Assignment 5

1. The following method of moment will help calibrating the asset value approach to default.

Let D_t denote the number of obligors that defaulted in period t, and N_t the number of obligors that belonged to the group at the start of period t. We will assume that one period corresponds to one year. Data is observed over T years. Assume that all obligors have the same default probability, i.e., we set $p_i = p_j = p$; our default threshold is then $c_i = c_j = d = \Phi^{-1}(p)$. Use data spd.csv for the following questions.

a) Calculate the average default probability by

$$\hat{p} = \frac{1}{T}\sum_{t=1}^{T}\frac{D_t}{N_t}$$

b) Consider obligor i's asset value A_i as a one factor model:

$$A_i = w_i Z + \sqrt{1 - w_i^2}\varepsilon_i, \quad \text{cov}(\varepsilon_i, \varepsilon_j) = 0, \quad i \neq j; \quad \text{cov}(Z, \varepsilon_i) = 0, \forall_i$$

Find the correlation ρ_{ij}^{asset}.

c) Under the setting that obligors have the same default probability, what happen to the previous correlation?

d) The distribution for joint defaults for year t will be estimated as follows:

$$\hat{p}_{2t} = \frac{\frac{D_t(D_t-1)}{2}}{\frac{N_t(N_t-1)}{2}} = \frac{D_t(D_t-1)}{N_t(N_t-1)}$$

Calculate the average probability for joint defaults, $\hat{p}_2$, over T years.

e) Furthermore, assume the joint distribution of asset values of obligor i and obligor j follow bivariate normal distribution. What is the correlation of the bivariate normal?

f) Let Φ_2 denote the cumulative distribution function of bivariate normal distribution, set

$$p_{ij} = \Phi_2(d_i, d_j, \rho_{ij}^{asset})$$

Use "uniroot" function in R to find out ρ_{ij}^{asset}

2. Predict LGD with data file lgd.csv
   a) Create a variable $LGD\_A_{j,\,t\text{-}1}$ for the average default rate of the same instrument type with the following info. (Show the last 5 values of this variable.)

Mean LGD for data until

	2006	2007	2008
Sr. Sec.	0.457	0.482	0.365
Sr. Unsec.	0.626	0.636	0.538
Sr. Sub.	0.672	0.681	0.703
Sub.	0.685	0.711	0.712

   b) Create a variable $I\_DEF_{i,\,t\text{-}1}$ as the average default rates of the corresponding industry for each obligor based on the following table: (Show the first 5 values of this variable)

Industry default rates (in %)

	2006	2007	2008
Cap Ind	1.285	0.715	3.071
Cons G	0.967	0.651	3.783
Energy	0	0	1.835
Media	1.415	0.92	4.147
Retail	1.183	1.802	2.247
Tech	0.743	0.486	1.164
Transp	2.353	0	2.963

   c) Estimate the LGD with LEV, LGD_A, and I_DEF in a regression model
   d) Predict the LGD for the following obligor:

Year	ID	Industry	Type	LGD	LEV
2009	446	Cons G	Sr. Sec.	0.748255543857217	0.607452818682007

   e) Use Beta(a, b) to fit the empirical distribution of LGD. Find a and b.
   f) Transform the LGD into a normal variable and conduct the regression analysis with LEV, LGD_A, and I_DEF again.
   g) Estimate the LGD for the obligor in d) again