

# Tabular results

Table 1: Forex rates inference results.

Method	RMSE		NLPD	
	Diffusion	Matérn	Diffusion	Matérn
Euclidean	$2.17 \pm 0.13$	$2.19 \pm 0.12$	$2.12 \pm 0.07$	$2.20 \pm 0.18$
Line-Graph	$2.43 \pm 0.07$	$2.46 \pm 0.07$	$2.28 \pm 0.04$	$2.32 \pm 0.03$
Non-HC	$2.48 \pm 0.07$	$2.47 \pm 0.08$	$2.36 \pm 0.07$	$2.34 \pm 0.04$
HC	$0.08 \pm 0.12$	$0.06 \pm 0.12$	$-3.52 \pm 0.02$	$-3.52 \pm 0.02$

Table 3: WSN inference results.

Method	Node Heads		Edge Flowrates	
	RMSE	NLPD	RMSE	NLPD
Diffusion, non-HC	$0.16 \pm 0.05$	$0.72 \pm 2.06$	$0.32 \pm 0.05$	$0.97 \pm 1.80$
Matérn, non-HC	$0.16 \pm 0.04$	$0.71 \pm 2.39$	$0.26 \pm 0.05$	$0.10 \pm 0.13$
Diffusion, HC	$0.15 \pm 0.04$	$-0.47 \pm 0.14$	$0.22 \pm 0.03$	$-0.20 \pm 0.13$
Matérn, HC	$0.15 \pm 0.04$	$-0.25 \pm 0.48$	$0.23 \pm 0.03$	$-0.45 \pm 0.49$

Table C.1: Ocean current inference results.

Method	RMSE			NLPD		
	Diffusion	Matérn	Hodge Laplacian	Diffusion	Matérn	Hodge Laplacian
Euclidean	$1.00 \pm 0.01$	$1.00 \pm 0.00$	—	$1.42 \pm 0.01$	$1.42 \pm 0.10$	—
Line-Graph	$0.99 \pm 0.00$	$0.99 \pm 0.00$	—	$1.41 \pm 0.00$	$1.41 \pm 0.00$	—
Non-HC	$0.35 \pm 0.00$	$0.35 \pm 0.00$	$0.35 \pm 0.00$	$0.33 \pm 0.00$	$0.36 \pm 0.03$	$0.33 \pm 0.01$
HC	$0.34 \pm 0.00$	$0.35 \pm 0.00$	$0.35 \pm 0.00$	$0.33 \pm 0.01$	$0.37 \pm 0.04$	$0.33 \pm 0.01$

# Sampling gradient and curl edge GPs

*Proof.* We focus on the case of gradient GPs. First, we can decompose the gradient kernel in terms of  $U_1 = [U_H \ U_G \ U_C]$  as

$$K_G = U_1 \begin{pmatrix} \mathbf{0} & & \\ & \Psi_G(\Lambda_G) & \\ & & \mathbf{0} \end{pmatrix} U_1^\top. \quad (\text{B.9})$$

From a vector  $\mathbf{v} = (v_1, \dots, v_{N_1})^\top$  of variables following independent normal distribution, we can draw a random sample of gradient function as

$$\mathbf{f}_G = U_1 \text{diag}([\mathbf{0}, \Psi_G^{\frac{1}{2}}(\Lambda_G), \mathbf{0}]) \mathbf{v} \quad (\text{B.10})$$

where  $\text{diag}([\mathbf{a}, \mathbf{b}, \mathbf{c}])$  is the diagonal matrix with  $(\mathbf{a}, \mathbf{b}, \mathbf{c})^\top$  on its diagonal.

Therefore, their curls are

$$\text{curl } \mathbf{f}_G = \mathbf{B}_2^\top U_1 \text{diag}([\mathbf{0}, \Psi_G^{\frac{1}{2}}(\Lambda_G), \mathbf{0}]) = \mathbf{B}_2^\top U_G \Psi_G^{\frac{1}{2}}(\Lambda_G) = \mathbf{0}. \quad (\text{B.11})$$

Likewise, we can show the samples of a curl GP are div-free.