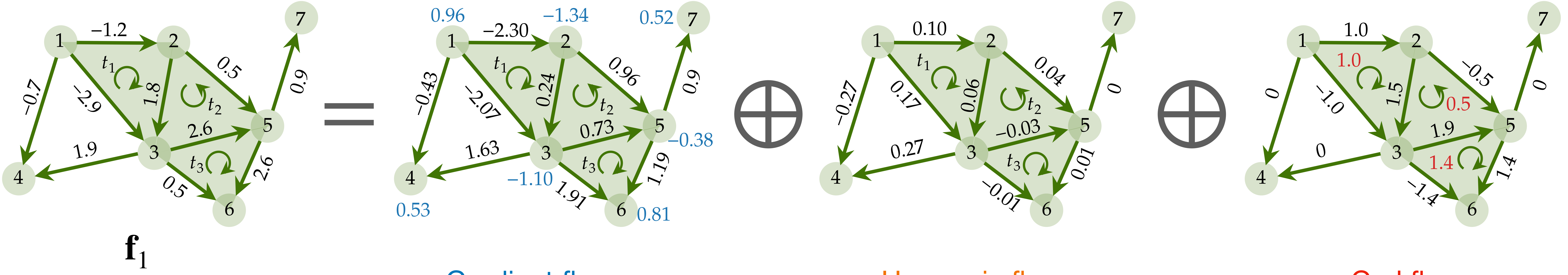
Hodge decomposition

Lovász et al. 2004; Lim et al. 2020

$$\mathbb{R}^{N_1} = \mathrm{im}(\mathbf{B}_1^\top) \oplus \ker(\mathbf{L}_1) \oplus \mathrm{im}(\mathbf{B}_2)$$
$$\mathbf{f}_1 = \mathbf{f}_G + \mathbf{f}_H + \mathbf{f}_C$$



