# Robust Dendritic Computations With Sparse Distributed Representations

Numenta

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## SUMMARY

The neocortex represents information everywhere using sparse distributed patterns.

How accurately can neurons recognize these sparse patterns? This poster shows:

1) Scaling laws for computing error probabilities.

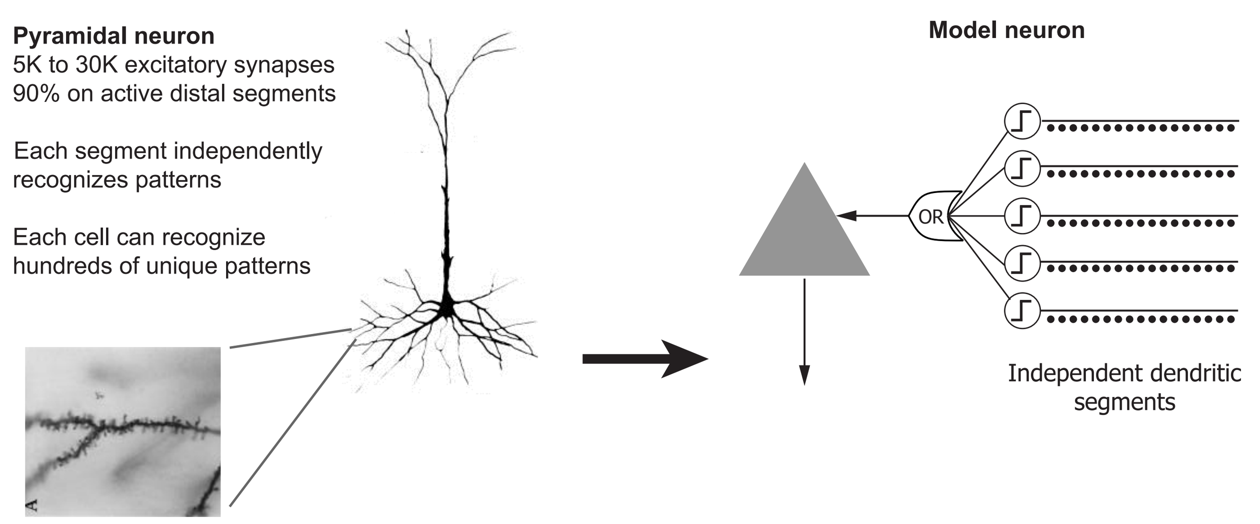
2) High dimensional sparse patterns can be classified extremely reliably, even in the presence of large amounts of noise and failures.

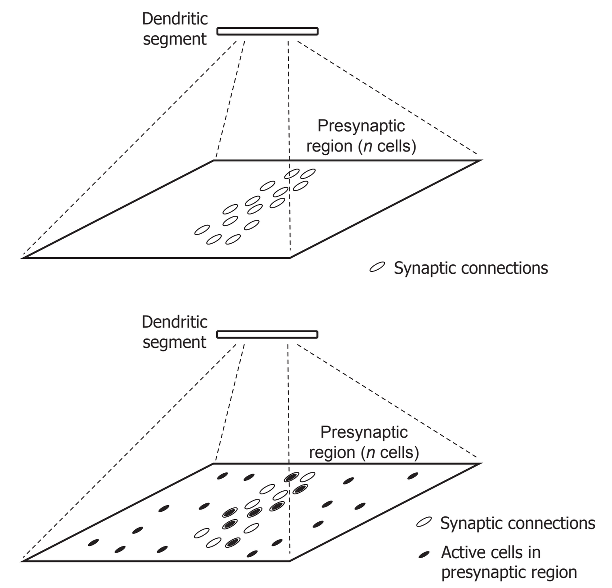
3) Active dendritic segments can reliably classify patterns using a tiny number of synapses.

4) The equations explain experimentally observed NMDA spike thresholds in active dendrites.

5) Behavior of Poirazi-Mel and HTM neuron models closely match theoretical predictions. Understanding the behavior can lead to dramatically improved accuracies.

## MODELING DENDRITES




**Dendritic pattern recognition**

The synapses on a dendrite subsample from a pattern in a presynaptic region containing *n* cells.

