Democratizing Soft Robots: Co-design of Soft Robotic Manipulators By Leveraging Model-based Control

Research proposal for Cultuurfonds Wetenschapsbeurzen

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1 Introduction

Soft robots are inspired by invertebrates and promise the ability to operate in close proximity to humans as they are inherently compliant and, with that, allow for safe interactions with the environment and humans [1, 2]. Throughout the last decade, there have been many soft robot designs proposed, such as soft fluidic elastomer robots [3], vine robots [4], tendon-driven soft robots [5, 2], or soft robots based on engineered metamaterials [6, 7]. However, there are no established design guidelines (e.g., number of actuators, choice of material, structure and topology, choice of sensor modalities, etc.) for these robots. This represents a barrier to entry for both newcomers and industrial adoptees, as designing a performant soft robot requires extensive experi-

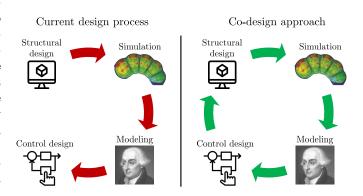


Figure 1: The traditional design process for soft robots is a one-way street: first, the structural design is done in CAD, then we simulate the soft robot's behavior (e.g., FEM). Subsequently, reduced-order kinematic and dynamical models are developed. Finally, the controller is designed to leverage model knowledge. Instead, we will establish a co-design procedure that involves structure, actuation, sensing, and control.

ence, know-how, and many iterations. Furthermore, the soft robotic development process is usually a **one-way street**: a mechatronics engineer comes up with a new, innovative design [6], then a modeling expert derives a dynamical model [8], and finally, a control theorist implements a controller that leverages the dynamical model [7, 9]. However, this often leads to **designs that are very challenging to model and control**.

Instead, we strive to **co-design the soft robot with a control law** and, with that, **democratize the design process**. While there has been some initial work on the co-design of soft robots [10, 11, 12, 13, 14], the existing methods are **too simplistic** as they discretize the continuum into passive, actuated, and sensorized **voxels** or particles. Therefore, the derived designs can only **very rarely be realized in practice** [15, 16]. Additionally, most works train a

learning-based (e.g., RL) controller from scratch for each design iteration, creating a computational bottleneck for the co-design process [11, 13, 14]. The alternative is to optimize a controller over a set of different designs [17], with the drawback being that the controller is not specialized in operating a given design. With that, the evaluation of the closed-loop system does not represent the actual performance that could be achieved with a specialized controller.

This project will alleviate the first issue of simplistic designs by leveraging the **monotone** theory of co-design that has been successfully used for co-designing autonomous [18, 19] and mobility systems [20] and will allow us to frame complex but realizable designs in a structured way as a Monotone Design Problem with Implementation (MDPI). Secondly, the issue of computationally efficiently deriving a specialized controller for a given soft robotic design will be addressed by taking a **model-based control** [21, 22, 23] approach: assuming that the manipulator structure can be modeled with slender rods will let us identify a suitable kinematic model and allow us to derive the **potential forces** that are then subsequently compensated within the control law.

2 Proposed innovation in a nutshell

In this work, we aim to formulate the co-design of soft robotic manipulators as a Monotone Design Problem with Implementation (MDPI) [18] that will enable us to **jointly optimize the structure**, **sensing**, **actuation**, **and control algorithms**. Within the MDPI, we will consider **functionalities** such as **task-centric performance**, the desired **safety level**, and resources such as **manufacturing costs**. We aim to leverage **model-based** strategies for devising **controllers** for a given design, allowing us to evaluate designs computationally efficiently in a **closed loop**. Exploring many different designs will let us analyze the **trade-offs** inherent to the design problem (e.g., design complexity vs. performance vs. interaction safety).

3 Technical details

3.1 Problem setting

We consider the problem setting of **designing a soft robotic manipulator** that can meet given **task-centric performance and safety requirements** while minimizing **manufacturing costs** (and, with that, complexity). The design incorporates the **structure** (parametrized by $\pi_b \in \mathbb{R}^{n_{\pi,b}}$), the **actuation** (parametrized by $\pi_a \in \mathbb{R}^{n_{\pi,a}}$), the **sensing** (parametrized by $\pi_s \in \mathbb{R}^{n_{\pi,s}}$), and the **control** (parametrized by $\pi_c \in \mathbb{R}^{n_{\pi,c}}$) of the soft robotic manipulator.

