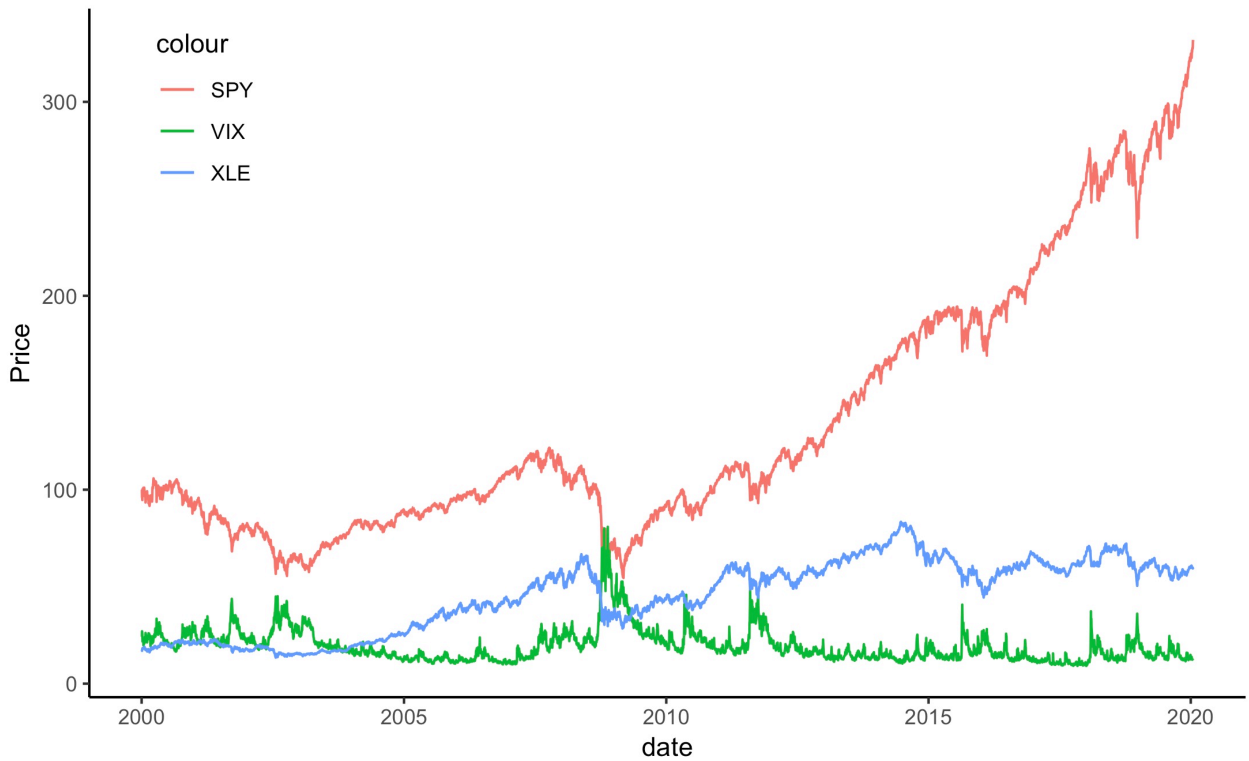
HW 3&4

This notebook is created by Kaiyi Li at 2020.03.08 for the homework 3 and 4 of volatility class. Before we get started, let's load the dataset and library.

```
library(rugarch)
library(ggplot2)
library(reshape2)
library(rmgarch)
raw_data <- read.table("spyxlevix20.TXT",header = T)
#Covert the date to date object
raw_data$date <- as.Date(raw_data$date)
raw_data = na.omit(raw_data)
```

Next, we plot all these series.

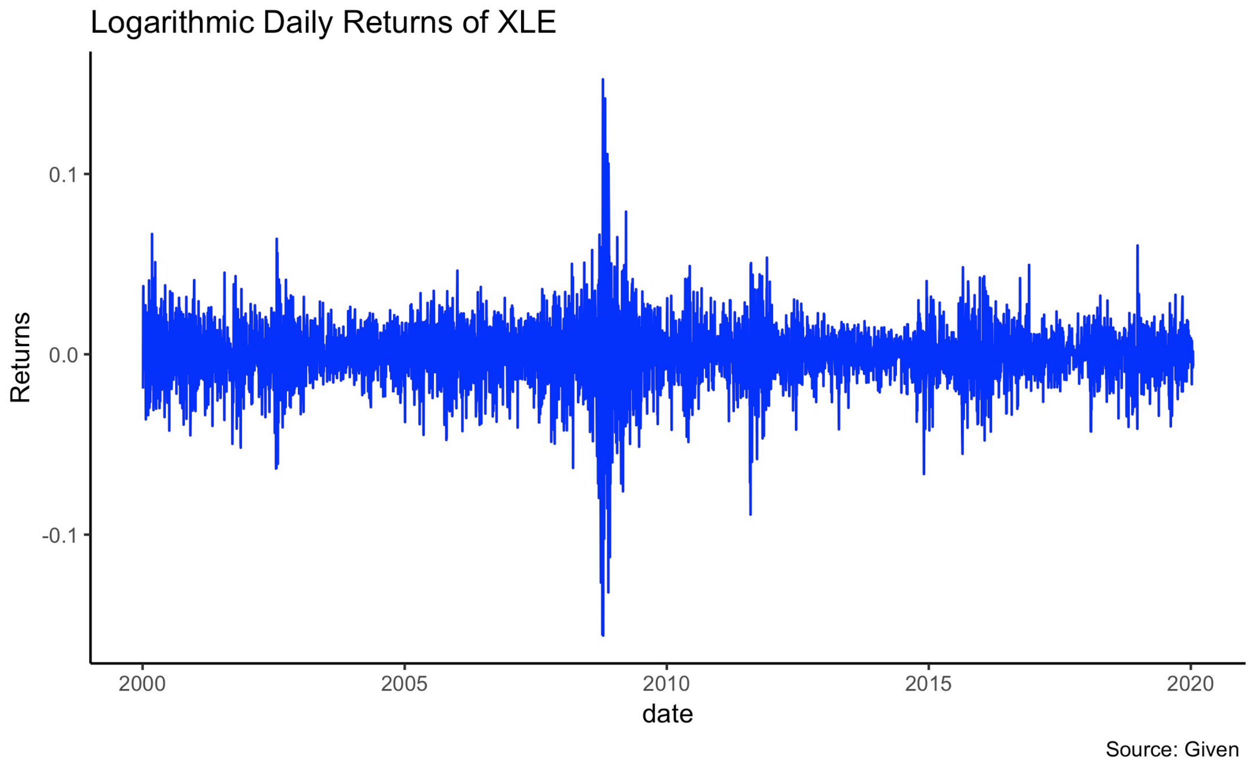
```
theme_set(theme_classic())
ggplot(raw_data, aes(x=date)) +
  ylab('Price')+
  geom_line(aes(y=raw_data$spy,col='SPY'),alpha=1) +
  geom_line(aes(y=raw_data$vix,col='VIX'),alpha=1) +
  geom_line(aes(y=raw_data$xle,col='XLE'),alpha=1) +
    theme(legend.position=c(.1,.85))
ggplot2::ggsave("PricePaths.jpg")
```



1. You have a portfolio of $1,000,000 of stock all in XLE which is an energy ETF. What is the 1% one day Value at Risk for this portfolio on Monday Jan 20, 2020? Use the attached dataset.

