

# Towards Event Prediction in Temporal Graphs

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## ABSTRACT

This paper proposes a class of temporal association rules, denoted by TACOs, for event prediction. As opposed to previous graph rules, TACOs monitor updates to graphs, and can be used to capture temporal interests in recommendation and catch frauds in response to behavior changes, among other things. TACOs are defined on temporal graphs in terms of change patterns and (temporal) conditions, and may carry machine learning (ML) predicates for temporal event prediction. We settle the complexity of reasoning about TACOs, including their satisfiability, implication and prediction problems. We develop a system, referred to as TASTE. TASTE discovers TACOs by iteratively training a rule creator based on generative ML models in a creator-critic framework. Moreover, it predicts events by applying the discovered TACOs. Using real-life and synthetic datasets, we experimentally verify that TASTE is on average 31.4 times faster than conventional data mining methods in TACO discovery, and it improves the accuracy of state-of-the-art event prediction models by 23.4%.

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## 1 INTRODUCTION

Event prediction is to predict a real-world occurrence that relates to a particular topic and will take place at a specific time [109]. Events range from large-scale (e.g., disease outbreaks), to medium-scale (e.g., system failures), to small-scale (e.g., fraud detection). Event prediction is important in a variety of domains such as disease control and business intelligence. To illustrate, consider online recommendation, a special case of event prediction (“sale events”), to recommend items to users. Recommendation models are mainly categorized into collaborative filtering (CF) by learning from user-item interactions, content-based (CB) by assessing the similarity of content features of users and items, and hybrid by integrating the two [107]. In the real world, however, e-commerce companies often monitor changes to transaction graphs, and employ rules to catch users’ temporal interests and detect fraudulent behaviors.

**Example 1:** Below are two rules from an e-commerce platform.

(1) If a movie is nominated for a film award and if a user watches the movie within two weeks after its nomination, then recommend

the movie to the friends of the user between the nomination date and the date of the award ceremony. Here the **nomination** indicates a “change” that triggers the recommendation in a time interval. Such cases are beyond conventional CF and CB models.

(2) If a user searched “barbecue” at least  $m$  times in June, then recommend meat to the user in the next two months.

Temporal changes are also used to predict other events.

(3) If at least  $m$  cases of an infectious disease  $z$  are reported in an area within the past 2 weeks, then offer vaccines for  $z$  to the people there.

(4) If device  $M$  is used to access  $k$  accounts only once within a short period of one hour, and if each of these accounts has been regularly accessed by other devices at least  $m$  times in the past month, then device  $M$  is likely committing account-takeover attacks. □

**Challenges.** No matter how important, event prediction is challenging. While rules have been studied for graphs, e.g., graph functional dependencies (GFDs) [12], graph association rules (GARs) [21] and Horn rules [25], they are defined on *static* graphs and cannot express temporal changes. **There are several open questions.** How can we specify patterns of changes to graphs and temporal interests of users in logic rules? Can we improve the accuracy of event-prediction ML models with temporal conditions, and interpret their predictions? How expensive is it to reason about rules with temporal conditions and embedded ML models? How can we efficiently discover such rules from real-life graphs? Is it possible to scale with large graphs when we apply the rules to predict events?

**Contributions & organization.** This paper tackles these issues.

(1) *Temporal rules* (Section 2). We propose a class of temporal association rules, referred to as TACOs (TemporAl event prediCtiOn rules), to enrich event-prediction ML models with temporal conditions. TACOs are defined on temporal graphs in terms of change patterns, temporal conditions and ML prediction models (as predicates). In contrast to previous graph rules, TACOs are applied to *updates* to temporal graphs, which are typically much smaller than the entire graphs, to monitor changes and predict events.

(2) *Complexity* (Section 3). We study classical problems for TACOs, including (a) satisfiability to check whether a set of TACOs has no conflicts, (b) implication to decide whether a set of TACOs entails another TACO, and (c) prediction to forecast whether an event will occur. We show that they are  $\Sigma_2^P$ -complete,  $\Pi_2^P$ -complete and NP-complete, respectively. That is, despite the increased expressivity, TACOs do not make our lives much harder compared with [21, 22].

(3) *TASTE system* (Section 4). Despite the intractability, we develop a system called TASTE (TemporAl SysTem). TASTE (a) discovers high-quality TACOs with generative ML models, and (b) applies the discovered TACOs for temporal event prediction in parallel.

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(4) *Rule discovery* (Section 5). From the perspective of probability distribution learning, we develop a *creator-critic* framework to discover TACOs in TASTE, as a major contribution. The rule creator adopts GAN and LSTM language models to generate TACOs with candidate patterns and temporal conditions, while the critic evaluates such TACOs rules on temporal graphs, collects high-quality rules, and provides feedback to the creator for improving its quality in the next round. Different from the traditional discriminator in GAN, the “critic” is a predefined scoring algorithm that requires no training. This method avoids the exhaustive levelwise search in an exponentially large space [20] and is able to find high-quality TACOs with large graph patterns.

(5) *Parallel prediction* (Section 6). We develop a parallel algorithm for temporal event prediction with TACOs. In contrast to existing graph partitioning methods such as edge-cut or vertex-cut [3, 77], we propose to partition a temporal graph based on temporal locality such that event prediction can be made communication-free. We show that the parallel event prediction algorithm guarantees to run faster when given more processors, *i.e.*, parallel scalability.

(6) *Experimental study* (Section 7). Using real-life and synthetic datasets, we verify the accuracy, efficiency and scalability of TASTE. We find the following. On average, (1) TASTE is 23.8% and 23.0% more accurate than prior methods for temporal event prediction and dynamic recommendation, respectively. (2) The creator-critic discovery method of TASTE is more than 31 times faster than the levelwise algorithms. It finds TACOs with patterns of 20 edges in 1639s, in contrast to days by the conventional methods. (3) 84.76% of rules mined by levelwise methods can be found by TASTE using a small amount of training data and training rounds. (4) The event prediction algorithm in TASTE is parallelly scalable, *e.g.*, its runtime is reduced by 3.2 times when using 32 processors instead of 4.

**Novelty.** The paper proposes an approach towards event prediction by enriching ML models with temporal logic conditions in logic rules, and a practical method to disc over rules with large graph patterns. The novelty of the work consists of (1) TACOs, a class of rules for event prediction on *dynamic* graphs; (2) the complexity bounds of the satisfiability, implication and prediction problems for TACOs; (3) a creator-critic framework, the first that is able to discover high-quality graph rules with large patterns; (4) a strategy for partitioning temporal graphs based on temporal locality; (5) an algorithm for temporal event prediction that guarantees the parallel scalability; and (6) TASTE, a system based on new approaches to discovering TACOs and predicting events with TACOs.

**Related work.** We categorize the related work as follows.

*Temporal event prediction.* A variety of ML models have been explored for temporal event prediction, including (1) regression [24, 48, 64, 67]; (2) point processes [73, 99]; (3) survival analysis [16, 94]; and (4) embedding via, *e.g.*, tensor decomposition [18, 105], recurrent neural network (RNN) [36, 56, 92, 93], graph neural network (GNN) [11, 50, 58, 71, 81], autoencoder [29, 30, 78] and diachronic encoder [15, 26, 28, 101]. In particular, as a special type of event prediction, (temporal) recommendation has attracted a great deal of interest. For example, dynamic recommender systems (DRS) have been developed to improve the recommendation quality. They

can incorporate temporal factors of user preferences [75] using time-dependent neighborhood models [23, 60, 89], matrix factorization [42, 55, 59, 76] and tensor models [33, 96, 97].

Temporal association rules have been developed to specify time-dependent correlations of transaction data in relational tables [95, 102]. In addition, association rules have also been studied on graphs [5, 21, 65]. GARs [21] are defined on static graphs to specify regularities among entities. GTARs (Graph Temporal Association Rules) [65] specify connections between events by means of two event patterns that share a common focus node and a single constant time interval. Similarly, GERs (Graph Evolution Rules) [5] are defined with two connected sub-patterns that are decomposed from a single pattern, to represent local changes, *e.g.*, relabeling [83].

This work differs from the prior work in the following.

- (1) As an effort to unify rule-based and ML-based methods for event prediction, we propose TACOs by embedding event-prediction ML models as predicates. This allows us to not only leverage existing ML models, but also refine the ML models with logic conditions.
- (2) Unlike existing ML-based prediction strategies (*e.g.*, DRS) that adopt unexplainable blackbox models, our method with TACOs offers a logic interpretation of the ML predictions for temporal events.
- (3) TACOs are more expressive than GARs, GTARs and GERs. As will be seen in Section 2, these rules can all be expressed as TACOs. TACOs may embed prediction ML methods beyond GTARs and GERs, and support various temporal conditions on different events, while GTARs and GERs can only express constant time intervals.
- (4) We establish the complexity of classical problems for temporal graph rules. While these problems were studied for GARs on static graphs, no prior work has considered these for GTARs or GERs.

*Rule learners.* Prior graph rule discovery methods can be classified as follows: (1) levelwise data mining approaches that iteratively enumerate all possible candidate rules, and prune non-interesting rules with criteria such as support/confidence [5, 20, 65], head coverage and PCA confidence [25]; and (2) rule learners that typically enumerate paths as candidate rules and learn a weight for each candidate to quantify its quality. The weight learning methods include Markov logic networks [40, 79], path ranking [44], relational dependency networks [66, 68], neural logic programming [80, 82, 103], and reinforcement learning [14, 52, 62, 74, 87, 100].

Different from existing rule learners, (a) we study rule discovery in temporal graphs, while the prior learners focus on static graphs. (b) We propose to discover TACOs with general topological structures, beyond merely paths. (c) We discover TACOs that may carry ML predicates. (d) We propose a deep-generative-model-based approach to learning graph rules with large graph patterns. In contrast, conventional mining methods for graph rules [20] can hardly find rules with patterns of 7 edges or more.

