

Reviews and Simulation of Probabilistic Data Association Techniques for Target Tracking in Clutter

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Abstract—This paper will provide analysis and simulation of the probabilistic data association filter(PDAF) method, to solve the problem of targeting tracking with low observable measuring targets, applied in areas such as a radar tracker. Regardless of the detailed problems mentioned by Kirubarajan and Bar-Shalom, the main focus is to compare with Kalman Filter(KF). In the following part of the study, simulated results of the KF is compared to PDAF, the main goal is to understand and discuss the constraints and trade-offs between such two popular and effective approaches. In addition to the full interpretation and discussion on PDAF, we will finish by giving some real world examples and popular applied areas applying PDAF.

I. INTRODUCTION

Through the studying on Bayesian estimation and learning, we have encountered and solved target maneuvering problem by fitting a hidden Markov model, and later on also viewed the problem as a Markov Auto-regression Time Series. PDFA has provided a new perspective to view the data we have collected. Unlike the algorithms we have covered in class, PDFA focus on a statistical focus to the problem, which all of the potential candidates for association to a tracker are combined.

With an update of a single statistically most probable estimation. Then, the probability distribution of the tracking errors and clutter are taken into account, only one of the observed candidates is considered to match as the target, and unmatched data points are regarded as false alarms. Before we get into the methodology, we would like declare that the study is focused on linear state space model and linear filtering for PDFA and KF, although PDFA can be used to solve non-linear model problems.

II. PROBABILISTIC DATA ASSOCIATION SETTING

In this section, the basic setting of Probabilistic Data Association is discussed, with comparison to Kalman Filter, which shares similarity in problem scenario and computation process.

A. Assumptions

As background information, to give a full definition of the problem, we are investigating target motion, our goal is to find the best observation and to predict the target location. To be noticed, the assumptions is almost as simple as Kalman filter, below they are both linear filter and linear state space model.

The major assumption difference among KF and PDAF is that PDAF takes every single observed point in the sliding window into consideration, and there exists an additional case that the target we are tracking can be missing with a certain probability, in the current gating process.

The information below are obtained from observation or prior knowledge. All noises are treated as zero-mean Gaussian white noise.

- $X(k)$: State of the target
- $Z(k)$: True measurement of the target
- $v(k)$: noise in state target, with covariance matrix $Q(k)$
- $w(k)$: noise in measurement, with covariance matrix $R(k)$
- $F(K)$: transition matrix of the real state;
- $H(k)$: measurement or observation matrix;

B. Data Association

Here we want to illustrate data association in target tracking under Bayesian approach, "Data association is the process of associating uncertain measurements to known tracks", we compute the the distribution in data association space from priors, posterior and observations. In KF, we do not have such windowing produce and the big amount of data in observation, although KF can be combined with windowing for implementing a KF gating algorithm.

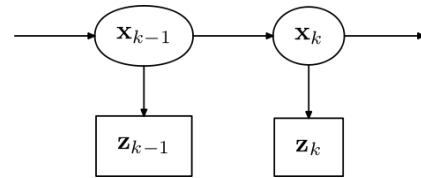


Fig. 1. Kalman Filter State Space Model Single State Transition

Some common data association scenarios can be evaluated by using different models. These estimation methods are extensively used to track features as location, shape (2D position), velocity, acceleration and appearance, and a score can be yield for evaluating such features.

For example, a most direct way is to match a constant position and bound to a mark region. One corresponding example is estimating the position of a constant speed moving object in the space, where no external forces are

involved, the object is moving in straight line with non-changing speed. More advanced method in data association can be applied to calculate the overlapping region in determining the similarity between the 2 frames or shapes.

$$\beta = \frac{2(A+B)}{\text{area}(A) + \text{area}(B)} \quad (1)$$

Global Nearest Neighbor(GNN)is quite intuitive as the name of the method, which choose the nearest result based on the Euclidean or the Mahalanobis distance, and gave a score for based on the distance, Mahalanobis takes uncertainty into consideration to add onto the result of Euclidean, and we choose the highest score among each. Further, all of these compared results can be gathered together as a matrix of received scores. And the goal is to maximize the summed score.

