Eigenspace of $\mathbf{L}_1$ spans Hodge subspaces

- **Nonzero** Eigenspace of down Laplacian spans the gradient space

Simplicial Fourier transform

Frequency — eigenvalues

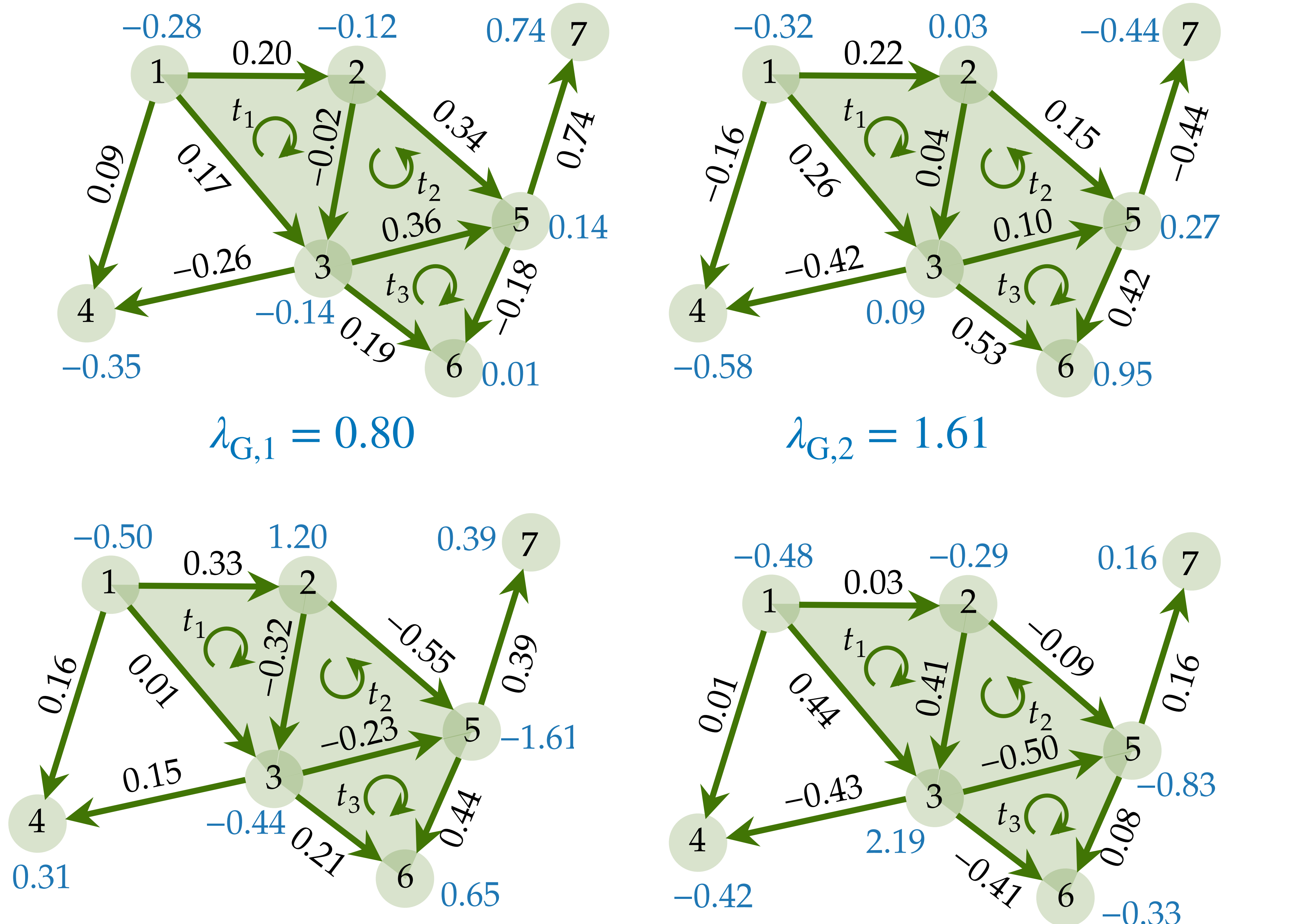
Fourier basis — eigenvectors

$$\lambda_G = \|\mathbf{B}_1 \mathbf{u}_G\|_2^2$$

Gradient eigenvector

Fourier basis reflecting divergent properties

$$\lambda_C = \|\mathbf{B}_2^\top \mathbf{u}_C\|_2^2$$



$\lambda_{G,1} = 0.80$ $\lambda_{G,2} = 1.61$

$\lambda_{G,5} = 5.12$ $\lambda_{G,6} = 6.08$

$\lambda_{C,1} = 1.59$ $\lambda_{C,2} = 3.00$

$\lambda_{C,3} = 4.41$ $\lambda_{H,1} = 0$

EVD: $\mathbf{L}_1 = \mathbf{U}_1 \mathbf{\Lambda}_1 \mathbf{U}_1^\top$

$$\boxed{\begin{array}{c}\mathbf{U}_1 = [\mathbf{U}_H\ \mathbf{U}_G\ \mathbf{U}_C] \\ \mathrm{span}(\mathbf{U}_G) = \mathrm{im}(\mathbf{B}_1^\top)\end{array}}$$

