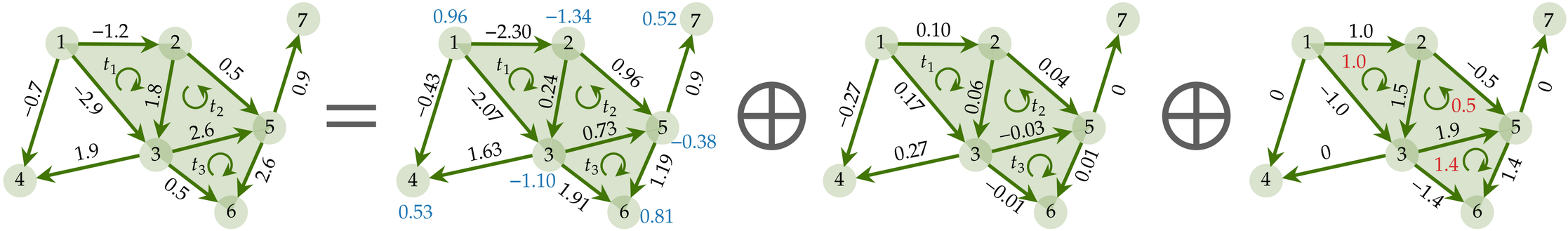
Hodge decomposition

Lovász et al. 2004; Lim et al. 2020

$$\mathbb{R}^{N_1} = \text{im}(\mathbf{B}_1^\top) \oplus \ker(\mathbf{L}_1) \oplus \text{im}(\mathbf{B}_2)$$
$$\mathbf{f}_1 = \mathbf{f}_G + \mathbf{f}_H + \mathbf{f}_C$$

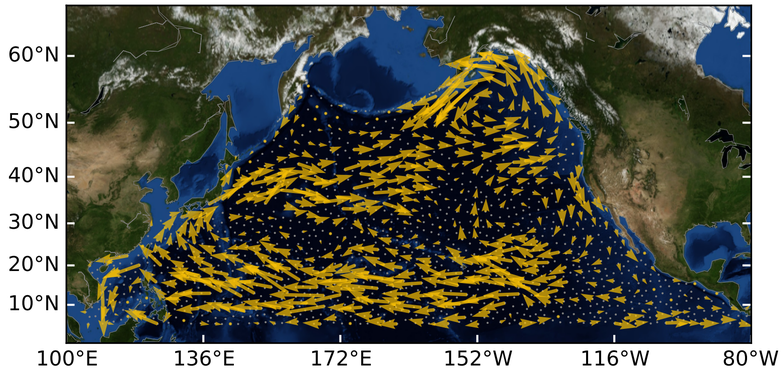


Edge flow = Gradient flow (Curl-free, irrotational) + Harmonic flow (Div- and curl-free) + Curl flow (Div-free, solenoidal)

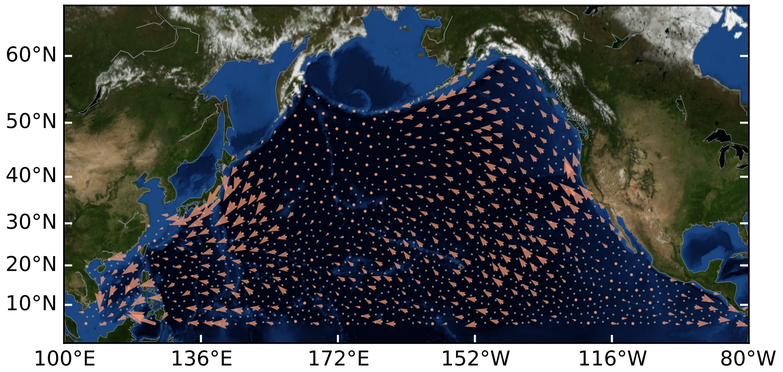
Hodge-compositional Edge GP

$$\mathbf{f}_G \sim \text{GP}(\mathbf{0}, \mathbf{K}_G)$$
$$\mathbf{f}_H \sim \text{GP}(\mathbf{0}, \mathbf{K}_H)$$
$$\mathbf{f}_C \sim \text{GP}(\mathbf{0}, \mathbf{K}_C)$$

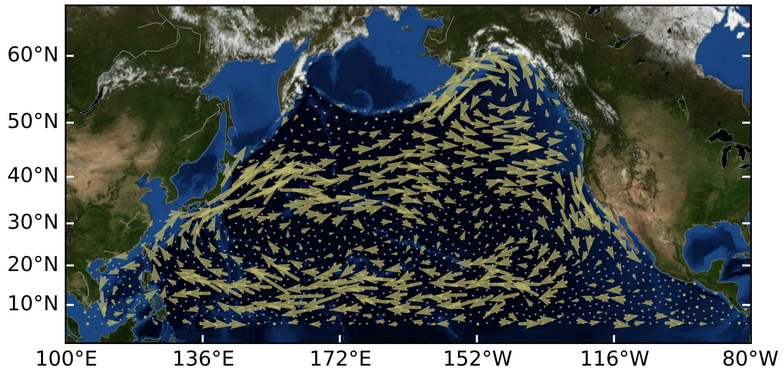
Applications of Hodge decomposition



Ocean currents



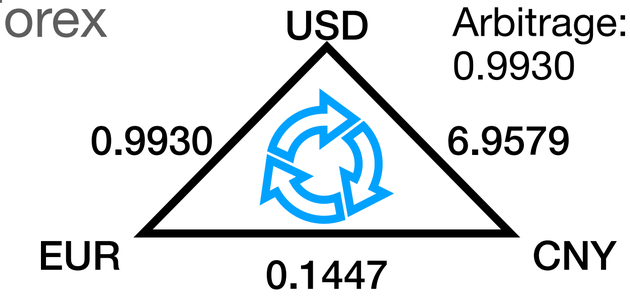
Gradient flow
Curl-free, irrotational



Curl flow
Div-free, solenoidal

Chen, Yu-Chia et al. (2021) "Helmholtzian Eigenmap."

Forex



Arbitrage:
0.9930

$$r^{a/b} r^{b/c} = r^{a/c}$$ Arbitrage-free

$$f_{[a,b]} + f_{[b,c]} - f_{[a,c]} = 0$$ Curl-free

- Water flows (div-free)
- Electrical currents (KCL), voltages (KVL)

- Brain networks (Anand et al. 2022)
- Game theory (Candogan et al. 2011)
- Ranking problems (Jiang et al. 2011)
- Random walks (Strang et al. 2020)
- ...