$$\beta_{m1} = \max\{\beta_{11}, \beta_{21}, \beta_{31}, \beta_{41}\} \quad (2)$$

$$\hat{\beta} = \sum_N \sum_{i=1}^a \beta_{ai} \quad (1)$$

There are effective solution on how to yield the highest sum of the scores fast, but we will skip this part, and we now look back to PDFA, PDFA is the a Bayesian perspective to view the observation as samples under a distribution, and β_{mn} is the likelihood that yield such result. It is more conservative compared to GNN, unlike GNN, we look for the probability, in such way it gives much better result as cluttered data samples and noisy models.

C. Covariance and State Update

Both Kalman filter and PDAF are on-line learning models. Covariance matrix computation can be done off-line. The state and covariance matrix estimation is similar to the idea of a mean value and variance of a given clutter of data, which is equivalent to the idea of identifying an unbiased estimator based observations.

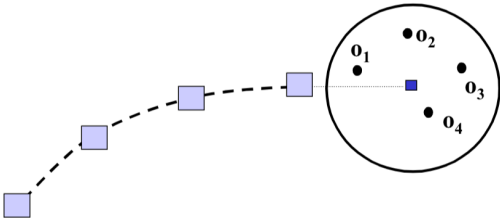


Fig. 2. Target Tracking Data Association Filter

Kalman filter update is based on residual vector (the difference between the predicted location and observed location) when using the single best observation chosen based on previous update, and PDAF can have or choose as many as observations to validate the sliding window region.

$$\tilde{y} = z_{max} - H(k)\hat{x}_{x-1} \quad (1)$$

while PDAF uses weighted combination of observations

$$\tilde{y} = \sum_{i=1}^{m(k)} \beta_i(k)(z_i - H(k)\hat{x}_{x-1}) \quad (2)$$

To be noticed and partially mentioned in assumption above, if we are not taking no match β_0 as one of the outcomes, this representation would exactly equivalent to the the sum of differences, between the weighted center of mass from observations and the predicted location.

In KF, to incorporate weighted matches and possibility of no match, the computation of Kalman posterior covariance $P(k|k-1)$ must update when a single match is used, a typical computation is applied as below

$$P(k|k) = (I - K(k)H(k))P(k|k-1) \quad (3)$$

In PDAF, the update of the covariance with state is

$$x(k+1) = F(k)x(k) + v(k) \quad (4)$$

$$P(k|k) = \beta_0(k)P(k|k-1) + (1 - \beta_0(k))(I - K(k)H(k))P(k|k-1) + K(k) + \hat{P} \quad (5)$$

D. Validation and Innovation

Although KF and PDAF comes from the Bayesian model which try to capture prior given posterior, here exist a major difference in PDFA and KF that PDFA has a threshold value γ in the gating window, which is controlled by our design. PDAF algorithms capture an elliptical region in the sliding window, and γ is also playing a factor in updating the target state:

$$(z_i(k) - \hat{z}(k|k-1))^T S(k)^{-1} (z_i(k) - \hat{z}(k|k-1)) < \gamma$$

III. ALGORITHM AND SIMULATION

In this section, the basic algorithm of Probabilistic Data Association is implemented and simulated on the 2-D maneuvering target. The process of basic algorithm, which is based on a Poisson model when the probability mass function (pmf) of the number of false measurements (FAs or clutter) in the validation region, is shown in 2.