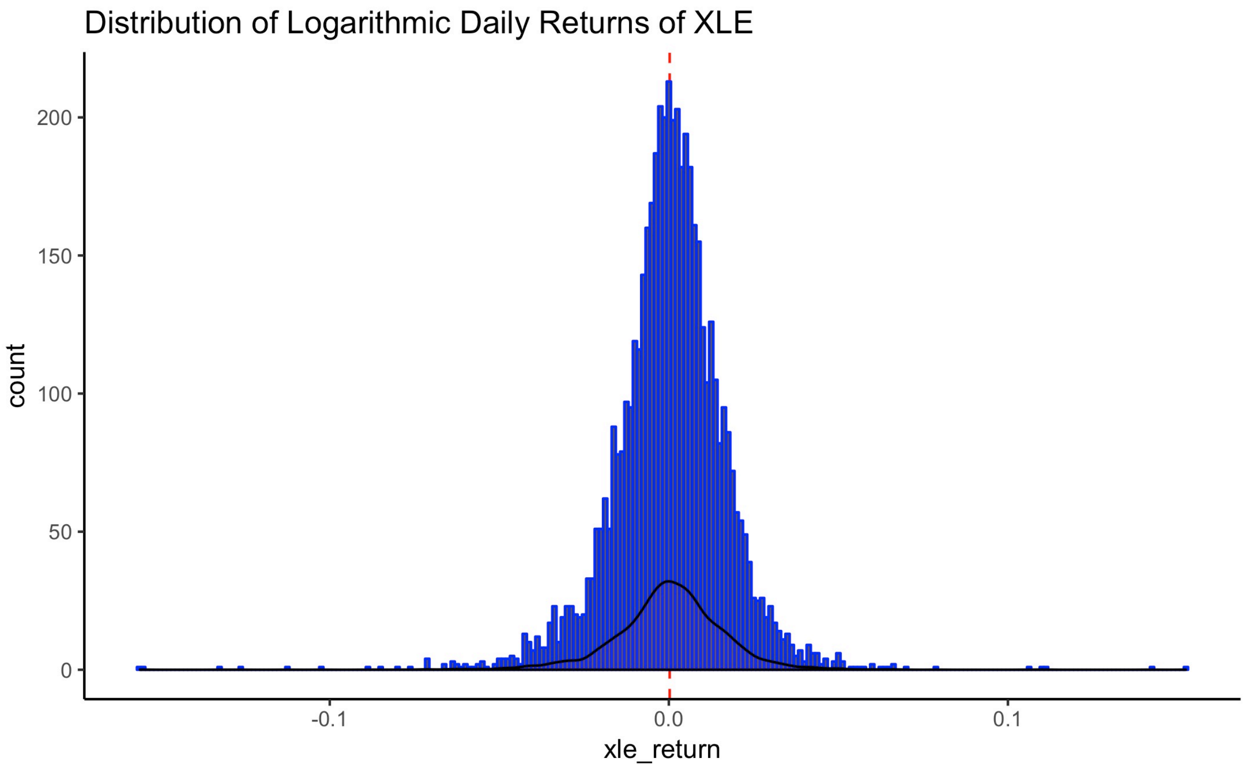
First, let's plot the path of XLE's return.

```
xle_returns <- data.frame(
  date = raw_data$date[-1],
  xle_return = diff(log(raw_data$xle))
)
ggplot(xle_returns, aes(x=date)) +
  geom_line(aes(y=xle_return),group = 1,colour = "blue") +
  labs(title="Logarithmic Daily Returns of XLE",
       caption="Source: Given ",
       y="Returns")
ggplot2::ggsave("XLE_Daily_Returns.jpg")
```



The distribution of XLE's return.

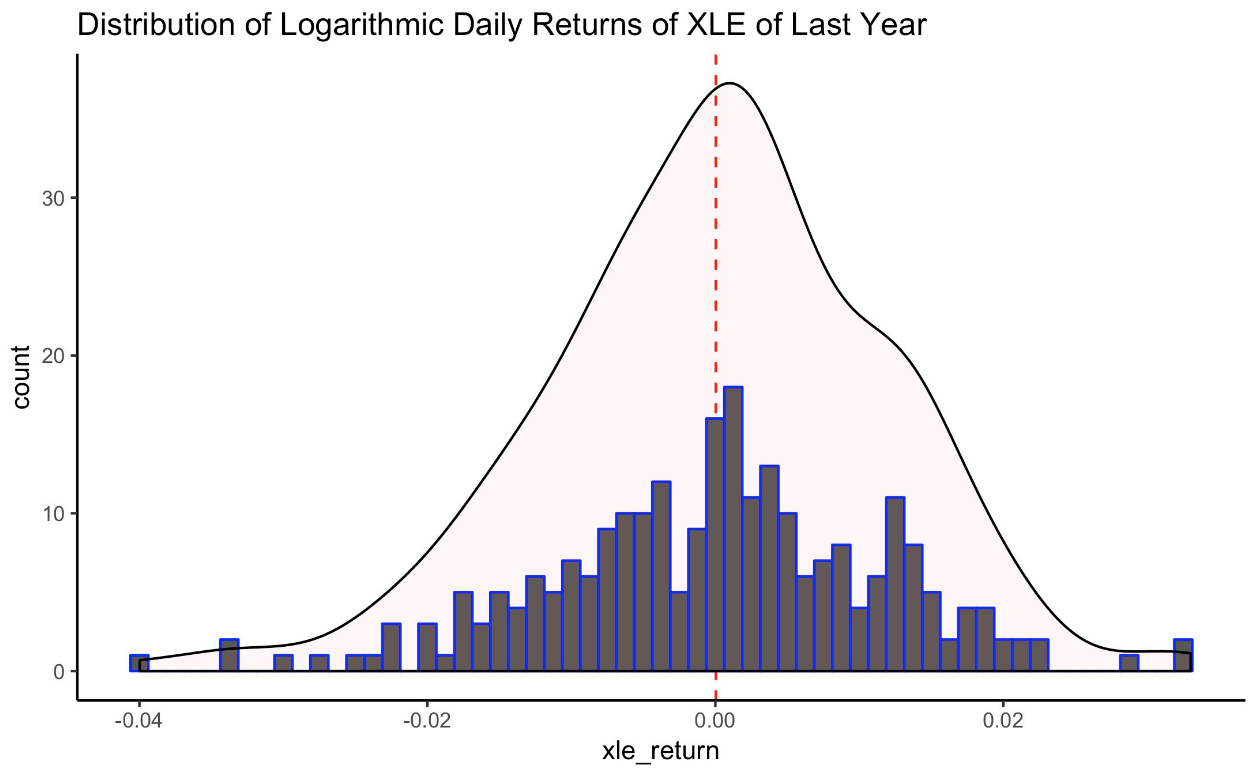
```
ggplot(xle_returns,aes(x=xle_return))+
  geom_vline(aes(xintercept=mean(xle_return)),colour =
"red",linetype="dashed")+geom_histogram(binwidth = 0.00125,colour = "blue")+
  geom_density(alpha=0.05, fill="#FF6666") +
  labs(
  title = "Distribution of Logarithmic Daily Returns of XLE"
)
ggplot2::ggsave("Distribution_of_Logarithmic_Daily_Returns_of_XLE.jpg")
```



a) Using the quantile from one year history it is?

