Lecture 09

Example: the product space $\mathbb{R}^\omega$ with box topo or uniform topo is not connected

$\mathbb{R}^\omega = \{\text{all } \underbrace{\text{bounded sequence}}_{B}\} \cup \{\text{all unbounded} \dots\}$

let $(r_n)_{n\in\mathbb{N}} \in \mathbb{R}^\omega$ is bounded sequence

then $\prod_{n\in\mathbb{N}} (r_n - 1, r_n + 1)$ a neighbour of (r_n). in the box topo which consists of bounded seqhence

Similarly. unbounded $\mathbb{R}^\omega - B$ is open

in uniform topo. choose $B_{\bar{\rho}}((r_n), 1)$ as a neighbour of (r_n)

then similarly.

Def: An ordered set X is a linear continuum if:

- X has the least upper bound property
  which means any non-empty bounded subset of X has a least upper bound $\in X$
- If $x < y$. $\exists\, z$ s.t. $x < z < y$. $(\forall x, y \in X)$

Example: $\mathbb{R}$ is linear continuum.

$\mathbb{Q}$ is not ....

$\mathbb{Z}$ is not.

Theorem 24.1 if X is a linear continuum in the order topology. then X is connected and so are intervals and rays Y in X (may be $Y = X$)

proof: let A, B be disjoint nonempty open subsets of Y.

we prove that $A \cup B \subsetneq Y$ $\Rightarrow$ Y has no separation

choose $a \in A$. $b \in B$ and $a < b$

$\Rightarrow$ $[a, b] \subset Y$

let $A_0 = A \cap [a, b]$ $B_0 = B \cap [a, b]$

let $c = \sup(A_0) \in [a, b]$

case 1: suppose that $c \in B_0$

so $a < c \le b$

since B is open in Y B_0 is open in $[a, b]$

so $\exists$ $d < c$. $(d, c] \subset B_0$

$\Rightarrow$ d is a smaller upper bound of A_0

contradiction

Case 2: suppose that $c \in A_0$

A_0 is open

$\Rightarrow \exists$ $d > c$. $[c, d) \subset A_0$

so $\exists e$ s.t. $c < e < d$

$\Rightarrow$ c is not the upper bound. contradiction

$\Rightarrow$ Y has no separation

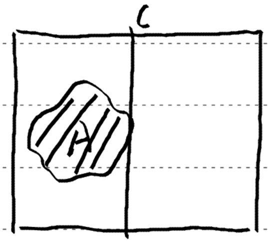
Corollary 24.2 $\mathbb{R}$ is connected and also each interval and ray in $\mathbb{R}$

Remark: The ordered sequence $I_o^2 = I \times I$ (with the dictionary topo) is linear continuum.

let A be any non-empty bounded subset of I_o^2

Consider $\pi_1(A)$ projection on to the first coordinate

let $c = \sup(\pi_1(A))$ in I



if $B = \{ x \in I_o^2 : \pi_1(x) = c \}$ is empty

then $c \times 0$ is the sup of A

if $B \neq \phi$ let $d = \sup(\pi_2(B))$

the $c \times b$ is the sup of A

the second condition is trivial

Theorem 24.3 (Intermediate value theorem)

Let $f: X \to Y$ be continuous. where X is connected and Y is ordered with the order topo.

if $f(a) < r < f(b)$. Then $\exists$ $c \in X$. $f(c) = r$

proof: suppose r is not a value ($r \notin f(X)$)

Then $X = f^{-1}((-\infty, r)) \cup f^{-1}((r, +\infty))$

$a \in f^{-1}((-\infty, r))$. $b \in f^{-1}((r, +\infty))$. not empty.

is a seperation of X. contradiction

Def: A path in a space X is a continuous map $f: [a,b] \to X$ for given pair of points x, y in X. s.t. $f(a) = x$ $f(b) = y$

if $\forall\, x, y \in X$ $\exists$ a path f then X is path connected.