3.2 MDPI of a Soft Robotic Manipulator

We consider the **monotone theory of co-design** [24]. We first introduce a few definitions: A map $f: P \to Q$ between two partially ordered sets (posets) $\langle P, \preceq_P \rangle$, $\langle Q, \preceq_Q \rangle$ is monotone iff $x \preceq_P y$ implies $f(x) \preceq_Q f(y)$, which is also the case for composed maps.

We now move on to defining MDPIs, which act on the functionalities \mathcal{F} and the resources \mathcal{R} : An MDPI d is a tuple $\langle \mathcal{I}_d, \text{prov}, \text{req} \rangle$, where \mathcal{I}_d is a set of implementations and prove, req are functions mapping \mathcal{I}_d to \mathcal{F} and \mathcal{R} , respectively. MDPIs can solved for their implementation \mathcal{I}_d by leveraging Kleene's fixed point theorem [24] and determining $h'_d: \mathcal{R} \to \mathcal{A}\mathcal{R}$ that map a resource $r \in \mathcal{R}$ to the maximum antichain of functionalities provided by r.

In Fig. 2(a), we propose an **MDPI** for a soft robotic manipulator. For a given task, the functionalities include the performance at completing the task, the safety level (i.e., the

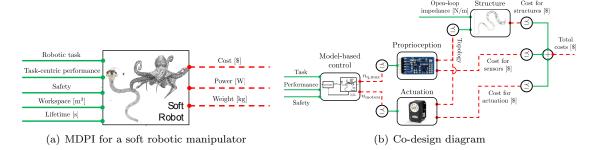


Figure 2: **Panel (a):** Monotone Design Problem with Implementation (MDPI) for a soft robotic manipulator: we distinguish between functionalities such as task-centric performance and resources such as the manufacturing costs. **Panel (b):** The co-design diagram zooms in on the *Soft Robot* block and reveals its sub-tasks (e.g., model-based control, proprioception, design, etc.). For visual clarity, we focus on the functionalities task, performance, and safety, and the resource manufacturing costs. The co-design diagram can also be extended to include workspace, lifetime, power, and weight.

opposite of injury risk) of the closed-loop system during the execution of the task, the workspace the manipulator can cover, and finally, the expected lifetime of the soft robot (e.g., higher stresses lead to a reduced lifetime). As **resources**, we consider the manufacturing costs, the power requirements (for computation, sensing, and actuation), and the weight of the soft robot.

We show in Fig. 2(b) a **co-design diagram for the soft robotic manipulator** that includes several required subtasks. Importantly, "co-design diagrams are a formalization of logical dependencies and not to be understood as signal-flow diagrams." [18] To improve the visual clarity, we focus on a few selected resources and functionalities in Fig. 2(b). The structure, proprioception, and actuation subtasks contribute to the soft robot's total (manufacturing) costs. For simplicity, we require the robot's state to be fully observable. Therefore, increasing the state space for generating a more accurate kinematic model will also require more sensors motivating the resource/functionality $n_{\rm q,max}$. Similarly, adding more actuators (i.e., incrementing $n_{\rm motors}$) will also increase the number of statically feasible robot configurations and influence which robot shapes we can target with our controller.

3.3 Parametrization of Design

To allow for an optimization procedure over the design space to be possible, we need first to parameterize the design. Important parameters of the **structural design** (included in π_b) are the topology of the manipulator consisting of slender rods, including their respective lengths and diameters, the material density, and the elastic and shear moduli of the soft material. As **actuation** parameters π_a , we first consider the type of actuation (e.g., tendon-driven, fluidic-driven, shape-memory alloys, metamaterial, etc.) and the number of actuators. Each actuation type will have additional parameters associated with it, such as, in the case of tendon-driven, the topology of the cable routing and the end-point where the tendon is attached to the structure. Finally, we list the parameters π_s involved in the **proprioception** task: analog to actuation, we consider the sensor type and the number of sensors. Furthermore, the parametric model considers each sensor's SE(3) pose w.r.t. the undeformed soft robotic structure.

3.4 Simulation and Model-based Control

For simulating a derived, parametrized design, we will leverage **Discrete Cosserat-rod Models** (**DCMs**) [25], which are suitable for simulating slender, cylindrical objects (i.e., $L \gg D$). DCMs consider continuous curves in 3D space and discretize them in a FEM fashion into many nodes and segments [25]. This allows us to simulate the dynamic behavior of complex topologies consisting of slender rods.