Plot the last year's return of XLE.

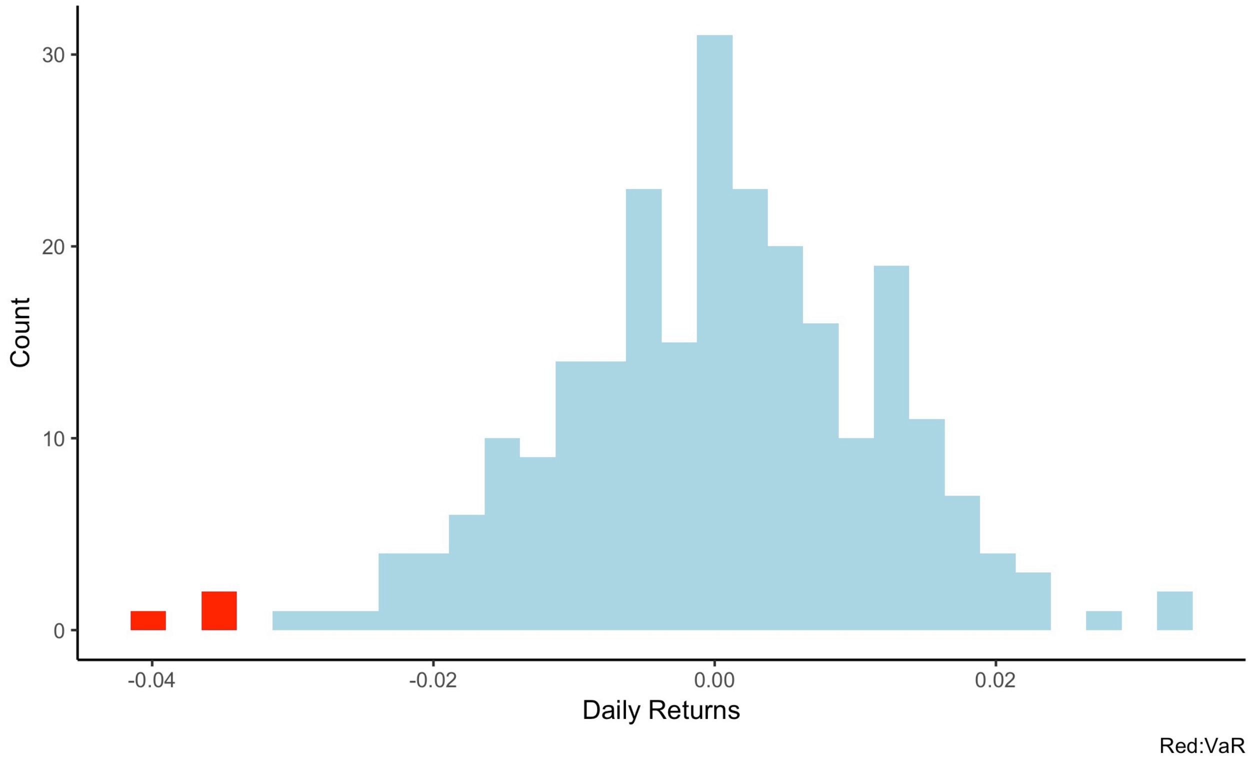
```
# Extract returns of last year.
last_year_xle_returns <- tail(xle_returns,252)
# last_year_xle_returns
ggplot(last_year_xle_returns,aes(x=xle_return))+
  geom_vline(aes(xintercept=mean(xle_return)),colour =
"red",linetype="dashed")+geom_histogram(binwidth = 0.00125,colour = "blue")+
  geom_density(alpha=0.05, fill="#FF6666") +
  labs(
  title = "Distribution of Logarithmic Daily Returns of XLE of Last Year"
)
ggplot2::ggsave("Distribution_of_Logarithmic_Daily_Returns_of_XLE_Of_Last_Year.jpg")
```




```
rets <- last_year_xle_returns$xle_return
quantile(rets , 0.01)
```

```
          1%
-0.03212911
```

```
qplot(rets , geom = 'histogram') + geom_histogram(fill = 'lightblue' , bins = 30) +
    geom_histogram(aes(rets[rets < quantile(rets , 0.01)]) , fill = 'red' , bins = 30) +
    labs(x = 'Daily Returns',y='Count',caption = "Red:VaR" )
ggplot2::ggsave("Last_Year_VaR.jpg")
```



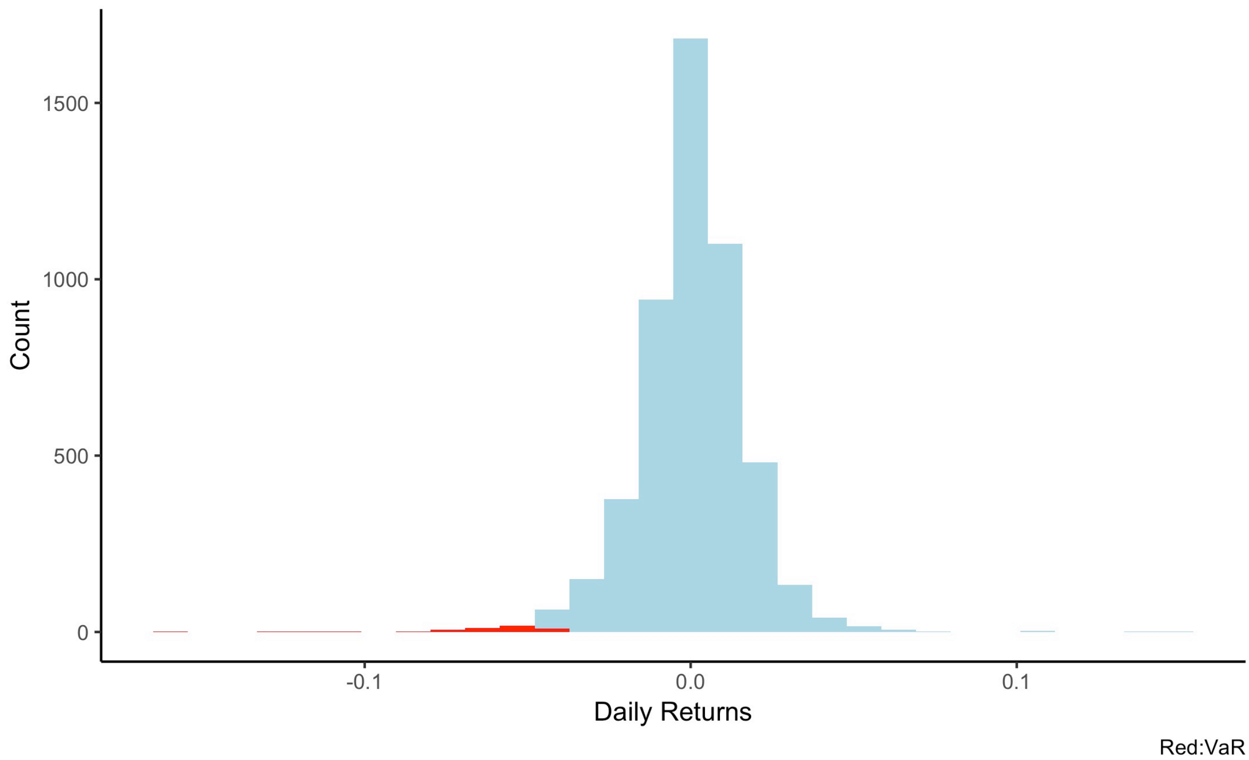
Using one year data, the VaR is $32129.11.

