Simplicial Signal Smoothness

Variations in terms of faces and cofaces

- For edge flows

Total divergence $\|\mathbf{B}_1\mathbf{f}\|_2^2 = \mathbf{f}^\top\mathbf{L}_d\mathbf{f}$

Total curl $\|\mathbf{B}_2^\top\mathbf{f}\|_2^2 = \mathbf{f}^\top\mathbf{L}_u\mathbf{f}$

- For node signal:

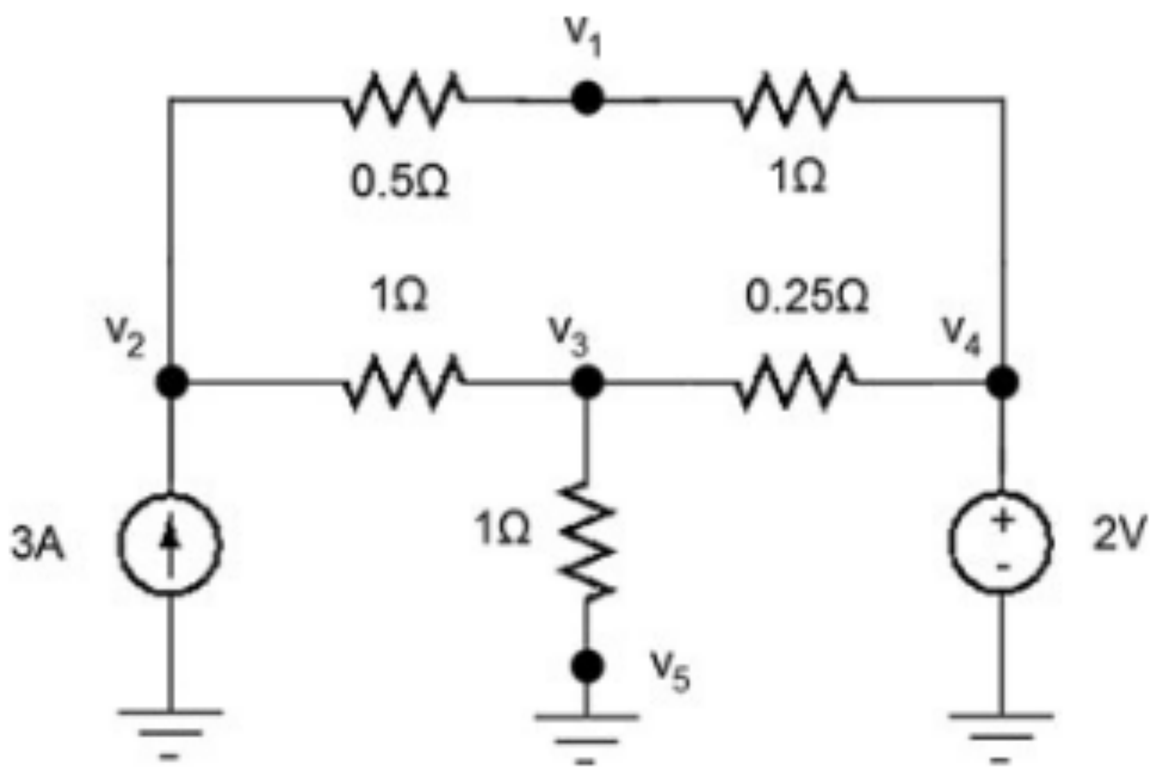
$$\|\mathbf{B}_1^\top\mathbf{v}\|_2^2 = \mathbf{v}^\top\mathbf{L}_0\mathbf{v}$$