The covariance of the state updated with the correct measurement is

$$P^c(k|k) = P(k|k-1) + W(k)S(k)W(k)^T. \quad (7)$$

The final equations of the association probability with the Poisson cluster model are

$$\beta_i(k) = \begin{cases} \frac{e_i}{b + \sum_{j=1}^{m(k)} e_j}, & i = 1, \dots, m(k) \\ \frac{b}{b + \sum_{j=1}^{m(k)} e_j}, & i = 0 \end{cases} \quad (8)$$

Algorithm 1 Probabilistic Data Association (PDAF)

Input:

- 1: $F(k)$, $H(k)$, $Q(k)$, $R(k)$
- 2: $\{\bar{z}(k)\}_{k=0}^n$: cluttered measurement input;
- 3: γ : validation threshold

Output: $\{\hat{x}(k|k)\}_{k=0}^H$

- 4: initialize $S(0) = I$ and $P(0|0) = I$;
 - 5: **for** $k = 1 : N$ **do**
 - 6: **predict:**
 - 7: $\hat{x}(k|k-1) = F(k) \hat{x}(k-1|k-1)$;
 - 8: $\hat{z}(k|k-1) = H(k) \hat{x}(k|k-1)$;
 - 9: $P(k|k-1) = F(k) P(k-1|k-1) F(k)^T + Q(k)$
 - 10: **update:**
 - 11: $S(k) = H(k) P(k|k-1) H(k)^T + R(k)$;
 - 12: $v_i(k) = z_i(k) - \hat{z}(k|k-1)$, $i = 1, \dots, m(k)$
 - 13: **if** $(z_i(k) - \hat{z}(k|k-1))^T S(k)^{-1} (z_i(k) - \hat{z}(k|k-1)) < \gamma$ **and** $z_i(k) \in \{\bar{z}(k)\}$ **then**
 - 14: $z(k) = \{z_i(k)\}_{i=1}^{m_k}$
 - 15: **end if**
 - 16: Compute β based on (8);
 - 17: $v(k) = \sum_{i=1}^{m(k)} \beta_i(k) v_i(k)$;
 - 18: $\hat{x}(k|k) = \hat{x}(k|k-1) + W(k) v(k)$;
 - 19: $P(k|k) = \beta_0(k) P(k|k-1) + (1 - \beta_0(k)) P^c(k|k) + \tilde{P}(k)$;
 - 20: **end for**
-

where

$$e_i = e^{-0.5 v_i(k)^T S(k)^{-1} v_i(k)} \quad (9)$$

$$b = \lambda |2\pi S(k)|^{1/2} \frac{1 - P_D P_G}{P_D} \quad (10)$$

A. PDAF Target Tracking Simulation

The parameters and variables involved with the target and PDAF are labeled in I. While the target data is generated by maneuvering target modeling, the measurements are generated based on the parameters: P_D and the number of samples. In the following experiments, P_D is the probability of detecting the true target, or the chances that the measurements include the only original recorded real target.

All the rest of our measurements, presented in the data set, are false measurements (False Alarms(FA) or clutter), these values are generated by uniformly randomized process, or Gaussian white noise. Number of sample also affects P_{FA} , which means the rate of clutter in the validation region. In the cluttered measurements, the real target value, if once has been detected, it is randomly placed between the false measurements.

B. Target Tracking with Kalman Filter

The first analysis is to track the target is by a Kalman Filter without clutter or FA. In this setting, Kalman Filter gives good performance in tracking the target.

TABLE I
SIMULATION PARAMETERS

| Parameters | Descriptions | Value |
|------------|----------------------------------|--|
| $F(k)$ | Transition Matrix of Maneuvering | $\begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}$ |
| $H(k)$ | Measurement Matrix of the Target | $\begin{bmatrix} 2 & 0.01 & 0.2 & -0.1 \\ -0.2 & -0.02 & 1 & 0.02 \end{bmatrix}$ |
| $Q(k)$ | Maneuvering Noise Covariance | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ |
| $R(k)$ | Measurement Noise Covariance | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ |
| P_D | Probability of Detection | 0.1 ~ 0.9 |
| N_s | Number of Measurement per state | 1 ~ 100 |
| γ | Validation Threshold | 10000 |
| λ | Spatial Density of Poisson Model | 0.001 |