b) Using the quantile from the full data set it is?

```
rets<- xle_returns$xle_return
quantile(rets , 0.01)
```

```
         1%
-0.04551611
```

```
qplot(rets , geom = 'histogram') + geom_histogram(fill = 'lightblue' , bins = 30) +
    geom_histogram(aes(rets[rets < quantile(rets , 0.01)]) , fill = 'red' , bins = 30) +
    labs(x = 'Daily Returns',y='Count',caption = "Red:VaR" )
ggplot2::ggsave("Full_Dataset_Year_VaR.jpg")
```



Using full data, the VaR is $4516.11.

c) Using the normality assumption and a GARCH(1,1), it is?

```
garchSpec <- ugarchspec(
  variance.model=list(model="sGARCH", garchOrder=c(1,1)),
  mean.model=list(armaOrder = c(0,0),include.mean=F))
fit = ugarchfit(spec = garchSpec,data = xle_returns$xle_return)
garch_vol_pred <- ugarchforecast(fit, n.ahead = 1)
-qnorm(0.01)*sigma(garch_vol_pred)*10^6
```

```
   1983-10-22 08:00:00
T+1            20468.25
```

Under normality assumption, using GARCH(1,1), the VaR is $20468.25 .

d) Using the GJR-GARCH model model with normality, it is?

```
GJRgarchSpec <- ugarchspec(
  variance.model=list(model="gjrGARCH", garchOrder=c(1,1)),
  mean.model=list(armaOrder = c(0,0),include.mean=F),
  distribution.model="norm")
fit = ugarchfit(spec = GJRgarchSpec,data = xle_returns$xle_return)
GJR_garch_vol_pred <- ugarchforecast(fit, n.ahead = 1)
-qnorm(0.01)*sigma(GJR_garch_vol_pred)*10^6
```

```
    1983-10-22 08:00:00
T+1            21697.15
```

Under normality assumption, using GJR-GARCH(1,1), the VaR is $21697.15

e) Using the GJR-GARCH model with bootstrapped residuals, it is?

```
resid = xle_returns$xle_return/fit@fit$sigma
-quantile(resid,0.01)*sigma(GJR_garch_vol_pred)*10^6
```

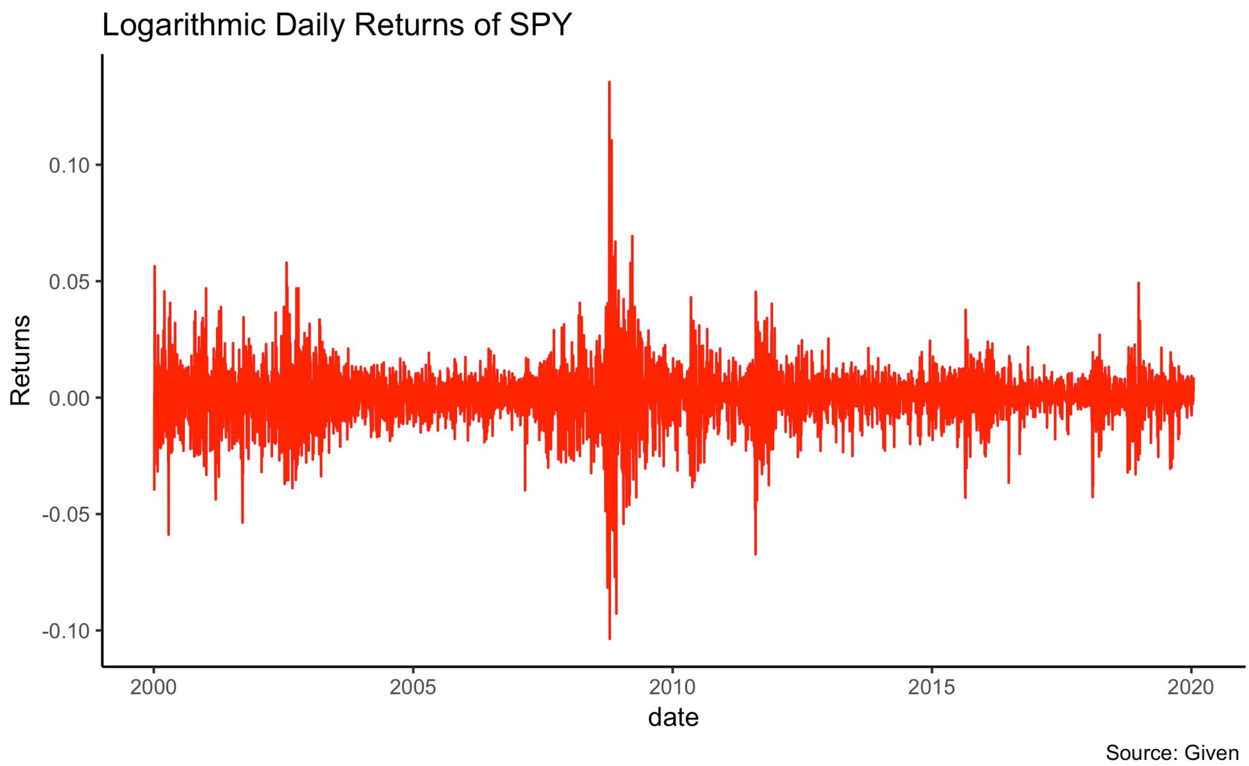
```
1983-10-22 08:00:00
T+1            24342.48
```

