Matérn Edge GPs

Derived from SDEs on the edge set

- $\mathbf{f}_1 \sim \mathrm{GP}(\mathbf{0}, \mathbf{K}_1)$

- Matérn graph kernel

$$\Phi(\mathbf{L}_1)\mathbf{f}_1 = \mathbf{w}_1, \text{ with}$$

$$\Phi(\mathbf{L}_1) = \left(\frac{2\nu}{\kappa^2}\mathbf{I} + \mathbf{L}_1\right)^{\frac{\nu}{2}} \text{ and } \mathbf{w}_1 \sim N(\mathbf{0}, \sigma^2\mathbf{I})$$

- The solution gives edge GPs

$$\text{Matérn: } \mathbf{f}_1 \sim \mathrm{GP}\left(0, \left(\frac{2\nu}{\kappa^2}\mathbf{I} + \mathbf{L}_1\right)^{-\nu}\right)$$

$$\text{Diffusion: } \mathbf{f}_1 \sim \mathrm{GP}\left(0, e^{-\frac{\kappa^2}{2}\mathbf{L}_1}\right)$$

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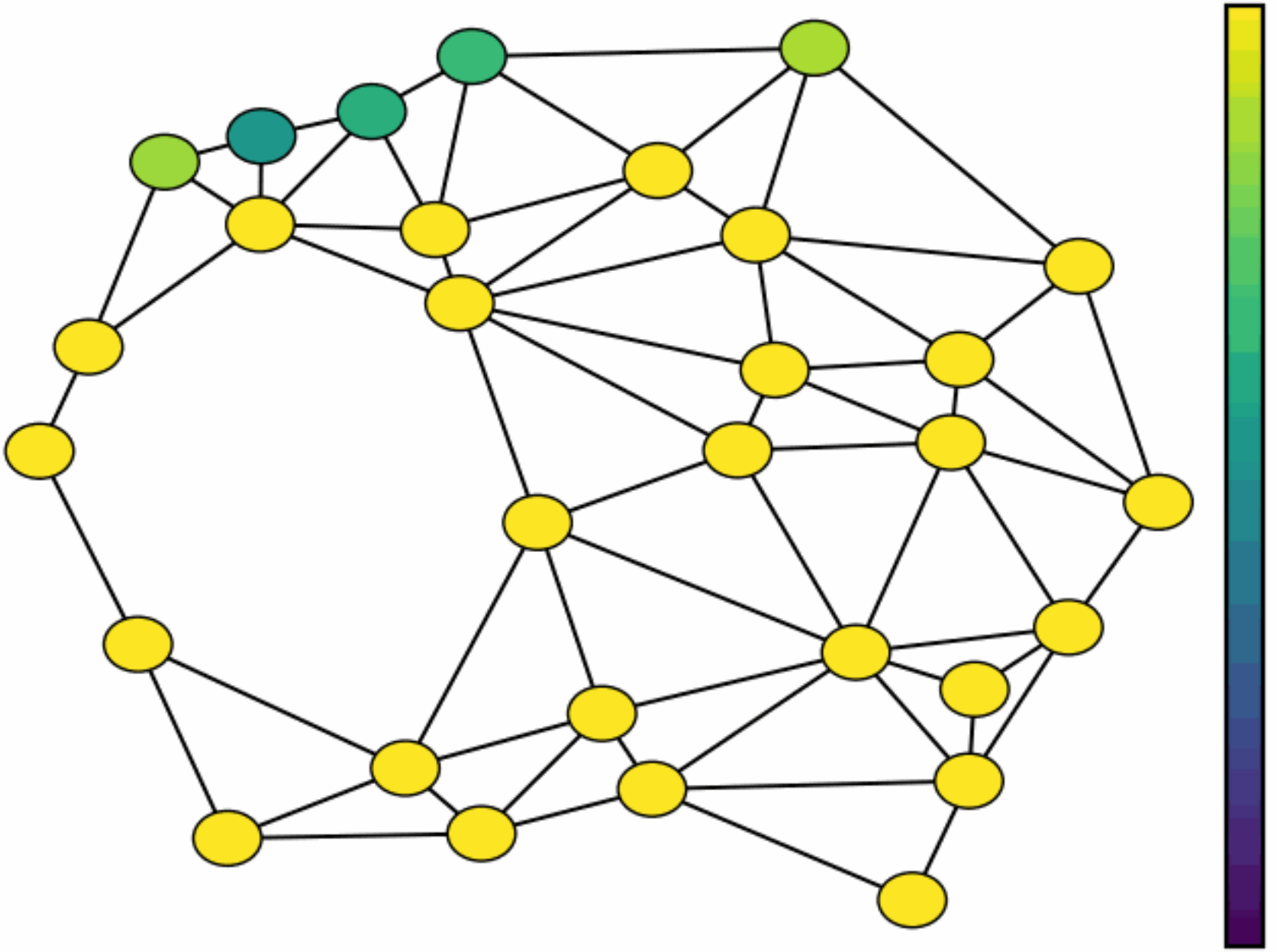
