Alpha Model of Fundamental Factors, The Explanatory Power of Statistical Factors in Stock Return

Li Yu

Mathematical Finance Program, Rutgers University

16:643:625:01: Portfolio Theory

December 11th, 2021

Abstract: In this project, I am told to construct a portfolio with a tracking error constrain as 3 percent. Instead of using the most common alpha model which is Fama-Macbeth model, I will construct one alpha model consisted with fundamental factors. My fundamental alpha model is composed of company performance, including return on assets (ROA), basic earnings per share (EPS), net profit margin, current ratio (CR), dividends per share (DPS), and net fix assets to the total asset (FA/TA). Statistical factors such as the coefficient of variation (CV) of stock returns, stock return skewness and stock return kurtosis are chosen to be test on their predictive power. This study aims to figure out whether a fundamental alpha model performs better than the benchmark and whether statistical factors have explanatory power to the future return. Information ratio (IR) and information coefficient (IC) of my strategy will also be included in my results. In addition, since only annually fundamentals data are available, alphas calculated by annual data is not precious. In order to make good use of monthly data, I also construct a CAPM alpha model to calculate individual stocks alphas, then construct an optimal portfolio.

Keywords: Tracking error constrain, portfolio, alpha model, fundamental factor,

statistical factor, IR, IC, CAPM model

Introduction and literature review:

Nowadays, thousands or even millions of active portfolio managers are trying to find the best alpha model that can realize the highest excessing return of the market. Investors also eager to find specific factors that bring with higher return than the market. Understanding how and why certain stock prices grow up while others go down is a remarkable problem for equity analysis. There is a considerable literature on predicting future returns. Fundamental and technical information can be used as a basis for investors to predict the return, risk or uncertainty, the amount, timing, and other factors associated with investment activities in the capital market (Husnan, 2004). If the fundamental indexes such as the return on assets and liquidity of a company are a way more prosperous than the other companies in the same market, the company's equity price is considered to grow up because of its outperformance than the other competence. Overall, eventually, growing earning and growing economy will be reflected in the share price growth (Teweless and Bradley, 1998).

As for statistical factors, specifically, the statistical factors that we will take into consideration are the coefficients of variation (CV), skewness, and kurtosis of the stock return. The coefficient of variation shows stock return volatility while skewness and kurtosis can also be closely related to returns. Conventionally, financial analysis has idiosyncratic preferences regarding the first and second moments of stock return while my intuition proposes that traders should also concentrate more on right skewed stocks or stocks with high coefficient of variation. In fact, skewness can explain the excess returns found in portfolios of low-size and high book-to-market in traditional models. (José Manuel Cueto, Aurea Grané and Ignacio Cascos)

Methodology:

Alpha Model with Fundamental Factors

Why would I choose the following six fundamentals?

Return on Asset (ROA)

If a company's can constantly generate profits, then the price of the company's equity will certainly grow up, which result in increased stock returns in the future. Return on Assets (ROA) is a ratio that measures a company's ability of generation profits by its own assets.

Current Ratio (CR)

The current ratio is the index that measures the company's ability to repay short-term liabilities with its current assets.

Net Income Margin (NIM)

The idea of including net income margin is relatively the same as using return on assets as a factor. Net income margin is a ratio that measures the level of return net income to net sales.

Earnings per Share (EPS)

Earnings per share (EPS) is calculated as a company's profit divided by the outstanding shares of its common stock. The resulting number serves as an indicator of a company's profitability.

Dividends per Share (DPS)

Dividend per share (DPS) is the sum of declared dividends issued by a company for every ordinary share outstanding. A higher DPS indicate a better ability of earning profits.

Net Fix Assets to Total Assets (FATA)

Net fix assets to the total assets ratio measure the level of fix assets to the total assets. A company that has more fix assets tends to have less diversifying profit generating methods.

In this paper, my portfolio is constructed by top thirty stocks in S&P500 ranked by capitalization which are AAPL UW Equity, ABT UN Equity, ADBE UW Equity, AMZN UW Equity, BAC UN Equity, CMCSA UW Equity, CRM UN Equity, CSCO UW Equity, DHR UN Equity, DIS UN Equity, GOOGL UW Equity, HD UN Equity, JNJ UN Equity, JPM UN Equity, KO UN Equity, LLY UN Equity, MA UN Equity, MSFT UW Equity, NFLX UW Equity, NKE UN Equity, NVDA UW Equity, ORCL UN Equity, PG UN Equity, TMO UN Equity, TSLA UW Equity, UNH UN Equity, V UN Equity, VZ UN Equity, WMT UN Equity and XOM UN Equity. As what have been discussed by Adamily and Taufik, they implement a single stocks' return regression model as:

$$Y\ Stock\ \text{Return} = \beta 1 + \beta 2 * EPS + \beta 3 * DPS + \beta 4 * FAtoTA + \beta 5 * ROE + \beta 6 * (ROA)$$

Regression model

The modification from a single stock to a portfolio requires the calculation of the corresponding weight for each stock in our portfolio. Given a 3% tracking error constraint, I made a use of a Python package calling PyPortfolio0pt to solve the optimal weight of my portfolio. The optimal weight of year t is calculated from the stock return in year t-2 and year t-1 and will be updated yearly. In the end, I get 13 yearly optimal weight from 2010 to 2021. You can find the code in Appendix A. My fundamental alpha model is

Portfolio Return =

Intercept + ROA*β1 + NIM*β2 + EPS*β3 + CR*β4 + DPS*β5 + FATA*β6

My intuition is that all beta values are positive due to all these six fundamentals have a positive relationship with a company's ability to make profits, which means the higher the fundamentals are, the more profits the company earns. I will discuss this in the discussion part if the result is not the same as my instinct.

