldsymbol{B}_2^\top \boldsymbol{U}_1 \mathrm{diag}([\mathbf{0}, \Psi_G^{\frac{1}{2}}(\boldsymbol{\Lambda}_G), \mathbf{0}]) = \boldsymbol{B}_2^\top \boldsymbol{U}_G \Psi_G^{\frac{1}{2}}(\boldsymbol{\Lambda}_G) = \mathbf{0}. \tag{B.11}$$

Likewise, we can show the samples of a curl GP are div-free.