

Matérn Edge GPs

Derived from SDEs on the edge set

- $\mathbf{f}_1 \sim \text{GP}(\mathbf{0}, \mathbf{K}_1)$
- Matérn kernels on the edge space

$$\text{EVD: } \mathbf{L}_1 = \mathbf{U}_1 \mathbf{\Lambda}_1 \mathbf{U}_1^\top$$

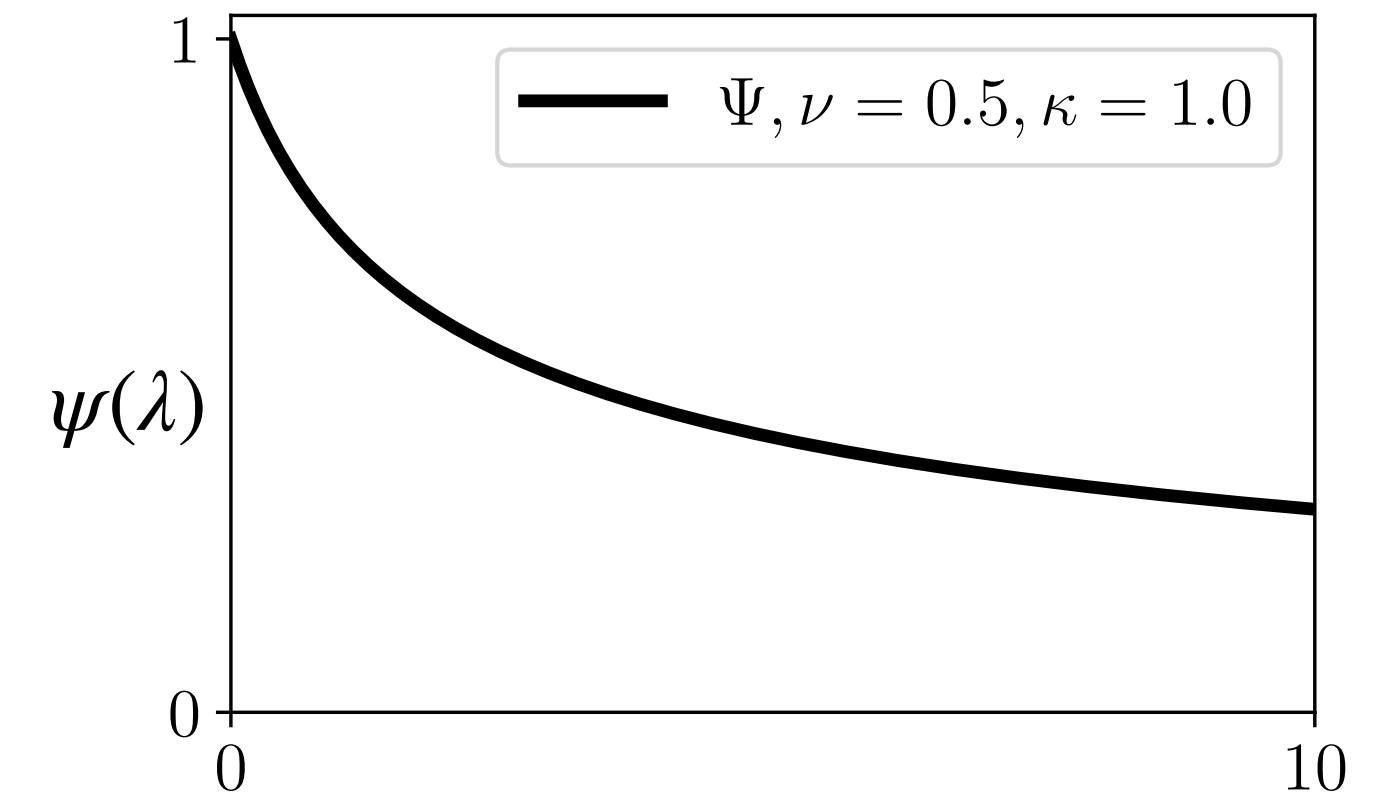
$$\Phi(\mathbf{L}_1) \mathbf{f}_1 = \mathbf{w}_1, \text{ with}$$

$$\Phi(\mathbf{L}_1) = \left(\frac{2\nu}{\kappa^2} \mathbf{I} + \mathbf{L}_1 \right)^{\frac{\nu}{2}} \text{ and } \mathbf{w}_1 \sim N(\mathbf{0}, \sigma^2 \mathbf{I})$$

