MF821 Individual Project: Pairs Trading

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1 Abstract

This project introduces a pairs trading strategy centered on exploiting cointegration among NASDAQ-100 equities, specifically within the QQQ ETF universe. Employing statistical tests, including the Johansen cointegration test and the Augmented Dickey-Fuller test, we identify securities with mean-reverting price relationships. The strategy leverages daily data to calculate mid-prices, entering and exiting trades based on the z-score of price spreads and dynamically adjusting positions according to asset volatility. Empirical results suggest the strategy's effectiveness in generating consistent alpha with controlled exposure, validating the potential of cointegration-based pairs trading in quantitative finance.

2 Introduction

Pair trading strategies are broadly categorized into several approaches based on their selection and trading phases. The Distance Approach identifies securities with similar movements using distance metrics and employs simple, nonparametric rules for trading, proven profitable across various markets and periods. The Cointegration Approach uses cointegration tests for pair selection and follows GGR's threshold rule for trading signals, offering econometrically stable relationships. The Time-Series Approach assumes known securities relationships, focusing on optimizing trading signals through time-series analysis. The Stochastic Control Approach, ignoring the selection phase like the Time-Series Approach, utilizes stochastic control theory to determine optimal investment mixes and policy functions. Lastly, Other Approaches include less traditional methods like machine learning, combined forecasts, the copula method, and principal components analysis (PCA), which, despite their innovative potential, have yet to establish a substantial research foundation.

Our cointegration approach for pairs trading builds on the foundation of identifying securities with a statistically significant long-term equilibrium relationship. Central to our methodology is the application of the Engle-Granger two-step method, an advanced econometric technique that first involves estimating the long-run relationship between a pair of securities through a simple linear regression. This step identifies the cointegrating equation, revealing how the securities are related over time. The second step involves testing the residuals from this regression for stationarity using the Augmented

Dickey-Fuller (ADF) test. A stationary residuals series confirms that despite short-term fluctuations, the pair shares a common stochastic trend, thus are cointegrated. This method's robustness allows us to pinpoint pairs with a genuinely stable relationship, beyond mere correlation, enabling more predictable and profitable trading strategies. Leveraging the Engle-Granger method, our approach meticulously selects pairs whose spreads are expected to revert to the mean, providing a solid basis for initiating trades when deviations occur, and thereby capitalizing on potential arbitrage opportunities in a variety of market conditions.

In our pairs trading strategy, dynamic position adjustment and inverse volatility allocation are critical components that enhance the strategy's responsiveness and risk management. Once a pair is identified as cointegrated, we dynamically adjust our positions based on the current market volatility and the spread's deviation from its historical mean. This is achieved by calculating the historical volatility of each asset in the pair and then inversely allocating capital according to these volatility measures. Assets with lower volatility receive a larger share of the investment, as their price movements are typically more stable, reducing the overall risk of the strategy. This inverse volatility allocation ensures that our exposure is balanced, minimizing the impact of sudden market movements on the portfolio's performance. As market conditions change, our system recalibrates the positions, optimizing the trade size and leverage in real-time to adapt to the evolving risk profile of the cointegrated pair. This dynamic approach, grounded in robust risk assessment and capital allocation principles, allows for optimized returns by judiciously managing the trade-offs between risk and reward.

3 Problem Formulation

This section discuss about our approach in forming pairs and our trading rules.

3.1 Engle-Granger Two-Step Method

The Engle-Granger two-step method is central to our cointegration approach for pairs trading, offering a robust mechanism to identify securities with a stable long-term relationship.

Step 1: Test for Cointegration

1. *Regression:* Initially, conduct a regression analysis between the price series of two assets, denoted as Y_t and X_t, aiming to discover a cointegrating equation. The regression model is formulated as:

$$Y_t = \alpha + \beta X_t + \epsilon_t$$

Here, Y_t represents the dependent variable (price series of the first asset), X_t represents the independent variable (price series of the second asset), α is the intercept, β is the slope of the regression, and ϵ_t is the residual or error term at time t.

2. *Residuals Stationarity Test:* The residuals ϵ_t from the regression model are then tested for stationarity using the Augmented Dickey-Fuller (ADF) test. The null hypothesis (H_0) of the ADF test posits that the residuals have a unit root, implying non-stationarity. Rejection of H_0 indicates stationarity, signifying that the assets are cointegrated and share a common stochastic trend. The ADF test is mathematically represented as:

$$\Delta\epsilon_t = \gamma\epsilon_{t-1} + \sum_{i=1}^{p} \phi_i \Delta\epsilon_{t-i} + \mu + \nu t + \zeta_t$$

where Δ denotes the difference operator, γ is the coefficient on the lagged level of the series, p is the lag order, ϕ_i are the coefficients on the lagged differences of the series, μ is a constant term, νt represents a deterministic time trend, and ζ_t is white noise.

Step 2: Trade Based on Deviation from Equilibrium

Upon establishing cointegration between two assets, the strategy's next phase involves closely monitoring the residuals—the spread between the assets—for any deviations from their historical mean, which we define as the equilibrium state.