*Pattern generators.* Graph generative models have been extensively studied to build graph patterns. Such models usually ensure that the generated patterns sustain certain properties, *e.g.*, the power-law node degree distribution [1], community structures [84, 98], distribution over random walks [7, 110], distribution of node and edge sequences [51, 54, 104], graphlets distribution [35, 106] and time-invariant conditions and time-variant conditions [108].



**Example 3:** The rules of Example 1 can be expressed as TACOs.

(1)  $\varphi_1 = \Delta Q_1[\bar{x}](X_1 \rightarrow (\text{recommend}(y, x), [0, t_1]))$ , where  $X_1 = \emptyset$  and  $t_1$  is a timestamp in weeks. The TACO says that if user  $x_1$  watches movie  $y$  in 2 weeks after the nomination of  $y$  for award  $z$  and if  $x$  is a friend of  $x_1$ , then recommend  $y$  to  $x$  by the date  $t^* + t_1$  of the award ceremony. Note that the condition that user  $x_1$  watches the movie in 2 weeks is specified by function  $T_Q$  in  $\Delta Q_1$  (see Fig. 1).

(2)  $\varphi_2 = \Delta Q_2[\bar{x}](X_2 \rightarrow (\text{recommend}(y, x), [0, 60]))$ , where  $X_2$  is  $(\bigwedge_{i \in [1, m-1]} e_i.t < e_{i+1}.t) \wedge (\bigwedge_{i \in [1, m]} e_i.t \in [t_{\text{june}}, t'_{\text{june}}])$ . Here  $e_i.t < e_{i+1}.t$  ensures that searches  $e_i$  and  $e_{i+1}$  are distinct and  $t_{\text{june}}$  (resp.  $t'_{\text{june}}$ ) indicates the date of June 1 (resp. June 30). It says that if  $x$  searches barbecue at least  $m$  times in June (specified in  $X_2$ ), then recommend meat to  $x$  in the next two months (60 days).

(3)  $\varphi_3 = \Delta Q_3[\bar{x}](X_3 \rightarrow (\text{offer}(y, x), [2, 6]))$ . Precondition  $X_3$  is  $\bigwedge_{i, j \in [1, m], i \neq j} (x_i.\text{id} \neq x_j.\text{id})$ , to enforce that the cases  $x_i$  and  $x_j$  are distinct. Observe that in the  $\Delta$ -pattern  $\Delta Q_3$ , the time window  $[0, 2]$  indicates that person  $x_i$  contracts disease  $z$  in 2 weeks after the current time  $t^*$ , i.e., the earliest case confirmed. It says that if at least  $m$  cases of disease  $z$  are confirmed in area  $w$  within 2 weeks and if person  $x$  lives in  $w$ , then offer vaccine  $y$  for disease  $z$  to  $x$  in a month.

(4)  $\varphi_4 = \Delta Q_4[\bar{x}](X_4 \rightarrow (x.\text{status} = \text{fraud}, [0, 0]))$ , where  $X_4$  is  $\bigwedge_{i \in [1, k]} ((\bigwedge_{j, l \in [1, m], j \neq l} z_i^j.\text{id} = z_l^j.\text{id} \wedge e_i^j.t \neq e_l^j.t) \wedge \mathcal{M}(x, y_i, \text{attack}, e_i.t))$ . Here  $\mathcal{M}$  is an ML model suspecting that  $x$  attacked account  $y_i$  at time  $e_i.t$ . The time windows in  $\Delta Q_4$  are in hours. The TACO says that if device  $x$  accesses  $y_1, \dots, y_k$  within one hour,  $\mathcal{M}$  suspects them as attack behaviors, and if each  $y_i$  has been accessed by other devices  $m$  times in the past month, then  $x$  is likely a fraud.  $\square$

**Remark.** (1) GARs [21], GTARs [65] and GERs [5] are special cases of TACOs. (a) GARs are TACOs when all time windows are  $[-\infty, \infty]$  and preconditions  $X$  carry no temporal predicates. We adopt a single predicate  $p_0$  for the event in each TACO to simplify the discussion; this does not lose expressive power compared to GARs [20]. (b) A GTAR can be expressed as multiple TACOs, where each TACO (i) has a pattern that extends the antecedent pattern  $P_1$  of the GTAR with the nodes in its consequent pattern  $P_2$ , and (ii) encodes a single edge from  $P_2$  as the event  $p_0$  with  $\tau = [t_\alpha, t_\alpha]$ , where  $t_\alpha$  is the constant time interval specified in the GTAR. (c) Similarly, a GER can be encoded as a set of TACOs, one for each update indicated by its patterns with the maximum timestamp [5].

(2) As demonstrated by  $\varphi_4$  of Example 3, one can plug a well-trained ML prediction model  $\mathcal{M}$  in precondition  $X$ , and enrich  $\mathcal{M}$  with temporal and logic conditions. A TACO of the form  $\Delta Q[\bar{x}](X \rightarrow (\mathcal{M}(x, y, l, t), \tau))$  interprets ML predictions with logic predicates.

(3) One can associate TACOs with a probability for the predicted events. However, this would incur higher complexity (#P-hard) as indicated by probabilistic logic programming [41].

(4) TACOs can readily handle interval-timestamped graphs  $G$  [4]. The only difference is that when pattern edge  $e$  of a TACO matches edge  $e'$  in  $G$ , we require that the time range of  $e$  overlaps with at least one time interval of  $e'$ , instead of covering a single timestamp.

**Semantics.** Denote by  $h(\bar{x})$  a match of  $\Delta Q[\bar{x}]$  in a graph  $G$ , and  $p$  a predicate of  $\Delta Q[\bar{x}]$ . We say that match  $h(\bar{x})$  satisfies a predicate  $p$ , denoted by  $h(\bar{x}) \models p$ , if the following condition is satisfied: (a) if  $p$  is

**Table 1: Notations**

Notations	Definitions
$G, \Delta Q[\bar{x}]$	graph and $\Delta$ -pattern, respectively
$\Delta G_Q$	updates to $G$ relative to pattern $\Delta Q$
$\varphi$	TACO $\Delta Q[\bar{x}](X \rightarrow (p_0, \tau))$
$\text{supp}(\varphi, G)$	the support of TACO $\varphi$ in graph $G$
$\text{conf}(\varphi, G)$	the confidence of TACO $\varphi$ in graph $G$
$\alpha, \beta, \gamma, \delta$	thresholds (node, support, confidence, time window)

$x.A$ , then node  $h(x)$  carries attribute  $A$ ; (b) if  $p$  is  $l(x, y)$ , then there is an edge from  $h(x)$  to  $h(y)$  labeled  $l$ ; (c) if  $p$  is  $\mathcal{M}(x, y, l, t)$ , then the ML classifier  $\mathcal{M}$  predicts an association between  $h(x)$  and  $h(y)$  with label  $l$  at time  $t^* + t$ ; (d) if  $p$  is  $x.A \oplus y.B$ , then attributes  $A$  and  $B$  exist at  $h(x)$  and  $h(y)$ , respectively, and  $h(x).A \oplus h(y).B$ ; similarly for  $x.A \oplus c$ ; and (e) if  $p$  is  $e_1.t \oplus e_2.t$ , then the timestamps of edges matching  $e_1$  and  $e_2$  have the  $\oplus$  relationship; similarly for  $e.t \oplus c$ .

For a conjunction  $X$  of predicates, we write  $h(\bar{x}) \models X$  if  $h(\bar{x}) \models p$  for all  $p$  in  $X$ . We write  $h(\bar{x}) \models \varphi$  for TACO  $\varphi = \Delta Q[\bar{x}](X \rightarrow (p_0, \tau))$  if  $h(\bar{x}) \models X$  implies  $h(\bar{x}) \models p_0$  and  $p_0$  occurs within time window  $\tau$ .

We say that graph  $G$  satisfies TACO  $\varphi = \Delta Q[\bar{x}](X \rightarrow (p_0, \tau))$ , denoted by  $G \models \varphi$ , if for all matches  $h(\bar{x})$  of  $\Delta Q[\bar{x}]$  in  $G$ ,  $h(\bar{x}) \models \varphi$ . We say that  $G$  satisfies a set  $\Sigma$  of TACOs, denoted by  $G \models \Sigma$ , if for all TACOs  $\varphi \in \Sigma$ ,  $G \models \varphi$ , i.e.,  $G$  satisfies every TACO in  $\Sigma$ .

**Example 4:** Consider the temporal graph  $G$  illustrated in Figure 1, where each edge is annotated a timestamp. Then  $G$  satisfies the TACO  $\varphi_1$  of Example 3. Observe that there only exists a single match  $h(\bar{x})$  of  $\Delta Q_1$  in  $G$ :  $x_1 \mapsto u_2$ ,  $x_1 \mapsto u_1$ ,  $x_2 \mapsto a$  and  $y \mapsto m_1$ . As  $u_2$  is linked from  $m_1$  via a recommend edge with timestamp  $t_1$  in  $G$ , we know that  $h(\bar{x}) \models (\text{recommend}(y, x), [0, t_1])$ , and  $G \models \varphi_1$  follows.  $\square$

The notations of the paper are summarized in Table 1.

### 3 REASONING ABOUT TEMPORAL RULES

In this section we study fundamental problems for TACOs, including the satisfiability, implication and prediction problems. We show that although TACOs are more expressive than GARs [21], these problems for TACOs are not much harder than for GARs.

**Satisfiability.** The *satisfiability problem* is as follows.

- Input: A set  $\Sigma$  of TACOs.
- Question: Does there exist a graph  $G$  such that  $G \models \Sigma$  and for each TACO  $\Delta Q[\bar{x}](X \rightarrow (p_0, \tau)) \in \Sigma$ ,  $\Delta Q$  has a match in  $G$ ?

