Supplement simulation results for different road conditions:

We will provide simulations of different system parameters to demonstrate the robustness of the controller developed in "Low-Complexity Quantized Prescribed Performance Control for Constrained Steer-by-Wire Systems with Input Nonlinearity and Bandwidth Limitations" more intuitively.

- ***Controller design***

1) Steer-by-Wire (SbW) system:

SbW system model:

$$\mathcal{J}_e\ddot{\theta}_f + \mu^2\mathcal{B}_m\dot{\theta}_f + \mathcal{H}_f(\theta_f, \dot{\theta}_f) = \mu(\tau_m(t) + \tau_d(t)) \tag{1}$$

where $\mathcal{J}_e = \mathcal{J}_f + \mu^2\mathcal{J}_m$ is the equivalent moment of inertia with $\mathcal{J}_f$ and $\mathcal{J}_m$ being the rotational inertia of the front wheel and steering motor, $\mathcal{H}_f(\theta_f, \dot{\theta}_f) = \tau_e(x) + \tau_f(x)$ denotes the uncertain nonlinearity including the self-aligning torque and friction torque of the front wheel, θ_f denotes the steering angle of the front wheel, μ and $\mathcal{B}_m$ are the ratio of motor output shaft angle to front wheel angle and the viscous friction coefficient of the motor, respectively. $\tau_m(t)$ and $\tau_d(t)$ are the output torque of steering motor assembly and the motor torque pulsation disturbance [1].

Input nonlinear model:

$$\tau_m(t) = \ell(t)\tau_c^* + \varsigma(t) \tag{2}$$

with τ_c^* being the control signal formed by the controller, $\tau_m(t)$ being the output of the steering motor, and $\ell(t) > 0$ and $\varsigma(t)$ being unknown time-varying parameters.

2) State and input quantizers:

State quantization using a uniform quantizer:

$$Q_x(\chi) = \begin{cases} L_i, & Case\ I\colon L_i-\psi/2\le\chi(t)<L_i+\psi/2 \\ 0, & Case\ II\colon -\psi/2\le\chi(t)<\psi/2 \\ -L_i, & Case\ III\colon -L_i-\psi/2\le\chi(t)<-L_i+\psi/2 \end{cases} \tag{3}$$

Input quantization using hysteresis quantizer:

$$Q_v(v)=\begin{cases} v_i, & Case\ I\colon \dfrac{v_i}{1+\varpi} < v \le v_i, Q_v^- \ge v_i \\ & or\ v_i \le v < \dfrac{v_i}{1-\varpi}, Q_v^- \le v_i \\ v_i(1+\varpi), & Case\ II\colon v_i\le v\le\dfrac{v_i}{1-\varpi}, Q_v^-\ge v_i(1+ \\ & \varpi) or \dfrac{v_i}{1-\varpi}\le v< v_i,\ Q_v^-\le v_i(1+\varpi) \\ 0, & Case\ III\colon 0\le v\le\dfrac{v_i}{1+\varpi}, Q_v^-\ge v_i(1+ \\ & \varpi)\ or\ \dfrac{v_i}{1+\varpi}<v<v_1, Q_v^-=0 \\ -Q_v(-v), & Case\ IV\colon v<0. \end{cases} \tag{4}$$

3) Control law and event-triggering mechanism:

The following error transformation is given as

$$z = \chi(t) - \lambda y_d \tag{5}$$

which $\chi(t) = \lambda x_1 + x_2$.