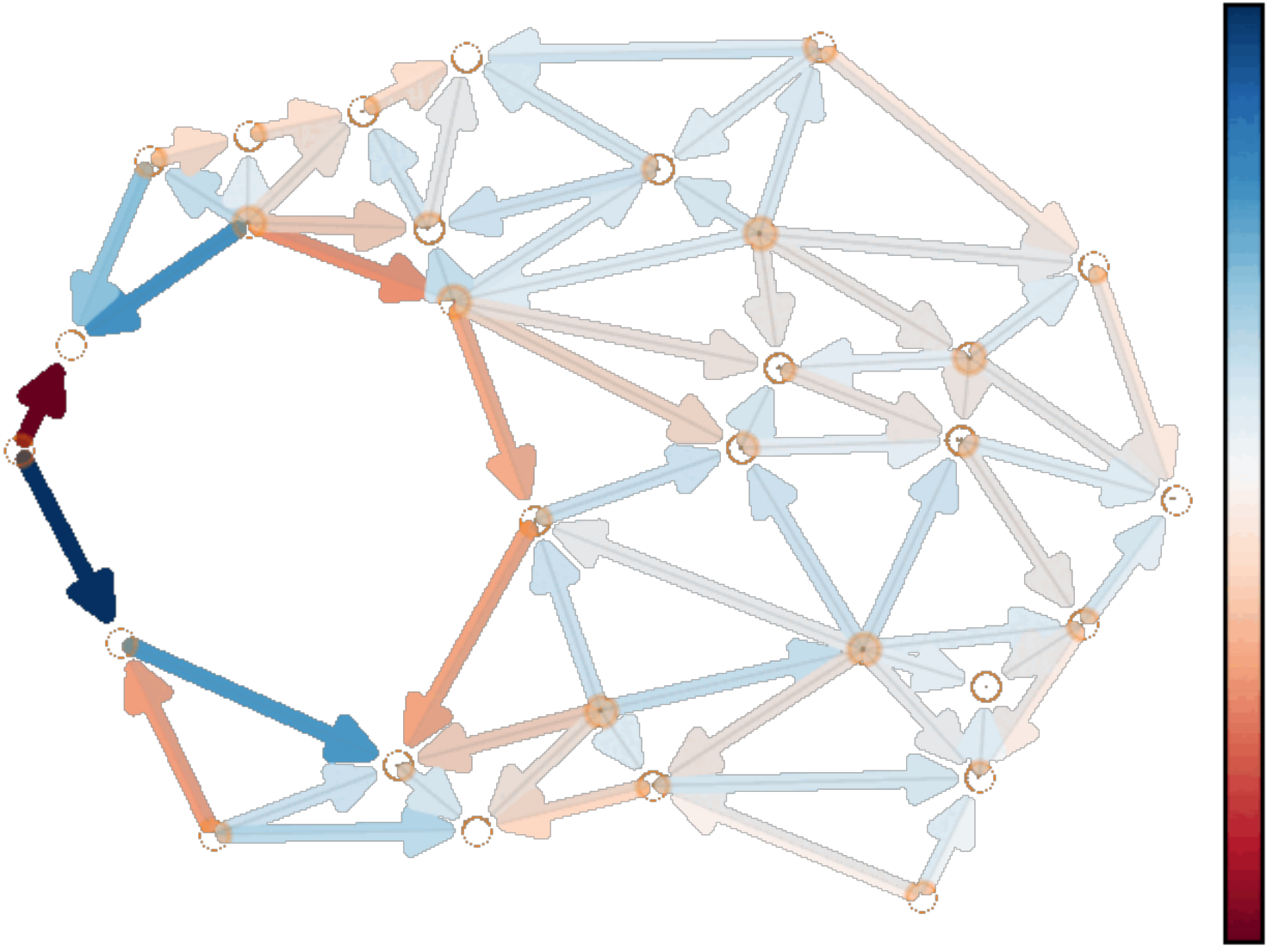
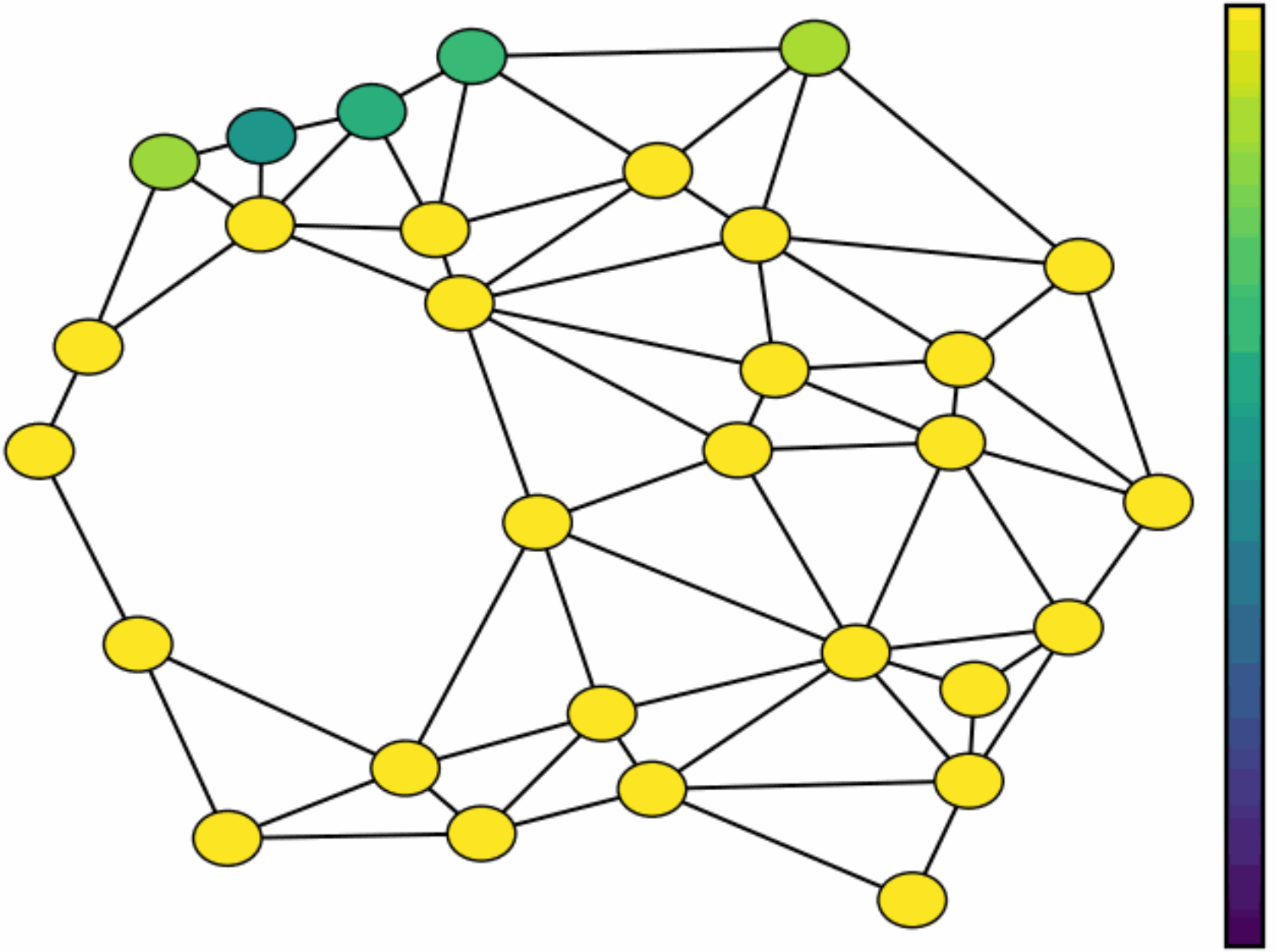
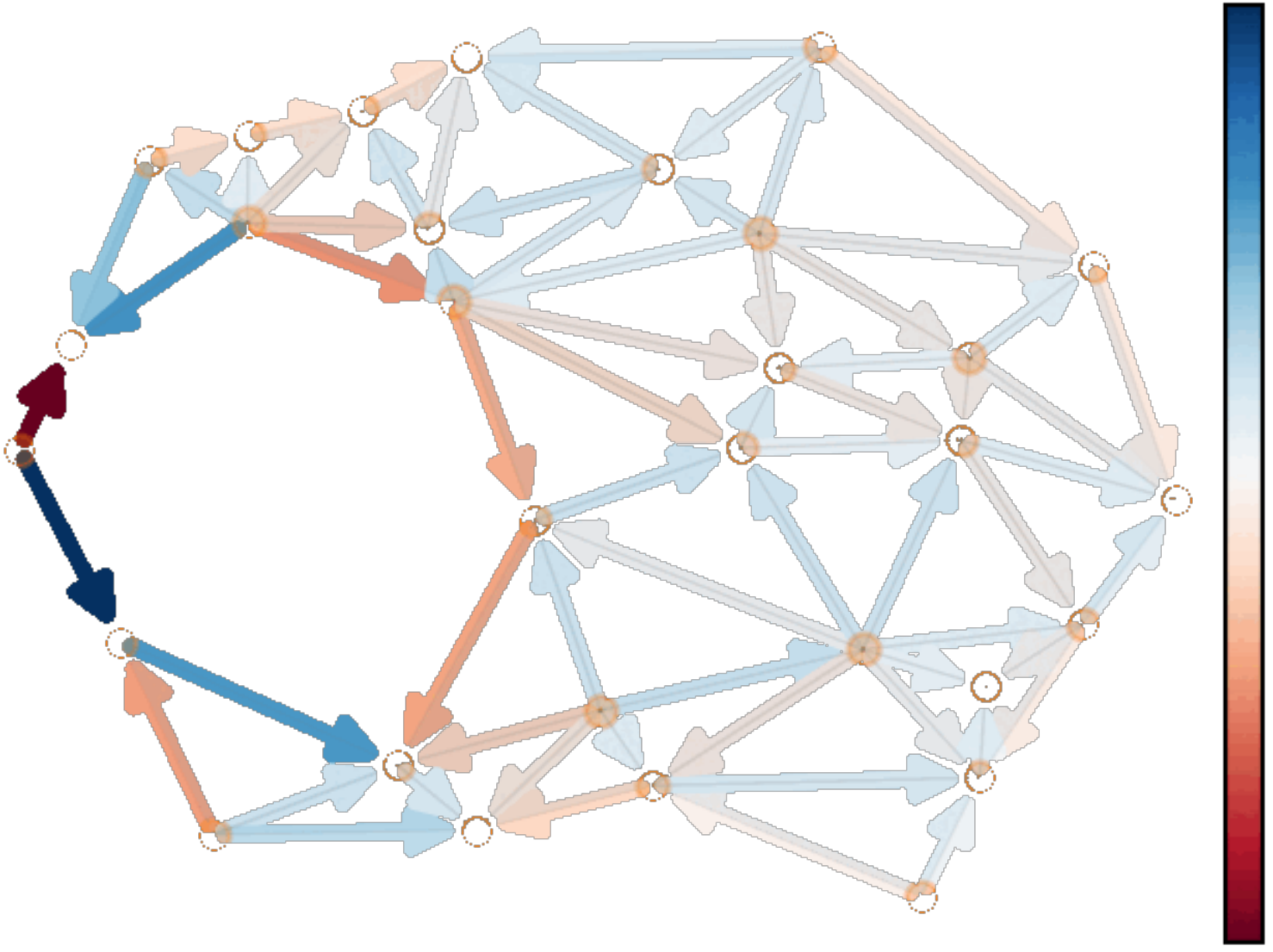
EVD: $\mathbf{L}_1 = \mathbf{U}_1 \mathbf{\Lambda}_1 \mathbf{U}_1^\top$