Proposition X path-connected $\Rightarrow$ X connected

proof: Suppose $X = A \cup B$ I: interval

Choose $a \in A$, $b \in B$. let $f: I \to X$.

then $[0,1] = f^{-1}(A) \cup f^{-1}(B)$

$0 \in f^{-1}(A)$, $1 \in f^{-1}(B)$ not empty a seperation

contradiction since $[0,1]$ is connected

find a space which is connected but not path-connected

$\mathbb{R}^\omega$ with product topo is path-connected.

$\mathbb{R}^\omega$ with box topo is even not connected

Example. I_o^2 is connected but not path-connected.

"connected" is already proved.

suppose $f: I \to I_o^2$ from 0×0 to 1×1

I connected. then by Intermediate theorem:

$\Rightarrow f$ is surjective

let $x \in I$. $x \times (0,1)$ is open in I_o^2

$U_x = f^{-1}(x \times (0,1))$ is open in I and non-empty

$\forall x$. $x \to U_x$. if $x \neq y$. $U_x \cap U_y = \phi$

since $(x \times (0,1)) \cap (y \times (0,1)) = \phi$

U_x is open. $\exists$ $q_x \in \mathbb{Q}$ and $q_x \in U_x$

so $x \to q_x$ is in injective

I is uncountable. but $\{q_x \in \mathbb{Q},\ q_x \in I\}$ is countable

contradiction

Lecture 10

Example: topologist's sine curve

$S = \{ x \times \sin(\frac{1}{x}) \mid 0 < x \le 1 \}$

S is the image of the connected set $(0,1]$ under a continuous map. S is connected.

then its closure $\bar{S}$ is also connected in $\mathbb{R}^2$.

but it is not path-connected.

$\bar{S} = S \cup (0 \times [-1,1])$

suppose $\exists$ a path from $f(0) \in (0 \times [-1,1])$ to $f(1) \in S$

we can assume that $f(t) \in S$ for $t > 0$

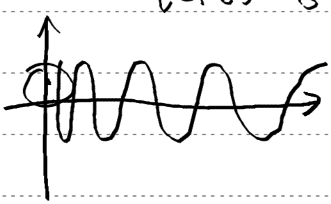
$f(0) = p \in (0 \times [-1,1])$

f is continous in 0 $\quad U = B(p, \varepsilon) \cap \bar{S}$

then $\exists$ a neighbour $[0, \delta)$ of 0 in I.

s.t. $f([0,\delta)) \subset U$

$[0,\delta)$ is connected.



Clearly $f([0,\delta))$ is not connected.

Contradiction

Another proof: $f(t) = (x(t), y(t)) \Rightarrow x(0) = 0$

$t > 0 \quad y(t) = \sin(\frac{1}{x(t)})$

there is sequence $t_n \to 0$. s.t. $y(t_n) = (-1)^n$. not converges.

Given n. choose u with $0 < u < x(\frac{1}{n})$ s.t. $\sin(\frac{1}{u}) = (-1)^n$.

then use intermediate theorem find $0 < t_n < \frac{1}{n}$ s.t. $x(t_n) = u$ ...

Def: A space X is locally connected in a point $x \in X$ if for every neighbour U of x, there is a connected neighbour V of X contained in U.

Example: $S = \{(x, \sin(\frac{1}{x})) \mid 0 < x \leq 1\}$

$\bar{S}$ is connected

but not locally connected in all points of $(0 \times [-1, 1])$

Def: X: topo space: a component of X is a maximal connected subset of X.

a path-component of X is ......

fact: 2 components are either equal or disjoint.

2 path-componets are ......

another definition:

Def: an equivalence relation on X by setting $x \sim y$ if there is a connected subset of X containing x and y.