Intuitively, this is to make sure that the discovered TACOs have no conflicts and can be applied to a graph at the same time.

The problem is coNP-complete for GARs [21] and  $\Sigma_2^P$ -complete for graph denial constraints (GDCs) [22]. We show the following.

**Theorem 1:** The satisfiability problem is  $\Sigma_2^P$ -complete for TACOs.  $\square$

Here  $\Sigma_2^P$  denotes the class of decision problems that can be checked in NP using an NP oracle, i.e.,  $\Sigma_2^P = \text{NP}^{\text{NP}}$ . Similarly, the class  $\Pi_2^P = \text{coNP}^{\text{NP}}$  is defined (see [70] for more details).

We assume w.l.o.g. the following about ML predicates in TACOs; (a) ML prediction (i.e., testing with pre-trained  $\mathcal{M}$ ) takes polynomial time (PTIME), and (b) there exists a “small” range of values such that for any node pairs  $(x, y)$ , their attribute values can be encoded by values in the small range and yield the same truth value for  $\mathcal{M}$ . These are the case for ML models used by TACOs in practice.



**Proof sketch:** We first show a small model property: if a set  $\Sigma$  of TACOs is satisfiable, then there exists a temporal graph  $G_\Sigma$  such that  $|G_\Sigma| \leq 4|\Sigma|^3$  and  $G_\Sigma \models \Sigma$ . Based on this, we develop an  $\Sigma_2^P$  algorithm to check whether a given set  $\Sigma$  of TACOs is satisfiable.

The lower bound follows from the  $\Sigma_2^P$ -completeness of the satisfiability problem for GDCs [22], since it is easy to verify that GDCs are also a special case of TACOs.  $\square$

**Implication.** A set  $\Sigma$  of TACOs *implies* a TACO  $\varphi$ , denoted by  $\Sigma \models \varphi$ , if for all graphs  $G$ , if  $G \models \Sigma$  then  $G \models \varphi$ .

The *implication problem* is stated as follows.

- Input: A set  $\Sigma$  of TACOs and a TACO  $\varphi$ .
- Question: Does  $\Sigma \models \varphi$ ?

The implication analysis helps us remove redundant rules.

The implication problem is NP-complete for GARs [21] and  $\Pi_2^P$ -complete for GDCs [22]. For TACOs it is no harder than for GDCs.

**Theorem 2:** The implication problem is  $\Pi_2^P$ -complete for TACOs.  $\square$

**Proof sketch:** We prove a small model property for the problem: if  $\Sigma \not\models \varphi$ , then there exists a temporal graph  $G_\varphi$  such that  $|G_\varphi| \leq 8|\varphi|(|\varphi| + |\Sigma|)^2$ ,  $G_\varphi \models \Sigma$  but  $G_\varphi \not\models \varphi$ . Based on this, we design an  $\Sigma_2^P$  algorithm to check whether  $\Sigma \models \varphi$ . The lower bound directly follows from the  $\Pi_2^P$ -complete implication problem for GDCs [22].  $\square$

**Prediction.** We also study the *prediction problem*.

- Input: A temporal graph  $G$ , a set  $\Sigma$  of TACOs, a time window  $\tau$ , a label  $l$ , and two nodes  $u$  and  $v$  in  $G$ .
- Question: Does there exist edge labeled  $l$  from  $u$  to  $v$  in  $\tau$  by  $\Sigma$ ?

This is to study the complexity of temporal prediction with TACOs.

A similar problem (deduction problem) was shown NP-complete for GARs [21]. It is no harder to predict events with TACOs.

**Theorem 3:** The prediction problem is NP-complete for TACOs.  $\square$

**Proof sketch:** For the upper bound, we define a notion of proof trees and show that  $(\diamond)$  the prediction  $(p_0, \tau)$  can be deduced in  $G$  if and only if there exists a proof tree  $\mathcal{T}$  with  $|\mathcal{T}| \leq 7|G|^2(|G| + |\Sigma|)^2(|G| + |\tau|)^3$  that witnesses the prediction. Based on  $(\diamond)$ , we design an NP algorithm for the problem: guess a tree  $\mathcal{T}$  such that  $|\mathcal{T}| \leq 7|G|^2(|G| + |\Sigma|)^2(|G| + |\tau|)^3$  and check whether it is a proof tree of the prediction  $(p_0, \tau)$  in PTIME.

We show the lower bound by reduction from the graph homomorphism problem, which is NP-complete (cf. [27]). The latter is to decide, given two undirected graphs  $G_1$  and  $G_2$ , whether there exists a homomorphism  $h$  from  $G_1$  to  $G_2$ . We use TACO  $\varphi$  to encode  $G_1$ , temporal graph  $G$  to encode  $G_2$ , and  $(p_0, \tau)$  to represent  $h$ .  $\square$

**Remark.** The complexity bounds remain unchanged for interval-timestamped graphs [4]. For the prediction problem, we only need to check whether a time interval overlaps with another, which takes PTIME. For the other two problems, the small model properties remain intact and so does the complexity. Moreover, the results also hold when timestamps are chosen from densely ordered set.

## 4 TEMPORAL SYSTEM

In this section we introduce TASTE (Temporal SysTEM), a system for discovering TACOs and predicting events with TACOs. We first state the corresponding discovery and prediction problems (Section 4.1). We then present the architecture of TASTE (Section 4.2).

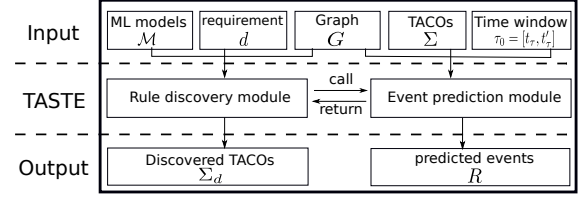


Figure 2: Architecture, inputs and outputs of TASTE

### 4.1 TACO Discovery and Event Prediction

We first define quality measures for TACOs.

**Quality metrics.** We use support and confidence to quantify the quality of TACOs, for frequency and reliability, respectively.

**Support.** The *support* of TACO  $\varphi = \Delta Q[\bar{x}](X \rightarrow (p_0, \tau))$  in graph  $G$  is

$$\text{supp}(\varphi, G) = |\Delta Q(e_x, \varphi, G)|,$$

where  $\Delta Q(e_x, \varphi, G)$  denotes the set of edges  $h(e_x)$  in those matches  $h(\bar{x}) \in \Delta Q(G)$  such that  $h(\bar{x}) \models X$ ,  $h(\bar{x}) \models p_0$ , and  $p_0$  occurs in  $\tau$ .

Intuitively, the support counts the number of distinct “satisfiable” matches in regards to the designated edge  $e_x$ . One can verify that this measure is anti-monotonic w.r.t. a partial order  $\leq$  over TACOs, i.e.,  $\text{supp}(\varphi, G) \geq \text{supp}(\varphi', G)$  for any  $G$  if  $\varphi \leq \varphi'$ .

**Confidence.** We define the *confidence* of a TACO  $\varphi = \Delta Q[\bar{x}](X \rightarrow (p_0, \tau))$  in graph  $G$ , denoted by  $\text{conf}(\varphi, G)$ , as follows:

$$\text{conf}(\varphi, G) = \frac{\text{supp}(\varphi, G)}{|\Delta Q(e_x, X, G)|}.$$

Here  $\Delta Q(e_x, X, G)$  is the set of edges  $h(e_x)$  in all matches  $h \in \Delta Q(G)$  that satisfy  $X$ . The ratio quantifies the likelihood that event  $p_0$  happens in  $\tau$  with matches satisfying prediction  $X$ , i.e., reliability.

**TACO discovery.** The discovery problem for TACOs is as follows.

- *Input:* A temporal graph  $G$ , a positive integer  $\alpha$ , a support threshold  $\beta > 0$ , a confidence threshold  $\gamma \in [0, 1]$ , and a bound  $\delta > 0$  on the lengths of time windows.
- *Output:* A set  $\Sigma$  of TACOs such that for each  $\varphi \in \Sigma$ ,  $\text{supp}(\varphi, G) \geq \beta$ ,  $\text{conf}(\varphi, G) \geq \gamma$ ,  $t_2 - t_1 \leq \delta$  for each time window  $[t_1, t_2]$  in  $\varphi$ , and there are at most  $\alpha$  nodes in the pattern of  $\varphi$ .

We call the tuple  $(\alpha, \beta, \gamma, \delta)$  a *discovery requirement*  $d$ . The problem aims to discover the set of all TACOs that conform to  $d$ , and have the expected number of pattern nodes bounded by  $\alpha$ .

**Temporal event prediction.** The problem is stated as follows.

- *Input:* A graph  $G$ , a set  $\Sigma$  of TACOs and a time window  $\tau_0$ .
- *Output:* A set  $R$  of edges for  $G$  predicted by  $\Sigma$  such that the occurrence of the event encoded by each edge in  $R$  is within  $\tau_0$ .

To simplify the discussion, we consider events  $p_0$  represented by edges  $l(x, y)$ , as commonly found in practice.

### 4.2 Overview of TASTE

The architecture of TASTE is depicted in Figure 2. TASTE supports two modules for TACO discovery and event prediction, as follows.

*(1) Rule discovery.* The discovery module is responsible for mining a set  $\Sigma_d$  of TACOs that conform to the given requirement  $d$  from temporal graph  $G$ , referred to as *high-quality* rules. Apart from  $d$ , it takes pre-trained ML models  $\mathcal{M}$  (Section 2) as additional input and embeds them in TACOs when discovering  $\Sigma_d$ .