Each segment has *s* synapses and represented by a binary vector *D* with *n* components and *s* "1" bits:

Activity in presynaptic region at time *t* represented by a binary vector $A_t$ with *n* components and $a_t$ active cells:

*D* recognizes the pattern in $A_t$ if at least θ presynaptic inputs are on: $\mathbf{A}_t \cdot \mathbf{D} \geq \theta$

## PROBABILITY OF FALSE POSITIVES

Probability of a random input matching a dendrite:

$$P(\mathbf{A}_t \cdot \mathbf{D} \geq \theta) = \frac{\sum_{b=\theta}^{s} |\Omega_{\mathbf{D}}(n, a_t, b)|}{\binom{n}{a_t}}$$

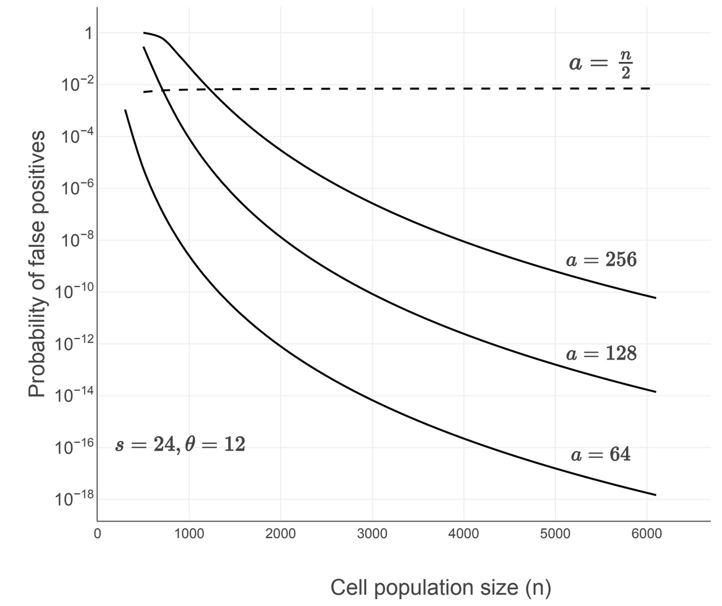
$|\Omega_{\mathbf{D}}(n,a,b)|$ counts the number of input vectors that exactly match *b* synapses on the dendrite:

$$|\Omega_{\mathbf{D}}(n,a,b)| = \binom{s}{b} \times \binom{n-s}{a-b}$$

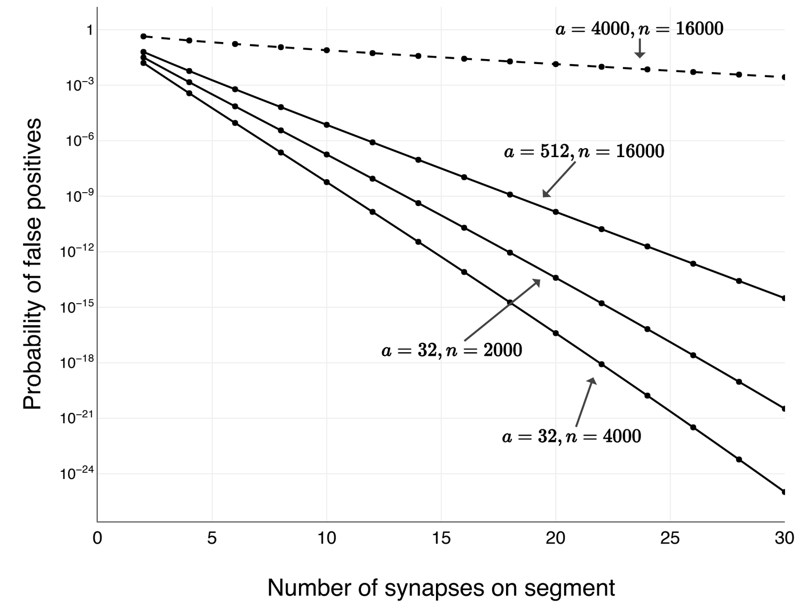
Number of ways to select exactly *b* out of *s* synapses

Number of vectors that have *a-b* bits on and no overlap with dendrite.