Gradient flow
Curl-free, irrotational

Harmonic flow
Div- and curl-free

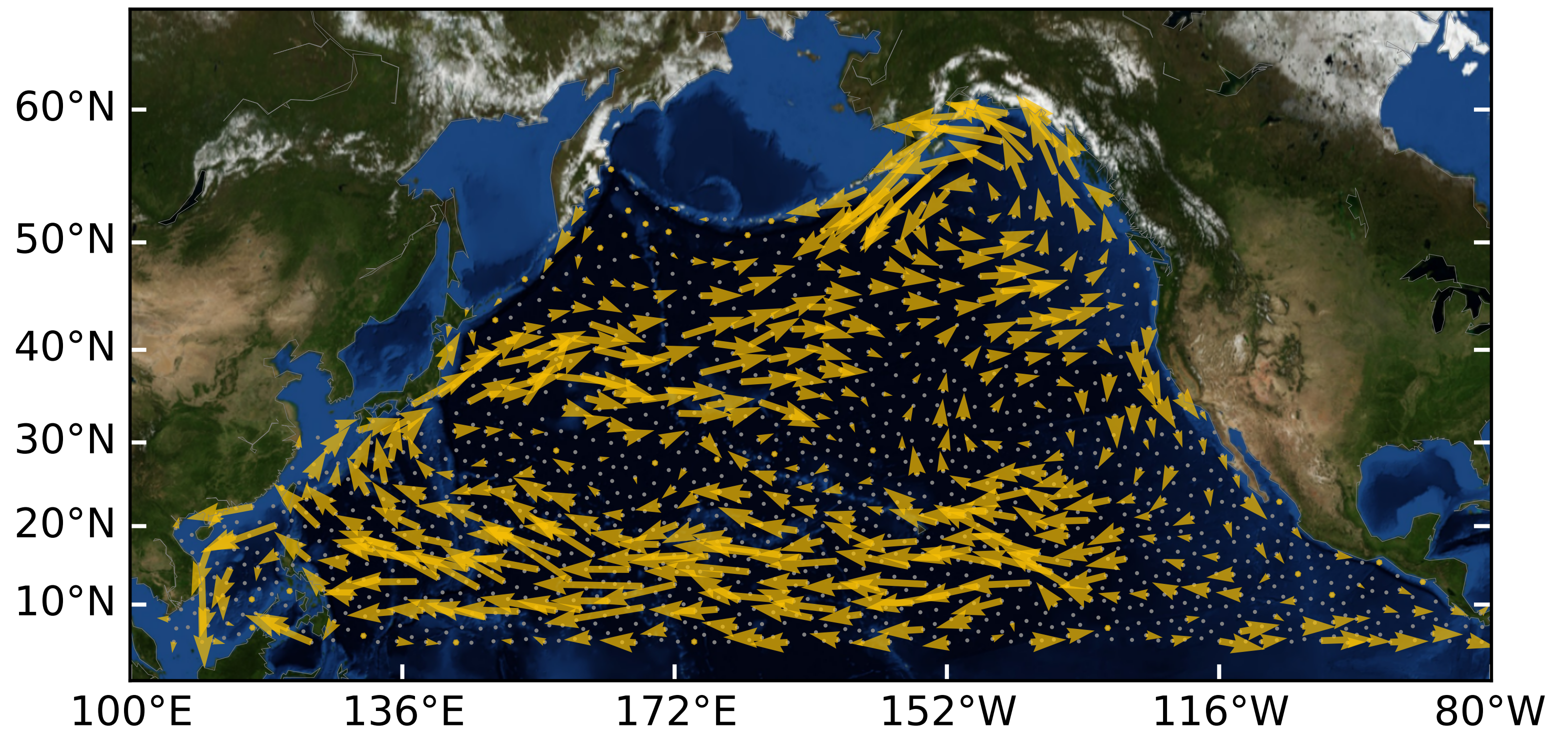
Curl flow
Div-free, solenoidal

- This holds for any simplex order k