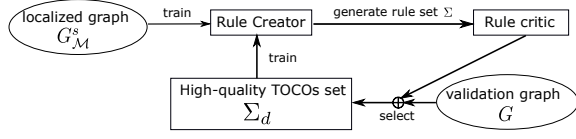


Figure 3: Dataflow of the iterative TACO discovery.

(2) *Event prediction*. The prediction module employs the discovered TACOs in  $\Sigma_d$  to predict all events (or a particular event) by running a parallel algorithm. It should be mentioned that this module can also accept TACOs provided by domain experts if available.

We will present algorithms underlying the discovery and prediction modules in Sections 5 and 6, respectively.

## 5 GENERATION-BASED TACO DISCOVERY

In this section we propose a Creator-Critic Discovery approach, denoted by CCD, to discovering high-quality TACOs  $\Sigma_d$ . We develop such an algorithm for the rule discovery module of TASTE.

Below we first introduce the generation-based approach and justify the method from the perspective of probability distribution learning. We then present CCD and its functions and models. Finally we formally prove the performance guarantees of CCD.

**Generation-based discovery.** One might be tempted to extend the existing levelwise search-based methods, e.g., [5, 20], to enumerate candidate TACOs and identify the required ones. However, these approaches need to search in a lattice and take exponential time, despite pruning strategies to reduce the cost [5, 20, 65].

In view of this, we propose CCD, which discovers high-quality TACOs by employing generative adversarial networks (GAN), and using interactive learning with weak supervision between a rule creator and a rule critic. This GAN-based framework is inspired by the latest progress in drug discovery and compound design [6, 88], where a GAN is trained to generate promising candidates without enumerating all possible combinations in the large search space. Such generative approach significantly accelerates the drug discovery process and produces more promising compounds.

As shown in Figure 3, in each iteration, the creator of CCD first employs ML models to generate a set  $\Sigma$  of candidate TACOs. Then the critic selects high-quality rules from  $\Sigma$  and adds them to  $\Sigma_d$ . Taking rules in  $\Sigma_d$  as feedback from the critic, the creator trains itself again to improve the probability of generating high-quality TACOs in the next iteration. CCD returns  $\Sigma_d$  when the number of iterations reaches a user-specified bound  $I$ .

From the perspective of probability theory, each TACO  $\varphi$  can be viewed as an event and the union of all TACOs constitutes a discrete countably infinite sample space  $\mathcal{A}$ . Given requirement  $d$ , the discovery aims to find a probability distribution  $\mathbb{P}$  described by a probability mass function  $P: \mathcal{A} \rightarrow \mathbb{R}$  (real numbers), where

$$\begin{cases} P(\varphi) > 0, & \text{if } \varphi \text{ is a high-quality TACO w.r.t. } d, \\ P(\varphi) = 0, & \text{if } \varphi \text{ is not a high-quality TACO w.r.t. } d. \end{cases} \quad (1)$$

In order to find the distribution, conventional levelwise methods build a searching lattice to enumerate each rule  $\varphi$  in  $\mathcal{A}$  and check whether it meets the requirement  $d$ . In contrast, CCD interactively trains deep generative models [63, 110] in the creator to approximate the target distribution  $\mathbb{P}$ , avoiding costly search in the exponentially large sample space.

---

### Algorithm 1: Creator-Critic Discovery (CCD)

---

**Input:** A temporal graph  $G$ , discovery requirement  $d=(\alpha, \beta, \gamma, \delta)$ , ML models  $\mathcal{M}$ , sample size  $N$ , and iteration number  $I$ .

**Output:** A set  $\Sigma_d$  of TACOs mined from  $G$  such that every rule in  $\Sigma_d$  conforms to the requirement  $d$ .

```

1   $\varepsilon \leftarrow 0$ ;  $\Sigma_d \leftarrow \emptyset$ ;  $G_M \leftarrow \text{MLExp}(G, \mathcal{M})$ ;
2   $G_M^s \leftarrow \text{LocalizedSample}(G_M, \alpha, N)$ ;
3   $\text{TrainCreator}(G_M^s, \alpha, \delta)$ ; /* Pretrain the rule creator */
4   $\Sigma \leftarrow \text{GenerateRule}(\alpha, \delta, \Sigma_d)$ ;
5   $\Sigma_d \leftarrow \text{SelectRule}(\Sigma, \beta, \gamma, G)$ ;
   /* Generate TACOs via interactive training */
6  while  $\varepsilon < I$  do
7     $G_M^s \leftarrow \text{LocalizedSample}(G_M, \alpha, N)$ ;
8     $\text{TrainCreator}(G_M^s \cup \Sigma_d, \alpha, \delta)$ ;
9     $\Sigma \leftarrow \text{GenerateRule}(\alpha, \delta, \Sigma_d)$ ;
10    $\Sigma' \leftarrow \text{SelectRule}(\Sigma, \beta, \gamma, G)$ ;
11    $\Sigma_d \leftarrow \Sigma_d \cup \Sigma'$ ;  $\varepsilon \leftarrow \varepsilon + 1$ ;
12 return  $\Sigma_d$ ;

```

---

This approach is feasible since deep generative models, e.g., GAN, are able to approximate various distributions [13]. Moreover, empirical research verifies that it is practical to train such models to approximate a target distribution  $\mathbb{P}$  within a limited number of iterations [31], where the training loss of the model converges. CCD also optimizes rule discovery in a data-driven manner, since generative models in the rule creator are able to generate rules that are topologically and semantically similar to rules from the training data [31, 47, 110]. Meanwhile, the creator can retrain its generative models using high-quality rules selected by the critic as feedback to improve the quality of approximation.

**Algorithm.** Implementing the creator-critic method, CCD discovers high-quality TACOs in three phases, as shown in Algorithm 1.

(1) It first calls function `MLExp` to prepare graph data  $G_M$  from  $G$ , for facilitating the discovery of TACOs with ML predicates  $\mathcal{M}$ .

(2) Then the rule creator is pretrained using `TrainCreator` with the localized graph  $G_M^s$  sampled by `LocalizedSample` (lines 2-3), such that it starts to generate rules that are likely to reach the thresholds of support and confidence in the input requirement  $d$ . The creator generates a set  $\Sigma$  of candidate TACOs by `GenerateRule`; the critic next evaluates  $\Sigma$  on temporal graph  $G$  and saves high-quality TACOs in a set  $\Sigma_d$  via function `SelectRule` (lines 4-5).

(3) In the third phase, algorithm CCD iteratively discovers high-quality TACOs and adds more such rules to  $\Sigma_d$  via an interactive training process of  $I$  iterations (lines 6-11). Each iteration is similar to the second phase, except that both  $G_M^s$  and  $\Sigma_d$  are used to train and improve the creator (line 8). Here the TACOs in  $\Sigma_d$  are feedback from the critic. Finally  $\Sigma_d$  is returned (line 12).

**Parameters.** In addition to the discovery requirement  $d$  of four thresholds and ML models  $\mathcal{M}$ , CCD takes another two input parameters: sample size  $N$  and iteration number  $I$ . Here  $N$  strikes a balance between the probability of generating high-quality rules and the workload of each iteration; and  $I$  determines the size of  $\Sigma_d$  and the cost of model training, where larger  $I$  usually helps generate more high-quality TACOs and train the model better.

We next present the functions and models adopted in CCD.

**Assistance functions.** We start with two functions MLExp and LocalizedSample, which are invoked by CCD to prepare graph data for model training. Initially, MLExp expands graph  $G$  to  $G_M$  by faithfully adding edges predicted by the input ML models  $M$ . This allows the creator to incorporate  $M$  as ML predicates and accelerates TACO discovery since there is no need to repeatedly apply  $M$  during the discovery process.

Besides, LocalizedSample collects a set  $G_M^s$  of  $N$   $\Delta$ -patterns by sampling localized graph structures from  $G_M$ , as training data for the creator in each iteration (lines 2 and 7). When deducing a pattern  $\Delta Q_i$ , LocalizedSample applies temporal random walk [69] with a randomly selected source node  $v$  to sample temporal paths, where the timestamps of all edges on a temporal path fall into a given time window. It finds top  $\alpha - 1$  frequently sampled nodes around  $v$ . Then  $\Delta Q_i$  is formed by these nodes and the edges connecting them. Guided by the  $\Delta$ -patterns in  $G_M^s$  during pretraining (line 3), the rule creator can learn to generate patterns that are more likely to find matches in the graph. This is because each  $\Delta Q_i$  must have matches as it is obtained by random walk in  $G_M$ , and the generative models learn to generate “new” patterns that are semantically and structurally similar to  $\Delta Q_i$ . Without pretraining, randomly initialized generative models in the creator may create meaningless TACOs.

In fact, a larger sample size  $N$  allows more sampling trails with higher probability of generating good patterns and more efficient ML training with batch optimizations [63, 110]. If  $N$  is small, CCD needs to run more iterations to find adequate number of TACOs.

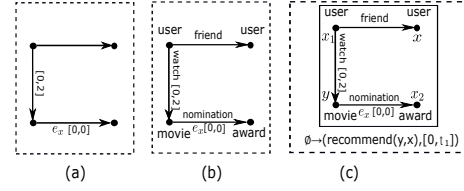
**Rule creator.** When generating a TACO  $\varphi$ , the creator first generates its  $\Delta$ -pattern, and then the dependency (lines 4 and 9).

**$\Delta$ -pattern generation.** The creator generates patterns in two steps: structure generation and semantic label generation, by employing temporal graph GAN and LSTM networks, respectively.

(1) In the first step, the creator takes each  $\Delta$ -pattern  $\Delta Q_i$  from  $G_M^s$  and  $\Sigma_d$  as input; it adopts TagGen [110], an end-to-end deep generative framework for temporal graphs, to deduce candidate  $\Delta$ -patterns  $\Delta Q_i^g$ . Due to the GAN module in TagGen [110], each generated  $\Delta Q_i^g$  and input  $\Delta Q_i$  have the same number of nodes (at most  $\alpha$ ), and bear similar topology and time constraints. No labels were generated in this step as TagGen does not support labels.

