Posterior distribution of Hodge components

$$
\begin{bmatrix} \boldsymbol{f}_H(\boldsymbol{x}) \\ \boldsymbol{f}_H(\boldsymbol{x}^*) \\ \boldsymbol{f}_G(\boldsymbol{x}) \\ \boldsymbol{f}_G(\boldsymbol{x}^*) \\ \boldsymbol{f}_C(\boldsymbol{x}) \\ \boldsymbol{f}_C(\boldsymbol{x}^*) \\ \boldsymbol{f}_1(\boldsymbol{x}) \\ \boldsymbol{f}_1(\boldsymbol{x}^*) \end{bmatrix} \sim \mathcal{N}\left(\mathbf{0}, \begin{bmatrix} \boldsymbol{K}_H & \boldsymbol{K}_H^* & & & & & \boldsymbol{K}_H & \boldsymbol{K}_H^* \\ \boldsymbol{K}_H^{*\top} & \boldsymbol{K}_H^{**} & & & & & \boldsymbol{K}_H^* & \boldsymbol{K}_H^{**} \\ & & \boldsymbol{K}_G & \boldsymbol{K}_G^* & & & \boldsymbol{K}_G & \boldsymbol{K}_G^* \\ & & \boldsymbol{K}_G^{*\top} & \boldsymbol{K}_G^{**} & & & \boldsymbol{K}_G^* & \boldsymbol{K}_G^{**} \\ & & & & \boldsymbol{K}_C & \boldsymbol{K}_C^* & \boldsymbol{K}_C & \boldsymbol{K}_C^* \\ & & & & \boldsymbol{K}_C^{*\top} & \boldsymbol{K}_C^{**} & \boldsymbol{K}_C^* & \boldsymbol{K}_C^{**} \\ \boldsymbol{K}_H & \boldsymbol{K}_H^{*\top} & \boldsymbol{K}_G & \boldsymbol{K}_G^{*\top} & \boldsymbol{K}_C & \boldsymbol{K}_C^{*\top} & \boldsymbol{K}_1 & \boldsymbol{K}_1^* \\ \boldsymbol{K}_H^{*\top} & \boldsymbol{K}_H^{**} & \boldsymbol{K}_G^{*\top} & \boldsymbol{K}_G^{**} & \boldsymbol{K}_C^{*\top} & \boldsymbol{K}_C^{**} & \boldsymbol{K}_1^{*\top} & \boldsymbol{K}_1^{**} \end{bmatrix}\right) \tag{B.26}
$$

where we represent the kernel matrices by $\boldsymbol{K}_1 = k_1(\boldsymbol{x}, \boldsymbol{x}), \boldsymbol{K}_1^* = k_1(\boldsymbol{x}, \boldsymbol{x}^*)$ and $\boldsymbol{K}_1^{**} = k_1(\boldsymbol{x}^*, \boldsymbol{x}^*)$, and likewise for the other kernel matrices. Given this joint distribution, we can obtain the posterior distributions of the three Hodge components as follows

$$
\boldsymbol{f}_H(\boldsymbol{x}^*)|\boldsymbol{f}_1(\boldsymbol{x}) \sim \mathcal{N}\left(\boldsymbol{K}_H^{*\top}\boldsymbol{K}_1^{-1}\boldsymbol{f}_1(\boldsymbol{x}), \boldsymbol{K}_H^{**} - \boldsymbol{K}_H^{*\top}\boldsymbol{K}_1^{-1}\boldsymbol{K}_H^*\right) \tag{B.27a}
$$