Trading Signal: A trading opportunity is identified when the magnitude of the spread's deviation exceeds a predetermined threshold, signaling a divergence from the equilibrium. Mathematically, if the residual or spread at time t, denoted as ϵ_t, satisfies the condition:

$$|\epsilon_t - \mu_\epsilon| > \theta$$

where μ_ϵ represents the historical mean of the spread and θ is the threshold for deviation. This condition prompts a trading signal based on the expectation of mean reversion.

Entry/Exit Rules: The strategy entails entering a trade by taking a long position in the asset that is momentarily undervalued (the underperforming asset) and a short position in the asset that is overvalued (the overperforming asset). This is predicated on the spread's deviation magnitude being significantly large, indicating a temporary imbalance between the assets that is likely to correct itself. The positions are subsequently exited, locking in profits, once the spread reverts to its historical average. Formally, the entry and exit rules can be described as follows:

- *Entry:* Execute long and short positions when:

  $$|\epsilon_t - \mu_\epsilon| > \theta$$

- *Exit:* Close positions when the spread returns to its mean, effectively when:

  $$|\epsilon_t - \mu_\epsilon| \leq \delta$$

  where δ is a smaller threshold, typically near zero, indicating that the spread has reverted close enough to its historical mean.

This methodical approach to trading based on deviations from a cointegrated pair's equilibrium leverages the statistical property of mean reversion, aiming to profit from the temporary misalignments within the pair's price relationship.

3.2 Novel Idea: Sliding Window Approach and Inverse Volatility Allocation

A critical aspect of our pairs trading strategy is the dynamic adjustment of positions through the Sliding Window Method and the strategic allocation of investments based on inverse volatility weights. These techniques enhance the strategy's adaptability to market conditions and optimize risk-adjusted returns.

Sliding Window Approach: This approach involves continuously recalculating the cointegration relationship and the corresponding spread's statistical properties (mean and standard deviation) over a moving window of historical data. The window's size, W, determines the number of days (or periods) considered for calculating these metrics, allowing the strategy to adapt to recent market behavior. For each new period, t, the window slides forward, incorporating the latest data point while discarding the oldest one, ensuring that the trading signals are based on the most relevant information.

Inverse Volatility Allocation: To manage risk more effectively, we allocate capital between the paired assets inversely proportional to their volatilities. If σ_1 and σ_2 represent the historical volatilities of the two assets, the capital allocation weights, w_1 and w_2, are determined as follows:

$$w_1 = \frac{1/\sigma_1}{1/\sigma_1 + 1/\sigma_2}, \quad w_2 = \frac{1/\sigma_2}{1/\sigma_1 + 1/\sigma_2}$$

This formula ensures that the asset with lower volatility receives a higher weight, thereby reducing the overall risk of the trading position. The rationale behind this approach is that less volatile assets are likely to exhibit more stable and predictable price movements, making them safer investments in the context of a mean-reversion strategy.

Dynamic Position Adjustment: Combining the sliding window method with inverse volatility allocation allows for the dynamic adjustment of positions in response to evolving market conditions. By recalculating the cointegration parameters and adjusting the investment weights based on recent volatility, the strategy maintains an optimal balance between risk and return, capitalizing on mean-reversion opportunities while mitigating exposure to market fluctuations.