The performance of my strategy is evaluated by sharp ratio measurement. If the left-hand side is greater than the right-hand side, we conclude that the portfolio return is statistically better than benchmark return.

$$\frac{E(r_P - r_f)}{\sigma_P} - \frac{E(r_B - r_f)}{\sigma_B} > 2\sqrt{\frac{2}{N}}$$

(N = Number of observations)

Predictive power of statistical factors

The aim of next part of my project is to figure out whether better statistics have inclination on better stock return performance. I use three statistics in my portfolio selection which are coefficient of variation (CV), Skewness (Skew) and Kurtosis. (Kurt) I will construct eight portfolios as follows, (each portfolio has a constraint of 3% tracking error)

	Portfolio	CV	Kurt	Skew
(1)	1-1-1	Low	Low	Low
(2)	1-1-2	Low	Low	High
(3)	1-2-2	Low	High	High
(4)	1-2-1	Low	High	Low
(5)	2-1-1	High	Low	Low
(6)	2-1-2	High	Low	High
(7)	2-2-1	High	High	Low
(8)	2-2-2	High	High	High

Coefficient of variation of price for each company:

$$\text{CV} = \frac{S_n}{\bar{x}}$$

Where $S_n^2 = standard\ deviation$ and $\bar{x}$ is the sample mean. CV represents the relative spread for positive random variables.

Skewness (Skew) of prices for each year and company:

$$\text{Skew} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^3}{S_n^3}$$

Positive stock return skewness indicates that the distribution of stock return is right skewness, which means certain stock has greater opportunity of getting positive return. Lower of even negative skewness demonstrate a larger probability of negative return.

Excess Kurtosis (Kurt) of prices for each year and company:

$$\text{Kurt} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^4}{S_n^4} - 3$$

Kurtosis measure the fatness of the stock return distribution. Positive kurtosis indicates that the stock returns have less extreme return while the contract have more extreme return than normal distribution.

(Remark: The largest four index are considered as high and the lowest four index are consider as low. I extend the number of largest and lowest index to five when constructing portfolio 2-2-2 for the reason that number of stocks are not enough to compute a semidefinite covariance.)

Finally, I use student-t test to test if the portfolio i-i-i return exceeds benchmark return. My null hypothesis and alternative hypothesis are: (95% significant level)

H_0: The return of portfolio i-i-i is less than or equal to benchmark return.

H_A: The return of portfolio i-i-i is greater than benchmark return.

My instinct tells me higher CV, skewness or kurtosis of stock return leads to stable positive return, in other words, those portfolios with more 2-2-2 will enjoy higher returns.

CAPM Alpha Model

For traditional CAPM model, it is familiar to us that

$$Return_{stock} - alpha = r_f + beta * (Return_{benchmark} - r_f)$$

So, we have:

$$alpha = \ Return_{stock} - r_f - beta * (Return_{benchmark} - r_f)$$

r_f = Risk free rate

Beta = $\frac{Cov(Return_{i,}Return_{benchmark})}{Var(Return_{benchmark})}$

(For convenience, r_f equals to 0 to this point since risk free rate varies day to day and it is hard to give a reasonable estimation of it through 10 years)

After calculating individual stocks alphas from 134-time windows, we use lambda, covariance matrix and alphas as inputs to compute our optimal portfolio active weights. In order to find optimal portfolio weights, we want to maximize

w_a'\alpha - 0.5 \lambda w_a'\Sigma w_a

subject to

sum w_a = 0,

-0.3 <= w_a' \beta <= 0.3

-w_b <= w_a <= 1 - w_b

We use cvxopt package in python to solve the optimization problem. Then we can easily get optimal portfolio weight and get both real returns and predictive returns of our portfolio. Finally, IC and IR can be computed out easily.

Data description:

Fundamental data are from Bloomberg FA and S&P500 stock prices are downloaded from Yahoo Finance. Since fundamental data are fully shown annually, I choose annual stock return but not daily or monthly stock return. However, there are numerous of miss fundamental data before 2009 so I only collect data after 2009. To this point, there will be a problem that I only have twelve observations in our OLS model, which is not sufficient for our alpha model.

The OLS regression model is used in this project. Note that all the fundamentals in my model equals to the sum of product of the optimal weight and the corresponding individual stock's fundamentals. For instance, $ROA_p = \sum W_i * ROA_i$.

(W_i is the optimal weight and ROA_i is ROA of the individual stock) After regressing the weighted fundamentals on the portfolio return, I use the coefficient vector dot product with the fundamentals in t year to get the annual portfolio return prediction for t+1 year. Then alpha equals to the annual portfolio return prediction of t+1 year minus the benchmark return of t+1 year. (Benchmark return is S&P500 annual return)

In the second part, I calculate monthly stock return CV, Skew and Kurt and classify stocks by high CV, low CV, high Skew, low Skew, high Kurt and low Kurt. Source of data is the same as part one.

In the CAPM alpha model part, I use monthly returns and all of the rolling OLS are using a time window of 24 months. Consequently, we will get 134 time periods results for all alphas, predictive returns and IRs.

Result:

Alpha Model with Fundamental Factors

Table one is the result summary of OLS regression and table two is the corresponding β values for six fundamental factors. Note that P> |t| value is too big, which is caused by the lack of yearly data. From the coefficient result, factor CR has the greatest coefficient which is 13.7735. We have evidence that current ratio contributes the most to stock returns. Surprisingly, the coefficient of ROA is -1.3813, which contradicts our initial guess. If more data were given, the coefficient of ROA might be greater than zero.