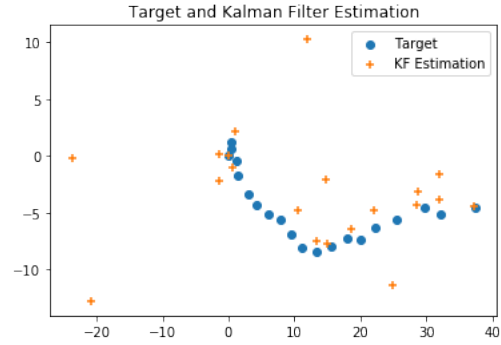


Fig. 3. Kalman Filter Tracking without clutter

C. PDAF

The target is now tracked by PDAF and KF with data points contain cluttered measurements. Different P_D are set in generating cluttered measurements based on the same noisy data set.

During each state, 100 measurements are obtained at each state, blended with white noise from our sliding gate, it may or may not contain the true measure of the real target.

Under the same data set, the Kalman Filter is then applied to the cluttered measurements. We used the GNN as mentioned above to select the closest one as the validated measurement, and it is chosen to be the "real" measurement to process onto the Kalman Filter state space process.

Shown in figure 7, we can tell that the tracking of a single best observation for Kalman Filter model is not as comprehensive as PDFA, PDFA takes much more data into consideration(samples marked in green). With such large amount of FAs, KF cannot precisely track the target, and the error is expected to grow with larger amount of data, since it is accumulating errors with the growth of k , we will look into the errors in the following part and elaborate on

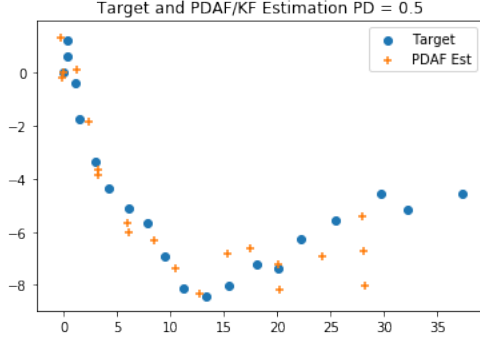


Fig. 4. PDAF Tracking with $P_D = 0.5$

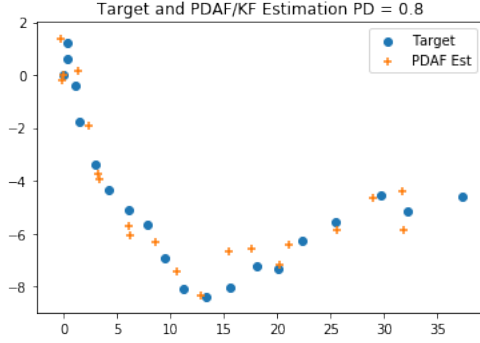


Fig. 5. PDAF Tracking $P_D = 0.8$

some interesting findings.

D. Error Analysis

Corresponding to the simulation, we now validate the received results to the ground truth states, the errors are analyzed with varying P_D and N_s , separately for PDAF and Kalman Filter under comparison. P_D is set between 0.1 to 0.99 and N_s is set between 1 to 100. From the error plots, Figure. 8 and Figure. 11, it is obvious enough that the PDA out-performs Kalman Filter with outstanding performance almost entirely thorough out the rage of P_D and N_s , under the disturbance of FAs.