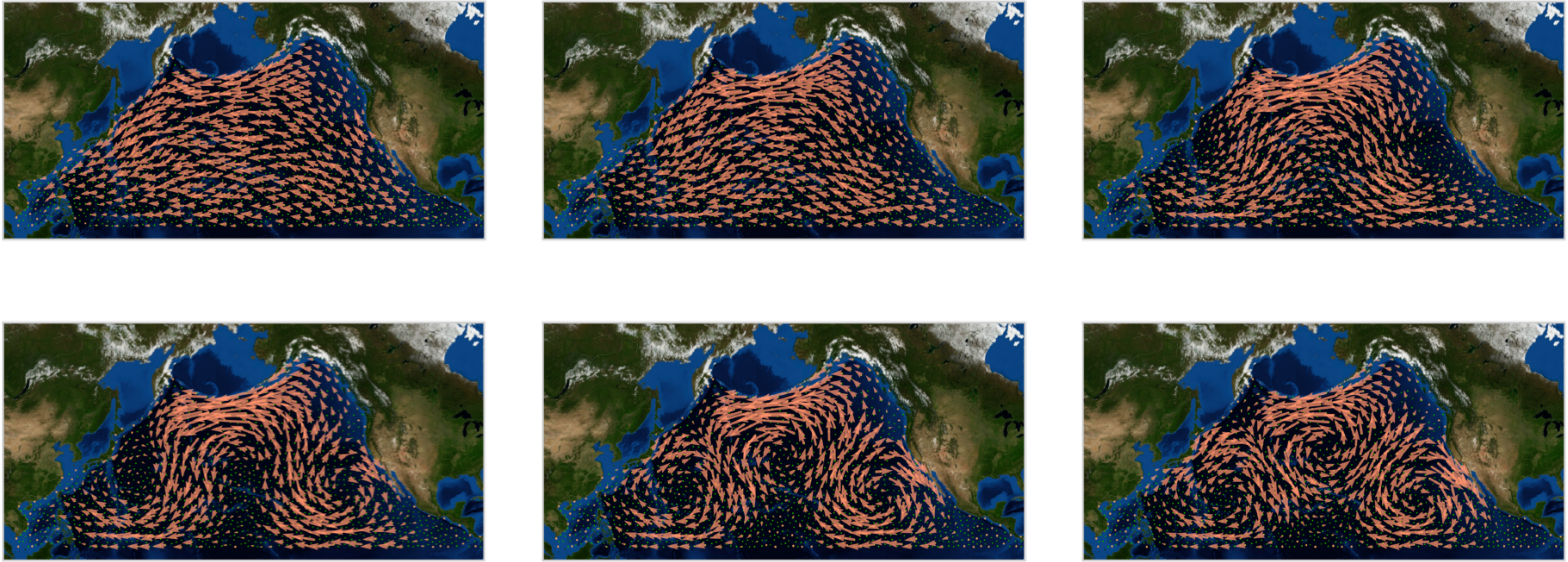
$$
\boldsymbol{f}_G(\boldsymbol{x}^*)|\boldsymbol{f}_1(\boldsymbol{x}) \sim \mathcal{N}\left(\boldsymbol{K}_G^{*\top}\boldsymbol{K}_1^{-1}\boldsymbol{f}_1(\boldsymbol{x}), \boldsymbol{K}_G^{**} - \boldsymbol{K}_G^{*\top}\boldsymbol{K}_1^{-1}\boldsymbol{K}_G^*\right) \tag{B.27b}
$$

$$
\boldsymbol{f}_C(\boldsymbol{x}^*)|\boldsymbol{f}_1(\boldsymbol{x}) \sim \mathcal{N}\left(\boldsymbol{K}_C^{*\top}\boldsymbol{K}_1^{-1}\boldsymbol{f}_1(\boldsymbol{x}), \boldsymbol{K}_C^{**} - \boldsymbol{K}_C^{*\top}\boldsymbol{K}_1^{-1}\boldsymbol{K}_C^*\right) \tag{B.27c}
$$

From these posterior distributions, we can directly obtain the means and the uncertainties of the Hodge components of the predicted edge function.

Learning dynamics of edge flows

e.g., from gradient to curl



$$(1-t)\mathbf{f}_{t=0} + t\mathbf{f}_{t=1}$$