Table 1:

OLS Regression Results

Dep. Variable:	Annual Returns	**R-squared:**	0.647
Model:	OLS	**Adj. R-squared:**	0.224
Method:	Least Squares	**F-statistic:**	1.530
Date:	Fri, 10 Dec 2021	**Prob (F-statistic):**	0.329
Time:	13:53:52	**Log-Likelihood:**	21.423
No. Observations:	12	**AIC:**	-28.85
Df Residuals:	5	**BIC:**	-25.45
Df Model:	6		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	0.0770	0.221	0.349	0.742	-0.491	0.645
Portfolio ROA	-1.3812	1.886	-0.732	0.497	-6.229	3.467
Portfolio NIM	0.3501	1.102	0.318	0.763	-2.481	3.182
Portfolio EPS	2.5238	2.716	0.929	0.395	-4.457	9.505
Portfolio CR	13.7735	8.279	1.664	0.157	-7.507	35.054
Portfolio DPS	1.0516	12.586	0.084	0.937	-31.301	33.404
Portfolio FATA	-0.4264	0.520	-0.819	0.450	-1.764	0.912

Omnibus:	7.217	**Durbin-Watson:**	2.366
Prob(Omnibus):	0.027	**Jarque-Bera (JB):**	3.534
Skew:	1.266	**Prob(JB):**	0.171
Kurtosis:	3.807	**Cond. No.**	796.

Table 2:

	0
ROA	-1.3812
NIM	0.3501
EPS	2.5238
CR	13.7735
DPS	1.0516
FATA	-0.4264

At this point in time, our fundamental alpha model looks like:

Portfolio Return$_{t+1}$ ***=***

0.077 + ROA*(-1.3812) + NIM*0.3501 + EPS*2.5238 + CR*13.7735 + DPS*1.0516 + FATA*(-0.4264)

The following table (Table 3) contains predicting portfolio return, real portfolio return, benchmark return, alpha and information ration, starting from 2011. Notice that no predictions are made in 2010. From 2011 to 2021, the predictive and real returns of my portfolio are surprisingly all greater than zero, which means my portfolio is making money every year. Benchmark returns have both negative and positive values, showing that the equity market is fluctuating. When it comes to alpha, there's only four negative alphas among eleven alphas, where table 5 shows that the mean alpha over 12 years is 0.455 and the information ratio (IR) is close to 1.518 hence the overall performance of my portfolio is outstanding. Additionally, table four shows that my portfolio information coefficient (IC) is close to 0.388, which is also a great evidence of successful portfolio construction. Graph 1 shows the cumulation of benchmark return, my fundamental alpha model prediction returns and my portfolio real return.

Table 3:

	Prediction	Real return	Benchmark return	Alpha	IR
2011	0.100675	0.145591	-0.000032	0.100707	3.356906
2012	0.160146	0.106709	0.134057	0.026089	0.869639
2013	0.140345	0.227401	0.296012	-0.155668	-5.188932
2014	0.246569	0.307439	0.113906	0.132663	4.422090
2015	0.254122	0.147848	-0.007266	0.261388	8.712930
2016	0.135819	0.089220	0.095350	0.040469	1.348960
2017	0.119031	0.161775	0.194200	-0.075169	-2.505630
2018	0.203075	0.248446	-0.062373	0.265448	8.848260
2019	0.241207	0.166799	0.288781	-0.047574	-1.585799
2020	0.182042	0.281178	0.162589	0.019453	0.648430
2021	0.179121	0.234480	0.247780	-0.068659	-2.288626

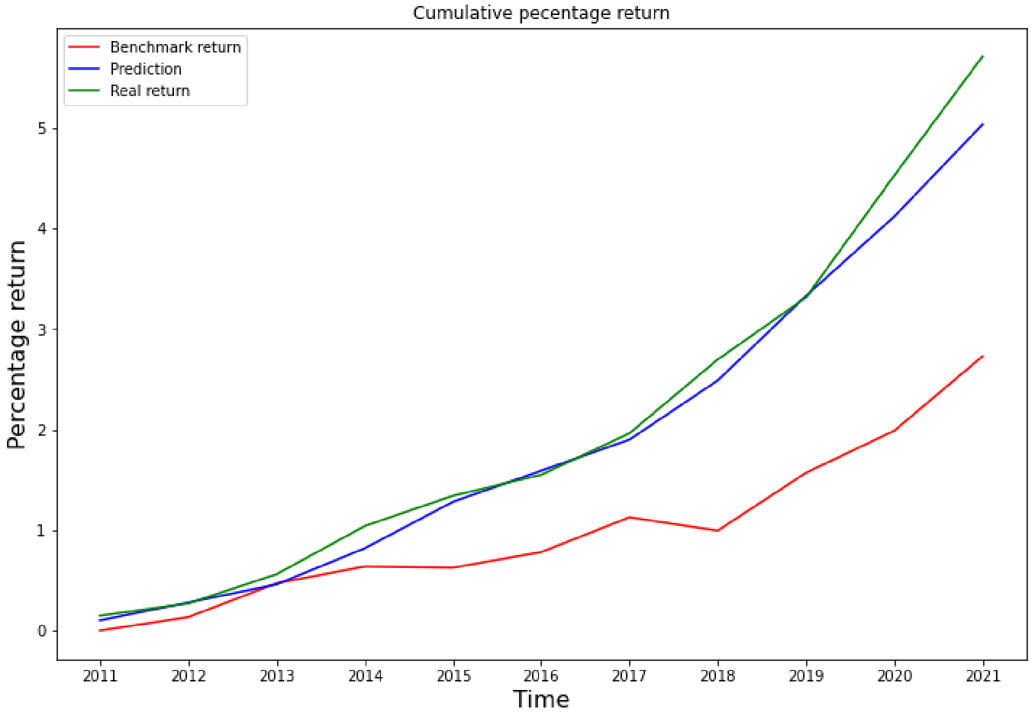
Table 4:

	Prediction	Real return
Prediction	1.000000	0.387669
Real return	0.387669	1.000000

Table 5:

```
The mean of alpha is: 0.04553756900438961
The overal portfolio IR is: 1.5179189668129869
```

Graph 1:



In spite of the fact that the result of alpha, IR and IC is satisfactory, statistical evaluation is required. Since even monthly risk-free rate is fluctuating from 2010 to 2021, it is hard to calculate a reasonable risk-free rate through twelve years. For convenience, an assumption is made that the risk-free rate is zero. Recall back my evaluation method:

$$\frac{E(r_P - r_f)}{\sigma_P} - \frac{E(r_B - r_f)}{\sigma_B} > 2\sqrt{\frac{2}{N}}$$

```
The left hand side value is: 2.263638032271383
The right hand side value is: 0.8528028654224418
```

Since the left-hand side value is greater than the right-hand side, we conclude that the performance of my fundamental alpha model is statistically better than the benchmark.

