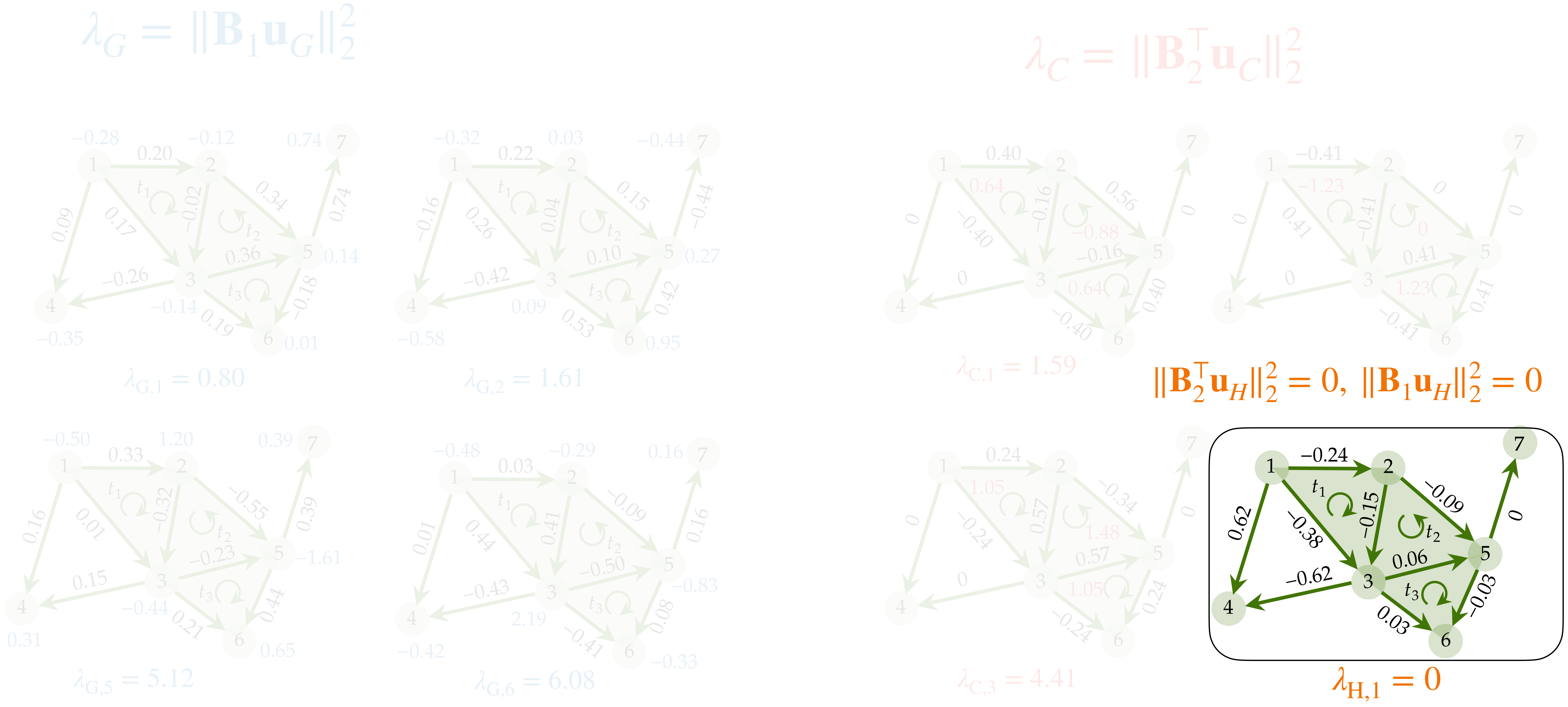
Eigenspace of $\mathbf{L}_1$ spans Hodge subspaces

- Kernel of Laplacian spans the harmonic space

Simplicial Fourier transform

Frequency — eigenvalues

Fourier basis — eigenvectors



EVD: $\mathbf{L}_1 = \mathbf{U}_1 \boldsymbol{\Lambda}_1 \mathbf{U}_1^\top$

$$\boxed{\begin{array}{c} \mathbf{U}_1 = [\mathbf{U}_H \ \mathbf{U}_G \ \mathbf{U}_C] \\ \mathrm{span}(\mathbf{U}_H) = \ker(\mathbf{L}_1) \end{array}}$$

Eigenspace of $\mathbf{L}_1$ spans Hodge subspaces

- **Nonzero** Eigenspace of down Laplacian spans the gradient space
- **Nonzero** Eigenspace of up Laplacian spans the curl space
- Kernel of Laplacian spans the harmonic space