With bootstrap residuals, using GJR-GARCH(1,1), the VaR is $24342.48

2. Which asymmetric form of ARCH model works best for SPY, (an ETF for sp500) in the attached data set?

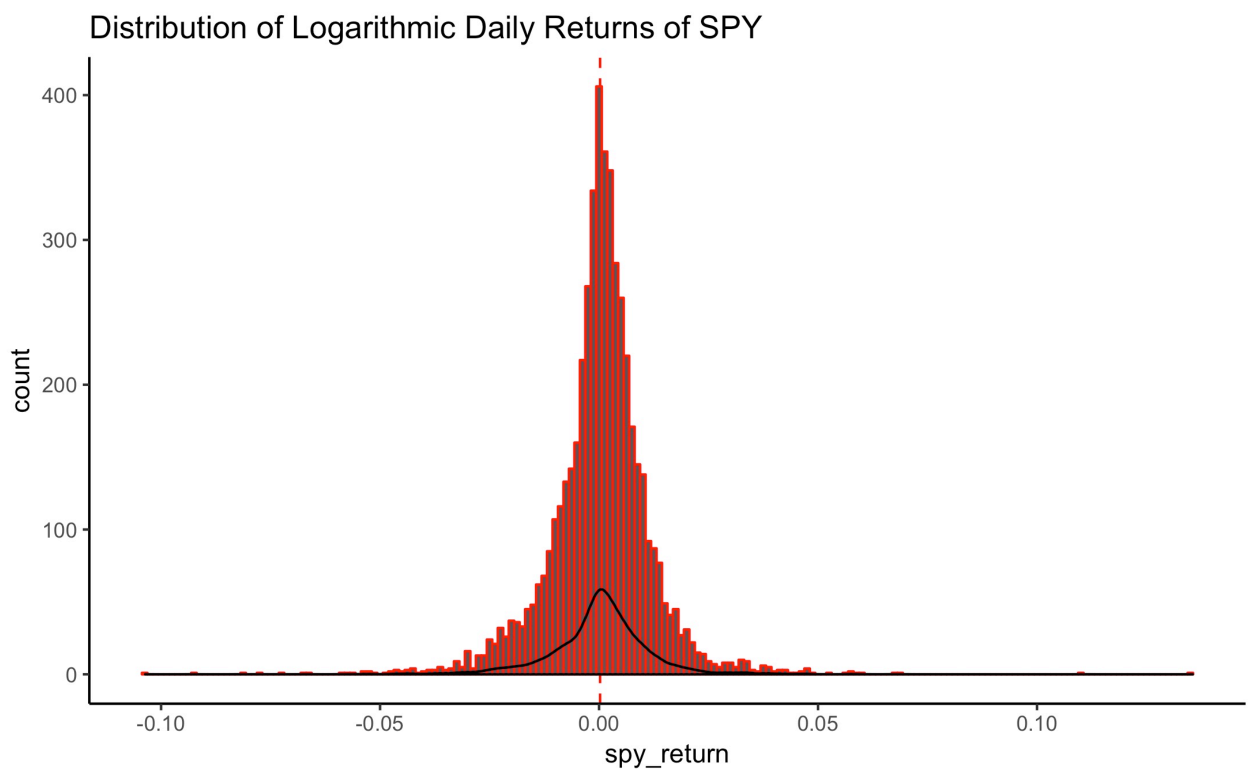
Plot the return path of SPY.

```
spy_returns <- data.frame(
  date = raw_data$date[-1],
  spy_return = diff(log(raw_data$spy))
)
ggplot(spy_returns, aes(x=date)) +
  geom_line(aes(y=spy_return),group = 1,colour = "red") +
  labs(title="Logarithmic Daily Returns of SPY",
       caption="Source: Given ",
       y="Returns")
ggplot2::ggsave("SPY_Daily_Returns.jpg")
```



Plot the distribution of SPY returns.

```
ggplot(spy_returns,aes(x=spy_return))+
  geom_vline(aes(xintercept=mean(spy_return)),colour =
"red",linetype="dashed")+geom_histogram(binwidth = 0.00125,colour = "red")+
  geom_density(alpha=0.05, fill="#FF6666") +
  labs(
  title = "Distribution of Logarithmic Daily Returns of SPY"
)
ggplot2::ggsave("Distribution_of_Logarithmic_Daily_Returns_of_SPY.jpg")
```



a) Compare GJR-GARCH and EGARCH with GARCH using the Schwarz criterion.

Specify and fit data to each model first.

```
GJRgarchSpec <- ugarchspec(
  variance.model=list(model="gjrGARCH", garchOrder=c(1,1)),
  mean.model=list(armaOrder = c(0,0),include.mean=F),
  distribution.model="norm")
EgarchSpec <- ugarchspec(
  variance.model=list(model="eGARCH", garchOrder=c(1,1)),
  mean.model=list(armaOrder = c(0,0),include.mean=F),
  distribution.model="norm")
garch <- ugarchspec(
  variance.model=list(model="sGARCH", garchOrder=c(1,1)),
  mean.model=list(armaOrder = c(0,0),include.mean=F),
  distribution.model="norm")
#Fit data into models
GJR_fit <- ugarchfit(
  spec = GJRgarchSpec,
  data = spy_returns$spy_return
)
E_fit <- ugarchfit(
  spec = EgarchSpec,
  data = spy_returns$spy_return
)
GARCH_fit <- ugarchfit(
  spec = garch,
  data = spy_returns$spy_return
)
```

Then we check the Schwarz criterion of each model.

```
infocriteria(GJR_fit)
```

```
Akaike       -6.542949
Bayes        -6.537772
Shibata      -6.542950
Hannan-Quinn -6.541135
```

The Schwarz criterion of GJR-GARCH is -6.537772.

```
infocriteria(E_fit)
```

```
Akaike       -6.550925
Bayes        -6.545748
Shibata      -6.550926
Hannan-Quinn -6.549111
```

The Schwarz criterion of E-GARCH is -6.545748.

```
infocriteria(GARCH_fit)
```

```
Akaike       -6.492660
Bayes        -6.488777
Shibata      -6.492660
Hannan-Quinn -6.491299
```

The Schwarz criterion of GARCH is -6.488777.

Both E-GARCH and GJR-GARCH are better than GARCH because their Schwarz criterion is smaller.

b) Can you find something even better?

Let's try a different distribution.

```
EgarchSpecT <- ugarchspec(
  variance.model=list(model="eGARCH", garchOrder=c(1,1)),
  mean.model=list(armaOrder = c(0,0),include.mean=F),
  distribution.model="sstd")
Et_fit <- ugarchfit(
  spec = EgarchSpecT,
  data = spy_returns$spy_return
)
infocriteria(Et_fit)
```

```
Akaike       -6.602090
Bayes        -6.594325
Shibata      -6.602093
Hannan-Quinn -6.599370
```

Using a skewed student distribution with E GARCH(1,1) model gives a lower Schwarz criterion and therefore is better.

c) Describe the strength of the asymmetry in SP as compared with XLE.

Fit the XLE data with EGARCH and GARCH to see the difference.

```
xle_garch_fit <- ugarchfit(garch,data = xle_returns$xle_return)
print(infocriteria(xle_garch_fit))
print("_____________________")
xle_egarch_fit <- ugarchfit(EgarchSpec,data = xle_returns$xle_return)
print(infocriteria(xle_egarch_fit))
```

```
Akaike       -5.660033
Bayes        -5.656150
Shibata      -5.660034
Hannan-Quinn -5.658673
[1] "____________________"

Akaike       -5.674970
Bayes        -5.669793
Shibata      -5.674971
Hannan-Quinn -5.673156
```

SPY has a stronger strength of the asymmetry than XLE. Here is the reason. The difference of Schwarz criterion between EGARCH and GARCH for XLE is -5.656150-(-5.669793) = 0.013643. However,the difference of Schwarz criterion between EGARCH and GARCH for XLE is -6.488777-(-6.545748) = 0.056971. A larger performance gap between GARCH and EGARCH of the SPY than that of XLE indicates that SPY has a stronger strength of the asymmetry than XLE.

3. Go to V-LAB and

A. find the volatility forecasts one day and one year ahead for the following assets using the GJR-GARCH model:

As of March 8, 2020.