Predictive power of statistical factors

The formations of my eight portfolios are as follows:

Portfolio 1-1-1 tickers are:

['CMCSA','CSCO','JNJ','KO','PG','V','VZ','XOM'].

Portfolio 1-1-2 tickers are:

['AAPL','GOOGL','JNJ','KO','NVDA','TSLA','VZ','XOM']

Portfolio 1-2-2 tickers are:

['AAPL','DHR','GOOGL','KO','NVDA','PG','TSLA','VZ','XOM']

Portfolio 1-2-1 tickers are:

['AAPL','CMCSA','CSCO','DHR','JNJ','KO','NVDA','PG','V','TSLA','VZ','XOM']

Portfolio 2-1-1 tickers are:

['ADBE','CMCSA','CSCO','JNJ','KO','NFLX','NVDA','TSLA','V','VZ','XOM']

Portfolio 2-1-2 tickers are:

['AAPL','ADBE','GOOGL','JNJ','KO','NFLX','NVDA','TSLA','VZ','XOM']

Portfolio 2-2-1 tickers are:

['AAPL','ADBE','CMCSA','CSCO','DHR','JNJ','NFLX','NVDA','V','TSLA']

Portfolio 2-2-2 tickers are:

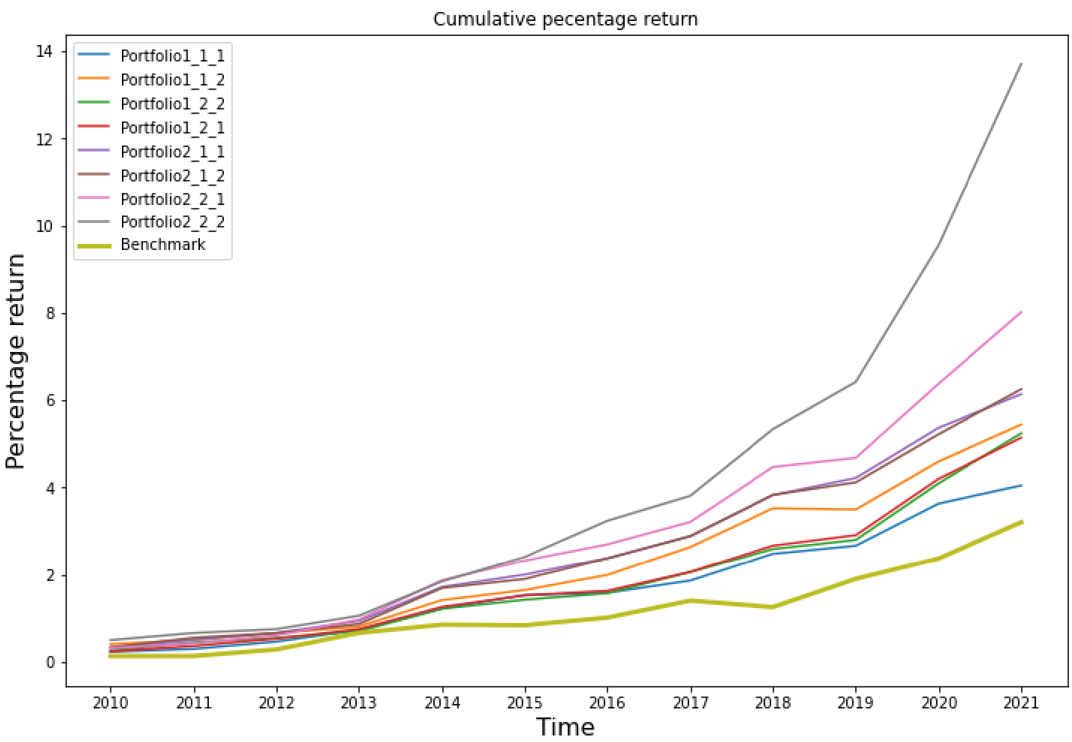
['AAPL','ADBE','AMZN','DHR','GOOGL','LLY','MSFT','NFLX','NVDA','TSLA']

Table 6 is benchmark return and my eight portfolios given a 3% tracking error constraint from 2010 to 2021.

