# TSB-based learning model

- Under the recent SB-based learning framework [Vargas 2021, De Bortoli 2021, Chen 2022]
- Learnable models $(Z_t(\theta), \hat{Z}_t(\hat{\theta}))$ for optimal policies $(Z_t, \hat{Z}_t)$
  - NNs, graph/simplicial NNs
- Trainable objective relating the TSBP objective and the models

$$\mathscr{L}_{TSB}(x_0) = \mathbb{E}\left[\log \nu_1(X_1)\right] - \int_0^1 \mathbb{E}\left[\frac{1}{2}\|Z_t\|^2 + \frac{1}{2}\|\hat{Z}_t\|^2 + \nabla \cdot (g_t \hat{Z}_t - f_t) + \hat{Z}_t^\top Z_t\right] dt$$

- Particular choices of models give topological variants
- diffusion models using score-matching [Song et al. 2021]

$$Z_t = 0, \quad \hat{Z}_t = g_t \nabla \log p_{t|0}$$

- Diffusion bridge models based on Doob's $h$-transform for a particular final distri.
- Probability flow ODE: flow-matching [Lipman et al. 2022]

**TSB-learning model**

| | |
|---|---|
| $Z_t \approx Z_t(\theta)$ | $l(x_0; \phi)$ |
| $\tilde{Z}_t \approx \tilde{Z}_t(\phi)$ | $l(x_1; \theta)$ |
| **Learnable** | **Trainable** |

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