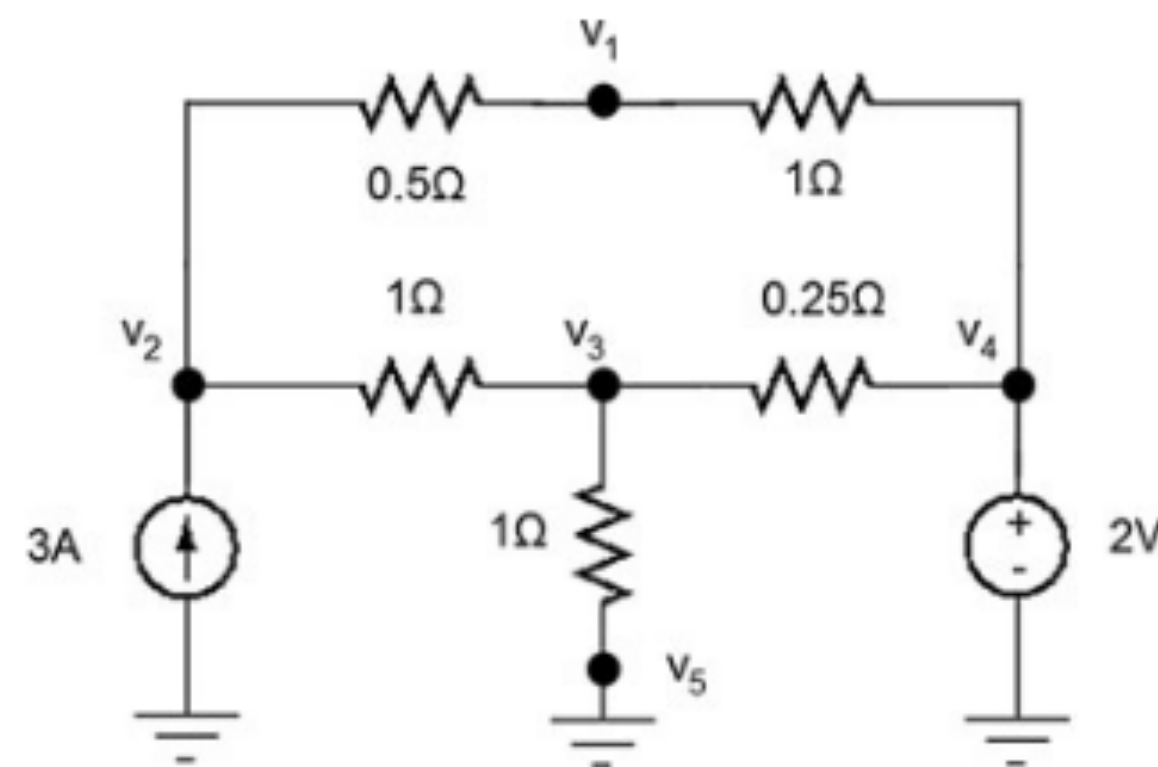


# Simplicial Signal Smoothness

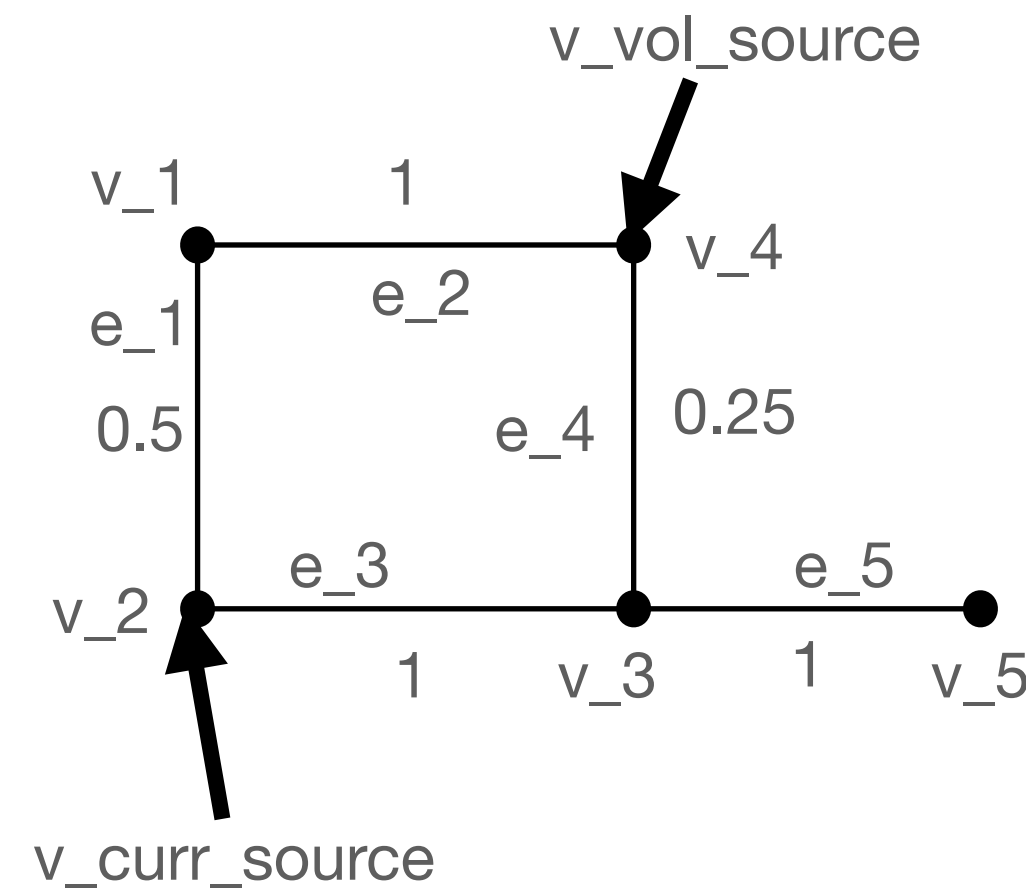
## Variations in terms of faces and cofaces

- For edge flows
  - Total divergence  $\|\mathbf{B}_1 \mathbf{f}\|_2^2 = \mathbf{f}^\top \mathbf{L}_d \mathbf{f}$
  - Total curl  $\|\mathbf{B}_2^\top \mathbf{f}\|_2^2 = \mathbf{f}^\top \mathbf{L}_u \mathbf{f}$
- For node signal:  $\|\mathbf{B}_1^\top \mathbf{v}\|_2^2 = \mathbf{v}^\top \mathbf{L}_0 \mathbf{v}$
- For general simplicial signals
  - Lower variation
  - Upper variation

# A Circuit toy example



(Grady et al. 2010)



$\mathbf{v} \in \mathbb{R}^{|\mathcal{N}|}$ : Electric potential on nodes

$$\mathbf{f}_{Vol} = \mathbf{B}_1^T \mathbf{v}: \text{(Kirchhoff's voltage law)}$$

$$\mathbf{f}_{currents} = \mathbf{G}^{-1} \mathbf{f}_{Vol}: \text{currents (Ohm's law)}$$

Diagonal resistance/conductance

$$\text{Kirchhoff's current law: } \mathbf{B}_1 \mathbf{f}_{currents} = \mathbf{0}$$

$$\text{Or } \mathbf{B}_1 \mathbf{f}_{currents} + \mathbf{v}_{curr\ source} = \mathbf{0}$$

$$\mathbf{B}_1 = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 \\ 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix} \quad \mathbf{v}_{vol} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ 2 \\ 0 \end{pmatrix}$$

$$\mathbf{B}_1 \mathbf{G}^{-1} \mathbf{B}_1^T \mathbf{v}_{vol} + \mathbf{v}_{curr\ source} = \mathbf{0}$$

Resistance — Metrics?  
Power, energy?