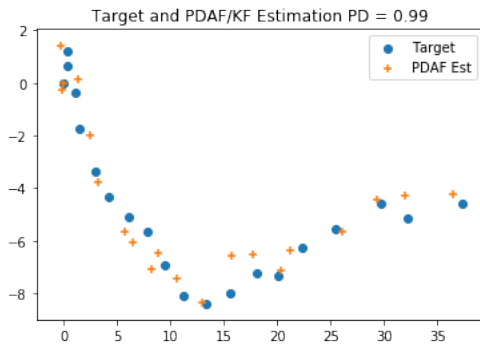


Fig. 6. PDAF Tracking $P_D = 0.99$

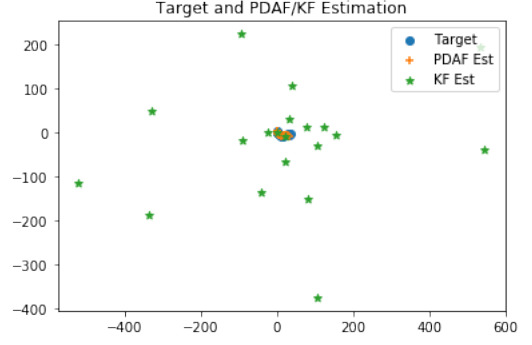


Fig. 7. PDAF Tracking vs KF Tracking, $P_D = 0.8$

Here we take a closer look at the errors of PDA Filters. On the one side, tracking errors decrease with P_D increases, which accords with our hypothesis in the algorithm section. And on the other side, the errors of PDA Filter tracking dose not show any obvious trend of increase or decrease when N_s is varying, which again accord with our hypothesis.

The essential scientific reasoning is that PDAF can rule out the measurements originated from the target with FA noises by computing the association probability $\{\beta_i\}_0^{m(k)}$. P_D , and it affects the weight of the error by determining the chance that if the true positive exists. On the contrary, the influence of the N_s is not influential in PDFA, because no matter how N_s can change, the true positive can still be detected.

The flaws and defect in KF is mentioned above in Figure 7. The performance is relatively unstable as both error plots against P_D and N_s , and we would rather conclude that the performance is coincidentally improved when P_D is around 0.6 to 0.8, it is not a common fact or a certain trend. With N_s increased it starts to show a decreased trend but stabilized around 200, still almost 10 times larger than the average performance to PDAF, it is pretty trivial because the loss of choosing a wrong target is much higher and the chance is much higher when N_s is small.

E. Simulation Discussion

To sum up the PDFA and KF clutter target tracking performance comparison above, we have verified the main idea is that PDAF out-performs Kalman Filter in clutter observations. However, our simulation is just a basic PDAF implement and there are some existing flaws do exist in the algorithm assumption.

With small P_D , PDAF is facing the problem of tracking the targets under high covariance, in another word, noisy environment. In the cases where P_D is too small, there is a larger possibility that PDAF cannot track the target.

In terms of the simulation process, which echoes to the result above, we did not test the robustness of our PDAF.

Due to the process have involved a limited number of states, the noise is set to be too simple (Gaussian). If robustness testing it required, the target simulation process should be designed under other types of mixed signal, and will add complexity to the assumption and modeling.

