Weekly Problem Assignment 1

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1. Empirical cdf:

$$E(Y) = \frac{1}{n}\sum_{k=1}^{n} p(X_k \leq x) = \frac{1}{n}\sum_{k=1}^{n} F_k(x)$$

$$E(Y^2) = \frac{1}{n^2}\sum_{k=1}^{n}\sum_{l=1}^{n} p(X_k \leq x, X_l \leq x)$$

$$var = \frac{1}{n^2}\sum_{k=1}^{n}\sum_{l=1}^{n}(p(X_k \leq x, X_l \leq x) - F_{xk}(x)F_{xl}(x))$$

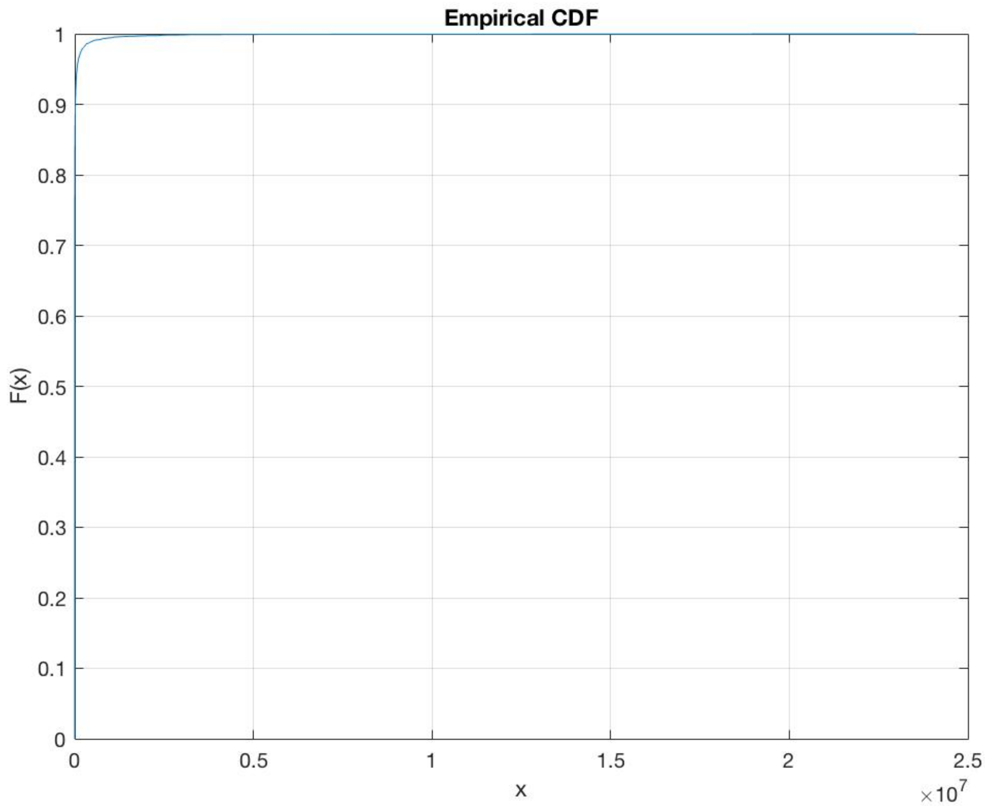
becasause the cdf is identical for itslef and distribution is iid