**The probability of error decreases dramatically with high dimensionality and input sparsity:**




**A tiny number of synapses, subsampling from a much larger pattern, is sufficient for robust recognition:**

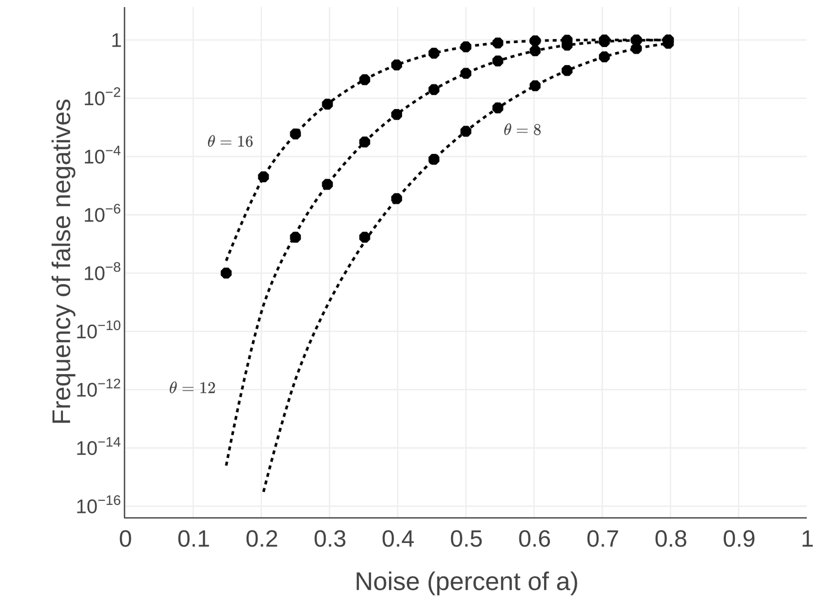


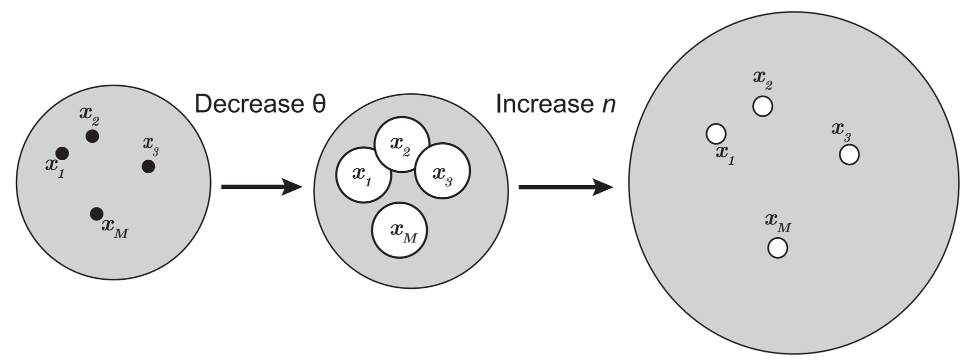

## FALSE NEGATIVES

Probability of a corrupted pattern *not* matching a dendrite:

$$P(\mathbf{A}_t^* \cdot \mathbf{D} < \theta) = \frac{\sum_{b=s-\theta+1}^{s} |\Omega_D(a_t^*, v, b)|}{\binom{a_t^*}{v}}$$

Here ***A**** is a corrupted version of the pattern represented in D, with *v* bits missing.

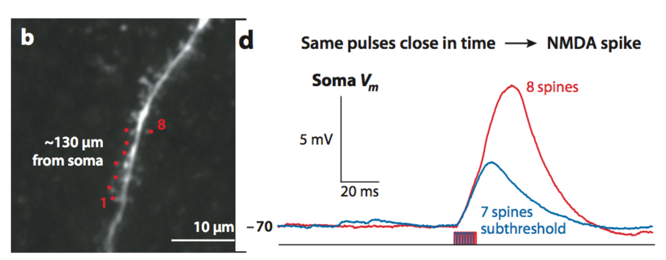



## PREDICTING NMDA SPIKE THRESHOLDS

Pyramidal neurons:
5K to 30K excitatory synapses
90% on distal "active" segments.