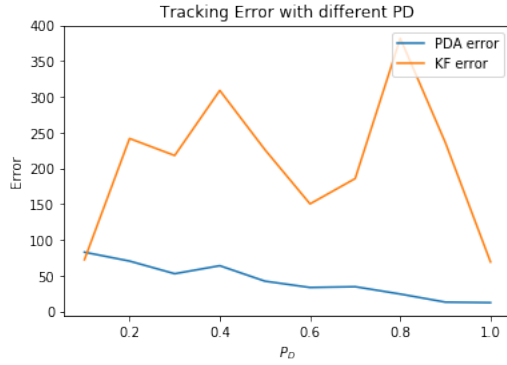


Fig. 8. Tracking Error, $P_D = 0.1 \sim 0.99$

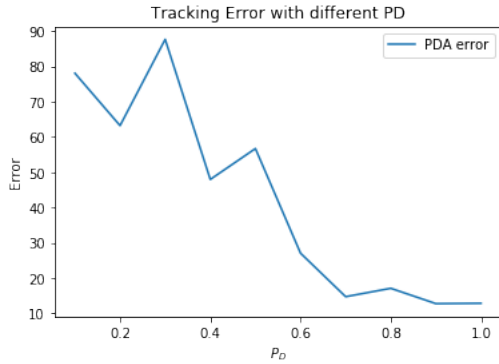


Fig. 9. PDA Tracking Error, $P_D = 0.1 \sim 0.99$

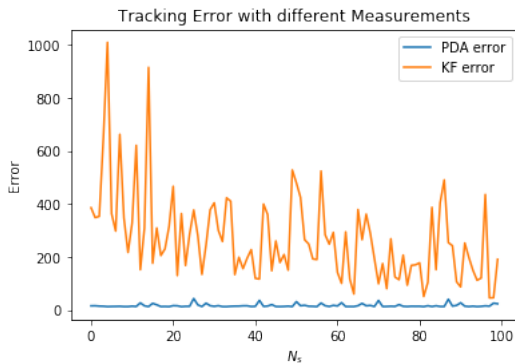


Fig. 10. Tracking Error, $N_s = 1 \sim 100$

IV. DISCUSSION

By we went through the assumption, algorithm, simulation and preliminary analysis of PDFA and KF result. We now combine the learning from earlier of this semester in HMM, KF and Bayesian estimation. We now broad the problem

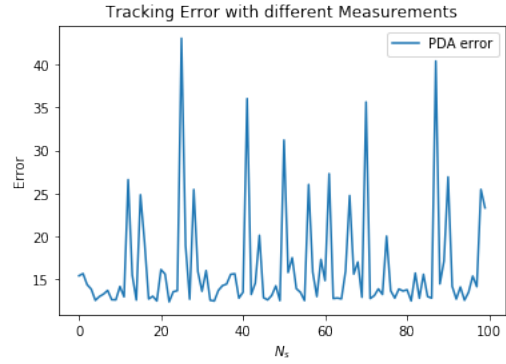


Fig. 11. PDA Tracking Error, $N_s = 1 \sim 100$

settings and take a look at the different cases in target tracking.

KF certainly has an advanced performance in a single target tracking problem, KF is optimal for correlated noise (linearly filtered white noise). We can not deny that KF is the most commonly used tool due to its performance and low cost in application, since it is a linear filter built on linear state space model. And we only need to take a few samples to give a relatively functioning result.

Then, PDFA can be interpreted as an upgrade in taking more data into consideration. While PDAF is much more robust to noises which are stronger and non-Gaussian which are discussed in the original paper by Kirubarajan and Bar-Shalom. With, assumptions and some conditions has been exploited above, and we now look back to review with other Bayesian Estimation methods.

The Bayesian study upgraded from Markov chain to first study in Hidden Markov Model(HMM), we have analyzed it as a noisy measured Markov model. Then, an HMM filter can be implemented as an online learning adaptive filter, which can solve the same problem PDFA and KF are addressing given same data set. However, HMM model depends crucially on iid noises, which has largely affected the performance under complex settings even it does not has resisted performance with non-Gaussian noise. And some more complex settings for FPDA has strong performance will be briefly mentioned in the following subsection.

A. JPDAF

One of the very commonly seen PDFA is the Joint Probability Data Association (JPDAF), JPDAF is a powerful tool in solving multiple target tracking problems, and quiet popular till today. The most studies we found are related to intelligent system, radar and low SNR data in general, they could also come from all different types of sensors.

It interests the group member because we have previously implemented an targeting tracking algorithm using OpenCV

package, and also in computer vision class offered at this moment. In the real world application, the problem was about how to make an effective tracking method to allow a robot to follow a human in factory or storage space environment, we have programmed the robot to move smoothly and coordinately turns without any flaws, the real challenge was the lighting condition, the lighting can vary largely from outdoor to indoor.

The original approach we had during undergraduate was to use a simple pattern for the robot to track and match, we have used web-cam and image processing to read the RGB value and the shape size. Then we have tested and measured a few points and calculate the average color RGB values by iterative testing, then we set threshold on these values.

Obviously, this model is not complex nor smart. And consequently, it is not effective, especially during the testing stage, the group member was turning to the opposite direction with the pattern attached on his or her back, the robot would be blind and can't determine the target anymore.

I found one of the studies tried to solve almost the exact same problem, and they have used a maneuvering robot to track a target as well. Instead of using a radar to receive signal, or a camera, the researchers used a laser scan to measure the probabilistic density of the true match. The researchers has also introduced other moving human as noise to add intensive amount of noise to the true observation.