(2) The creator next employs an LSTM language model  $M_L$  to generate labels for each  $\Delta Q_i^g$ . We adopt LSTM networks since it can model the rich semantics of labels on paths in knowledge graphs [47, 52, 53]. More specifically, it first trains  $M_L$  on a corpus  $C$  driven by the perplexity [63], where each word is a pair  $\langle L(e), L(v) \rangle$  of edge label and node label, named “label pair”, and  $v$  is the destination node of  $e$ . The corpus  $C$  is composed of label pair sequences for temporal paths, which are derived by applying temporal random walk on each  $\Delta Q_i^g$ . After the training, for every two nodes  $u$  and  $v$  in  $\Delta Q_i^g$  with the shortest temporal path  $\rho$  from  $u$  to  $v$ ,  $M_L$  generates a label pair sequence with a random seed, and attaches this sequence of labels to  $\rho$ . Only shortest paths are considered since they hold stronger associations [2, 39]. Finally the creator builds a  $\Delta$ -pattern  $\Delta Q_i'$  from  $\Delta Q_i^g$  by keeping the most frequent labels attached.

Note that the creator retrains itself using the latest  $\Sigma_d$  to increase the probability of generating high-quality  $\Delta$ -patterns



**Figure 4: TACO generation process.**

(line 8). The rationale behind this is that (1) in training data, the  $\Delta$ -pattern  $\Delta Q_i$  of a high-quality TACO in  $\Sigma_d$  has multiple matches; and (2) the GAN and LSTM networks enable the creator to generate new  $\Delta$ -pattern  $\Delta Q_i'$  that are similar to  $\Delta Q_i$  in terms of topological structure, temporal constraints and label semantics [47, 52, 110]. Therefore, the generated  $\Delta Q_i'$  is likely to find sufficient matches as well. The samples  $G_M^s$  returned by LocalizedSample are also used in retraining, to introduce disturbance to the creator, which prevents the ML generative models from converging at a local minimum where cliché patterns are repeatedly generated.

**Dependency generation.** Given a generated  $\Delta$ -pattern  $\Delta Q$ , the creator adapts the levelwise expansion process of [20] to construct a set of valid dependencies  $X \rightarrow (p_0, \tau)$ . More specifically, for each possible event  $(p_0, \tau)$ , it starts with  $X = \emptyset$  and iteratively extends the precondition  $X$  for  $\Delta Q$ . But unlike [20] that directly verifies the validity of each candidate TACO  $\varphi = \Delta Q[\bar{x}](X \rightarrow (p_0, \tau))$  after its generation, the creator first checks whether  $\varphi$  is redundant, i.e., whether  $\varphi$  is implied by the set  $\Sigma_d$  of TACOs discovered in the previous iterations. If  $\varphi$  is not redundant, i.e.,  $\Sigma_d \not\models \varphi$ , it proceeds with the validation of  $\varphi$  by the critic. We find that checking implication in advance helps reduce the discovery cost, since implication is conducted on  $\Sigma_d$ , which is much smaller than the graph  $G$ . In addition, we perform satisfiability checking of the TACOs in  $\Sigma$  and newly discovered candidate rules at this stage, to avoid further validation of inconsistent TACOs in  $G$ . This process terminates when no more predicates can be added. The termination is guaranteed since predicates in  $X$  are defined on (a) nodes in  $\Delta Q$  and (b) attributes and values (including timestamps) in updates  $\Delta G_Q$ ; hence the number of all possible predicates is bounded by the sizes of  $\Delta Q$  and  $\Delta G_Q$ .

**Rule critic.** For each generated candidate TACO  $\varphi$  in  $\Sigma$  that passes the implication and satisfiability checking, the rule critic computes  $\text{supp}(\varphi, G)$  and  $\text{conf}(\varphi, G)$ , and selects high-quality TACOs whose support and confidence are above the thresholds (SelectRule in lines 5 and 10). These rules are added to  $\Sigma_d$  and are provided to the rule creator for improving generative models in the next iteration. Computing  $\text{supp}(\varphi, G)$  and  $\text{conf}(\varphi, G)$  is efficient since the procedures can be parallelized with optimizations that are unique to temporal pattern matching (see Section 6) and we use existing efficient subgraph matching method, i.e., DAF [32] and its proposed CS structures to reduce redundant computation.

**Example 5:** Continuing with Example 4, Figure 4 shows the generation of TACO  $\varphi_1$  from temporal graph  $G$ . First, the creator calls TagGen to create a candidate  $\Delta$ -pattern (Fig. 4(a)) with time constraints. Then, as shown in Fig. 4(b), the LSTM model  $M_L$  is employed to add labels to nodes and edges with semantic meanings. The creator completes the generation by constructing dependencies with variables (Fig. 4(c)). Validated by the rule critic in  $G$ , this generated candidate rule is preserved in  $\Sigma_d$ , since its support and

confidence are both 1. Note that if the edge labeled with nomination is dropped from  $\phi_1$ , the confidence would reduce to 0.5.  $\square$

**Performance guarantees.** We next show that when the iteration number  $I$  is sufficiently large, CCD can return all TACOs that satisfy the requirement  $d$  with a high probability. Recall that when accumulating the training data in each round for the generative model, CCD conducts temporal random walk to sample subgraphs by function `LocalizedSample`. Based on this, we have the following.

**Theorem 4:** *Given graph  $G$ , requirement  $d$  and constant  $\epsilon \in (0, 1)$ , after  $\frac{|G|^\alpha}{N\beta} \left(1 - \ln \frac{\beta\epsilon}{|G|^\alpha} + \sqrt{\ln \frac{\beta\epsilon}{|G|^\alpha} (\ln \frac{\beta\epsilon}{|G|^\alpha} - 2)}\right)$  iterations, CCD can discover all TACOs satisfying  $d$  with probability at least  $1 - \epsilon$ .  $\square$*

**Proof sketch:** Since the levelwise expansion process of [20] is adapted to build dependencies, it suffices to show that CCD can sample all  $\Delta$ -patterns that satisfy  $d$  with probability  $1 - \epsilon$ , after sufficient iterations as given above. Using Chernoff–Hoeffding bound [17] and bonferroni inequality [9], we prove this by analyzing the expected number of  $\Delta$ -patterns mined across the iterations.  $\square$

**Cost analysis.** To see the cost of CCD, observe the following. Function `LocalizedSample` takes  $O(|G|)$  time for applying temporal random walk [69]. The cost for model training in `TrainCreator` is linear to the number of training samples, i.e.,  $O(N + |\Sigma_d|)$ . `GenerateRule` (`creator`) takes time polynomial in the size  $|G|$  to generate  $\Delta$ -patterns and dependencies. However, `SelectRule` (i.e., `critic`) takes  $O(|G|^\alpha)$  time to compute the support and confidence of each candidate TACO because of the graph homomorphism in temporal matching; nonetheless, we make use of parallelism (see Section 6) and the auxiliary structure in [32] to speed up the computation.

**Remark.** (1) While the theoretical iteration number  $I$  may be large, in practice, when sample size  $N$  is set 250 by default, the generative models converge within 25 iterations (i.e.,  $I=25$ ), where extra iterations add few novel TACOs to  $\Sigma_d$  (see Section 7). Therefore, we set  $I$  using a practical small value. This helps us reduce the overall cost and justifies the usage of deep generative models.

(2) The users may opt to inspect the generated TACOs in each round, select rules of their interest, and add to  $\Sigma_d$ . This incorporates user interests into discovery. They may also terminate the iterative process manually when they are satisfied with the TACOs in  $\Sigma_d$  so far.

## 6 PARALLEL EVENT PREDICTION

In this section we develop a parallel algorithm to support the event prediction module of TASTE. The algorithm is also used to compute support and confidence for the rule critic of algorithm CCD (Section 5). We propose a partitioning strategy for temporal graphs, and show that the algorithm guarantees the parallel scalability.

We start with a sequential prediction approach with TACOs.

**Sequential algorithm.** Given a temporal graph  $G$ , a set  $\Sigma$  of TACOs and a time window  $\tau_0$ , a sequential prediction algorithm, denoted as SeqEP, finds all edges (events)  $R$  predicted by  $\Sigma$  in  $G$  as follows. For each TACO  $\phi = \Delta Q[\bar{x}](X \rightarrow (p_0, \tau))$  in  $\Sigma$  with  $\tau = [t_1, t_2]$ , SeqEP (1) finds all matches of  $\Delta Q$  in  $\Delta G_Q$  via graph homomorphism; and (2) for each such match  $h(\bar{x})$ , it checks whether  $h(\bar{x}) \models X$  and the time window  $[t^* + t_1, t^* + t_2]$  deduced is a subin-

terval of  $\tau_0$ ; here  $t^*$  is the current time (Section 2.1); if so, SeqEP adds to  $R$  the edge that links the nodes matching the variables in  $p_0$ .

Note that SeqEP is applied to subgraphs  $\Delta G_Q$  of  $G$  for  $\Delta Q$  in  $\Sigma$ , which is typically much smaller than the entire graph  $G$ . In the sequel we refer to the union of such  $\Delta G_Q$ 's simply as  $G$ . Even so, the exponential cost of graph homomorphism [27] in temporal pattern matching (step (1)) motivates us to parallelize SeqEP.