- What is the case for $k = 0$?

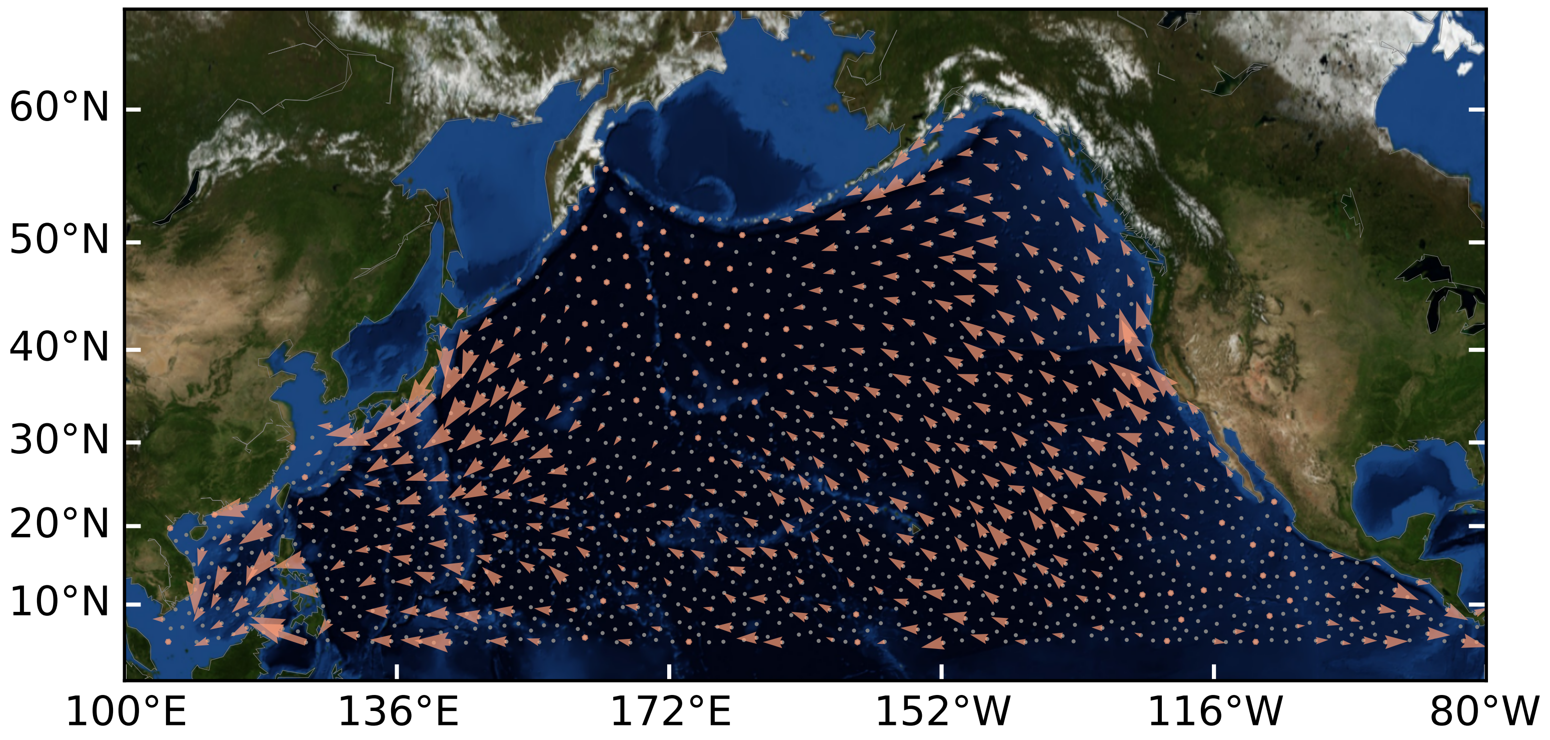
$$\mathbb{R}^{N_0} = \mathrm{im}(\mathbf{B}_1) \oplus \ker(\mathbf{L}_0)$$

