1. 最长子序列问题
   (a) 最长公共子序列
   Problem.
   (input) 序列 a, b

$$\begin{aligned} \max_{x} \quad & number\left(x\right) \\ s.t. \quad & x \subseteq a \\ & x \subseteq b \end{aligned}$$

   Algorithm.
   - 动态规划

$$f\left(a_{1:n}, b_{1:m}\right) = \begin{cases} f\left(a_{1:n-1}, b_{1:m-1}\right) + 1 & ; a_n = b_m \\ \max\left(f\left(a_{1:n}, b_{1:m-1}\right), f\left(a_{1:n-1}, b_{1:m}\right)\right) & ; a_n \neq b_m \end{cases}$$

$$f\left(a_1, b_1\right) = \begin{cases} 1 & ; a_1 = b_1 \\ 0 & ; a_1 \neq b_1 \end{cases} \qquad \text{(初始易知值)}$$

   Include.
   i. 最长连续公共子序列
   Algorithm.
   - 动态规划

$$f\left(a_{1:n}, b_{1:m}\right) = \begin{cases} f\left(a_{1:n-1}, b_{1:m-1}\right) + 1 & ; a_n = b_m \\ 0 & ; a_n \neq b_m \end{cases}$$

$$f\left(a_i, b_1\right) = \begin{cases} 1 & ; a_i = b_1 \\ 0 & ; a_i \neq b_1 \end{cases} \qquad \text{(初始易知值)}$$

$$f\left(a_1, b_i\right) = \begin{cases} 1 & ; a_1 = b_i \\ 0 & ; a_1 \neq b_i \end{cases}$$

   (b) 最长上升子序列
   Problem.
   (input) 序列 a

$$\begin{aligned} \max_{x \subseteq a} \quad & number\left(x\right) \\ s.t. \quad & x_i < x_{i+1} \quad ; i \in 1 : number\left(x\right) \end{aligned}$$

   Algorithm.

$$f\left(n\right) = max\left(f\left(i\right), max\left(f\left(j\right)\right) + 1\right) \quad ; j < i \ and \ a_j < a_i$$
$$f\left(1\right) = 1 \qquad \text{(初始易知值)}$$

   - $f\left(n\right)$: 以 a_n 为结尾的最长上升子序列的长度.

   (c) 最长公共前后缀
   Problem.
   (input) 序列 a

$$\begin{aligned} \max \quad & k \\ s.t. \quad & a_{1:k} = a_{n-k+1:n} \end{aligned}$$

Algorithm.

$$f(n)=\begin{cases} f(n-1)+1 & ;a_n=a_{f(n-1)+1} \\ f(f(n-1))+1 & ;a_n=a_{f(f(n-1))+1} \\ \vdots & \\ 0 & ;other \end{cases}$$

(d) 最长回文子序列

Problem.

(input) 序列 a

$$\begin{aligned} \max_{x\subseteq a} \quad & n_x = number(x) \\ s.t. \quad & x_i = x_{n_x-i+1} \quad ;i=1:n_x \end{aligned} \qquad \text{(回文约束)}$$

Algorithm.

- 动态规划

$$\begin{aligned} f(s,e) &= \begin{cases} f(s-1,e+1)+2 & f(s,e)>0 \ and \ a_{s-1}=a_{e+1} \\ 0 & other. \end{cases} \\ f(s,s) &= 1 \qquad \text{(初始易知值)} \\ f(s,s+1) &= 2 \quad ;a_s=a_{s+1} \end{aligned}$$

  - $f()$: $a_{s:e}$ 的回文字数, 不是回文序列则为 0.

(e) 序列匹配问题

i. 序列匹配

Problem.

(input) $a, b \quad ;b\subseteq a$

find min k, let $b=a_{k:k+n_b-1}$

Property.

- if $b_{1:i}=a_{k:k+i-1}$, and $l_i=$ 最长公共前后缀长度$(b_{1:i})$

$$\begin{aligned} &\Rightarrow b_{1:l_i} = a_{((k+i-1)-l_i+1):(k+i-1)} \\ &\quad b_{1:l'} \neq a_{((k+i-1)-l'+1):(k+i-1)} \quad ;l'>l \end{aligned}$$

Algorithm.

- KMP 算法

$$\begin{array}{lllllll} b_{1:i}=a_{k:k+i-1} & and & b_{i+1} & =a_{k+i} & \Rightarrow & b_{1:i+1} & = & a_{k:k+i} \\ b_{1:i}=a_{k:k+i-1} & and & b_{g(i)+1} & =a_{k+i} & \Rightarrow & b_{1:g(i)+1} & = & a_{(k+i)-g(i)+1:k+i} \\ b_{1:i}=a_{k:k+i-1} & and & b_{g(g(i))+1} & =a_{k+i} & \Rightarrow & b_{1:g(g(i))+1} & = & a_{(k+i)-g(g(i))+1:k+i} \\ & & & ... & \Rightarrow & ... & & \\ & & b_1 & =a_{k+1} & \Rightarrow & b_{1:1} & = & a_{k+i:k+i} \\ & & b_1 & \neq a_{k+1} & \Rightarrow & b_{1:1} & \neq & a_{k+i:k+i} \end{array}$$