Tickers	One-Day	One-Year
S&P 500	40%	24%
Merval	39.69%	40.84%
Barclays Aggregate Government Bond Index	14.93%	12.56%
Coca Cola	33.2%	31.07%
MBIA	41.27%	47.34%
Ruble Exchange rate	18.3%	18.69%
Cohen and Steers Realty Majors Index	38.29%	27.94%

(Couldn't find Cohen and Steers Realty Majors Index on vlab. Used iShares Cohen & Steers REIT ETF instead)

B. Are any of these out of line with your expectations from class? If so, can you say why?

For S&P 500, the forecasted volatility for next day is much larger than that of next year, which is out of line with my expectation.This might has something to do with the panic in market corresponding with the coronavirus.

4. If the government introduced a policy that was widely viewed as being able to reduce the future uncertainty in the stock market by requiring more transparency in accounting principles, what effect would this have on stock prices today? Relate this to the asymmetric volatility effect.

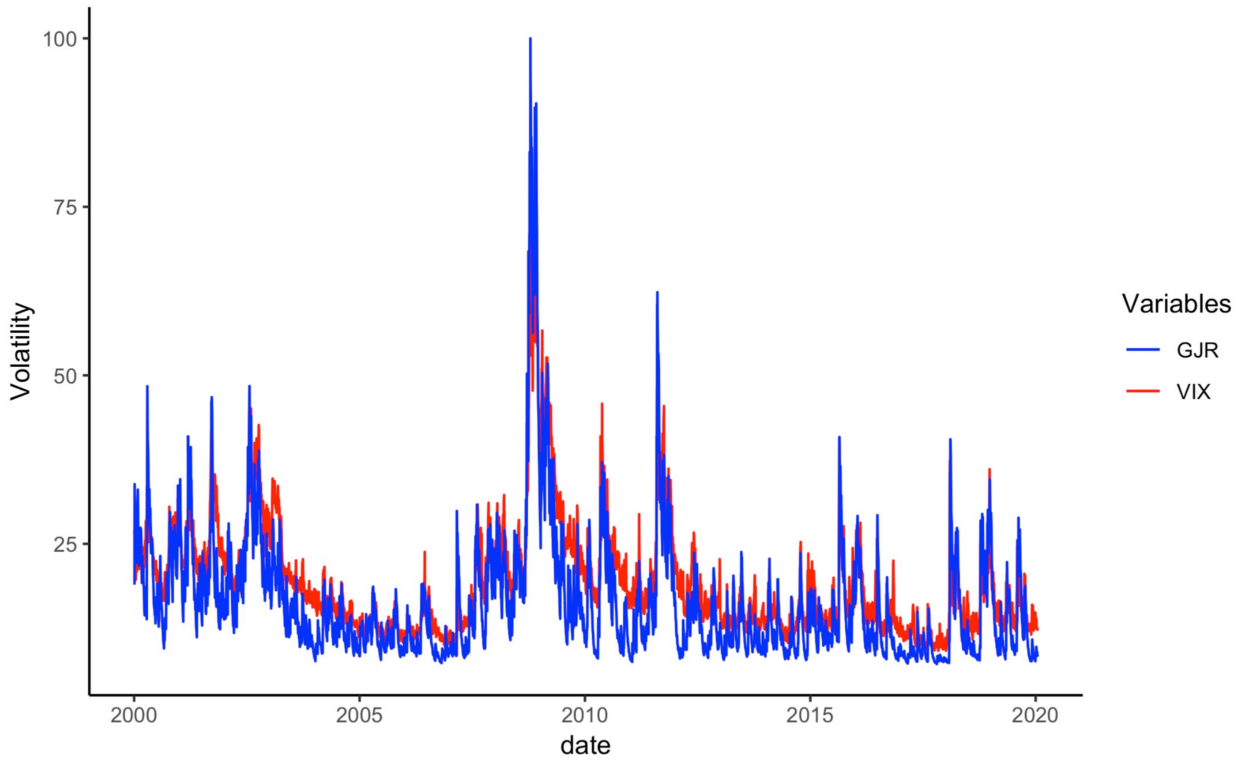
If a more transparent accounting principles were introduced, people would have more agreement on the valuation of these companies. According to Gregory R. Duffee's *Balance sheet explanations for asymmetric volatility* from Haas, UC Berkely, a change in firm's value has a correlation with firm's stock return. Balance effect was one of the source of asymmetric volatility. With a more transparent accounting principle, the asymmetric volatility should be reduced, the stock price should be less volatile.

5. Using the attached data on the VIX and SPY.

a) Plot the VIX and annualized GJR-GARCH volatilities to see whether there is a bias. What are two explanations for this bias?

Annualized the GJR-GARCH fitted volatilities, and plot them.

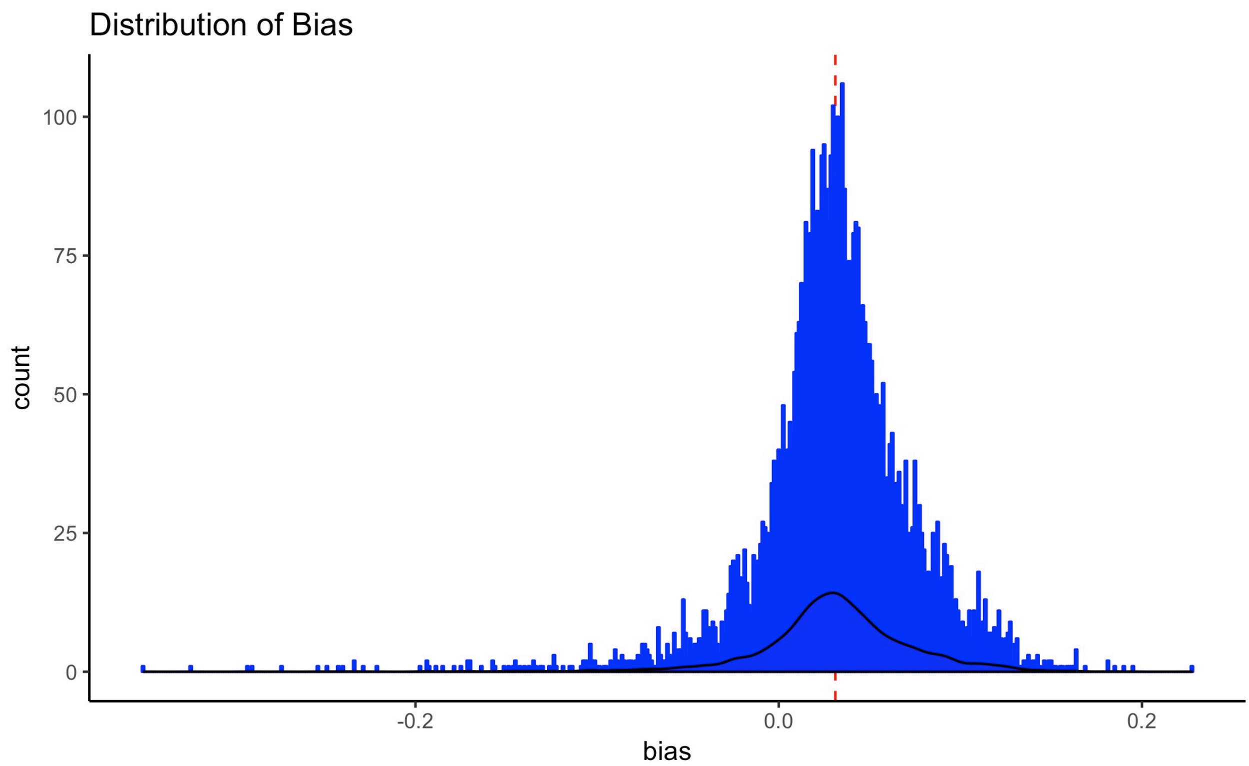
```
# annualized_GJR_vol <- 100*GJR_fit@fit$sigma*252^0.5
annualized_GJR_vol <- 100*GJR_fit@fit$sigma*sqrt(252)
#Join series.
join_vols <- data.frame(
  GJR_vols = annualized_GJR_vol,
  vix = raw_data$vix[-1],
  date = raw_data$date[-1]
)
ggplot(join_vols, aes(x=date)) +
  ylab('Volatility')+
  geom_line(aes(y=join_vols$vix,color='VIX'),alpha=1)+
  geom_line(aes(y= join_vols$GJR_vols,col='GJR'),alpha=1)+
  scale_color_manual(name = "Variables",
                     values = c("VIX" = "red", "GJR" = "blue"))
    theme(legend.position=c(.1,.85))
ggplot2::ggsave("volatilities.jpg")
```