The control law $v(t)$ is designed as

$$v(t) = -\eta S(z(t)), \quad S(z(t)) = \tan\left(\frac{\pi}{2}\frac{z(t)}{\rho(t)}\right) \tag{6}$$

The prescribed performance function $\rho(t)$ is

$$\rho(t)=\begin{cases}\xi_1+(\xi_0-\xi_1)e^{\left(-\frac{t}{t_\xi-t}\right)}, & Case\ I\colon 0\le t<t_\xi\\ \xi_1, & Case\ II\colon otherwise\end{cases}\tag{7}$$

The ETM is constructed as

$$u(t)=Q_v(v(t_k))\quad \forall t\in[t_k,t_{k+1}),\ t_1=0\tag{8}$$

$$t_{k+1}=\begin{cases}inf\left\{t>t_k:|e_v(t)|\ge\varrho|v(t)|+m\right\}, & \\ & Case\ I\colon if\ |v(t)|\le\kappa\\ inf\{t>t_k:|e_v(t)|\ge m\}, & Case\ II: otherwise\end{cases}\tag{9}$$

Please refer to the “Low-Complexity Quantized Prescribed Performance Control for Constrained Steer-by-Wire Systems with Input Nonlinearity and Bandwidth Limitations” for detailed parameter definitions on page 3-5.

- ***Simulation parameters selection***

1) The SbW system parameters selection

The specific simulation model of SbW systems can be found in [1]. The parameters of (1) are chosen as $\mathcal{J}_f=3.8\ \mathrm{kg}\cdot\mathrm{m}^2$, $\mathcal{J}_m=0.0045\ \mathrm{kg}\cdot\mathrm{m}^2$, $\mu=18$ and $\mathcal{B}_m=0.018\ \mathrm{Nm}\cdot\mathrm{s/rad}$. $\mathcal{H}_f(\theta_f,\dot{\theta}_f)=\tau_e+\tau_f$, in which the friction torque τ_f is considered as [1], i.e., $\tau_f=0.25(tanh(100x_2)-tanh(x_2))+30tanh(100x_2)+10x_2$, the self-aligning torque τ_e is considered as *Appendix B*, the drive torque $T_{fi}=50N\cdot m,\ i=r,l$, the initial vehicle speed and wheel rotation speed are $v_x=19\ m/s$ and $\omega=57\ rad/s$, respectively, the mechanical and pneumatic trail $t_p=0.023,\ t_m=0.016$, and the other parameters are given in [2] and Tab.I. The input nonlinearity of (2) are chosen as $\ell(t)=\ell_f\ell_r$ for $u(t)\ge-\varsigma_l$; $\ell(t)=\ell_f\ell_l$ for $u(t)<-\varsigma_l$, $\varsigma(t)=\varsigma_f(t)-\ell_f\ell_r\varsigma_r$ for $u(t)>\varsigma_r$; $\varsigma(t)=\varsigma_f(t)-\ell_f\ell_r u(t)$ for $-\varsigma_l\le u(t)\le\varsigma_r$; $\varsigma(t)=\varsigma_f(t)+\ell_f\ell_l\varsigma_l$ for $u(t)<-\varsigma_l$, where $\ell_l=1.2$, $\ell_r=1.4$, $\varsigma_l=40$ and $\varsigma_r=30$, and $\ell_f=1$, $\varsigma_f(t)=0$ for $t\in[0,5)s$; $\ell_f=0.75$, $\varsigma_f(t)=3sin(4t)$ for $t\in[5,10)s$; $\ell_f=0.5$, $\varsigma_f(t)=4sin(3t)$ for $t\in[10,15)s$; $\ell_f=0.25$, $\varsigma_f(t)=3sin(4t)$ for $t\in[15,20]s$. Besides, the disturbance is assumed as $d(t)=5\int[d_m-d(t)+2rand(1)]dt$ with $d_m=2cos(6t)$ for $t\in[0,5)$; $d_m=2.5cos(4t)$ for $t\in[5,10)$; $d_m=3cos(4t)$ for $t\in[10,15)$; $d_m=3.5cos(2t)$ for $t\in[15,20]$.

2) Controller parameters selection

The prescribed performance control design (5)-(9) in simulation, the positive constants $\lambda=60$, $\eta=50$, $\xi_0=10$, $\xi_1=0.09$, and the preseted steady-state time $t_\xi=0.2$. Finally, the parameters of the event-triggering mechanism are designed as $\varrho=0.04$, $m=4$, and $\kappa=10$. Besides, the desired signal is selected as $y_d=0.3sin(0.3t)\ rad$, with an initial value of $x=[0.1,0]^T$.