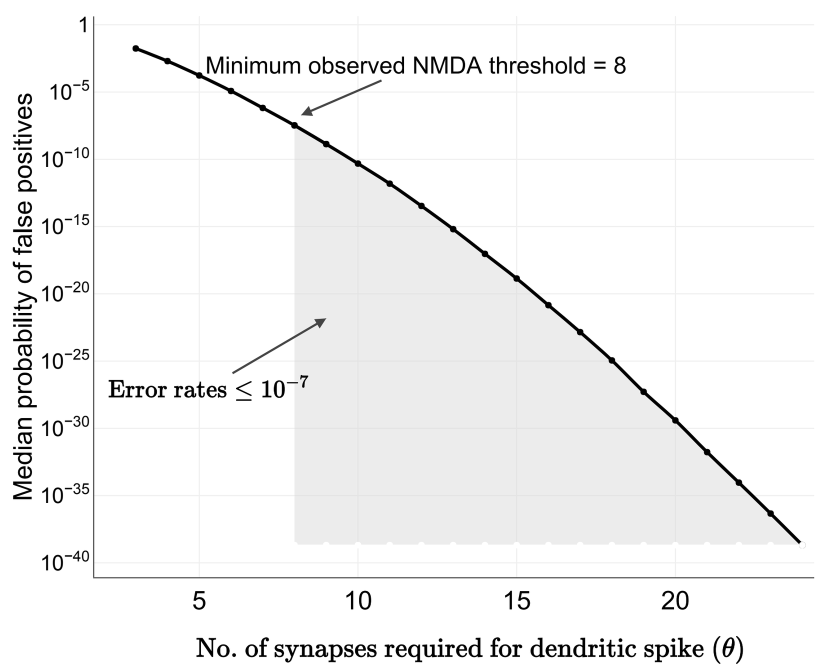
Each segment independently recognizes patterns: as few as 8 co-active clustered synapses can generate an NMDA spike.




Major, Larkum and Schiller 2013

*Neural activity is unreliable and noisy, so how can such a tiny number of synapses reliably detect patterns?*

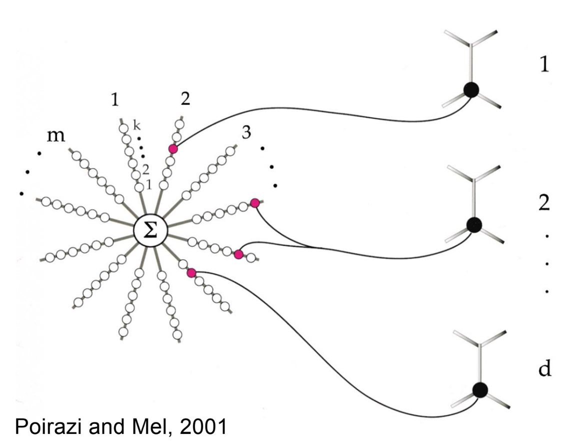
**Our equations show that the small NMDA thresholds observed in biology can be explained by high dimensional sparse representations.**




Predicted median false positive rate integrated over a range of values for n, sparsity, and s:
n :1,000 to 20,000, sparsity:0.5% to 3%, synapses:20 to 50

NMDA threshold of 8 leads to median error of less than 1 in 10 million with 50% noise.