First, the bias is due to a risk premium. VIX comes from options that are forward looking, and thus the volatility expectation for the future is included and there fore VIX is higher than GJR-GARCH volatilities. Also, the option traders has wider information and therefore has an expectation of future compared to other traders. Second, the bias is also due to traders has to pay a premium to buy volatility. And therefore a positive bias is persistent for VIX over fitted volatilities.

b) Plot the histogram of the bias. What is the interpretation of the mean?

```
bias <- (join_vols$vix-join_vols$GJR_vols)/100
ggplot(as.data.frame(bias),aes(x=bias))+
  geom_vline(aes(xintercept=mean(bias)),colour =
"red",linetype="dashed")+geom_histogram(binwidth = 0.00125,colour = "blue")+
  geom_density(alpha=0.05, fill="#FF6666") +
  labs(
  title = "Distribution of Bias"
)
ggplot2::ggsave("Distribution_of_bias.jpg")
```



The mean of the bias is postive. Which means there is a persistent positive bias. VIX is higher than the GJR-GARCH fitted volatilities most of the time.

c) Find the correlation between the log changes in VIX and the log changes in SP500.

```
log_changes_vix <- diff(log(raw_data$vix))
log_changes_spy <- diff(log(raw_data$spy))
cor(log_changes_vix,log_changes_spy)
```

```
-0.731414
```

The correlation between the log changes in VIX and the log changes in SP500 is -0.731414.

d) Describe whether this is consistent with an Asymmetric Volatility model.

The negative correlation between log changes in VIX and log changes in S&P 500 is consistent with an asymmetric volatility model. This is because during an uptrend of stock, stock returns are positive, however the volatility usually small during uptrend. When stock price decrease, stock return decrease and the negative correlation suggests that volatility increase.Therefore, negative correlation is consistent with asymmetric volatility.

e) Describe whether this correlation violates the Efficient Market Hypothesis.

This correlation violates the efficient market hypothesis. EMH suggests that generating profit from market timing is impossible. However, a correlation between vix changes and stock returns suggests that timing strategy could potentially generate profit, because VIX has a forward looking bias. The expectation of future volatility has a negative correlation with stock returns suggests that some timing strategy could exist.

f) Regress SPY returns on lagged VIX squared. Assume the errors are GJR-GARCH. What is the interpretation of this regression?

```
vix_squared <- (raw_data$vix[-1])**2
regress <- lm(spy_returns$spy_return ~ vix_squared)
summary(regress)
```

```
Call:
lm(formula = spy_returns$spy_return ~ vix_squared)

Residuals:
      Min        1Q    Median        3Q       Max
-0.089249 -0.005003 -0.000116  0.005107  0.144001

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.756e-03  2.219e-04   7.912 3.08e-15 ***
vix_squared -3.366e-06  3.257e-07 -10.334  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.01181 on 5040 degrees of freedom
Multiple R-squared:  0.02075, Adjusted R-squared:  0.02056
F-statistic: 106.8 on 1 and 5040 DF,  p-value: < 2.2e-16
```

Because that the lagged VIX squared is significant, we could conclude that the vix index of the previous trading day has explanation power for today's stock return. The coefficient is negative, thus the previous trading day's vix has a negative effect on today's stock return.

6. If you are long a 1 month variance swap on a market index,

a) What will happen to your position on a day of large absolute returns?

By long a 1 month variance swap, I paid a fix amount and receive a float amount based on variance. If a have a large absolute returns (which means larger than usual returns), the underlying variance of my variance swap increased and my position would increase if the swap is mark to market.

b) How will index option implied volatility change following a large positive return or a large negative return?

Both a large positive return or a large negative return would result in an increase in implied volatility.

c) Do you think variance swaps will respond differently to positive and negative returns? Why?

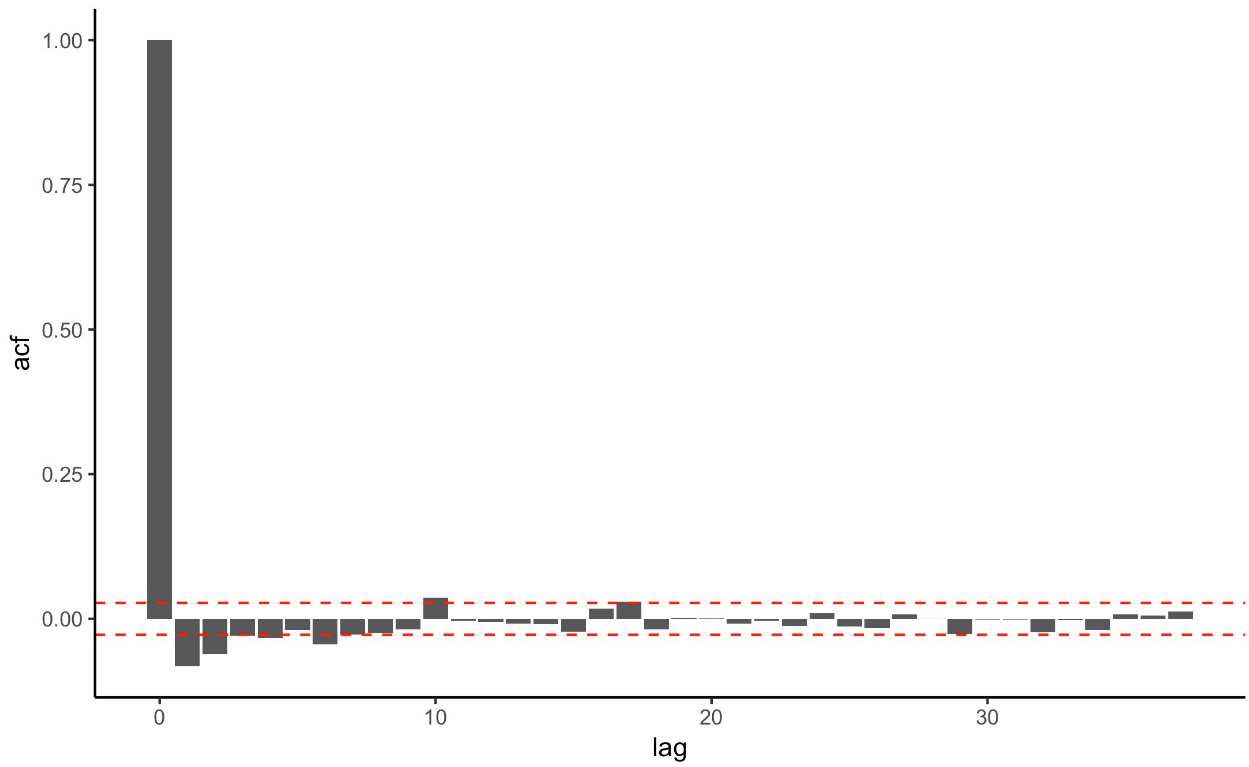
Variance swaps will respond differently to positive and negative returns. Because of asymmetric volatilities. Negative returns usually sharper and thus has larger variance. Therefore the variance swap would respond differently.

