

Number Theory: Proof Set 1

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Problem 1

Determine a formula for $\sum_{k=1}^n \frac{1}{\sqrt{k} + \sqrt{k+1}}$ as a function of n .

Solution

$$\begin{aligned}\sum_{k=1}^n \frac{1}{\sqrt{k} + \sqrt{k+1}} &= \sum_{k=1}^n \frac{1}{\sqrt{k} + \sqrt{k+1}} \cdot \frac{\sqrt{k} - \sqrt{k+1}}{\sqrt{k} - \sqrt{k+1}} \\&= \sum_{k=1}^n \frac{\sqrt{k} - \sqrt{k+1}}{k - (k+1)} \\&= \sum_{k=1}^n \sqrt{k+1} - \sqrt{k} \\&= (\cancel{\sqrt{2}} - \sqrt{1}) + (\cancel{\sqrt{3}} - \cancel{\sqrt{2}}) + \cdots + (\sqrt{n+1} - \cancel{\sqrt{n}}) \\&= \boxed{\sqrt{n+1} - \sqrt{1}}\end{aligned}$$