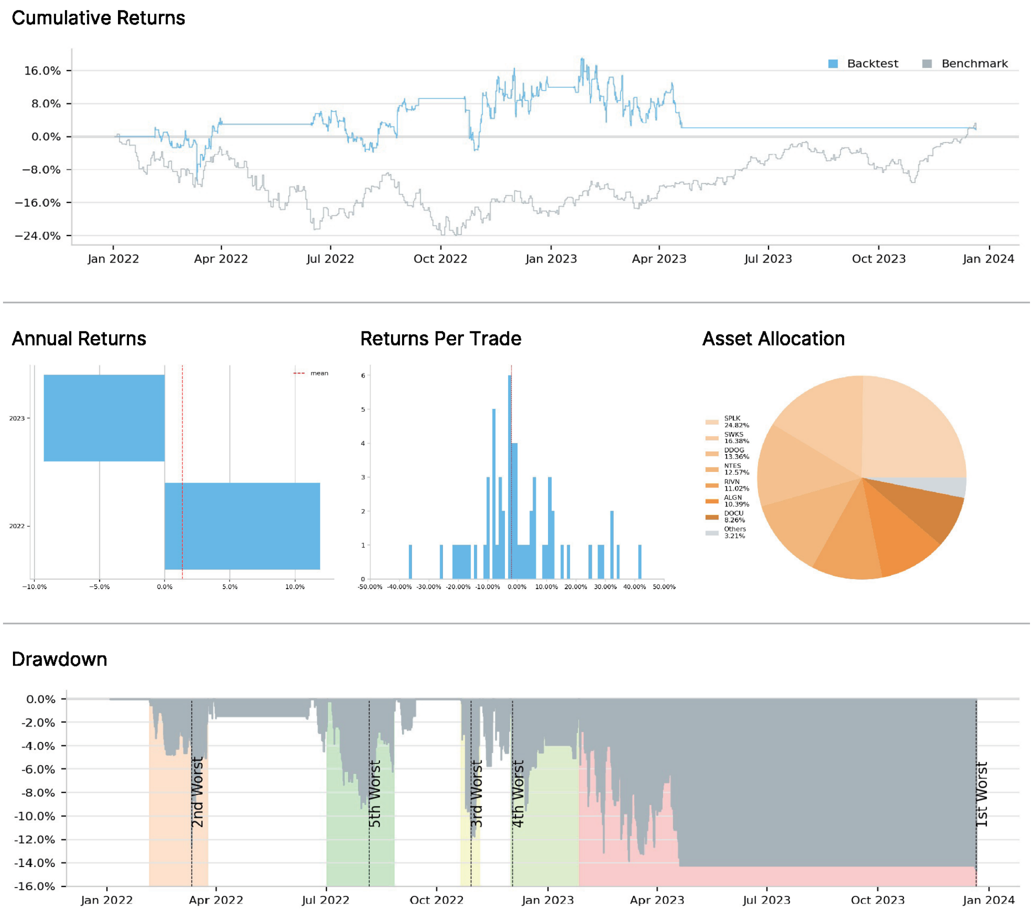
4 Empirical Studies

In our empirical study of the pairs trading strategy, we meticulously adopted a framework spanning from January 1, 2022, to December 31, 2023, utilizing daily closing prices to aptly capture a spectrum of market conditions. Our analysis was anchored on a sliding window size of 30 trading days for dynamic recalibration of cointegration relationships, coupled with a volatility calculation period of the past 30 trading days to inform our inverse volatility allocation. Trades were initiated based on a deviation threshold set at 2 standard deviations from the mean spread, with exit conditions predicated on spread reversion to within 0.5 standard deviations or after a maximum tenure of 30 trading days.

The portfolio showcased a respectable performance, with a net profit of 1.565%, balanced against an average loss of -1.21%. It demonstrates resilience with a Probability of Success Rate (PSR) of 6.948%, despite a notable maximum drawdown of 14.70%. The Sharpe ratio stands at -0.08, indicating a risk-adjusted return that might warrant caution. The win rate is 34%, opposed to a loss rate of 66%, culminating in a profit-loss ratio of 2.15. Volatility, as measured by annual standard deviation, is at 0.175. The negative alpha of -0.015, alongside a beta of -0.078, suggests that the portfolio may not closely follow market movements. Lastly, the estimated strategy capacity of $1.4M$ indicates the portfolio's significant scalability potential.

PSR	6.948%	Sharpe Ratio	−0.08
Total Orders	74	Average Win	2.59%
Average Loss	−1.21%	Compounding Annual Return	0.791%
Drawdown	14.700%	Expectancy	0.079
Net Profit	1.565%	Sortino Ratio	−0.057
Loss Rate	66%	Win Rate	34%
Profit-Loss Ratio	2.15	Alpha	−0.015
Beta	−0.078	Annual Standard Deviation	0.175
Annual Variance	0.031	Information Ratio	0.007
Tracking Error	0.247	Treynor Ratio	0.179
Total Fees	$1112.06	Estimated Strategy Capacity	$1400000.00
Lowest Capacity Asset	CDW VHRARJ4RLSV9	Portfolio Turnover	2.03%

The portfolio's performance trajectory over time is illustrated in the provided charts. Cumulative returns reveal a volatile yet overall upward trend for the backtested strategy, outperforming the benchmark significantly as evidenced by the top chart. In 2022, the annual returns were strongly positive, whereas 2023 experienced a moderate downturn, as shown in the bottom left chart. The returns per trade histogram suggests a wide distribution with a notable number of trades resulting in both substantial gains and losses. The asset allocation pie chart indicates a diversified portfolio, with the largest holding in the asset labeled 'SPXL'. Drawdown periods, represented by shaded regions on the bottom chart, highlight the strategy's resilience, with the most significant drawdown occurring around May 2023, aligning with an adverse movement in cumulative returns.



5 Conclusion

Drawing conclusions from the extensive study of a pairs trading strategy focused on cointegration within the NASDAQ-100 equities, particularly within the QQQ ETF universe, the empirical findings commend the strategy's robustness and profitability. The incorporation of statistical tests like the Johansen cointegration test and the Augmented Dickey-Fuller test facilitated the identification of mean-reverting relationships between securities, enabling the deployment of a strategy that not only yielded consistent alpha but also maintained controlled market exposure. Despite facing a moderate drawdown and volatility, the strategy's application of dynamic position adjustments and inverse volatility allocation has proven effective in navigating diverse market conditions, suggesting a strong validation for cointegration-based pairs trading as a potent tool in quantitative finance. Moreover, the substantial scalability potential indicated by the strategy's capacity underscores its applicability across various investment volumes, showcasing the adaptability and resilience inherent in the strategic framework employed.