The equivalence class is called component.

path connected ...... similar definition

Example $\bar{S}$ has 2 path-components S and $(0 \times [-1, 1])$

I_o^2 The path components are $x \times [0, 1]$

an open covering of a space X is a collection of open subsets of X whose union is X

Def: A space X is compact if every open covering of X contains a finite sub-collection which cover X. (a finite sub-covering)

Lemma 26.1 let Y be a subspace of X. Then Y is compact if and only if every open covering of Y in X (that is. by open sets in X whose union contains Y) contains a finite subcollection covering Y.

proof: "$\Rightarrow$" Y is compact.

let $\mathcal{U} = \{ U_\alpha , \alpha \in J \}$ be an open covering of Y in X.

(U_α open in X. $Y \subset \bigcup_{\alpha \in J} U_\alpha$).

$\Rightarrow \mathcal{U}' = \{ U_\alpha \cap Y . \alpha \in J \}$ open covering of Y

Y compact. $\Rightarrow \exists$ finite subcovering

$U_{\alpha_1} \cap Y. \ U_{\alpha_2} \cap Y \cdots U_{\alpha_n} \cap Y$ cover Y.

finite many $U_{\alpha_1}. U_{\alpha_2} \cdots U_{\alpha_n}$ cover Y

"$\Leftarrow$" let $\mathcal{U}' = \{ U'_\alpha . \alpha \in J \}$ be an open covering of Y.

so U_α' is open in Y $\Rightarrow U_\alpha' = U_\alpha \cap Y$. U_α open in X.

$\mathcal{U} = \{ U_\alpha . \alpha \in J \}$ is an open covering of Y in X.

$\Rightarrow \mathcal{U}$ has a finite subcovering $U_{\alpha_1} \cdots U_{\alpha_n}$.

$\Rightarrow U'_{\alpha_1} \cdots U'_{\alpha_n} = U_{\alpha_1} \cap Y \cdots U_{\alpha_n} \cap Y$ is a finite subcovering of Y.

Example: $\{\frac{1}{n} : n \in \mathbb{N}\} \subset \mathbb{R}$ is not compact.

$\{\frac{1}{n} : n \in \mathbb{N}\} \cup \{0\}$ is compact.

(a sequence and its limits) is compact

$x_n \to x$. $Y = \{x\} \cup \{x_n : n \in \mathbb{N}\}$. is compact

let $\mathcal{U} = \{U_\alpha : \alpha \in J\}$ be an open covering of Y in X.

let $x \in U_{\alpha_0}$ $\alpha_0 \in J$. U_{α_0} neighbour of x.

$x_n \to x$. $\exists N \in \mathbb{N}$. $x_n \in U_{\alpha_0}$ for $n > N$.

let $x \in U_{\alpha_n}$ for $n < N$.

$\Rightarrow$ $U_{\alpha_0} . U_{\alpha_1} \cdots U_{\alpha_N}$ finite subcovering

Example: let X be a set with finite complement topo

(the closed subsets of X are the finite subsets and X)

proof: let $\mathcal{U} = \{U_\alpha , \alpha \in J\}$. be an open covering of Y in X. We assume $Y \neq \phi$.

let $y_0 \in Y$ fixed. let $y_0 \in U_{\alpha_0}$. $X - U_{\alpha_0}$ is finite.

so $Y \cap (X - U_0) = \{y_1, \cdots y_n\}$ is finite

let $y_n \in U_{\alpha_n}$. for $1 \leq n \leq N$.

$\Rightarrow$ $U_{\alpha_0} . U_{\alpha_1} \cdots U_{\alpha_N}$ is a finite subcovering

Theorem. the closed subspace of a compact space is compact.

proof: let $Y \subset X$. X is compact. Y is closed.

let $\mathcal{U}$ be an open covering of Y in X. then $X - Y$ open

Then $\mathcal{U} \cup \{X-Y\}$ is an open covering of X

X compact $\Rightarrow$ there is a finite subcovering

$\{U_1 \cdots U_n, X-Y\}$ of X.