Table 6:

	Benchmark_return	Portfolio1_1_1	Portfolio1_1_2	Portfolio1_2_2	Portfolio1_2_1	Portfolio2_1_1	Portfolio2_1_2	Portfolio2_2_1	Portfolio2_2_2
2010	0.127827	0.225916	0.399611	0.252995	0.233725	0.305706	0.329106	0.313357	0.493429
2011	-0.000032	0.052148	0.069740	0.146988	0.101100	0.147314	0.163971	0.075744	0.108829
2012	0.134057	0.130986	0.101559	0.073734	0.122163	0.106883	0.068246	0.133169	0.053665
2013	0.296012	0.183270	0.093996	0.098212	0.144389	0.164496	0.125920	0.225059	0.175523
2014	0.113906	0.282418	0.334911	0.304276	0.291211	0.403967	0.443617	0.460921	0.382984
2015	-0.007266	0.141486	0.096388	0.093514	0.117689	0.104951	0.077940	0.158108	0.200151
2016	0.095350	0.020163	0.131218	0.062100	0.041751	0.126756	0.163173	0.112836	0.243929
2017	0.194200	0.108658	0.215851	0.189690	0.165429	0.149562	0.153326	0.139359	0.135260
2018	-0.062373	0.216293	0.244592	0.173847	0.198299	0.242805	0.242904	0.299205	0.317187
2019	0.288781	0.053251	-0.005825	0.058619	0.066051	0.081649	0.059322	0.038288	0.170995
2020	0.162589	0.264540	0.244863	0.342177	0.330838	0.220459	0.215889	0.299560	0.422815
2021	0.247780	0.089249	0.150969	0.224678	0.181121	0.120984	0.165709	0.221827	0.393710

Graph 2: (Cumulative return of benchmark and my eight portfolios)



From Graph 2, the differences among all returns are progressively more remarkable after 2013. At the end of the year 2021, we can see that the benchmark cumulative return is at the bottom and portfolio 2-2-2 cumulative return is far greater than the others, proving that high CV, high skewness and high kurtosis contribute to a higher return. The pink line is portfolio 2-2-1 which is the second largest cumulative return, demonstrating high CV and high skewness but low kurtosis also generates excess cumulative return than the benchmark. Based on what we found in graph2, we have evidence to say that portfolios with higher CV, higher skewness or higher kurtosis performs better than those with lower CV, lower skewness and lower kurtosis.

In the last part, the student t test shows that only portfolio 2-2-2 rejects null hypothesis at 95% of significant level. We conclude that we are 95% confident that only portfolio 2-2-2 annual return is greater than benchmark annual return.

CAPM Alpha Model

The following table on the next page is the CAPM model alphas of thirty different stocks from November 2010 to December 2012.

Table 6:

	AAPL	MSFT	GOOGL	AMZN	NVDA	TSLA	ABT	V	JPM	JNJ	...	KO	CRM	NFLX	NKE	ORCL	CSCO	LLY	DHR	XOM	VZ
2010-11-30	0.045476	-0.005378	-0.001675	0.051377	-0.020743	-0.001682	0.056479	-0.005158	0.009444	0.006995	...	0.016680	0.007453	-0.004589	0.013933	-0.025731	0.006853	0.010333	-0.001365	-0.004710	-0.009498
2010-12-31	0.045218	-0.005171	-0.001262	0.051898	-0.020299	-0.001032	0.053485	-0.006052	0.010847	0.005533	...	0.018463	0.010242	-0.004237	0.016200	-0.035342	0.004779	0.007690	0.001020	-0.005694	-0.009137
2011-01-31	0.038254	-0.013082	-0.001836	0.037560	-0.002976	0.001585	0.060317	-0.007304	0.005412	0.004358	...	0.033660	0.006977	-0.001229	0.011445	-0.043981	-0.000322	0.008009	0.004065	0.003947	-0.006218
2011-02-28	0.029688	-0.001572	-0.005305	0.021742	0.006750	0.003751	0.051373	-0.026749	0.008242	0.012662	...	0.010656	0.000689	0.004068	0.003717	-0.055827	0.020451	-0.013026	0.002301	-0.007464	-0.000258
2011-03-31	0.026945	-0.000284	-0.008910	0.022340	-0.003585	0.002259	0.048603	-0.029608	0.009397	0.011704	...	0.000171	0.000978	0.003450	0.004235	-0.044079	0.022840	-0.010662	0.004548	-0.006999	-0.000559
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
2021-08-31	0.025451	0.008071	0.020973	0.014585	-0.005556	-0.003812	0.003719	-0.000976	0.025942	-0.013004	...	0.064444	0.015190	0.000193	0.020319	0.092451	0.012018	-0.009071	-0.007856	0.006088	-0.029407
2021-09-30	0.022413	0.007002	0.017276	0.015140	-0.001779	-0.005369	0.011121	-0.004893	0.023358	-0.009284	...	0.059519	0.013093	-0.000541	0.022754	0.100816	0.014051	-0.004563	-0.008619	0.002261	-0.020075
2021-10-31	0.018360	0.008967	0.019826	0.014144	-0.002802	-0.008778	0.010742	-0.003632	0.025514	-0.010984	...	0.059780	0.015867	-0.000590	0.024091	0.098823	0.011647	-0.008910	-0.009987	0.004372	-0.017462
2021-11-30	0.022362	0.007991	0.018485	0.016803	-0.006127	-0.007825	0.008662	-0.001391	0.025566	-0.021559	...	0.070778	0.012944	0.001527	0.023163	0.102023	0.007317	-0.012340	-0.011414	0.001670	-0.018418
2021-12-31	0.020001	0.009485	0.014826	0.016023	-0.008052	-0.008784	0.006315	-0.001266	0.022533	-0.018259	...	0.067160	0.014875	0.002618	0.021553	0.088538	0.007221	-0.010194	-0.011712	0.001028	-0.017631

134 rows × 30 columns

By inputting alphas above, I use code below as a active optimal weight calculator to calculate the optimal active weights.

```
n = 30 #number of stocks

def active_weight_calculator(lambd ,cov, alphas):

    cov_matrix = np.array(cov)
    Q = 2*lambd*cvxopt.matrix(cov_matrix)
    p = -1 * cvxopt.matrix(np.array(alphas.T).T)
    G = cvxopt.matrix(-np.eye(n))
    h = cvxopt.matrix(list(np.ones(n)/n))
    A = cvxopt.matrix(list(np.ones(n))).T
    b = cvxopt.matrix([0.0])
    sol= cvxopt.solvers.qp( Q,p, G, h, A, b)
    sol_str = np.array(sol['x'])


    return pd.DataFrame(sol['x'])
```