**Parallel scalability.** To measure the effectiveness of parallelization, we adapt the criterion introduced by [43] to graph computation. Consider a problem  $\mathcal{P}$  posed on a graph  $G$ . We denote by  $T_s(|I_{\mathcal{P}}|, |G|)$  the worst-case complexity of a sequential algorithm  $\mathcal{F}$  for handling an instance  $I_{\mathcal{P}}$  of problem  $\mathcal{P}$  over  $G$ . For a parallel algorithm  $\mathcal{F}_p$ , we denote by  $T_p(|I_{\mathcal{P}}|, |G|, k)$  the time taken by it for processing problem instance  $I_{\mathcal{P}}$  on  $G$  using  $k$  processors. We say that algorithm  $\mathcal{F}_p$  is *parallelly scalable relative to  $\mathcal{F}$*  if

$$T_p(|I_{\mathcal{P}}|, |G|, k) = O\left(\frac{T_s(|I_{\mathcal{P}}|, |G|)}{k}\right)$$

for any instance  $I_{\mathcal{P}}$ . That is, the parallel algorithm  $\mathcal{F}_p$  achieves a “linear” reduction in sequential running time of a yardstick algorithm  $\mathcal{F}$ , allowing us to process large graphs by adding resources.

**Parallelizing event prediction.** A typical strategy for parallelizing sequential graph computation is to first partition a graph into  $k$  small fragments, and then conduct the computation over fragments at  $k$  processors in parallel with necessary message passing, e.g., the deduction algorithm with GARs [21]. Following this paradigm, we could partition a temporal graph via an existing graph partitioning method, e.g., edge-cut or vertex-cut [3, 77]. However, this easily incurs a large amount of communication in the subsequent parallel prediction. This is because most of the previous partitioning methods aim to minimize the number of cut edges or vertices but overlook the timestamps, which are crucial to temporal pattern matching; the edges in a match of a  $\Delta$ -pattern are often partitioned into different fragments and demand communication.

To rectify this problem, we partition a temporal graph  $G$  based on *temporal locality*, a unique property of temporal pattern matching with TACOs. That is, the timestamps of edges in each match  $h(\bar{x})$  of  $\Delta$ -pattern  $\Delta Q$  are within the range of localized timespan  $[t^* + t_{\min}, t^* + t_{\max}]$ , where  $t_{\min}$  (resp.  $t_{\max}$ ) is the minimum (resp. maximum) timestamp in  $\Delta Q$  as stated in Section 2.1.

**Temporal partitioning.** We propose a temporal partitioning strategy with which parallel event prediction can be made communication-free. Intuitively, it divides a time interval into  $k$  subintervals and guarantees that every specific range of timestamps for finding match  $h(\bar{x})$  is *entirely* covered by one subinterval. Guided by the resulting subintervals, the temporal graph  $G$  is partitioned into  $k$  fragments  $F_1, \dots, F_k$ , such that each  $F_i$  consists of the edges whose timestamps are within one subinterval. By the temporal locality, temporal pattern matching and hence event prediction can be conducted in parallel over such  $F_i$ 's with no communication.

The cost of parallel event prediction is determined by the maximum size  $\max_{i \in [1, k]} |F_i|$  of fragments. Thus we want to find a good division of the time interval to minimize  $\max_{i \in [1, k]} |F_i|$ . To do this, we develop function `BTPart`, shown as part of Algorithm 2. It takes as input a candidate time interval  $[t_0, t_k]$  that is deduced from the TACOs  $\Sigma$  and time window  $\tau_0$  for matching designated



---

**Algorithm 2: Parallel Event Prediction (ParEP)**


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**Input:** The number  $k$  of processors, a  $k$ -way randomly partitioned temporal graph  $G$ , a set  $\Sigma$  of TACOs, a time window  $\tau_0$ .

**Output:** A set  $R$  of edges predicted by  $\Sigma$  with events occur within  $\tau_0$ .

```

1  $[t_0, t_k] \leftarrow \text{RefTime}(\Sigma, \tau_0); \quad (t_{\min}, t_{\max}) \leftarrow \text{ExtractTS}(\Sigma);$ 
2 collect the size  $|G_t|$  of  $t$ -graph  $G_t$  for  $t \in [t_0, t_k];$ 
3  $\{t_1, \dots, t_{k-1}\} \leftarrow \text{BTPart}([t_0, t_k], \{|G_t| \mid t \in [t_0, t_k]\}, k, t_{\min}, t_{\max});$ 
4  $F_i \leftarrow G_{[t_{i-1}+t_{\min}, t_i+t_{\max}]}$  for each  $i \in [1, k];$ 
5  $\text{RBalance}(\{F_i \mid i \in [1, k]\});$ 
6 run  $\text{SeqEP}(F_i, \Sigma, \tau_0)$  at each fragment  $F_i$  to get  $R_i$  for  $i \in [1, k];$ 
7 return  $\bigcup_{i \in [1, k]} R_i;$ 
```

**Function**  $\text{BTPart}([t_0, t_k], \{|G_t| \mid t \in [t_0, t_k]\}, k, t_{\min}, t_{\max})$ :

```

1 foreach  $t \in [t_0, t_k]$  do
2    $S[t][1] \leftarrow |G_{[t_0+t_{\min}, t+t_{\max}]}|;$ 
3 foreach  $i \in [2, k]$  do
4   foreach  $t' \in [t_0, t_k]$  do
5      $S[t'][i] \leftarrow \min_{t \in [t_0, t']} \max(S[t][i-1], |G_{[t+t_{\min}, t'+t_{\max}]}|);$ 
6 foreach  $i \in [2, k]$  in descending order do
7    $t_{i-1} \leftarrow \arg \min_{t \in [t_0, t_i]} \max(S[t][i-1], |G_{[t+t_{\min}, t_i+t_{\max}]}|);$ 
8 return  $\{t_1, \dots, t_{k-1}\};$ 
```

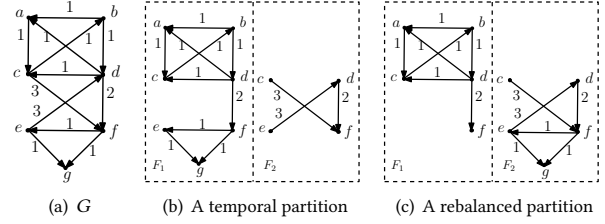
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edges in event prediction (see below), a set  $\{|G_t| \mid t \in [t_0, t_k]\}$  of sizes of  $t$ -graphs, the number  $k$  of fragments (processors), and the maximum (resp. minimum) timestamp  $t_{\max}$  (resp.  $t_{\min}$ ) in  $\Sigma$ . Here  $G_t$  refers to a  $t$ -graph that is composed of all edges in  $G$  bearing timestamp  $t$ . BTPart computes a set  $\{t_1, \dots, t_{k-1}\}$  of  $k-1$  partition points for the interval  $[t_0, t_k]$  such that fragment  $F_i$  becomes  $G_{[t_{i-1}+t_{\min}, t_i+t_{\max}]}$  for  $i \in [1, k]$ , which includes the edges of  $G$  with timestamps in the range  $[t_{i-1}+t_{\min}, t_i+t_{\max}]$ . Here  $t_{\min}$  and  $t_{\max}$  are to ensure the entire coverage as mentioned above.

Procedure BTPart adopts dynamic programming. It maintains a 2D array  $S$ , where  $S[t][i]$  records the minimum size of the largest fragments that are obtained by partitioning  $G_{[t_0+t_{\min}, t+t_{\max}]}$  into  $i$  subintervals. Hence  $S[t_k][k]$  is our objective value. BTPart first handles the base case  $i=1$  (lines 1-2), where  $S[t][1]=|G_{[t_0+t_{\min}, t+t_{\max}]}|$  since there is only one fragment w.r.t. the single subinterval. Here the size is derived from that of the input  $t$ -graphs. For cases where  $i>1$ ,  $S[t'][i]$  is determined by checking the values regarding all possible ranges  $[t_0, t']$  for  $t<t'$  and their  $i-1$  subintervals (lines 3-5). After  $S[t_k][k]$  is computed, the corresponding  $k-1$  partition points can be identified and are returned as the result (lines 6-8).

By induction on the iterations, one can verify that the returned partition points yield minimum  $\max_{i \in [1, k]} |F_i|$  (i.e.,  $S[t_k][k]$ ).

**Parallel algorithm.** Capitalizing on the temporal partitioning, we develop a parallel prediction algorithm, denoted by ParEP. As shown in Algorithm 2, initially ParEP uses function RefTime to deduce a time interval  $[t_0, t_k]$  for those edges in  $G$  that can potentially match designated edges in TACOs  $\Sigma$  (line 1). Since we only predict edges in time window  $\tau_0$ , a timestamp  $t$  is in  $[t_0, t_k]$  if and only if the gap between  $t$  and  $\tau_0$  is smaller than that of  $\tau$  for some  $(p_0, \tau)$  in  $\Sigma$ . It also extracts maximum and minimum timestamps from  $\Sigma$  via ExtractTS (line 1). Then ParEP performs temporal partitioning to get  $k$  fragments  $F_1$  to  $F_k$  and each  $F_i$  is assigned to a distinct processor (lines 2-4). It designates one processor as the coordinator to collect the sizes of  $t$ -graphs and apply function BTPart.



**Figure 5: Temporal partitioning and rebalancing**

To further balance the workload of parallel prediction, ParEP next redistributes the data in large fragments  $F_i$  having  $|F_i| > |G|/k$  by function RBalance (line 5). More specifically, for each large  $F_i$ , it evenly partitions those edges of  $F_i$  that can match the designated edges in  $\Sigma$ ; while the set of candidate matches in  $F_i$  for other pattern edges in  $\Sigma$  are replicated at all processors. Here candidate matches are identified via label comparison. Then SeqEP is invoked at all processors in parallel with no communication to get the predicted edges, and their union is finally returned (lines 6-7).