$\Rightarrow U_1 \cdots U_n$ is a subcovering of $\mathcal{U}$. covers Y.

Theorem: every compact subset of a Hausdorff space is closed.

proof: let $Y \subset X$. Y compact. X. Hausdorff

we prove that $X-Y$ is open

let $x_0 \in X-Y$. for each point $y \in Y$.

choose a disjoint neighb. U_y of x and V_y of y.

$\{V_y . y \in Y\}$ is an open covering of Y in X.

Y is compact. $\Rightarrow Y \subset V_{y_1} \cup \cdots \cup V_{y_n}$ finite covering.

let $U = U_{y_1} \cap \cdots U_{y_n}$. is a neighb. of x_0

$U \cap Y = \phi$. $U \subset X-Y$. is open.

$\forall x_0$. $\exists$ neighb. $U \subset X-Y \Rightarrow X-Y$ open.

Y closed.

Lecture 11.

Theorem 26.5. If $f: X \to Y$ is continuous and X is compact, then $f(X)$ is compact

(the image of a compact space under continuous map is compact)

proof: let $\mathcal{A}$ be an open covering of $f(X)$ in Y

then $f^{-1}(\mathcal{A}) = \{f^{-1}(A).\ A \in \mathcal{A}\}$ is an open covering of X.

X compact $\Rightarrow$ $f^{-1}(\mathcal{A})$ has a finite subcovering.

and hence also $\mathcal{A}$

Theorem 26.6. let $f: X \to Y$ be bijective and continuous

If X is compact and Y is Hausdorff.

then f is homeomorphism.

proof: We have to prove f^{-1} $Y \to X$ is continuous.

Since f is bijective continuous.

$\Leftrightarrow$ f is open map

$\Leftrightarrow$ f is closed map. since f is bijective.

let A be closed in X. X is compact

$\Rightarrow$ A is compact.

$\Rightarrow$ $f(A)$ is compact. by Theorem 26.5

$\Rightarrow$ $f(A)$ is closed in Y. since Y is hausdorff.

$\Rightarrow$ f is closed.

$\Leftrightarrow$ f^{-1} is continuous. f is homeomorphism.

Theorem 26.7 The product of finitely many compact space is compact.

proof: it suffices to prove the theorem for $X \times Y$ of 2 compact space

Step 1: (Tube lemma 26.8): consider $X \times Y$. with Y compact.

$\{x_0\} \times Y \cong Y$. $x_0 \in X$. $\{x_0\} \times Y \subset X \times Y$

if N is a neighbour of a slice $\{x_0\} \times Y$

Then there is a neighbour W of x_0 s.t. the tube $W \times Y \subset N$

proof of the tube lemma. we cover the slice $\{x_0\} \times Y$ by basis elements $U \times V \subset N$. U open in X. V open in Y.

$\{x_0\} \times Y \cong Y$ is compact. we can cover it by finite $U_1 \times V_1 \cdots U_n \times V_n$, every $U_i \times V_i$ intersects $\{x_0\} \times Y$.

define $W = U_1 \cap U_2 \cdots \cap U_n$ and $x_0 \in W$.

Since $\{U_i \times V_i\}$ covers $\{x_0\} \times Y$.

so $\{V_i\}$ covers Y.

so $\{U_i \times V_i\}$ covers $W \times Y$

$\Rightarrow W \times Y \subset \bigcup_i U_i \times V_i \subset N$.

Step 2: X. Y compact.

let $\mathcal{A}$ be an open covering of $X \times Y$. the slice $\{x\} \times Y \cong Y$ is compact. s.t. $\{x\} \times Y$ is covered by finitely many elements of $\mathcal{A}$: $A_1 \cdots A_n \in \mathcal{A}$

then $N_x = A_1 \cup \cdots \cup A_n$ is neighb. of $\{x\} \times Y$

hence there is a neighbour W_x of x. s.t. $W_x \times Y \subset N_x$

$\{W_x . x \in X\}$ is an open covering of X.