- The solution gives edge GPs

$$\text{Matérn: } \mathbf{f}_1 \sim \text{GP}\left(0, \left(\frac{2\nu}{\kappa^2} \mathbf{I} + \mathbf{L}_1\right)^{-\nu}\right)$$

$$\text{Diffusion: } \mathbf{f}_1 \sim \text{GP}\left(0, e^{-\frac{\kappa^2}{2} \mathbf{L}_1}\right)$$



- Low-pass in the eigen-spectrum

Smoothness

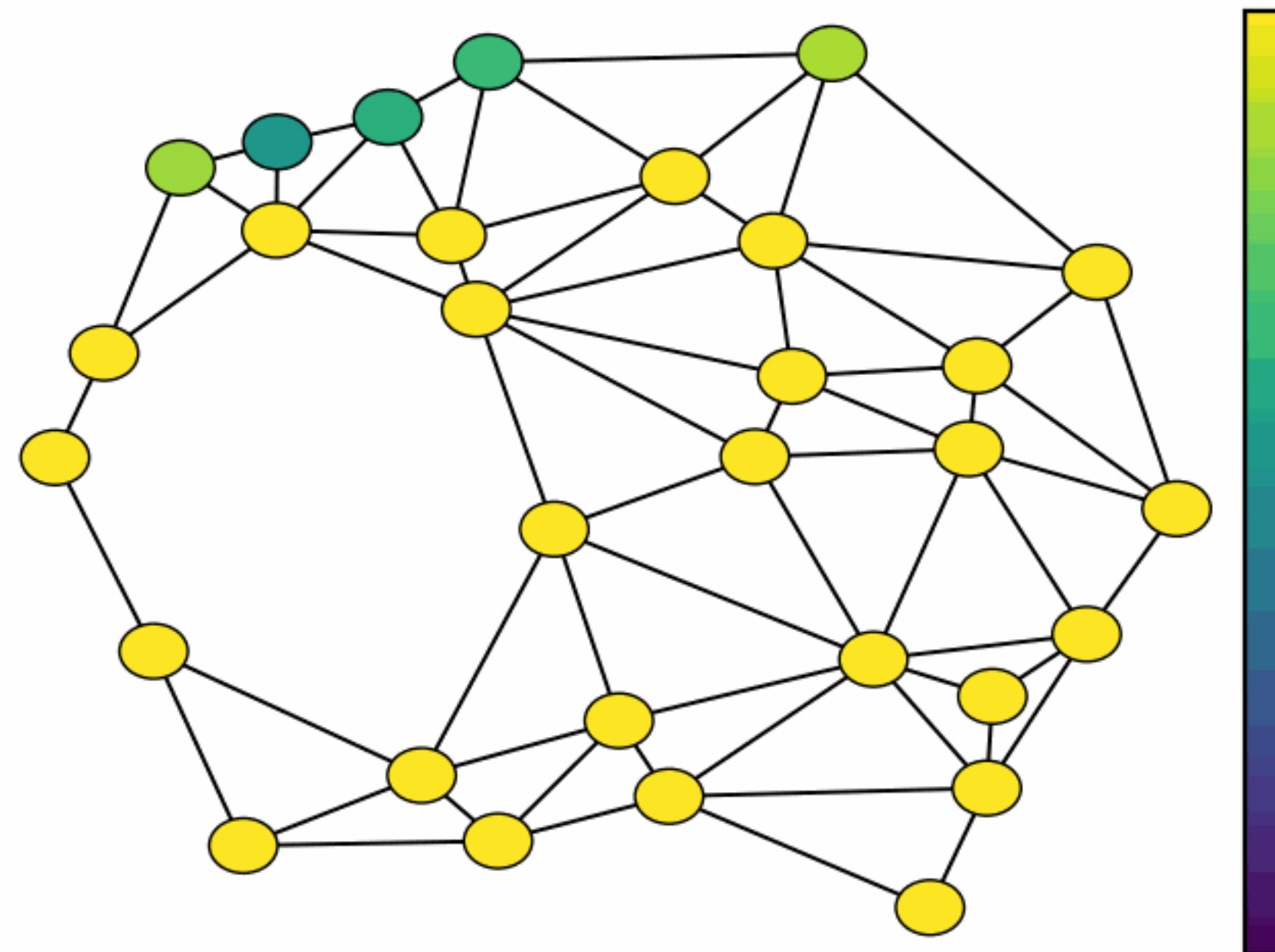
Node function — 0-form (scalar field)
Edge function — 1-form (vector field)

Divergence
Curl

Diffusion GPs

Diffusion processes on nodes and edges

Diffusion on nodes



Diffusion on edges