**Example 6:** Consider partitioning the temporal graph  $G$  shown in Figure 5(a) into 2 fragments, where only the timestamp of each edge is marked while labels are omitted to simplify the discussion. Suppose that  $(t_{\min}, t_{\max}) = (0, 1)$  for a given set  $\Sigma$  of TACOs and the candidate time interval  $[t_0, t_k]$  deduced from  $\Sigma$ , and the input time window  $\tau_0$  is  $[1, 4)$ . We can see the following.

(1) The temporal partition generated by BTPart is shown in Figure 5(b). Here the candidate interval is divided into  $[1, 2)$  and  $[2, 4)$ ; thus fragments  $F_1$  and  $F_2$  have edges whose timestamps are within  $[1, 2+1)$  and  $[2, 4+1)$ , respectively. Note that the edge  $(d, f)$  with timestamp 2 is replicated at both fragments. This ensures that all patterns in  $\Sigma$  can be matched on  $F_1$  or  $F_2$  locally.

(2) While Figure 5(b) is an optimal temporal partition, it is skewed, i.e., the edges with timestamp 1 in  $F_1$  make a large part of  $G$ . By moving edges  $(f, e)$ ,  $(e, g)$  and  $(f, g)$  to fragment  $F_2$  via function ReBalance, the partition becomes balanced (Figure 5(c)).  $\square$

The parallel scalability of ParEP is assured as follows, where the sequential SeqEP takes  $O(|\Sigma||G|^{|\Sigma|})$  time in the worst case and the cost of ParEP is bounded by  $O(|\Sigma| \frac{|G|^{|\Sigma|}}{k})$ .

**Theorem 5:** ParEP is parallelly scalable relative to SeqEP.  $\square$

**Remark.** (1) ParEP can be readily adapted to predict whether a particular event  $p_0$  will happen and when it will take place, by allowing users to set the range  $\tau_0$  and applying relevant TACOs in  $\Sigma$ . (2) We use ParEP to compute the support and confidence of TACOs, (i.e., the rule critic in CCD). To do these, the input time window  $\tau_0$  is defined as the smallest range that covers all the timestamps in  $G$ .

## 7 EXPERIMENTAL STUDY

Using real-life and synthetic graphs, we experimentally evaluated (1) the efficiency and (2) the quality of the creator-critic rule discovery method, (3) the accuracy of TASTE for event prediction and dynamic recommendation, and (4) the (parallel) scalability of ParEP.

**Experimental setting.** We start with the experimental setting.

**Datasets.** We used six real-life temporal graph benchmark datasets, shown in Table 2 and classified into three different classes: (1) event-

Table 2: Datasets

Dataset	V	E	Relation type	Timestamp interval
ICEWS18	23K	469K	256	1 day
GDELT	8K	2.2M	240	15 minutes
YAGO	11K	201K	10	1 year
WIKI	13K	670K	24	1 year
MovieLens	80K	10M	10	1 day
Amazon	12.2M	30.3M	5	1 day

based temporal knowledge graphs: ICEWS18 [36] from the Integrated Crisis Early Warning System [8], and GDELT [36] from the Global Database of Events, Language, and Tone [46]; (2) knowledge graphs with temporally associated facts: YAGO [57] and WIKI [45], e.g., YAGO records the time when a football player plays for a club; and (3) dynamic recommendation datasets: MovieLens [34] of movie ratings and Amazon [61] of product ratings, where original timestamps were reorganized such that the time granularity between two adjacent timestamps is one day.

As designed in [36, 50], each dataset has been divided into training, validation and test sets, with proportion of 80%, 10% and 10%, respectively, by timestamps. The training and validation sets were used for model training and rule discovery, while the test set was for accuracy test. Each dataset includes ground truth of event (temporal edge) prediction results [36, 50], and the testing set actually poses the set of “queries” w.r.t. predicting temporal events.

We also designed a graph generator to create larger synthetic datasets, for evaluating the scalability. The synthetic graphs had up to 10M nodes and 1B edges in the range of 10000 timestamps, with labels, attributes and values drawn from 200 symbols.

**Algorithms.** The creator and critic in the rule discovery module were implemented in Pytorch and C++, respectively, while the temporal event prediction module was implemented in C++. We compared TASTE with five baseline methods for event prediction: (1) AGER, which applies GERs [5] that capture local changes in temporal graphs for event prediction; (2) SACN [86], a convolution-based embedding approach for knowledge graph completion; (3) REGCN [50], a knowledge graph reasoning method based on Graph Convolution Network; (4) Caser [91], a sequential recommendation algorithm based on convolutional neural networks; and (5) SASRec [37], a transformer-based sequential recommender system. We adopted open-source codes of SACN [85], REGCN [49], Caser [90] and SASRec [38] with default configurations, and implemented AGER in C++. We have also implemented two levelwise search-based rule mining methods to discover GERs and TACOs in C++, denoted as GERMine and TACOMine, as discovery baselines.

The baselines SACN, REGCN, Caser and SASRec were parallelized with multi-threads by PyTorch. For AGER, GERMine and TACOMine, we applied the same parallelization method of ParEP (Section 6) to compute the corresponding matches, which dominate their costs. The thread number in parallelization is equal to the number  $k$  of cores used by TASTE, for a fair comparison.

**ML models.** REGCN and SASRec were adopted as the ML predicates in TACOs for temporal graph completion and dynamic recommendation, respectively. For models in the creator of CCD, we used the code of TagGen provided by the authors with default config-

Table 3: Quality of the creator-critic discovery on ICEWS18

$I \backslash N$	50	100	150	200	250	300
15	20.00%	23.81%	28.57%	39.05%	48.57%	65.71%
20	23.81%	25.71%	40.95%	55.24%	66.67%	73.33%
25	30.47%	41.90%	44.76%	71.43%	84.76%	96.19%
30	39.05%	44.28%	48.94%	76.15%	87.23%	97.14%

urations [111], and implemented the LSTM model as [63] with its default training configuration and two 650-wide layers.

We conducted experiments on a cluster of up to 72 Intel Xeon 3.1 GHz processing cores on two machines connected by 10Gbps links, with 256GB memory. By default we set  $\alpha=5$ ,  $\beta=100$ ,  $\gamma=0.8$  and  $\delta=20$  for discovery requirement; the number of iterations  $I=25$  and sample size  $N=250$  for discovery module CCD; and the number of cores  $k=32$  for prediction method ParEP, unless stated otherwise. All the experiments were repeated 5 times. The average is reported here.

**Experimental results.** We next report our findings. In every experiment, the results on at least one graph from each of the three classes are shown. We defer the other results to the full version.

**Exp-1: Efficiency.** We first compared the efficiency of CCD, GERMine and TACOMine for rule discovery. Since it is very costly for levelwise methods to discover rules with large patterns, in order to compare efficiency within bearable running time, we set a target of discovering 100 high-quality rules as benchmark. That is, each discovery process terminated when 100 rules had been discovered.

(1) *Varying  $\alpha$ .* We varied  $\alpha$  from 3 to 11 to study the impact of pattern node number on discovery methods using ICEWS18, WIKI and MovieLens. As shown in Figures 6(a) to 6(c), CCD is on average 9.1 and 14.3 times faster than GERMine and TACOMine when  $\alpha \leq 5$ , respectively. The computation time of levelwise search-based methods grows exponentially as  $\alpha$  gets larger, and both GERMine and TACOMine cannot terminate in 1.2 days when  $\alpha > 5$ , while the cost increase of CCD is mild. This is because larger number of pattern nodes incurs exponentially larger search space for levelwise mining, but little extra cost for ML generative models. In particular, CCD finds TACOs with patterns of more than 20 edges in 1639s.

(2) *Varying  $\beta$ .* We varied  $\beta$  from 50 to 150, to study the impact of support threshold over ICEWS18, YAGO and Amazon. As reported in Figures 6(d) to 6(f), the runtime of CCD does not change much, as it always takes multiple iterations for the creator to generate high-quality rules regardless of the value of  $\beta$ . In contrast, GERMine and TACOMine take less time with larger  $\beta$  in most cases, since higher bound on support prunes more candidates and reduces search space.

(3) *Varying  $\gamma$ .* Varying confidence  $\gamma$  from 0.7 to 0.9, we report the results on GDELT, WIKI and Amazon in Figures 6(g) to 6(i), respectively. It is shown that the all discovery algorithms take longer time with the increase of  $\gamma$ . However, the generation-based CCD is less sensitive to  $\gamma$ . This is because the levelwise methods have to expand their search space at an exponential scale to get the requested number of high-quality rules with higher confidence.