X is compact $\Rightarrow$ finitely many $W_{x_1} \cdots W_{x_n}$ suffice to cover X.

$X \times Y = (W_{x_1} \cup \cdots \cup W_{x_n}) \times Y$

$= (W_{x_1} \times Y) \cup \cdots \cup (W_{x_n} \times Y)$

since each $W_{x_i} \times Y$ can be covered by finitely many elements of $\mathcal{A}$. then

$X \times Y$ can be covered.

Theorem (Tychonoff) an arbitrary product of compact spaces is compact (in the product topo. not in the box topo. in general)

(吉洪诺夫定理)

(Alexander subbase theorem & Axiom of choice.)

Theorem 27.1 Let X be an ordered set with the least upper bount property.

In the order topo. each closed interval is compact.

proof: Let $a<b$ and $\mathcal{A}$ be an open covering of the closed interval $[a,b]$ (in the subspace topo. which is the same as the order topo on $[a,b]$)

let $C=\{y\in[a,b] : [a,y]$ is covered by finitely many elements of $\mathcal{A}\}$ with $a\in C, \neq\phi$

let $c=\sup(C)$

Claim: $c\in C$. ($[a,c]$ is covered by finitely many elements of $\mathcal{A}$).

Choose $A\in\mathcal{A}$ with $c\in A$.

assume $c>a$. $c\in A$. A is open $\Rightarrow$

there is $d<c$: $(d,c]\subset A$

then $[a,b]$ can be finitely covered.

then $[a,c]$ can be finitely covered since $(d,c]\subset A$

Claim $c=b$. ($\Rightarrow$ $[a,b]$ can be finitely covered)

suppose $c<b$. let $c\in A\in\mathcal{A}$. A is open.

$\Rightarrow$ there is $d>c$. $[c,d)\subset A$.

$[a,c]$ can be finitely covered.

$[a,d]$ can be finitely covered.

contradict with c is the $\sup\{C\}$.

So $c\geqslant b$ $\quad c=b$

Corollary: i) Each closed interval $[a,b]$ in $\mathbb{R}$ is compact.

2) the ordered square $I_0^2 = [0\times 0, 1\times 1]$ is compact.

Theorem 27.3 (Heine - Borel)

A subset A of $\mathbb{R}^n$ is compact if and only if A is closed and bounded in the euclidean or square metric

proof: "$\Rightarrow$" A is compact. $A \subset \bigcup_{k\in\mathbb{N}} B_d(0,k) = \mathbb{R}^n$.

$\Rightarrow$ A is compact. $A \subset \bigcup_{i\leq N} B_d(0,i)$ finite N.

$\Rightarrow$ A is bounded

A is closed as a compact subset of hausdorff space $\mathbb{R}^n$

"$\Leftarrow$" A is closed an bounded.

$\Rightarrow$ $A \subset [-N, N]^n \subset \mathbb{R}^n$

$[-N, N]$ is compact in $\mathbb{R}$.

$\Rightarrow$ $[-N, N]^n$ is compact.

A closed. A is compact in $[-N, N]^n$

Theorem 27.4 (Extrem value theorem)

Let $f: X \to Y$ be continuous where Y is ordered with the order topo.

If X is compact then $\exists$ c and d in X

s.t. $f(c) \leq f(x) \leq f(d)$ $\forall x$.

(" f has a max and a min ").

proof: X compact . f continuous

$\Rightarrow$ $A = f(X)$ is compact

suppose that f does not have a max

$\{(-\infty, a), a \in A\}$ is an open covering.

A is compact $\Rightarrow$ $A \subset (-\infty, a_0)$. $a_0 \in A$.

Contradiction.

Similar when the case min.