Simplicial Fourier transform

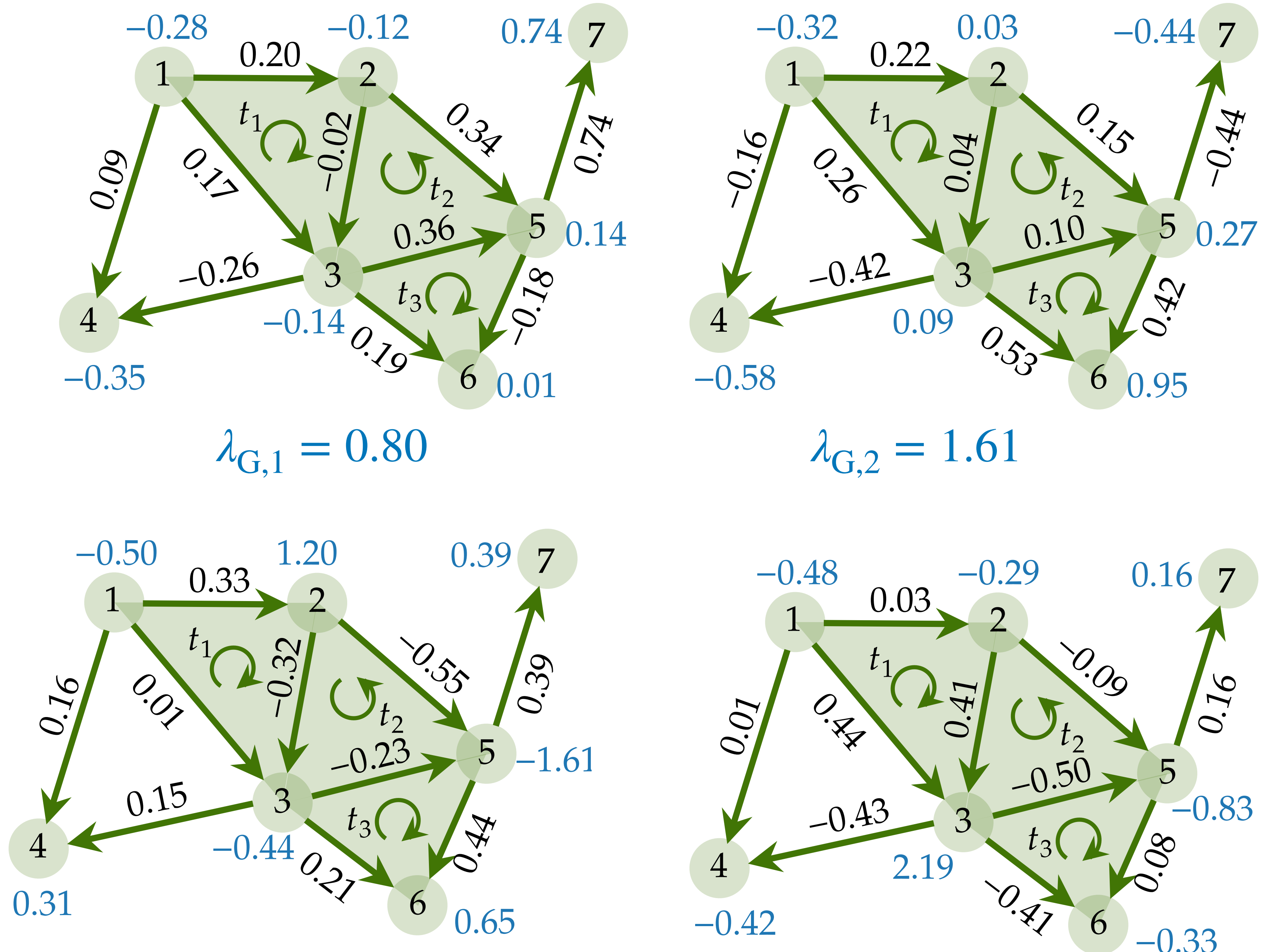
Frequency — eigenvalues

Fourier basis — eigenvectors

$\lambda_G = \|\mathbf{B}_1 \mathbf{u}_G\|_2^2$

Gradient eigenvector

Fourier basis reflecting divergent properties



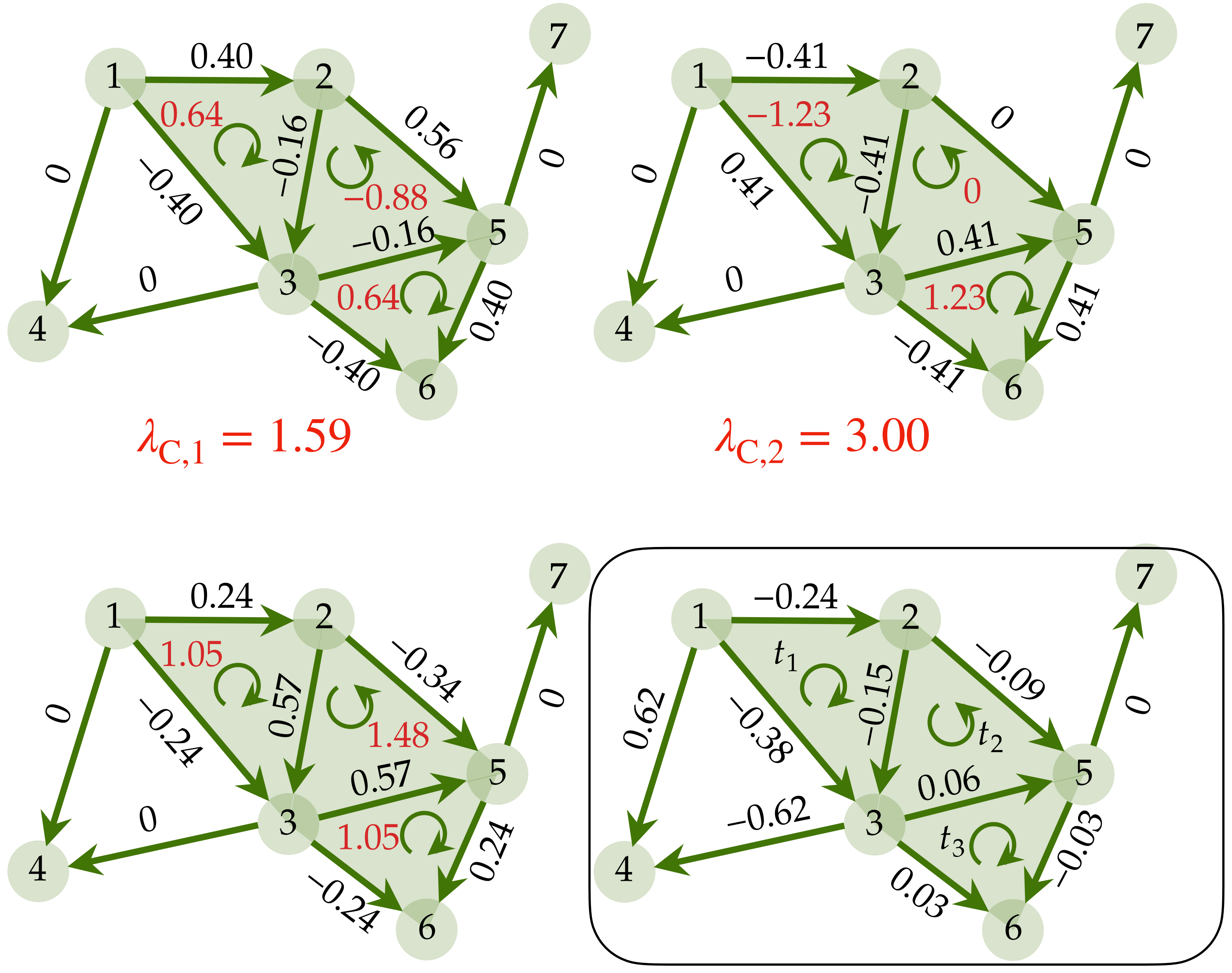
$\lambda_{G,1} = 0.80$ $\lambda_{G,2} = 1.61$ $\lambda_{G,5} = 5.12$ $\lambda_{G,6} = 6.08$

$k = 1$

$\lambda_C = \|\mathbf{B}_2^\top \mathbf{u}_C\|_2^2$

Curl eigenvector

Fourier basis reflecting rotational properties



$\lambda_{C,1} = 1.59$ $\lambda_{C,2} = 3.00$ $\lambda_{C,3} = 4.41$ $\lambda_{H,1} = 0$

EVD: $\mathbf{L}_1 = \mathbf{U}_1 \mathbf{\Lambda}_1 \mathbf{U}_1^\top$

$\mathbf{U}_1 = [\mathbf{U}_H \ \mathbf{U}_G \ \mathbf{U}_C]$
$\mathrm{span}(\mathbf{U}_H) = \ker(\mathbf{L}_1)$
$\mathrm{span}(\mathbf{U}_G) = \mathrm{im}(\mathbf{B}_1^\top)$
$\mathrm{span}(\mathbf{U}_C) = \mathrm{im}(\mathbf{B}_2)$

- $\tilde{\mathbf{x}}_k = \mathbf{U}_k^\top \mathbf{x}_k,\ k = 1$
- $\tilde{\mathbf{x}}_k = [\tilde{\mathbf{x}}_{k,\mathrm{H}}^\top, \tilde{\mathbf{x}}_{k,\mathrm{G}}^\top, \tilde{\mathbf{x}}_{k,\mathrm{C}}^\top]$