The results of this active weight calculator are like follows:

```
     pcost       dcost       gap    pres   dres
 0: -8.1926e-03 -1.0435e+00  1e+00  5e-17  6e+00
 1: -8.6453e-03 -4.9147e-02  4e-02  4e-17  2e-01
 2: -1.2567e-02 -2.5190e-02  1e-02  4e-17  6e-02
 3: -1.6207e-02 -1.9482e-02  3e-03  7e-17  7e-03
 4: -1.7805e-02 -1.8150e-02  3e-04  8e-17  3e-17
 5: -1.8024e-02 -1.8050e-02  3e-05  7e-17  3e-17
 6: -1.8044e-02 -1.8045e-02  5e-07  9e-17  3e-17
 7: -1.8045e-02 -1.8045e-02  5e-09  8e-17  3e-17
Optimal solution found.
     pcost       dcost       gap    pres   dres
 0: -9.6055e-03 -1.0456e+00  1e+00  9e-17  6e+00
 1: -1.0144e-02 -5.1212e-02  4e-02  5e-17  2e-01
 2: -1.4494e-02 -2.7510e-02  1e-02  5e-17  5e-02
 3: -1.8247e-02 -2.1584e-02  3e-03  5e-17  5e-03
 4: -1.9756e-02 -2.0214e-02  5e-04  7e-17  2e-04
 5: -2.0040e-02 -2.0069e-02  3e-05  4e-17  7e-06
 6: -2.0062e-02 -2.0062e-02  6e-07  4e-17  8e-08
 7: -2.0062e-02 -2.0062e-02  6e-09  7e-17  8e-10
Optimal solution found.
     pcost       dcost       gap    pres   dres
 0: -7.9729e-03 -1.0408e+00  1e+00  1e-17  6e+00
 1: -8.4190e-03 -4.9253e-02  4e-02  4e-17  2e-01
 2: -1.2187e-02 -2.4033e-02  1e-02  5e-17  5e-02
 3: -1.5475e-02 -1.8207e-02  3e-03  1e-16  2e-03
 4: -1.6704e-02 -1.6971e-02  3e-04  7e-17  9e-06
 5: -1.6863e-02 -1.6879e-02  2e-05  9e-17  2e-07
 6: -1.6872e-02 -1.6873e-02  8e-07  1e-16  2e-09
 7: -1.6873e-02 -1.6873e-02  6e-08  1e-16  1e-11
Optimal solution found.
```

To verify that the sum of active weight should be zeros, it is obvious that the sum of active portfolio weight are zeros for all time periods.

```
w_a.sum(axis = 1)
```

```
0    9.020562e-17
0    4.163336e-17
0   -6.245005e-17
0   -2.081668e-17
0    4.857226e-17
         ...
0   -4.163336e-17
0   -5.551115e-17
0    6.938894e-18
0   -3.469447e-17
0   -2.081668e-17
Length: 134, dtype: float64
```

The final optimal portfolio weight is a 30*134 matrix, so I only screenshot part of the final optimal weight here. The sum of all optimal portfolio weight are also ones.