- Characteristic decomposition

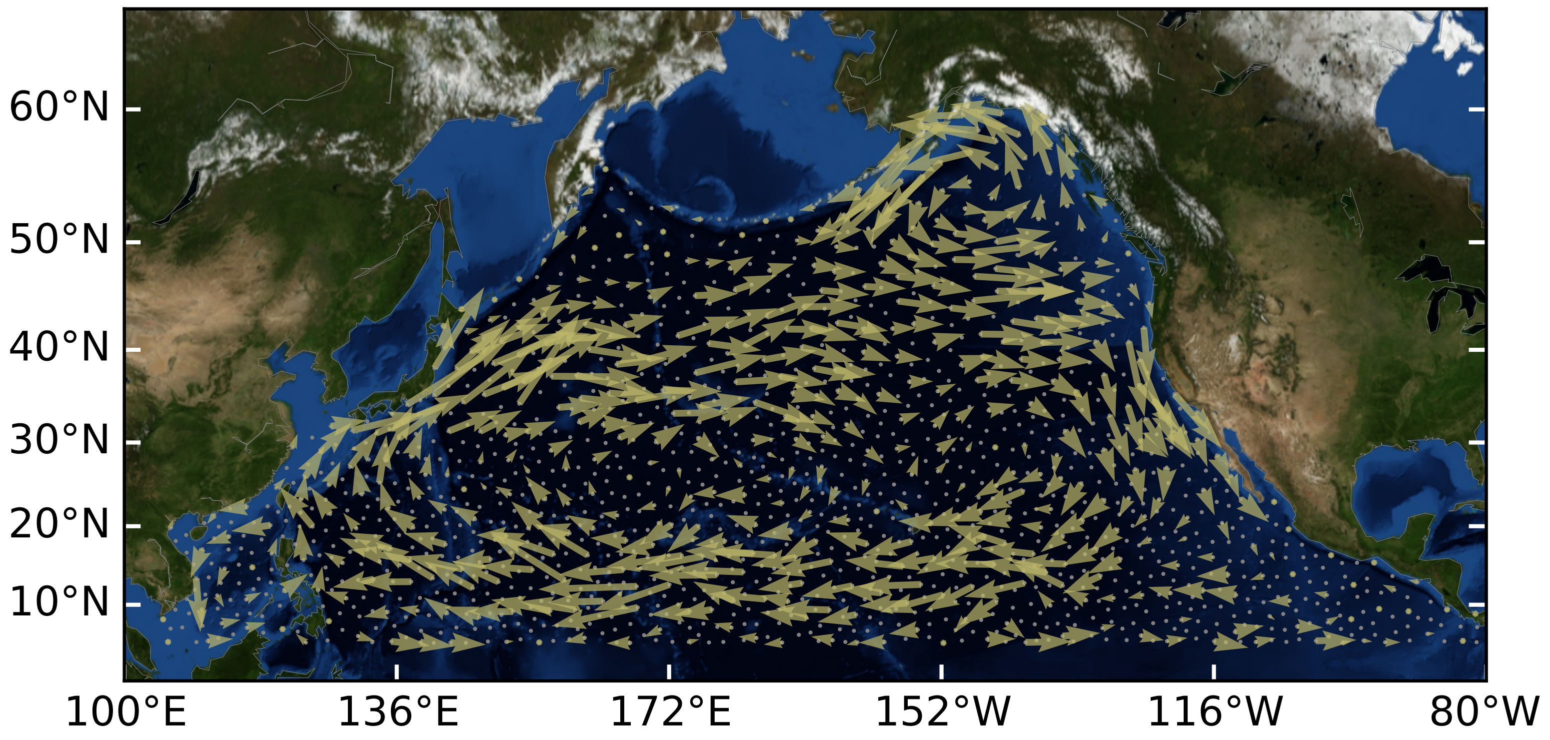
Applications of Hodge decomposition



Ocean currents



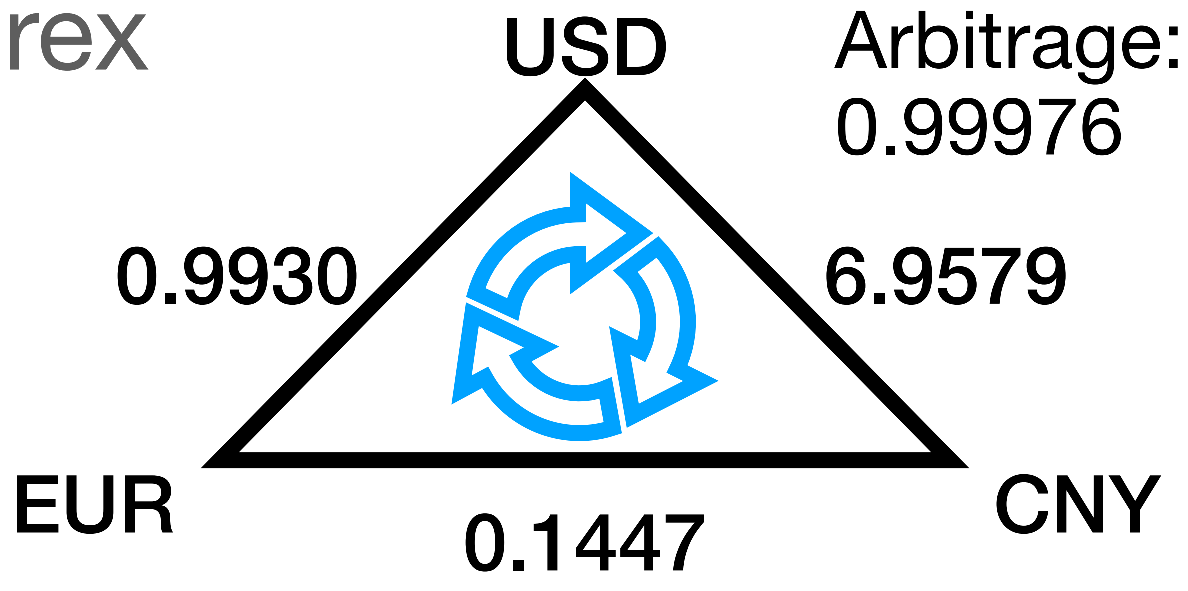
Gradient flow
Curl-free, irrotational



Curl flow
Div-free, solenoidal

Chen, Yu-Chia et al. (2021) ”Helmholtzian Eigenmap."

Forex



Arbitrage:
0.99976

$r^{a/b} r^{b/c} = r^{a/c}$ Arbitrage-free

$f_{[a,b]} + f_{[b,c]} - f_{[a,c]} = 0$ Curl-free

- Water flows (div-free)
- Electrical currents, voltages

- Brain networks (Anand et al. 2022)
- Game theory (Candogan et al. 2011)
- Ranking problems (Jiang et al. 2011)
- Random walks (Strang et al. 2020)
- …