Eigenspace of $\mathbf{L}_1$ spans Hodge subspaces

- **Nonzero** Eigenspace of up Laplacian spans the curl space

Simplicial Fourier transform

Frequency — eigenvalues

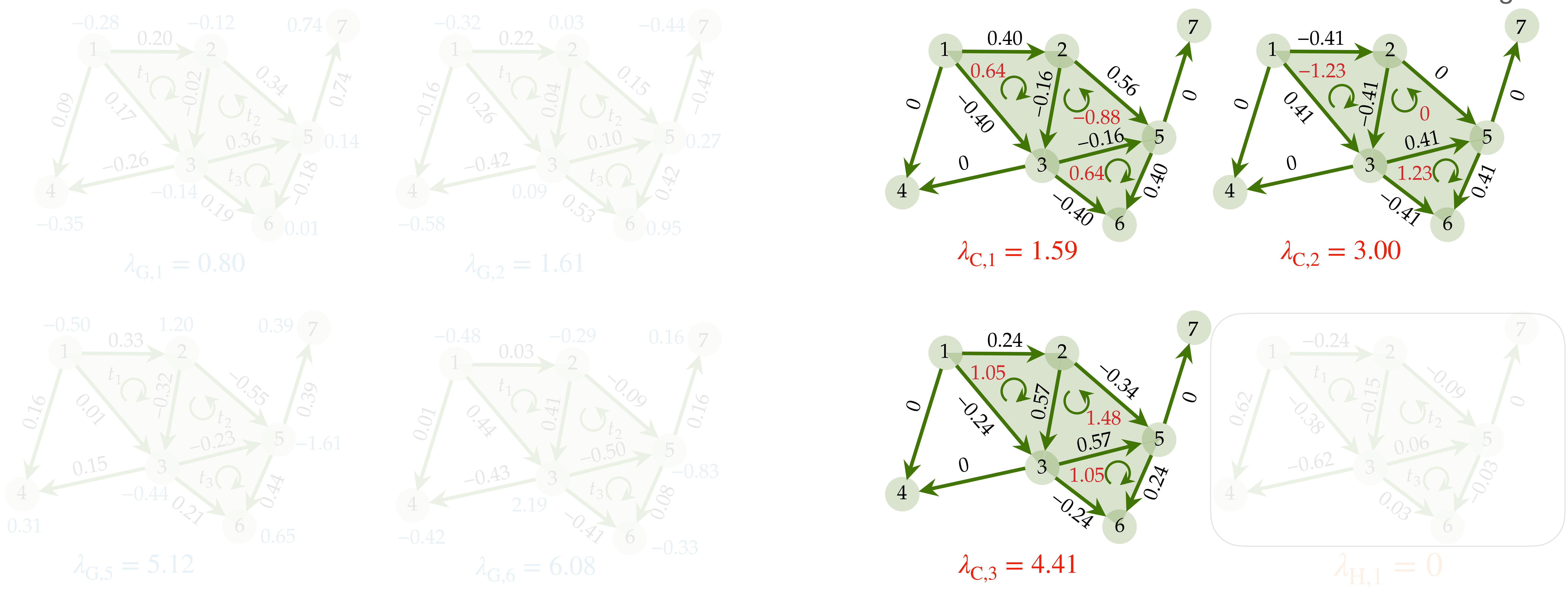
Fourier basis — eigenvectors

$\lambda_G = \|\mathbf{B}_1\mathbf{u}_G\|_2^2$

$\lambda_C = \|\mathbf{B}_2^\top\mathbf{u}_C\|_2^2$

Curl eigenvector

Fourier basis reflecting rotational properties



EVD: $\mathbf{L}_1 = \mathbf{U}_1\mathbf{\Lambda}_1\mathbf{U}_1^\top$

$$\mathbf{U}_1 = [\mathbf{U}_H\ \mathbf{U}_G\ \mathbf{U}_C]$$

$$\mathrm{span}(\mathbf{U}_C) = \mathrm{im}(\mathbf{B}_2)$$