Example: if is metric and has a countable dense subset Y. then X has a countable basis (X is second countable)

proof: $\mathcal{B} = \{ B_d(y, \frac{1}{n}) \quad y \in Y. \ n \in \mathbb{N} \}$ is countable.

claim: $\mathcal{B}$ is a basis of X.

suppose U is open in X. and $x \in U$

then $\exists$ $B_d(x, \frac{1}{m}) \subset U$. $m \in \mathbb{N}$

let $y \in B_d(x, \frac{1}{2m})$ since Y is dense

then $x \in B_d(y, \frac{1}{2m}) \subset B_d(x, \frac{1}{m}) \subset U$. for $\forall x$.

then $\mathcal{B}$ is a basis of X.

Corollary: $\mathbb{R}_l$ is not metrizable (but first countable)

proof: $\mathbb{Q}$ is dense and accountable.

but $\mathbb{R}_l$ is not second countable.

27节还得看.

Lecture 12

Def: 1) A space X is limit point compact if every infinite subset has a limit point

2) X is sequentially compact if every sequence in in X has a converged subsequence.

Theorem 28.1 X compact $\Rightarrow$ X limit point compact.

proof: Let A be a subset of X which has no limit point.

$\bar{A} = A \cup A' = A$

$\Rightarrow$ A is closed. $\Rightarrow$ A is compact since X is compact.

for each $a \in A$. a is not a limit point of A.

$\Rightarrow$ there is a neighb. of a s.t. $U_a \cap A = a$

$\{U_a : a \in A\}$ is an open covering of A. in X.

A compact. $\Rightarrow$ $A \subset U_{a_1} \cup U_{a_2} \cdots \cup U_{a_n}$

$\Rightarrow$ $A \subset \{a_1 \cdots a_n\}$ is finite.

Contradiction

but the converse of the theorem not always true.

Theorem 28.2 Let X be metrizable. then following equivalent:

(1) X is compact.

(2) X is limit compact

(3) X is sequentially compact.

proof: (1) $\Rightarrow$ (2) has been proved.

(2) $\Rightarrow$ (3) Given sequence x_n in X

if $S = \{x_n \mid n \in \mathbb{N}\}$ finite.

then there is a constant subsequence converged. trivial.

we assume $S = \{x_n \mid n \in \mathbb{N}\}$ infinite.

X is limit point compact. $\Rightarrow$

S has a limit point $x_0 \in X$.

$x_i \in B_d(x_0, \frac{1}{i}) \cap S$ and $x_i \neq x_0$ and $x_i \neq x_j$ $i \neq j$

since $B_d(x_0, \frac{1}{i}) \cap S$ is infinite because X is metrizable

($\Rightarrow$ hausdorff)

so $\exists$ subsequence $x_i \to x_0$

(3) $\Rightarrow$ (1):

step 1 Lemma: Suppose X is metric and sequentially compact. Then for each $\varepsilon > 0$, there is a finite covering of X consisting of ε-ball.

Lemma proof suppose there is $\varepsilon > 0$ and no such finite covering of X.

let $x_1 \in X$ $B_d(x_1, \varepsilon) \subsetneq X$

let $x_2 \in X - B_d(x_1, \varepsilon)$ $x_3 \in X - B_d(x_1, \varepsilon) - B_d(x_2, \varepsilon)$

these ε-ball can't cover X finitely

x_i is a sequence: $d(x_i, x_j) \geq \varepsilon$ $\forall i \neq j$.

$\Rightarrow$ x_i has no converged subsequence

$\Rightarrow$ X is not sequentially compact. contradiction.

step 2: let $\mathcal{A}$ be an open covering of X

(3) $\Rightarrow$ there is a lebesgue number $\delta > 0$

by the lemma: there is an open covering of X by finitely many ε-ball with $\varepsilon = \frac{\delta}{3}$

each of these balls has diameter $\frac{2\delta}{3} < \delta$.

so it contained in a element of $\mathcal{A}$.

so there is a finite subcovering of $\mathcal{A}$.