In order to make the kinematic model fully observable and tractable for control purposes, we identify a **reduced-order kinematic model** (e.g., Piecewise Constant Strain (PCS) [26], or Piecewise Affine Curvature (PAC) [27] models) consisting of $n_{\rm s}$ DOFs that is able to approximate the actual robot's shape best. We refer to the parameters of the reduced-order model as the robot's configuration $q \in \mathbb{R}^{n_{\rm q}}$, where $n_{\rm q} = n_{\rm s}$.

This allows us to derive the forward dynamics using the **Euler-Lagrangian** mechanism in the form [21]

$$B(q) \ddot{q} + C(q, \dot{q}) \dot{q} + G(q) + K(q) = A(q) \tau + P^{T}(q) \lambda$$

$$P(q) \ddot{q} + \dot{P}(q) \dot{q} = 0$$
(1)

where $B \in \mathbb{R}^{n_{\mathbf{q}} \times n_{\mathbf{q}}}$ is the mass matrix, $C(q,\dot{q}) \in \mathbb{R}^{n_{\mathbf{q}} \times n_{\mathbf{q}}}$ collects the Coriolis and centrifugal effects, $G(q) \in \mathbb{R}^{n_{\mathbf{q}}}$ and $K(q) \in \mathbb{R}^{n_{\mathbf{q}}}$ contributes the gravitational and elastic forces. The configuration-dependent actuation matrix $A(q) \in \mathbb{R}^{n_{\mathbf{q}} \times n_{\mathbf{a}}}$ maps the actuation $\tau \in \mathbb{R}^{n_{\mathbf{a}}}$ into configuration space, where we assume that the jth actuator has a scalar control input $\tau_j \in \mathbb{R}$. $\lambda \in \mathbb{R}^{n_c}$ are the Lagrange multipliers enforcing constraints dictated by the specified topology (for example closed-chains) [28]. Therefore, the Pfaffian constraints are formulated as $P(q) \dot{q} = 0$ with $P \in \mathbb{R}^{n_c \times n_q}$.

If $n_{\rm q} > n_{\rm a}$, the system is **underactuated**, and with that, the design of a controller is not straightforward [21]. However, often times, we can find a transformation $h(q) = \begin{bmatrix} h_{\rm a}(q)^{\rm T} & h_{\rm u}(q)^{\rm T} \end{bmatrix}^{\rm T}$ into the collocated coordinates $\theta = \begin{bmatrix} \theta_{\rm a}^{\rm T} & \theta_{\rm u}^{\rm T} \end{bmatrix}^{\rm T} \in \mathbb{R}^{n_{\rm q}}$, where $h_{\rm a}(q) = \int A(q)^{\rm T} \dot{q} \, dt \in \mathbb{R}^{n_{\rm a}}$ [29]. In these collocated variables, the **input** $\tau \in \mathbb{R}^{n_{\rm a}}$ acts directly on the actuated variables $\theta_{\rm a}$, and, therefore, the actuation matrix takes the form $A_{\theta} = \begin{bmatrix} \mathbb{I}^{n_{\rm a}} & 0^{n_{\rm a} \times n_{\rm q} - n_{\rm a}} \end{bmatrix}^{\rm T}$. In the collocated coordinates, the system dynamics are given by

$$B_{\theta}(\theta)\ddot{\theta} + \eta_{\theta}(\theta,\dot{\theta}) + G_{\theta}(\theta) + K_{\theta}(\theta) = A_{\theta}\tau$$
(2)

Next, we move towards designing a control law. As an example, we consider regulation, for which the goal is to move the robot towards a desired shape q^d . We can design a **P-satI-D** + feedforward controller [21, 7]

$$\tau = K_{\theta, \mathbf{a}}(\theta^{\mathbf{d}}) + k_{\mathbf{p}} \left(\theta_{\mathbf{a}}^{\mathbf{d}}(t) - \theta_{\mathbf{a}}(t)\right) - k_{\mathbf{d}} \dot{\theta}_{\mathbf{a}}(t) + k_{\mathbf{i}} \int_{0}^{t} \tanh(\gamma \left(\theta_{\mathbf{a}}^{\mathbf{d}}(t') - \theta_{\mathbf{a}}(t')\right)) dt'$$
(3)

where $k_{\rm p}$, $k_{\rm i}$, $k_{\rm d}$, and γ are the feedback control gains.