3) State quantizer and input quantizer parameters selection

The parameters of state quantizer (3) are selected as $\lambda=60$ and $\psi=0.01$. Besides, the parameters of the input quantizer (4) are selected as $\beta=0.8$ and $v_{min}=0.2$ in different simulations. The detailed simulation process and parameter calculation are shown in Tab. II.

4) Parameter selection under different road conditions

The tire stiffness is given for different road conditions:

$$\begin{cases}C_\alpha=30,000\ \mathrm{N/rad},\ C_s=50,000\ \mathrm{N/rad}, & \text{dry asphalt road}\\ C_\alpha=20,000\ \mathrm{N/rad},\ C_s=35,000\ \mathrm{N/rad}, & \text{wet asphalt road}\\ C_\alpha=10,000\ \mathrm{N/rad},\ C_s=21,000\ \mathrm{N/rad}, & \text{snowy road.}\end{cases}\tag{10}$$

The nominal friction coefficient between the tyre and the ground μ for different road conditions is given

$$\mu=\begin{cases}0.7, & \text{dry asphalt road}\\ 0.4, & \text{wet asphalt road}\\ 0.15, & \text{snowy road}\end{cases}\tag{11}$$

TABLE I
NOMENCLATURE

Notations	Descriptions	Value
m	Total mass of the vehicle	1298.9 kg
m_s	Sprung mass of the vehicle	1167.5 $kg \cdot m^2$
I_{zz}	Moment of inertia of the vehicle about the yaw axis	1627 $kg \cdot m^2$
I_{xx}	Moment of inertia of the vehicle about the roll axis	498.9 $kg \cdot m^2$
I_{xz}	Sprung mass product of the inertia	0 $kg \cdot m^2$
l_f	Distance of the centre of gravity from the front axle	1 m
l_r	Distance of the centre of gravity from the rear axle	1.454 m
d_f	Front track width	1.436 m
d_r	Rear track width	1.436 m
h	Height of the centre of gravity of the sprung mass	0.533 m
h_s	Distance of the centre of gravity of the sprung mass from the roll axes	0.4572 m
R_w	Radius of the wheel	0.35 m
I_w	Moment of inertia of the wheel	2.1 $kg \cdot m^2$
C_α	Cornering stiffness of one tyre	30,000 N/rad
C_s	Longitudinal stiffness of one tyre	50,000 $N/unit\ slip$
k_{rsf}	Front roll steer coefficient	-0.2
k_{rsr}	Rear roll steer coefficient	0.2
K_R	Ratio of the front roll stiffness to the total roll stiffness	0.552
c_φ	Torsional damping of the roll axis	3511.6 $N \cdot m/s$
k_φ	Torsional stiffness of the roll axis	66,185.8 $N \cdot m/rad$
ε_r	Road adhesion reduction factor	0.015 s/m
g	Acceleration due to gravity	9.81 m/s^2
μ	Nominal friction coefficient between the tyre and the ground	0.7
T_i	Driving torque of each wheel	50 $N \cdot m$
$v_x(0)$	Initial vehicle speed	19 m/s
$w_i(0)$	Initial rotation speed of each wheel	57 rad/s
t_p	Pneumatic trail	0.016
t_m	Mechanical trail	0.023

- ***Simulation results and analysis***

  The simulation results are shown in Fig.1 and Tab.III. Fig.1(a) shows the tracking error $y - y_d$ under different road conditions. Fig.1(b) shows the steering angle tracking performance under three road conditions. Fig.1(c) shows the output of the input nonlinear module of the SbW system, and Fig.1(d) gives the values of model uncertainty under different road surfaces. From Fig.1(a)-(b), it can be seen that the tracking effect under different road surfaces is relatively ideal, and the tracking error is constrained within the boundary. From Fig.1(c), it can be seen that the output of the input nonlinear module is different on different road surfaces. Compared to other road surfaces, the output of the input nonlinear module is smaller on snowy roads due to its smaller road adhesion coefficient. Fig.1(d) shows that the value of model uncertainty $f(x)$ varies under each condition. Combining Fig.1(a)-(b), we can conclude that the developed control instrument has good robustness. Moreover, Tab.III provides different tracking error indicator values for three sets of simulations.