This problem has also became highly demanded today in auto driving or smart city, which could worth looking into in solving a surveillance camera traffic and vehicles identification on the road, and decrease the error rate could help auto-driving vehicle to avoid crashing into other fast moving vehicles. The model is more complex than single target tracking, in such case, the tracking involved with other incoming data could mixed up in the gating window, and we will go through D. Schulz, W. Burgard, D. Fox, and A. B. Cremers's approach.

JPDAF is a powerful tool in solving multiple tracking problems, and quiet popular. And most studies we found are related to intelligent system, radar and low SNR data in general, they could also come from different sensors. I found one of the studies which could worth look into in solving surveillance camera for analyzing traffic and vehicles on the road.

People Tracking with Mobile Robots Using Sample-based Joint Probabilistic Data Association Filters, the most important idea is still we want to analyze all of the observations correspond a single target track for prediction. And in JPDAF we want to only choose the observation once for one track, any observation should not be counted more than once for accurate state update in PDAF.

And the data collection is in the same way as PDAF, we add an extra step to classify the overlapping sliding window region observations, we want the maximize the probability of assigned target adds up to the largest hypothesis likelihood.

Similar to our problem formulation. The states of the people (targets) are represented by quadruples (x, y, ϕ, v) .

k : target number, and time will be marked as t instead
 $X(t)$: State of the target
 (x, y) position relative to the robot
 ϕ : orientation
 v : walking speed of the person(tracking target)
 $\omega \in \Omega$ feasible joint association events at time t
 $\omega = (j, k)$ observation j is associated with target k
 $g(\omega)$ is the likelihood matrix of event (j, k) , $\sum_{k=1}^K g(\omega) \leq 1$
 B false alarm rate
 P_D probability of detection

B. Implementation of JPDAF

Assumptions of JPDAF

- The number of tracking targets with clutter is known.
- Measurements from a single target in the validation region with neighboring target from another track.
- The past state of the system is statistically obtained with state mean and covariance matrix, and each of the states is Gaussian distributed .
- Each targeting tracking can fit into different models.

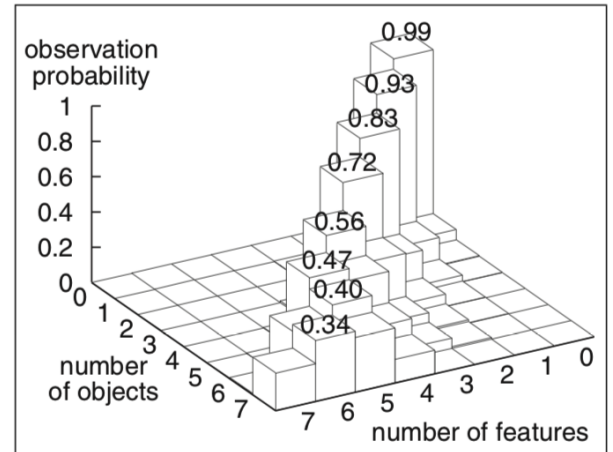


Fig. 12. Hypothesis Likelihood of Target Tracking in JPDAF

I want to highlight on the update of H , the observational matrix, in a JPDAF, we keep updating the findings in the likelihood and we maximize the conditional probability under time t , which joint probability is the largest that k and j can be assigned to the predicted track.

Algorithm 2 JPDA Algorithm under Poisson Pmf

Input:

- 1: $F(t)$, $H(t)$, $Q(t)$, $R(t)$
- 2: $\{\bar{z}(t)\}_{t=0}^n$: cluttered measurement input;
- 3: γ : validation threshold