14	1.397160e-01	1.446557e-01	1.773765e-01	9.688176e-02	1.095121e-01	1.284227e-01	3.912938e-02	9.970290e-02	1.131236e-01	1.424495e-01	...	2.833193e-08	2.544786e-09	1.408870e-08	3.462181e-09	2.536822e-09	3.315697e-09	5.846207e-09	2.964171e-09	3.824746e-09	8.445919e-09
15	1.357094e-08	7.032962e-10	1.266582e-09	1.316687e-08	6.020096e-09	6.122849e-08	1.619884e-07	2.668648e-07	2.667652e-07	1.279076e-07	...	1.304873e-01	1.423984e-01	2.164779e-01	2.744871e-01	2.765715e-01	2.746832e-01	2.447699e-01	2.514946e-01	2.457008e-01	2.179584e-01
16	1.174797e-07	1.582129e-09	1.759914e-08	2.814976e-09	1.533920e-09	2.076843e-08	1.240371e-08	5.937315e-07	2.493163e-08	2.309622e-02	...	2.622342e-06	1.535468e-07	1.560687e-08	5.140672e-09	4.786018e-09	3.175600e-09	1.121345e-08	3.746642e-09	3.866506e-09	8.438707e-09
17	1.332716e-08	6.359797e-10	8.002434e-10	4.081468e-09	1.629307e-09	1.599211e-08	5.245468e-09	3.143434e-08	4.573905e-09	2.078478e-09	...	1.727535e-01	1.331921e-01	1.016472e-01	9.445440e-02	9.543288e-02	7.668336e-02	7.960122e-02	1.102819e-01	9.752337e-02	1.048523e-01
18	1.125083e-01	1.101747e-01	1.156302e-01	1.386230e-01	1.413322e-01	1.331237e-01	1.408912e-01	1.293992e-01	1.229231e-01	1.157502e-01	...	1.640495e-02	1.008743e-02	1.363310e-02	1.416219e-02	1.643724e-02	2.592405e-02	4.082138e-02	3.924679e-02	2.748440e-02	3.003529e-02
19	6.558942e-03	1.774850e-08	3.096450e-08	1.021046e-07	2.993517e-09	3.307817e-08	3.010341e-08	3.460607e-02	3.449501e-02	5.497372e-02	...	1.045498e-02	2.115376e-07	3.955099e-02	5.693148e-02	5.260806e-02	3.076008e-02	1.320847e-07	7.614724e-03	1.153418e-02	1.128983e-02
20	2.049929e-06	2.545210e-07	5.325744e-02	3.001580e-06	4.339349e-09	5.071858e-08	1.337091e-07	3.833524e-08	3.020194e-09	1.774514e-09	...	8.486080e-02	9.440795e-02	1.508075e-01	1.635597e-01	1.565356e-01	1.695427e-01	1.547447e-01	1.550503e-01	1.874729e-01	1.785448e-01
21	1.009308e-04	3.799886e-02	1.073173e-02	2.345861e-08	1.868413e-08	1.862161e-07	1.504052e-07	8.384497e-08	7.246636e-09	4.611836e-09	...	4.855341e-08	7.648431e-09	2.208411e-05	2.440155e-08	1.124981e-07	3.090613e-04	1.561707e-02	2.912309e-02	6.272998e-07	3.987741e-02
22	2.220912e-08	1.121235e-09	4.741178e-09	3.151620e-03	7.061302e-08	1.265332e-05	6.087612e-02	8.788861e-05	4.470294e-08	6.589230e-08	...	3.673428e-07	1.337873e-08	2.396822e-07	1.288193e-08	5.146131e-09	5.591435e-09	1.423870e-08	6.482412e-09	1.424741e-08	4.626866e-08
23	1.009868e-06	2.014391e-05	7.620009e-07	1.319419e-08	8.503332e-09	7.010387e-08	1.599781e-07	3.079548e-07	8.027989e-09	2.819824e-09	...	7.301695e-03	2.411664e-08	4.832725e-07	7.463041e-08	4.029967e-06	1.527240e-08	2.663870e-06	5.871798e-07	1.940483e-06	2.827489e-03
24	3.281756e-09	1.794828e-10	1.402515e-10	7.983946e-10	4.723135e-10	4.758139e-09	1.884649e-09	1.195789e-08	1.324939e-09	6.296719e-10	...	1.060825e-01	1.123344e-01	1.066204e-01	1.035342e-01	9.905021e-02	9.830693e-02	1.095655e-01	1.075674e-01	1.087309e-01	9.834813e-02
25	5.292078e-02	2.792822e-02	2.845768e-08	1.945993e-01	2.101236e-01	2.321177e-01	2.029535e-01	2.682782e-01	2.200198e-01	1.829640e-01	...	3.879512e-02	8.347384e-02	1.749797e-02	3.524338e-07	4.299774e-08	1.961693e-08	1.654920e-02	1.594058e-07	2.917614e-08	6.671980e-08
26	6.967925e-08	1.396431e-09	4.281547e-09	2.566323e-09	1.300560e-09	1.430023e-08	5.925977e-09	8.390032e-08	1.077333e-08	4.422982e-09	...	1.289327e-07	3.732346e-08	1.994542e-08	5.810116e-09	5.546322e-09	3.968866e-09	1.498278e-08	3.833106e-09	3.871058e-09	8.626978e-09
27	1.126171e-07	1.381132e-08	5.902716e-02	3.736779e-02	4.939674e-02	2.416844e-02	6.804388e-02	2.065537e-02	6.737658e-07	2.447502e-02	...	2.731711e-08	3.753540e-09	2.619747e-08	4.259142e-09	3.232337e-09	3.763895e-09	6.799431e-09	2.657393e-09	3.360752e-09	7.304868e-09
28	2.847593e-08	1.049181e-09	4.297550e-03	6.179063e-09	2.371878e-09	3.011457e-08	1.489647e-08	1.130296e-07	1.137584e-08	3.755654e-09	...	4.645945e-02	7.882323e-03	3.211798e-05	1.678340e-08	9.365758e-09	7.069398e-09	1.456394e-08	8.871012e-09	7.467002e-09	1.431008e-08
29	2.984950e-08	1.219822e-09	6.609533e-09	8.780601e-07	8.730935e-08	7.864500e-04	2.682975e-06	4.021573e-07	1.596592e-07	1.377844e-08	...	4.972760e-09	3.615804e-10	3.206593e-09	1.216136e-09	7.958812e-10	1.619752e-09	3.128571e-09	1.518957e-09	1.938161e-09	4.648776e-09

30 rows × 134 columns

```
port_weight.sum()
```

```
0     1.0
0     1.0
0     1.0
0     1.0
0     1.0
     ...
0     1.0
0     1.0
0     1.0
0     1.0
0     1.0
Length: 134, dtype: float64
```

Portfolio alphas, tracking errors, information ratio and information coefficient are easily calculated given the optimal portfolio weights. The results are as follows.

Tracking errors:

```
TE = []
for i in range(len(w_a)):
    tracking_error = np.sqrt(np.dot(np.dot(w_a.iloc[i,:],cov) , w_a.iloc[i,:].T))
    TE.append(tracking_error)
TE
```

```
0. 02262353260490321,
0. 02092344745549299,
0. 013516250349664449,
0. 015412879887734742,
0. 01624578537291985,
0. 014848471366362798,
0. 01552358578615068,
0. 014334145652415607,
0. 015579053786408602,
0. 015426327465699882,
0. 015761022134158444,
0. 015187817771669792,
0. 014650279308303503,
0. 014269785647657284,
0. 011773204586439063,
0. 012543120758453416,
0. 012991707338048771,
0. 015496672498604269,
0. 015079933164610242,
0. 015954891659444758,
0. 01653890602681508,
0. 016051607129384266,
0. 017786177268550956,
0. 018128358948764488,
0. 020745866894303952,
0. 02143025109874787,
0. 02135166202525296,
0. 021015666936375327,
0. 021227993965489675,
0. 024231427208056495,
0. 022098374430396677,
0. 02038198010505817,
0. 022962398787484354,
0. 02476565426563623,
0. 026232215501042728,
0. 02554454964623814,
0. 0261379084342438,
0. 02608487648464849,
0. 025994261715728207,
0. 02807155577363088,
0. 025781715458023574]
```

The above tracking error vector is only part of all 134 tracking errors, but we can see that the tracking errors are strictly less than 0.03 as assigned. Now, it is easy to update our IR since we get both our portfolio tracking errors and portfolio alphas.