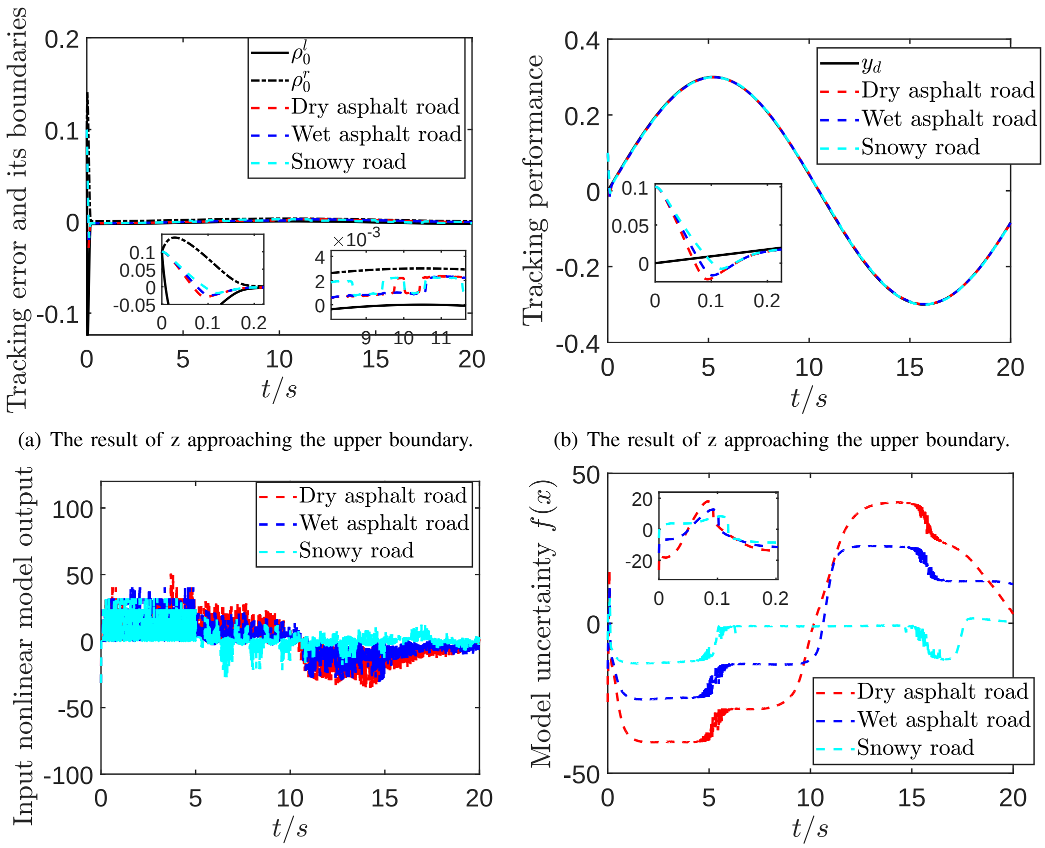
REFERENCES

[1] B. Ma and Y. Wang, "Adaptive output feedback control of steer-by-wire systems with event-triggered communication," *IEEE/ASME Transactions on Mechatronics*, vol. 26, no. 4, pp. 1968–1979, 2021.