Output: $\{\hat{x}(t|t)\}_{t=0}^H$

- 4: initialize $S(0) = I$ and $P(0|0) = I$;
 - 5: **for** $t = 1 : N$ **do**
 - 6: **predict:**
 - 7: $\hat{x}(t|t-1) = F(t) \hat{x}(t-1|t-1)$;
 - 8: $\hat{x}_t^k = \sum_w P(x_t^k | g(\omega)) g(\omega) \leq 1$
 - 9: $\hat{z}(t|t-1) = H(t) \hat{x}(t|t-1)$;
 - 10: $P(t|t-1) = F(t) P(t-1|t-1) F(t)^T + Q(t)$
 - 11: **update:**
 - 12: $S(t) = H(t) P(t|t-1) H(t)^T + R(t)$;
 - 13: $v_i(t) = z_i(t) - \hat{z}(t|t-1)$, $t = 1, \dots, m(t)$
 - 14: **if** $(z_i(t) - \hat{z}(t|t-1))^T S(t)^{-1} (z_i(t) - \hat{z}(t|t-1)) < \gamma$
and $z_i(t) \in \{\bar{z}(t)\}$ **then**
 - 15: $ztk = \{z_i(t)\}_{i=1}^{m_t}$
 - 16: **end if**
 - 17: $\beta = \sum^k g(\omega)$
 - 18: $v(t) = \sum_{i=1}^{m(t)} \beta_i(t) v_i(t)$;
 - 19: $P(H) = \prod g(\omega) P_D \prod (1 - P_D) \prod B$;
 - 20: $\hat{x}(t|t) = \hat{x}(t|t-1) + W(t)v(t)$;
 - 21: $P(t|t) = \beta_0(k)P(t|t-1) + (1 - \beta_0(t))P^c(t|t) + \tilde{P}(t)$;
 - 22: **end for**
-

C. MCMCDA

After we have discussed JPDA, some other common and relatively advanced methods are widely applied to implement target tracking. One of the intuitive way which was also applied in Bayesian Estimation is Monte Carlo Markov Chain(MCMC) model. We now can apply MCMC as a different perspective in modeling the question.

Instead of what we have done in Kalman or PDFA, we now capture the state transition for the underlying tracks that might exist, and we perform merging, deleting or switching for the tracks that have been detected in the last step using MCMC Bayesian filtering. We will not illustrate the details of MCMCDA, the benefit of MCMCDA is similar to PDFA, it is more robust to noisy data set and low detection rate. It outperforms JPDA in the cases where problems are more complex and unknown, MCMCDA can save time in processing based on the observation numbers and sample size. As a consequence, using JPDA will consume more time in classifying the joint samples if the dimension of the data is higher.

V. CONCLUSIONS

During the study in single maneuvering target tracking in clutter problem, we have fully understood and applied probabilistic data association filtering by going step by step in introducing the problem background, and we have built problem modeling assumptions, methodology and

implementation, we have then performed a through analysis on the performance with PDFA, combined with comparison to Kalman Filtering.

Then we have moved onto the earlier studying which had been covered earlier in this semester such as Hidden Markov Model, and then we have extended the learning to joint probabilistic data association filtering in solving multiple target tracking problem under much extensive amount of noise in measurements.

By reviewing and comparing some of the basic Bayesian method, we then expanded the learning to solve more complicated problem such as multiple target tracking. We have went through JPDA and then touched on MCMCDA, these methods are both good ways in application and will require more time in computing, and MCMCDA has an advantage in practice in general.

We have learned that the Bayesian approach is a major idea behind signal and data processing and machine learning, with a given data set, we view the data set as a distribution with a probability density function, and we model the problem based on the power and time limit given for doing a trade off between complexity and speed. In practice the model can vary due to different conditions such as the interference noise type or the potential existing target track for detection. And this study is a valuable experience for real word application from academic findings and gained the learning in implementation by computation in practice.

APPENDIX

Appendixes should appear before the acknowledgment.

ACKNOWLEDGMENT

The preferred spelling of the word acknowledgment in America is without an e after the g. Avoid the stilted expression, One of us (R. B. G.) thanks . . . Instead, try R. B. G. thanks. Put sponsor acknowledgments in the unnumbered footnote on the first page.

References are important to the reader; therefore, each citation must be complete and correct. If at all possible, references should be commonly available publications.

REFERENCES

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