$$= \frac{1}{n^2}\sum_{k=1}^{n} F_{xk}(x)(1 - F_{xk}(x))$$

Simulation Result var(y)
ans =0.0989

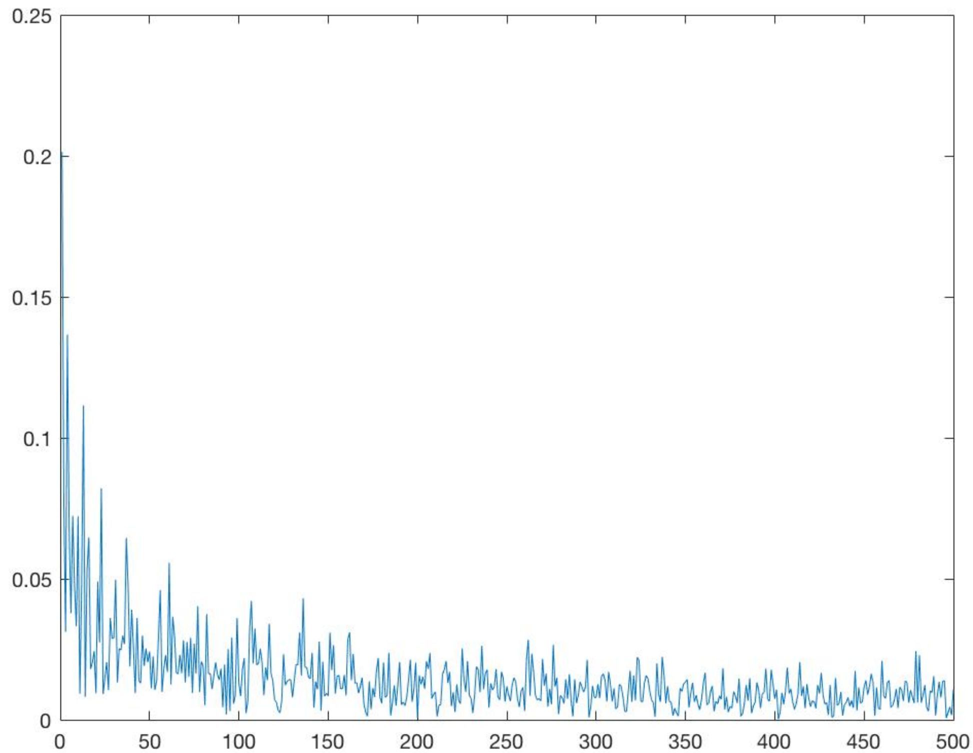
2. Simulation of F(x) using composition, we can simulate

$$\frac{x}{3}, \frac{x^3}{3}, \frac{x^5}{3}$$

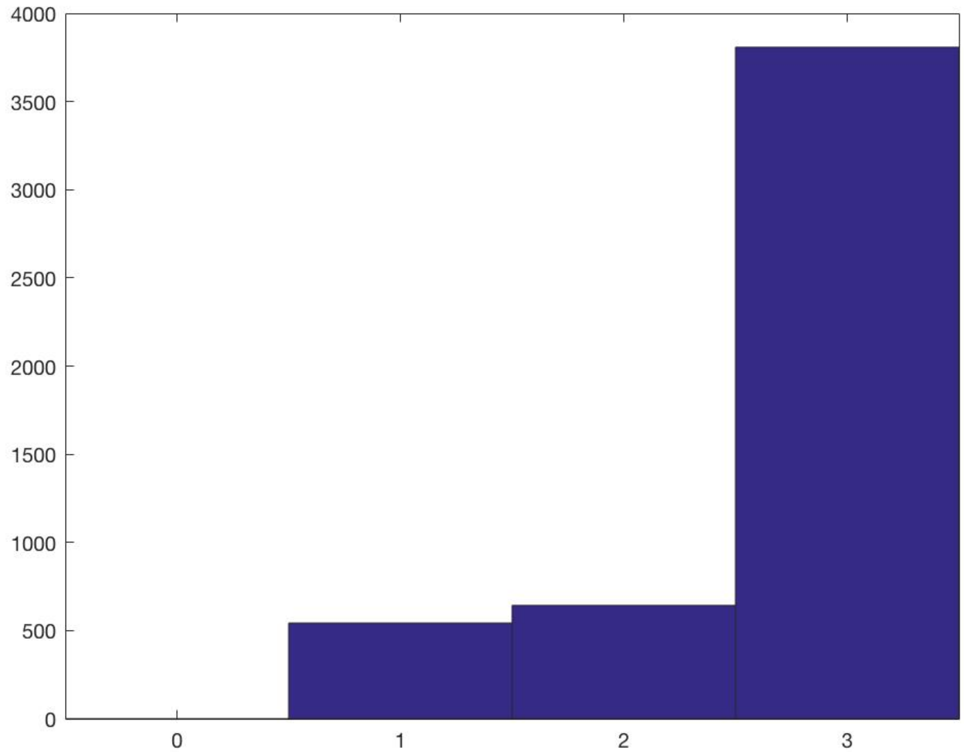


3. Simulate 3 state Markov Chain, using acceptance rejection method.
   The error decrease with the increased number of samples, which approves the law of

large numbers.



This plot is for state distribution with state 1 2 3 when 5000 times of the Markov process



4.

5.

6.

7. a) proof of Smoothing property of conditional expec- tations

$$
\begin{aligned}
&E\left\{E\left\{X|Y\right\}\right\} \\
&= E\left[\sum x \cdot P(X = x|Y)\right] \\
&= \sum_y [\sum_x x \cdot P(X = x|Y)] \cdot p(Y = y) \qquad [\\
&= \sum_y x \sum_x P(X = x, Y = y) \\
&= \sum_x x \cdot P(X = x) \\
&= E(X)
\end{aligned}
$$

b) Proof of Rao-Blackwellization

c)stratified sampling in stochastic simulation