7. Compute the log change in the VIX.

a) Does this show autocorrelation?

First, let's plot the acf of the log changes of vix.

```
significance_level <- qnorm((1 + 0.95)/2)/sqrt(sum(!is.na(log_changes_vix)))
vix_acf <- acf(log_changes_vix,plot = F)
vix_acf <- with(vix_acf,data.frame(lag,acf))
ggplot(data=vix_acf, mapping=aes(x=lag, y=acf))+
       geom_bar(stat = "identity", position = "identity")+
      geom_hline(yintercept = significance_level,colour = "red",linetype="dashed")+
  geom_hline(yintercept = -significance_level,colour = "red",linetype="dashed")
ggplot2::ggsave("ACF_Of_VIX_log_changes.jpg")
```



There is some significance (5%) autocorrelation in the first few lags.

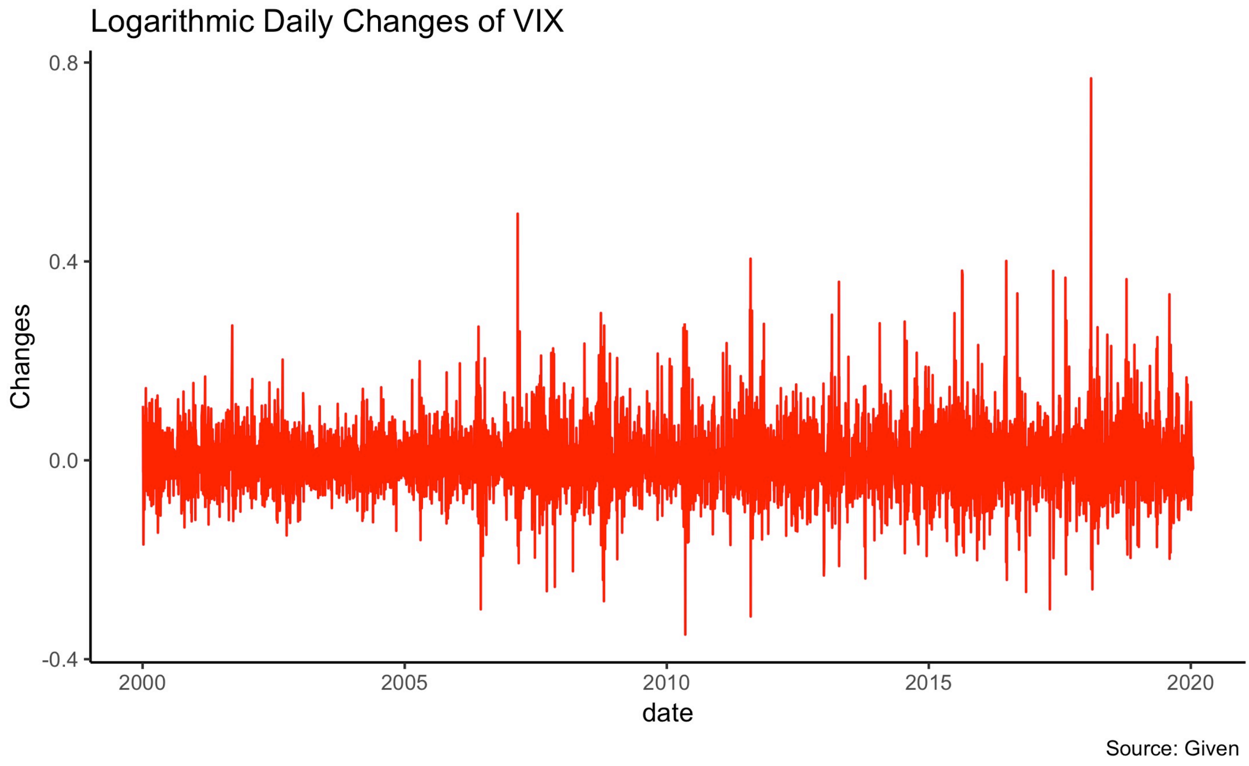
b) Does this have volatility clustering?

It has volatility clustering. Lag 1, 2 are significantly autocorrelated at 5% significance level, which indicates volatility clustering.

c) Build a model for the volatility of the vix that incorporates both these features. This is a model of the volatility of volatility.

First, plot the returns(log difference) of vix.

```
vix_changes <- data.frame(
  vix_returns = log_changes_vix,
  date = raw_data$date[-1]
)
ggplot(vix_changes, aes(x=date)) +
  geom_line(aes(y=vix_returns),group = 1,colour = "red") +
  labs(title="Logarithmic Daily Changes of VIX",
       caption="Source: Given ",
       y="Changes")
ggplot2::ggsave("VIX_Daily_Changes.jpg")
```



Fit a GARCH(1,1) model because there are autocorrelations.

```r
garch <- ugarchspec(
  variance.model=list(model="sGARCH", garchOrder=c(1,1)),
  mean.model=list(armaOrder = c(0,0),include.mean=F),
  distribution.model="norm")
fit <- ugarchfit(spec = garch,data = log_changes_vix)
infocriteria(fit)
```

```
Akaike       -2.640011
Bayes        -2.636128
Shibata      -2.640012
Hannan-Quinn -2.638651
```

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : sGARCH(1,1)
Mean Model  : ARFIMA(0,0,0)
Distribution   : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
omega   0.000428    0.000063   6.7753        0
alpha1  0.133013    0.012839  10.3599        0
beta1   0.779189    0.022226  35.0580        0

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
omega   0.000428    0.000124   3.4641 0.000532
alpha1  0.133013    0.025679   5.1799 0.000000
beta1   0.779189    0.043090  18.0830 0.000000

LogLikelihood : 6658.467
```

Plot the residuals.

```
standarized_residuals <- (fit@fit$residuals-
mean(fit@fit$residuals))/sqrt(var(fit@fit$residuals))
Residuals <-data.frame(
  residuals = standarized_residuals,
  Date = raw_data$date[-1]
)
ggplot(Residuals,aes(x=residuals))+
  geom_vline(aes(xintercept=mean(residuals)),colour =
"blue",linetype="dashed")+geom_histogram(binwidth = 0.0125,colour = "brown2")+
  geom_density(alpha=0.5, fill="#FF6666") +
  labs(
  title = "Distribution of Standarized Residuals"
)
ggplot2::ggsave("DistributionOfStandarizedResiduals.jpg")
```