Table 7:

```
port_alpha = []

for i in range(len(port_weight.T)):

    port_alpha.append(np.dot(port_weight.iloc[:,i],alpha.iloc[i,:]))

portfolio_alpha_df = pd.DataFrame(port_alpha)
portfolio_alpha_df
```

	0
0	0.045638
1	0.043868
2	0.041380
3	0.037430
4	0.036574
...	...
129	0.040951
130	0.038677
131	0.039902
132	0.042856
133	0.036782

134 rows × 1 columns

Table 8:

```
IR = portfolio_alpha_df/pd.DataFrame(TE)
IR
```

	0
0	1.462381
1	1.442793
2	1.389894
3	1.320575
4	1.323536
...	...
129	1.566727
130	1.482742
131	1.535043
132	1.526669
133	1.426675

134 rows × 1 columns

Table 9:

```
predictive_por_return_df = pd.DataFrame(np.dot(predict_df1,port_weight).sum(axis = 0))
real_por_return_df = pd.DataFrame(np.dot(stock_monthly_returns.iloc[23:,:],port_weight).sum(axis = 0))
IC = predictive_por_return_df.set_axis(['Predictive Portfolio Return'],axis=1, inplace=False)
IC['Real Portfolio Return'] = real_por_return_df
display(IC)
```

	Predictive Portfolio Return	Real Portfolio Return
0	1.916951	2.997439
1	1.842415	2.948287
2	1.815800	2.893874
3	1.883967	2.887574
4	1.878386	2.862408
...	...	...
129	1.961234	3.680795
130	1.985588	3.716533
131	1.936944	3.668667
132	1.996921	3.784794
133	1.898552	3.674560

134 rows × 2 columns

```
IC.corr()
```

	Predictive Portfolio Return	Real Portfolio Return
Predictive Portfolio Return	1.000000	0.609604
Real Portfolio Return	0.609604	1.000000

From the table 7 above on the top left, we can see that our portfolio seems to always have a positive alpha, which is good. Our monthly alphas are nearly 4% which is very high. On the right-hand side, table 8 shows that our portfolio information ratio varying from 1.3 to 1.5, which is also a relatively nice information ratio given a 3% tracking error constraint. As for information coefficient, our portfolio IC is 0.6 which is a satisfactory result.

Summary:

To sum up everything that have been stated so far, fundamental factor, current ratio proxy the most for future stock return with a coefficient of 13.7735. Problems are remained in the OLS regressing summary because of the limitation of observation, where P > |t| values is too large. In the future research, the problem would be solved if monthly or even daily fundamental data is available. Although number of observations are limited, the performance of my fundamental alpha model is still statistically better than benchmark. Statistical factors, coefficient of variance (CV), skewness and kurtosis do have explanatory power on stock return, where higher CV, higher skewness or higher kurtosis result in higher stock returns. The CAMP alpha model provides a portfolio with positive alphas for even 10 years and an IR at 1.5 level with tracking error less than 3 percent. Although this alpha model predictive returns are slightly less than the portfolio real return, the predictive returns are more steady and have relatively high information coefficient.

Appendix:

```
from pypfopt import expected_returns, risk_models, EfficientFrontier
import yfinance as yf

w = []

def TE(w, portfolio_returns, benchmark_returns):
    xi= (w @ portfolio_returns.T - benchmark_returns)
    mean = cp.sum(xi)/len(benchmark_returns)
    return cp.sum_squares(xi- mean)

for i in range(0,12):
    prices = data1.iloc[i*12:12*(i+2):,0:-1]
    spy_prices = data1.iloc[i*12:12*(i+2):,-1]

# Get returns
    rets = expected_returns.returns_from_prices(prices)
    spy_rets = expected_returns.returns_from_prices(spy_prices)


# Define constraint
#def TE(w, portfolio_returns, benchmark_returns):
    #xi= (w @ portfolio_returns.T - benchmark_returns)
    #mean = cp.sum(xi)/len(benchmark_returns)
    #return cp.sum_squares(xi- mean)

# Construct efficient frontier
    mu = expected_returns.mean_historical_return(prices)
    S = risk_models.sample_cov(prices)
    ef = EfficientFrontier(mu, S)
    ef.add_constraint(lambda w: TE(w, rets, spy_rets) <= 0.3**2)
    ef.min_volatility()

    dict(ef.clean_weights())
    data_dict = dict(ef.clean_weights())
    data_items = data_dict.items()
    data_list = list(data_items)
    w.append(pd.DataFrame(data_list))

display(w)
```

References:

Husnan, F. 2004a. The Effect of Stroe Name, Brand Name and Price Discounts on Consumers' Evaluations and Purchase Intentions', *Journal of Retailing* Vol.74(3):33152.

Teweless, R.J.& Edward, S.B(1998), The Stock Market, John Wiley & Sons, New York. Warren, E.; Fess, P.; Reeve, J.M. (1996), Accounting, International Thompson Publishing, Ohio.

José Manuel Cueto, Aurea Grané and Ignacio Cascos, Models for Expected Returns with Statistical Factors, *Journal of Risk and Financial Management*

Sonnia Cindy Tamuunu, Farlane Rumokoy, THE INFLUENCE OF FUNDAMENTAL FACTORS ON STOCK RETURN

Adamilyara Aqil Abdulmannan, Taufik Faturohman, THE RELATIONSHIP BETWEEN FUNDAMENTAL FACTORS AND STOCK RETURN: A CASE BASED APPROACH ON BANKING COMPANIES LISTED IN INDONESIA STOCK EXCHANGE, *Journal of Business and Management* Vol. 4, No.5, 2015:579-586