## POIRAZI-MEL NEURON: IMPROVED BINARY CLASSIFICATION



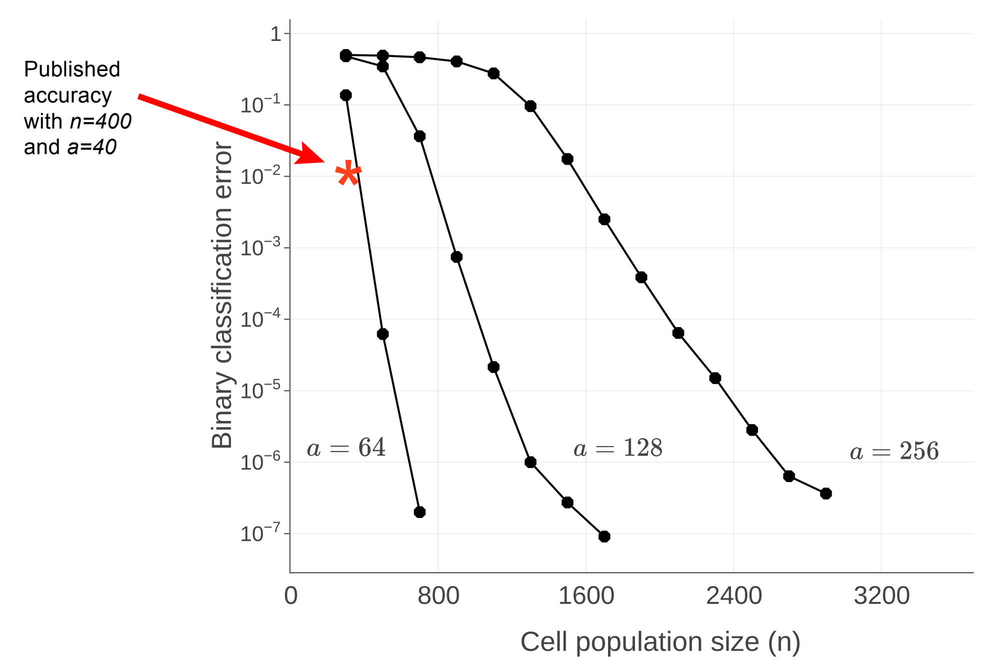
Poirazi and Mel, 2001

Simple model of neuron with *m* independent dendritic segments.

Original paper tested populations of these neurons on binary classification tasks, using input dimensionality *n=400*, of which 40 components were on (10% sparsity).

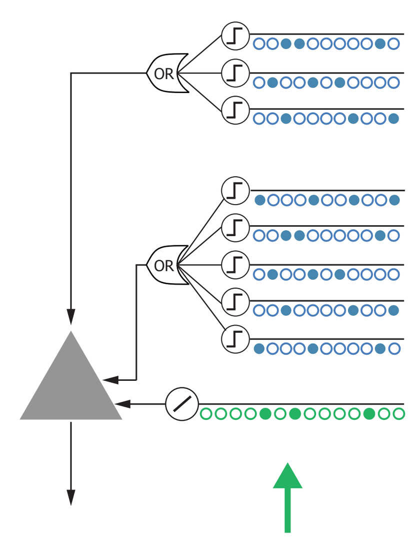
We tested this neuron model by varying the dimensionality and sparsity of the inputs.

**Large improvement in accuracy when high dimensional sparse representations are used, as predicted by our theory.**

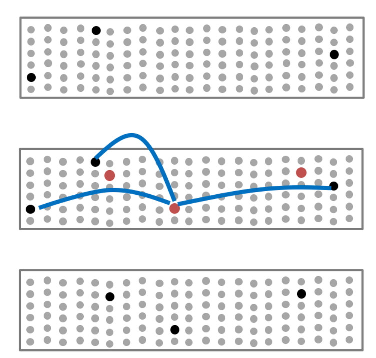



In all cases the total number of dendrites per neuron was kept at 500, and the number of synapses per segment was kept constant at 24. The threshold was 12.

## HTM NEURON: SEQUENCE MEMORY




(Hawkins and Ahmad, 2016)



$A_t$ = Pattern at time *t*

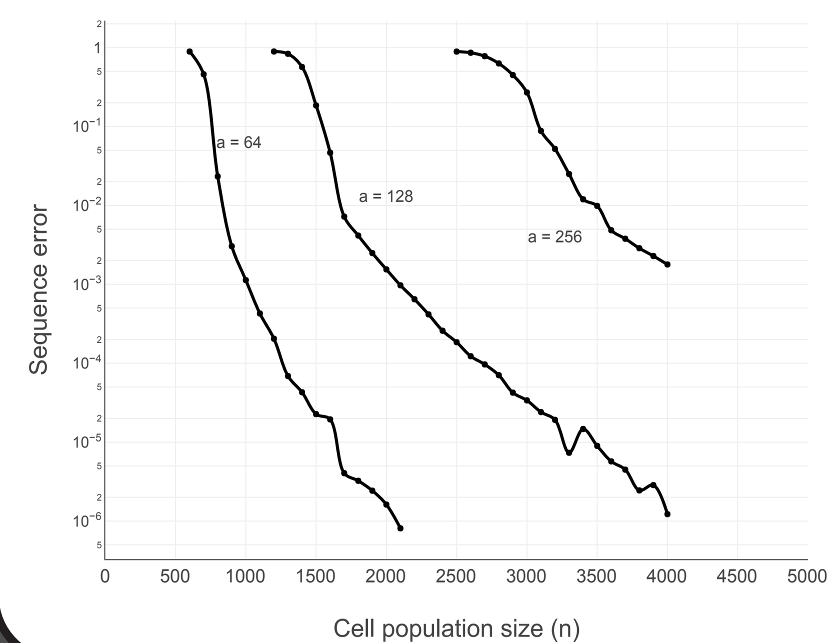
Cells which recognize $A_t$ on their active dendrite predict the next input.