3.5 A Metric for Measuring Safety

We need to define a metric that measures the *safety* functionality of the soft robot. For rigid robots, impact velocity and inertia (i.e., mass) are the two main factors influencing the contact force on impact with external objects or humans and, with that, dictate the injury risk [30]. For soft robots, the **natural elasticity** of the material reduces the stiffness and enables **increased**



Figure 3: Project timeline including work packages.

safety. Therefore, we will define a function

$$f_{\text{safety}}(\pi_{\text{b}}, \pi_{\text{a}}, \pi_{\text{b}}, \pi_{\text{c}}) := \frac{1}{\text{Momentum} \cdot \text{Impedance}} \approx \frac{1}{\left\|\frac{\partial \mathcal{L}_{\text{cl}}(q, \dot{q})}{\partial \dot{q}}\right\|_{\infty} \cdot \left\|J^{+^{\text{T}}}(q)\frac{\partial \mathcal{L}_{\text{cl}}(q, \dot{q})}{\partial q}J(q)\right\|_{\infty}}$$
(4)

measuring the safety level, where J(q) is the Jacobian mapping the configuration-space velocity \dot{q} to task-space, and $\mathcal{L}_{\rm cl}(q,\dot{q})$ is the Lagrangian of the closed-loop system. $f_{\rm safety}(\pi_{\rm b},\pi_{\rm a},\pi_{\rm b},\pi_{\rm c})$ scales inversely with the inertia, maximum velocity along the trajectory, and task-space stiffness of the manipulator.

4 Impact

This project will provide soft robotic researchers and practitioners with a structured approach for designing controllable continuum soft manipulators that achieve the required performance and safety levels. Furthermore, based on the various designs explored during this research project, we expect to formulate best practices for designing soft robots (e.g., how many actuators are necessary for a given manipulator length) and trade-offs for design decisions (e.g., how does the relationship between task performance and safety look like?). This will democratize the design of soft robots, as fewer hardware iterations and know-how collected over a long period of time are necessary to create a high-performing design. To maximize the scientific outreach of the discoveries, the applicant will make all the project outputs openly available: software frameworks for identifying suitable kinematic parametrization of a given structure, derivation of model controllers, and the co-design optimization procedure, interesting soft robotic designs identified during the research project and the data underlying the trade-off analysis.

5 Feasibility and project timeline

The applicant plans to pursue the proposed research project in collaboration with Prof. Gioele Zardini during a four-month visit to the Laboratory for Information & Decision Systems (LIDS) at M.I.T. in Cambridge, USA.

The project is very ambitious, but its path is already well-defined. The project is divided into work packages: **WP1:** crafting the MDPI formulation/co-design diagram, **WP2:** defining metrics for functionalities and resources (e.g., cost, safety level, and performance), **WP3:** parametrization of the design space (e.g., parametrization of the soft robot structure, the actuation, the integrated sensing capabilities, etc.), **WP4:** implementing a simulator that is capable

of behaving as a digital clone of the real soft robot, **WP5**: derivation of a model-based controller for a given design, and **WP6**: analyzing the co-design trade-offs and identifying best-practice design strategies.

Nonetheless, the **potential high impact** comes with a **considerable risk** of failing. For example, it is a possible pitfall to spend considerable effort on developing a very accurate model/simulator for the considered soft robotic manipulator designs only to find out at a later stage of the project that **optimizing over such a fine-grain model is infeasible**. Therefore, as visualized in Fig. 3, we aim to develop a **Minimal Viable Product (MVP)** of the full co-design procedure within the **first two and a half months** of the project.

The applicant has extensive experience with the topics touched in this research proposal, such as sensing for soft robots [31, 32], kinematic models and simulations based on Cosserat rods [8], and model-based control of soft robots [23, 22, 8, 9]. During his Ph.D., **Prof. Zardini** led the ground-breaking research into the co-design of embodied and mobility systems, investigating both the mathematical foundations [33] and their application to practical problem settings [20, 18, 19]. Under the mentorship of **Prof. Zardini**, the applicant is therefore well prepared to tackle this challenging endeavor.

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