TABLE II
SIMULATION PROCESS AND PARAMETER CALCULATION

Step	Calculate
State quantizer (3)**:** $x_1 \longrightarrow Q_x(\chi)$	
Step 1	Input of state quantizer: $\chi = 60x_1 + \dot{x}_1$
Step 2	Case I: $Q_x(\chi) = L_i$, Case II: $Q_x(\chi) = 0$, Case III: $Q_x(\chi) = -L_i$
Designed parameters	$\lambda = 60$, and $\psi = 0.01$.
Computational control law (5)-(7)**:** $Q_x(\chi) \longrightarrow v(t)$	
Step 3	Error transformation: $z = Q_x(\chi) - 60y_d$
Step 4	Prescribed performance function: Case I: $\rho(t) = 0.08 + (7 - 0.08)exp(-t/(0.1 - t))$, Case II: $\rho(t) = 0.08$
Step 5	Control law: $v(t) = -100\tan\left((\pi z(t))/(2\rho(t))\right)$
Designed parameters	$\lambda = 60$, $\eta = 50$, $\xi_0 = 10$, $\xi_1 = 0.09$ $t_\xi = 0.2$, $y_d = 0.3sin(0.3t)$ rad, and $x = [0.1, 0]^T$.
Input quantizer (4)**:** $v(t) \longrightarrow Q_v(v)$	
Step 6	Case I: $Q_v(v) = v_i$, Case II: $Q_v(v) = v_i(1 + \varpi)$, Case III: $Q_v(v) = 0$, Case IV: $Q_v(v) = -Q_v(-v)$
Designed parameters	$\beta = 0.8$, and $v_{min} = 0.02$.
ETM (8)-(9)**:** $Q_v(v) \longrightarrow u(t)$	
Step 7	Case I: $\|e_v(t)\| \geq \varrho\|v(t)\| + m$, Case II: $\|e_v(t)\| \geq m$
Designed parameters	$\varrho = 0.04$, $m = 4$, and $\kappa = 10$.
The SbW system (1)**:** $u(t) \longrightarrow x_1$	
Step 8	$\mathcal{H}_f(x) = \tau_e + \tau_f$
Step 9	$\mathcal{J}_e = 3.8 + 18^2 \times 0.0045$
Step 10	$f(x) = -(18^2 \times 0.018x_2 + \mathcal{H}_f(x))/\mathcal{J}_e$
Step 11	$g(t) = 18/\mathcal{J}_e$
Step 12	$d(t) = 5\int[d_m - d(t) + 2rand(1)]dt$
Step 13	$\ddot{x}_1 = \dot{x}_2 = f(x) + g(t)\ell(t)u(t) + d(t)$
τ_e, τ_f, d_m, $\ell(t)$, $\varsigma(t)$	Please see *The SbW system parameters selection.*

[2] H. Du, J. Lam, K.-C. Cheung, W. Li, and N. Zhang, "Side-slip angle estimation and stability control for a vehicle with a non-linear tyre model and a varying speed," *Proceedings of The Institution of Mechanical Engineers Part D-journal of Automobile Engineering*, vol. 229, no. 4, pp. 486–505, 2015.



(a) The result of z approaching the upper boundary.

(b) The result of z approaching the upper boundary.

(c) The result of z approaching the lower boundary.

(d) The result of z approaching the lower boundary.

Fig. 1. The worst-case dynamics of $z(t)$ are considered in this paper.

TABLE III
TRACKING PERFORMANCE IN DIFFERENT ROAD SURFACES

	Indicator	$[0,5)s$	$[5,10)s$	$[10,15)s$	$[15,20]s$
Dry asphalt road	IAE	0.0136	0.0029	0.0093	0.0029
	RMSE	0.0081	0.0008	0.0019	0.0007
	SD	0.0080	0.0007	0.0004	0.0007
Wet asphalt road	IAE	0.0134	0.0026	0.0088	0.0024
	RMSE	0.0082	0.0006	0.0018	0.0006
	SD	0.0082	0.0005	0.0005	0.0006
Snowy road	IAE	0.0131	0.0051	0.0084	0.0043
	RMSE	0.0084	0.0012	0.0017	0.0010
	SD	0.0083	0.0008	0.0005	0.0009