X is compact.

Theorem 27.5. (Lebesgue number lemma)

Let $\mathcal{A}$ be an open covering of compact metric space X. There exists a $\delta > 0$ (a Lebesgue number associated to the open covering $\mathcal{A}$) such that every subset of X with diameter less than δ. is contained in at least $A \in \mathcal{A}$

proof: we prove that the theorem holds when X is metric and sequentially compact

Suppose that no such $\delta > 0$ exists. then for each $\delta > 0$. there is a subset A_δ of X whose diameter is less than δ. but A_δ is not contained in any $A \in \mathcal{A}$

Consider $\delta = \frac{1}{n}$. $n \in \mathbb{N}$. $A_\delta = A_{\frac{1}{n}}$

Let $x_n \in A_{\frac{1}{n}}$. there is a sequence x_n in X

X is sequentially compact.

$\Rightarrow$ the sequence x_n has a converged subsequence

$$x_{n_i} \to x_0 \text{ in } X$$

let $x_0 \in A \in \mathcal{A}$ A is open.

there is $B_d(x_0, \varepsilon) \subset A$

Choose n_i s.t. $d(x_{n_i}, x_0) < \frac{\varepsilon}{2}$

and $\frac{1}{n_i} < \frac{\varepsilon}{2}$

Let $y \in A_{\frac{1}{n_i}}$. $d(y, x_0) \leq d(y, x_{n_i}) + d(x_{n_i}, x_0) < \varepsilon$

$A_{\frac{1}{n_i}} \subset A$. Contradiction.

Def: An ordered set is well ordered if every subset has a minimum.

Let X be an ordered set and $\alpha \in X$. $S_\alpha = (-\infty, \alpha)$ is a section $\{x \in X \mid x < \alpha\}$ of α

axiom of choice $\Leftrightarrow$ Zorn's Lemma

$\Leftrightarrow$ well ordered theorem of zermalar

well ordered theorem:

Every set X can be well ordered

Lemma 1.2

Theorem: There is an uncountable well ordered set. s.t every section is countable

(" the minium well ordered uncountable set")

proof: Using the well-ordered theorem

let X be any uncountable well-ordered set.

If every section S_x $x \in X$ is countable then we are done.

otherwise $Y = \{x \in X . S_x$ is not countable$\} \neq \emptyset$.

X is well-ordered $\Rightarrow$

Y has a minium $\Omega \in Y \subset X$

then S_Ω is uncountable

$\Omega \notin S_\Omega$

but every element of S_Ω. $x<\Omega$.

$\Rightarrow$ S_x should be countable ∵

then take S_Ω

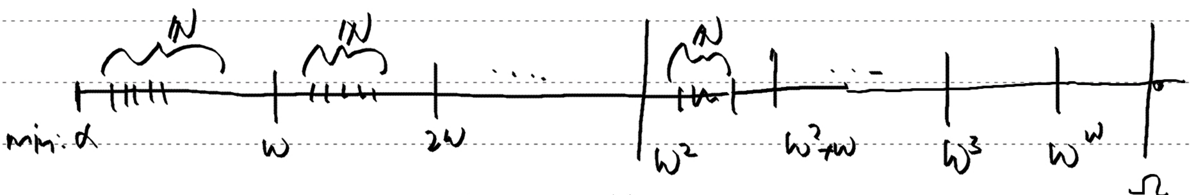
in every case. call the set of theorem S_Ω

S_Ω with the order topo

$\bar{S}_\Omega \equiv S_\Omega \cup \{\Omega\}$ Ω: the largest of $\bar{S}_\Omega$

then $\bar{S}_\Omega$ is well ordered also

in a well-ordered set. each element has an immediate successor if it is not the largest



it is still countable before Ω. S_Ω

S_Ω: uncountable.

well-ordered.

选择公理(良序定理)可以证明 S_Ω 存在.

"但似乎不能具体构造出来"