If these cells get feedforward input, they become active and inhibit their neighbors. This activity becomes $A_{t+1}$.

HTM Sequence Memory has been shown to work very well in real-world applications. Reliably detecting sparse patterns on active dendrites is key to their accuracy.

Here we examine the effects of sparsity and dimensionality on HTM's sequence prediction accuracy.

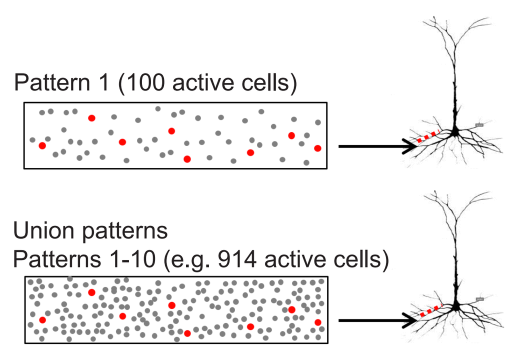
**Improvement in accuracy with high dimensional sparse representations.**




Sequence memory is trained on 1,000 sequences, with 20 elements per sequence.

Error is plotted as the average Jaccard similarity across all sequences between the predicted cells at time *t-1* and the actual active cells at time *t*.

## UNCERTAINTY RESOLVED WITH UNIONS



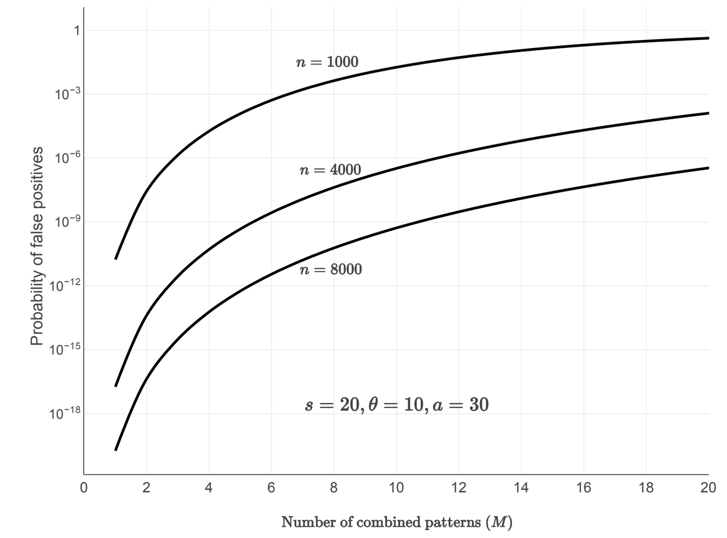

Uncertainty and surprise might result in a union (superposition) of multiple patterns representing multiple possibilities.

If $A_t$ contains a union of *M* patterns, the overall activity will be denser. However the expected number of active cells is smaller than $M{\cdot}A_t$:

$$E[|\mathbf{A}_t^M|] = (1-(1-(a/n))^M)n$$

For example, a union of 10 patterns with n=4000 and a=100 will lead to 914 active cells.

$$\frac{\sum_{b=\theta}^{s} |\Omega_{\mathbf{D}}(n, E[|\mathbf{A}_t^M|], b)|}{\binom{n}{E[|\mathbf{A}_t^M|]}}$$




The overall probability of an error is higher with unions.

However, with high dimensional sparse representations, individual dendrites can still very robustly recognize individual patterns even in the presence of unions.

## References

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Major, G., Larkum, M.E., & Schiller, J. (2013) Active properties of neocortical pyramidal neuron dendrites. Annu. Rev. Neurosci., 36, 1–24.

Poirazi, P. & Mel, B.W. (2001) Impact of active dendrites and structural plasticity on the memory capacity of neural tissue. Neuron, 29, 779–796.