- For general simplicial signals

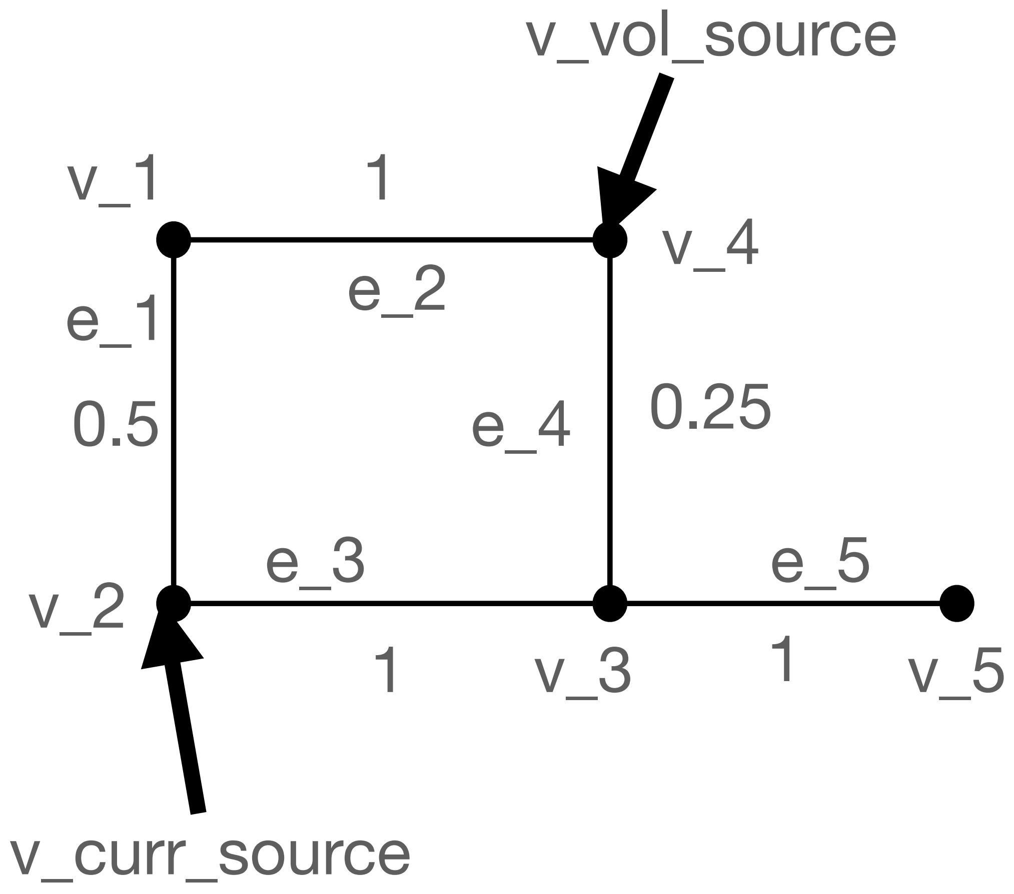
Lower variation

Upper variation

A Circuit toy example



(Grady et al. 2010)



$\mathbf{v} \in \mathbb{R}^{|\mathcal{N}|}$: Electric potential on nodes

$\mathbf{f}_{Vol} = \mathbf{B}_1^\top \mathbf{v}$: (Kirchhoff's voltage law)

$\mathbf{f}_{currents} = \mathbf{G}^{-1}\mathbf{f}_{Vol}$: currents (Ohm's law)

Diagonal resistance/conductance

Kirchhoff's current law: $\mathbf{B}_1 \mathbf{f}_{currents} = \mathbf{0}$

Or $\mathbf{B}_1 \mathbf{f}_{currents} + \mathbf{v}_{curr\,source} = \mathbf{0}$

$$\mathbf{B}_1 = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 \\ 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix} \qquad \mathbf{v}_{vol} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ 2 \\ 0 \end{pmatrix}$$

$$\mathbf{B}_1 \mathbf{G}^{-1} \mathbf{B}_1^\top \mathbf{v}_{vol} + \mathbf{v}_{curr\,source} = \mathbf{0}$$

Resistance — Metrics?
Power, energy?