# Ball Sinking into a Viscous Fluid

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# 1 Motion of a Ball Sinking into a Viscous Fluid

一个刚体在液体中的运动由下列的运动方程描述:

$$\begin{aligned} \rho_p V_c \dot{\boldsymbol{u}}_c &= \rho_f \oint_{\partial S} \boldsymbol{\tau} \cdot \boldsymbol{n} \mathrm{d}\sigma + (\rho_p - \rho_f) V_c \boldsymbol{g}, \\ I_c \dot{\boldsymbol{\omega}}_c &= \rho_f \oint_{\partial S} \boldsymbol{r} \times (\boldsymbol{\tau} \cdot \boldsymbol{n}) \mathrm{d}\sigma, \end{aligned} \tag{1}$$

其中 $\rho_p$ 是刚体的密度,$V_c$ 是刚体的体积,$\boldsymbol{u}_c$ 是刚体的质心速度,$\rho_f$ 是流体的密度,$\boldsymbol{\tau}$ 是应力张量,$\boldsymbol{n}$ 是单位法向量,$\boldsymbol{g}$ 是重力加速度,$I_c$ 是刚体的转动惯量,$\boldsymbol{\omega}_c$ 是刚体的角速度,$\boldsymbol{r}$ 是刚体上的点到质心的矢量,$S$ 是刚体的表面. 对于不可压流体, 应力张量 $\boldsymbol{\tau}$ 可以表示为

$$\boldsymbol{\tau} = -pI + \nu(\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T)$$

# 2 Coupling of the Solid and the Fluid

固液边界的耦合条件为无滑移边界条件, 即液体和固体之间的相对速度为 0. 固体的速度可以表示为平动和转动的合成:

$$\boldsymbol{u}(\boldsymbol{x}) = \boldsymbol{u}_c + \boldsymbol{\omega}_c \times (\boldsymbol{x} - \boldsymbol{x_c}), \tag{2}$$

# 3 Processing CutCell

按照 Cell-Liniking method[2, 1] 的思路, 对于这些 CutCell, 首先需要计算出处于液体区域的体积和质心, 然后考虑在质心处的多重 Taylor 展开, 通过最小二乘法来计算展开式的系数 [3].

## 3.1 Volume and Centroid Calculation

在计算时, 为了不失一般性, 我们选择根据 CutCell 的不同情况, 选择合适的 cell 顶点来构建一个曲边三角形 [4], 并通过映射将其映射到参考单元上, 然后计算参考单元上的积分.

$$\begin{aligned} \boldsymbol{\psi} : R = [\lambda_1^e, \lambda_2^e] \times [0, 1] &\longrightarrow \Omega_e \\ \boldsymbol{\lambda} = (\lambda, \vartheta) &\longmapsto \boldsymbol{\psi}(\boldsymbol{\lambda}) := \boldsymbol{C}(\lambda)(1 - \vartheta) + \vartheta \boldsymbol{x}_3, \end{aligned} \tag{3}$$

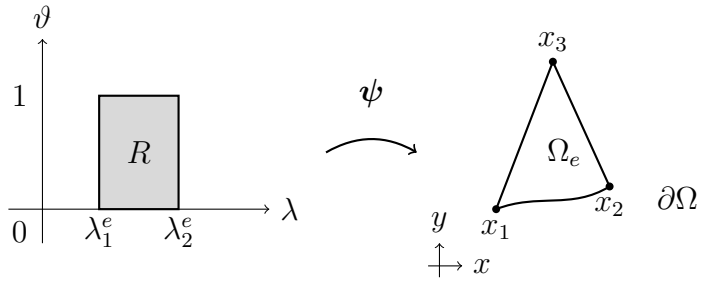


Figure 1: Mapping of the reference element $R$ to the physical element $\Omega_e$

于是, 体积和质心的计算可以表示为:

$$\begin{aligned} V_c &= \int_R |J_{\boldsymbol{\psi}}(\boldsymbol{\lambda})| \mathrm{d}\boldsymbol{\lambda} \\ \boldsymbol{x}_c &= \frac{1}{V_c} \int_R \boldsymbol{\psi}(\boldsymbol{\lambda}) |J_{\boldsymbol{\psi}}(\boldsymbol{\lambda})| \boldsymbol{\lambda} \end{aligned} \tag{4}$$

以上的计算采用 $R$ 上的 Gauss-Legendre 型积分公式.

## 3.2 Diffculties in the specific case

在大多数情况下, 边界曲线与 Cell 只会切出一个部分, 但是在某些特殊情况下, 边界曲线会切出两个部分, 如图2所示, 阴影部分表示固体区域, 在图中以虚线标记计算单元, 实线表示边界曲线, 蓝色实心点表示边界和单元的交点, 可以看到边界曲线切出了两个部分.

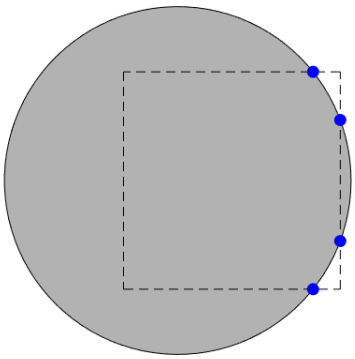

Figure 2: Example: A cell is cut by the curve twice.

# References


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