(4) *Varying  $\delta$ .* We varied  $\delta$  from 10 to 30. Here  $\delta$  counts the number of discrete timestamps in the required time window. As shown in Figures 6(j) to 6(l) on ICEWS18, YAGO and MovieLens, respectively,

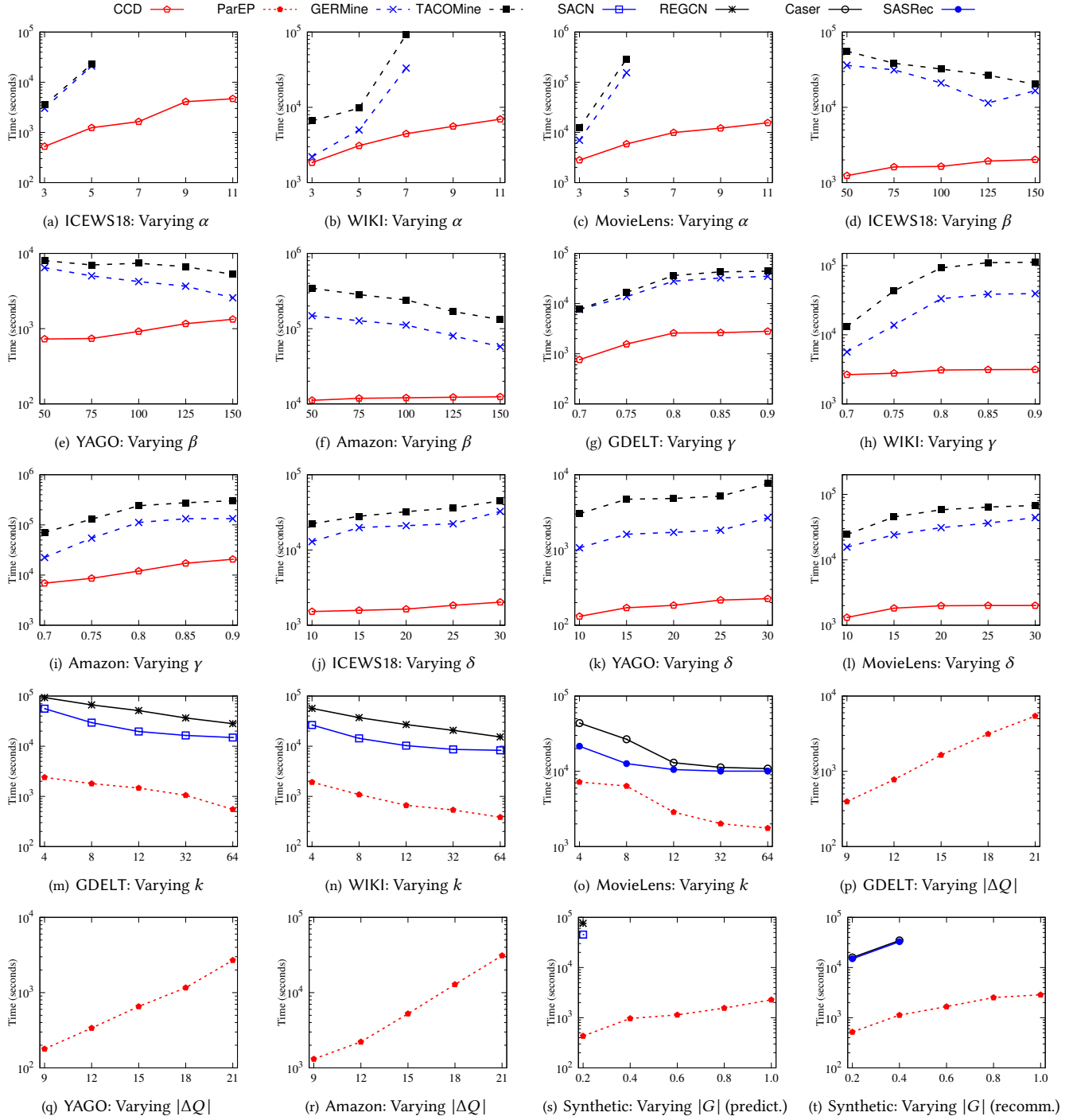


Figure 6: Performance evaluation

the runtime for all three increases as  $\delta$  grows, since a longer time window bound gives more generation workload for TagGen in the creator and expands the search space for GERMine and TACOMine.

(5) *Impact of  $N$  and  $I$ .* We also tested the impact of the sample size  $N$  and iteration number  $I$  on the efficiency of CCD. It exhibits a moderately runtime increase with the increase of  $N$  or  $I$  (not shown).

This is as expected, since a larger  $N$  (resp.  $I$ ) causes more work for the generator (resp. more rounds of the entire computation).

**Exp-2: Quality of discovery.** Recall that TACOs obtained by the generation-based CCD are a subset of those returned by levelwise search algorithms. Thus we checked how many rules in the com-

**Table 4: Event prediction/recommendation accuracy**

Dataset	AGER	SACN	REGCN	Caser	SASRec	TASTE
ICEWS18	59.32%	63.28%	68.58%	-	-	75.24%
GDELT	54.60%	62.32%	66.31%	-	-	73.41%
YAGO	61.32%	64.03%	74.73%	-	-	84.14%
WIKI	58.56%	63.58%	71.32%	-	-	82.53%
MovieLens	65.80%	-	-	75.83%	80.45%	87.30%
Amazon	59.30%	-	-	63.50%	73.10%	73.10%

plete set found by levelwise method TACOMine can be discovered by CCD in all the datasets, *i.e.*, the coverage of complete TACOs. As reported in Table 3, the coverage becomes higher with larger size  $N$  of samples or more training iterations  $I$ . This is because enlarging either  $N$  or  $I$  could increase the possibility of approximating the target distribution and hence generating high-quality rules, as discussed in Section 5. We can also see that small  $N$  and  $I$  suffice to get a large portion of high-quality TACOs, *e.g.*, the coverage reaches 84.76% when  $N = 250$  and  $I = 25$ , while the time of TACOMine is reduced by 18.5 times simultaneously (see Exp-1).

In addition to the adopted GAN model, we tested the performance of using classic graph generation models, *i.e.*, Erdős-Rényi (ER) [72] and Barabási-Albert (BA) [1] models, in CCD. We find that when  $N = 300$  and  $I = 30$ , the coverage values of the TACOs found with ER and BA are merely 33.07% and 50.89%, respectively, much lower than that by GAN. This is because these classical models cannot iteratively learn from the graph data and generate high-quality patterns in an adaptive manner.

We also manually checked the discovered TACOs. Besides the typical ones that include logic and temporal predicates only, some TACOs can also help enrich or interpret ML predictions.

**Exp-3: Accuracy.** As shown in Table 4, we evaluated the accuracy of TASTE with the TACOs and baselines on two tasks: temporal event prediction and dynamic recommendation [37, 50]. Note that there was no result for SACN and REGCN (resp. Caser and SASRec) on dynamic recommendation (resp. temporal event prediction) datasets since they are not designed for the task. TASTE (ParEP) applied the discovered TACOs with confidence above 0.9, in which graph patterns have at most 9 nodes. We find that very few TACOs with more than 9 pattern nodes have high confidence and support, similar to the findings of frequent pattern mining [19]. We adopted Hit Rate@10, the fraction of times that the ground-truth item is among the top 10 items [37, 50], to evaluate the accuracy.

For event prediction on ICEWS18, GDELT, YAGO and WIKI, TASTE on average outperforms AGER, SACN and REGCN by 34.9%, 24.5% and 12.2%, respectively. As for the dynamic recommendation on MovieLens and Amazon, TASTE is 35.8%, 22.5% and 10.6% more accurate than AGER, Caser and SASRec, respectively. These show that by combining rules and ML models, TASTE beats the state-of-the-art deep-learning-based REGCN and SASRec in accuracy on both tasks, while none of REGCN and SASRec works on both.

**Exp-4: Scalability.** We finally evaluated (1) the parallel scalability of the prediction module ParEP in TASTE system by varying the number  $k$  of processors, (2) the impact of pattern size on its efficiency, and (3) the scalability of ParEP over larger synthetic graphs. Since GERs are a special case of TACOs, ParEP is also applicable to GERs and the runtime of AGER is not shown.

(1) *Parallel scalability.* Varying  $k$  from 4 to 64, Figures 6(m)-6(o) report the results on GDELT and WIKI for event prediction, and on MovieLens for recommendation in the same setting as Exp-3, respectively. As shown there, (a) ParEP is parallelly scalable. When  $k$  increases from 4 to 32, it is on average 3.2 times faster on the three graphs. This verifies the effectiveness of ParEP under data-partitioned parallelism. (b) In addition to its higher accuracy, ParEP performs better in efficiency than SACN and REGCN (resp. Caser and SASRec), *e.g.*, when  $k = 64$ , it is on average 24.2, 45.4, 6.2 and 5.7 times faster than SACN, REGCN, Caser and SASRec, respectively.

(2) *Impact of pattern size.* Varying the size  $|\Delta Q|$  of  $\Delta$ -patterns in TACOs, which is measured as the sum of the pattern node and edge numbers in each  $\Delta Q$ , we report the performance of different methods in Figures 6(p) to 6(r). The results show that ParEP becomes slower with the increase of  $|\Delta Q|$ , as expected. Nonetheless, it is still efficient when handling relatively large  $\Delta$ -patterns. For instance, it needs 1645 seconds on GDELT when  $|\Delta Q| = 15$ , which is better than 16380 seconds by SACN (see Figure 6(m)).

(3) *Scalability.* Fixing  $k = 32$ , we varied the size  $|G| = |V| + |E|$  of synthetic graphs  $G$  using a scale factor from 0.2 to 1.0, and evaluated all approaches, where ParEP applied 100 TACOs. As shown in Figures 6(s) and 6(t) for prediction and recommendation, respectively, ParEP outperforms the baselines in all cases. On average it takes 1403s when  $|G| = 810M$ , while the others cannot finish in 1 day.

**Summary.** We find the following. (1) On average the generative ML method of CCD outperforms the levelwise algorithms in efficiency by more than 31 times. It is able to discover TACOs with patterns of 20 edges in 1639s from temporal graphs, while the levelwise methods could not finish in 1.2 days. (2) CCD is able to find as high as 84.76% of the complete rules derived by levelwise method, using 250 samples and 25 training attempts. (3) By combining rules and ML models, on average the discovered TACOs improve the existing approaches by 23.8% and 23.0% in accuracy, for event prediction and dynamic recommendation, respectively. (4) Our algorithm ParEP is parallelly scalable and scales well with the datasets, it takes less than 1403s on graphs with 810M nodes and edges using 32 processors.

## 8 CONCLUSION

We have proposed a new approach towards event prediction, from foundation (rules and complexity) to a system (supported by scalable algorithms), with novelty summarized in Section 1. We have experimentally verified that TASTE is promising in event prediction.

One topic for future work is to evaluate TASTE for predicting events of other types, *e.g.*, finance crisis. Another topic is to make TASTE “real-time” by incrementally discovering rules and predicting events in response to updates to temporal graphs.

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