Diffusion on nodes vs on edges





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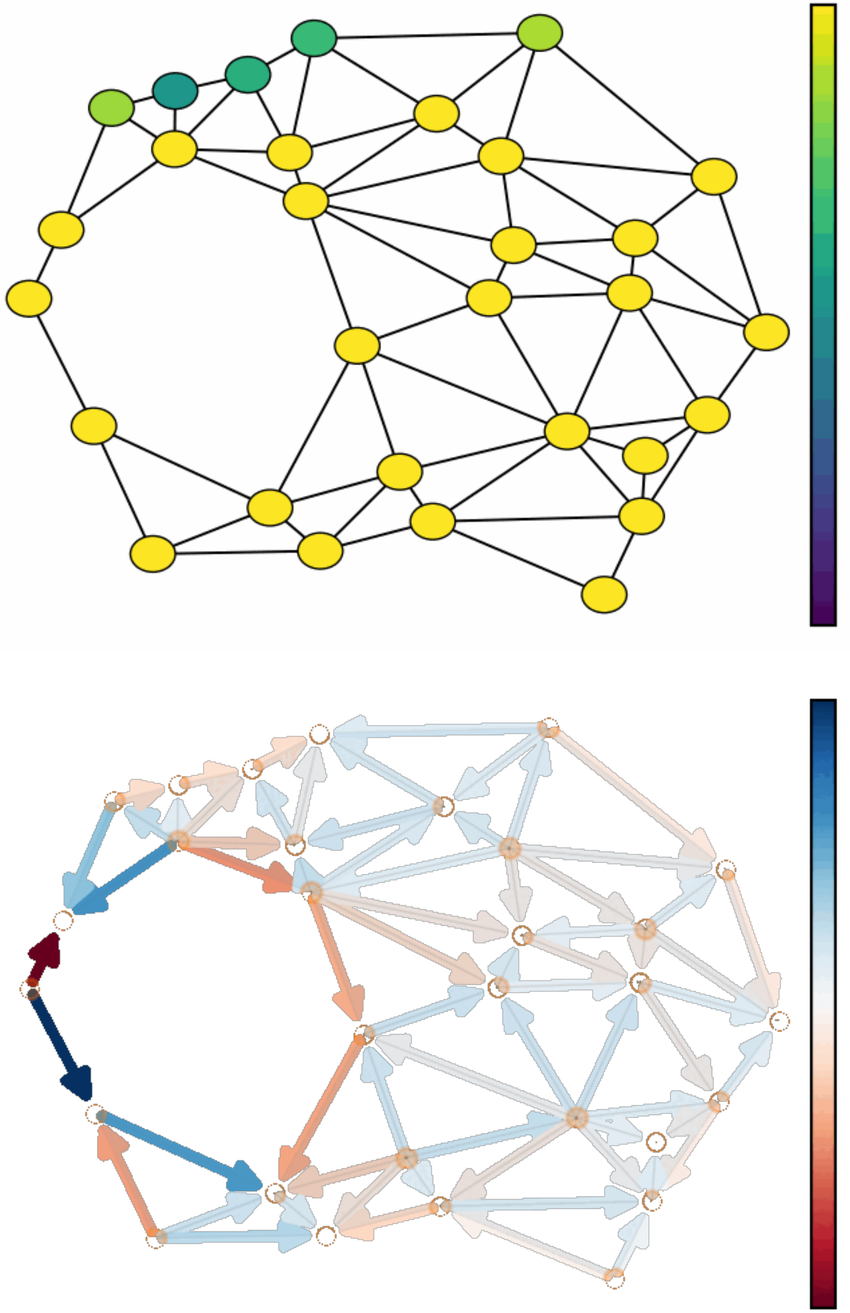

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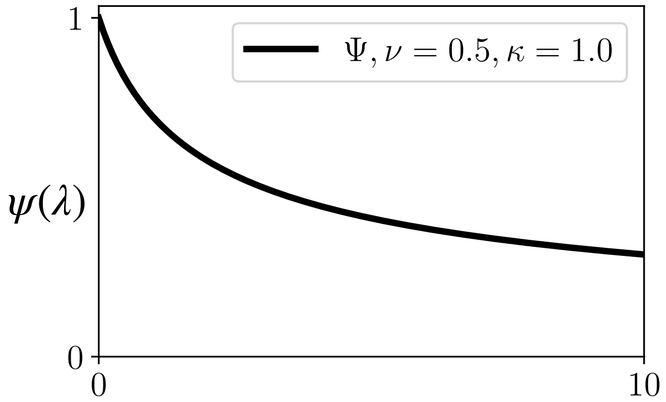

- Low-pass in the eigen-spectrum

Smoothness

Node function — 0-form (scalar field)

Edge function — 1-form (vector field)

Divergence

Curl