

Ball Sinking into a Viscous Fluid

Li Ruixiang

`liruixiang@zju.edu.cn`

Contents

1	Motion of a Ball Sinking into a Viscous Fluid	2
2	Coupling of the Solid and the Fluid	2
3	Processing CutCell	2
3.1	Volume and Centroid Calculation	2
3.2	Cell Division	3
3.3	Difficulties in the specific case	3

1 Motion of a Ball Sinking into a Viscous Fluid

一个刚体在液体中的运动由下列的运动方程描述：

$$\begin{aligned}\rho_p V_c \dot{\mathbf{u}}_c &= \rho_f \oint_{\partial S} \boldsymbol{\tau} \cdot \mathbf{n} d\sigma + (\rho_p - \rho_f) V_c \mathbf{g}, \\ I_c \dot{\boldsymbol{\omega}}_c &= \rho_f \oint_{\partial S} \mathbf{r} \times (\boldsymbol{\tau} \cdot \mathbf{n}) d\sigma,\end{aligned}\tag{1}$$

其中 ρ_p 是刚体的密度, V_c 是刚体的体积, \mathbf{u}_c 是刚体的质心速度, ρ_f 是流体的密度, $\boldsymbol{\tau}$ 是应力张量, \mathbf{n} 是单位法向量, \mathbf{g} 是重力加速度, I_c 是刚体的转动惯量, $\boldsymbol{\omega}_c$ 是刚体的角速度, \mathbf{r} 是刚体上的点到质心的矢量, S 是刚体的表面. 对于不可压流体, 应力张量 $\boldsymbol{\tau}$ 可以表示为

$$\boldsymbol{\tau} = -p\mathbf{I} + \nu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T)$$

2 Coupling of the Solid and the Fluid

固液边界的耦合条件为无滑移边界条件, 即液体和固体之间的相对速度为 0. 固体的速度可以表示为平动和转动的合成:

$$\mathbf{u}(\mathbf{x}) = \mathbf{u}_c + \boldsymbol{\omega}_c \times (\mathbf{x} - \mathbf{x}_c),\tag{2}$$

3 Processing CutCell

按照 Cell-Liniking method[2, 1] 的思路, 对于这些 CutCell, 首先需要计算出处于液体区域的体积和质心, 然后考虑在质心处的多重 Taylor 展开, 通过最小二乘法来计算展开式的系数 [3].

3.1 Volume and Centroid Calculation

在计算时, 为了不失一般性, 我们选择根据 CutCell 的不同情况, 选择合适的 cell 顶点来构建一个曲边三角形 [4], 并通过映射将其映射到参考单元上, 然后计算参考单元上的积分.

$$\begin{aligned}\boldsymbol{\psi} : R = [\lambda_1^e, \lambda_2^e] \times [0, 1] &\longrightarrow \Omega_e \\ \boldsymbol{\lambda} = (\lambda, \vartheta) &\longmapsto \boldsymbol{\psi}(\boldsymbol{\lambda}) := \mathbf{C}(\lambda)(1 - \vartheta) + \vartheta \mathbf{x}_3,\end{aligned}\tag{3}$$

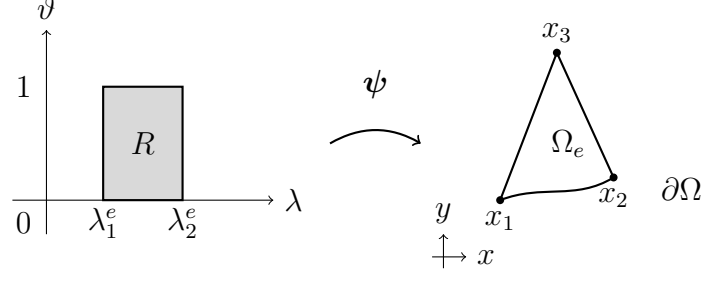


Figure 1: Mapping of the reference element R to the physical element Ω_e

于是, 体积和质心的计算可以表示为:

$$\begin{aligned} V_c &= \int_R |J_\psi(\boldsymbol{\lambda})| d\boldsymbol{\lambda} \\ \mathbf{x}_c &= \frac{1}{V_c} \int_R \psi(\boldsymbol{\lambda}) |J_\psi(\boldsymbol{\lambda})| d\boldsymbol{\lambda} \end{aligned} \quad (4)$$

以上的计算采用 R 上的 Gauss-Legendre 型积分公式.

3.2 Cell Division

根据 cell 与边界曲线的位置关系, 将其分为不同的类别.

3.3 Difficulties in the specific case

在大多数情况下, 边界曲线与 Cell 只会切出一个部分, 但是在某些特殊情况下, 边界曲线会切出两个部分, 如图3所示, 阴影部分表示固体区域, 在图中以虚线标记计算单元, 实线表示边界曲线, 蓝色实心点表示边界和单元的交点, 可以看到边界曲线切出了两个部分.

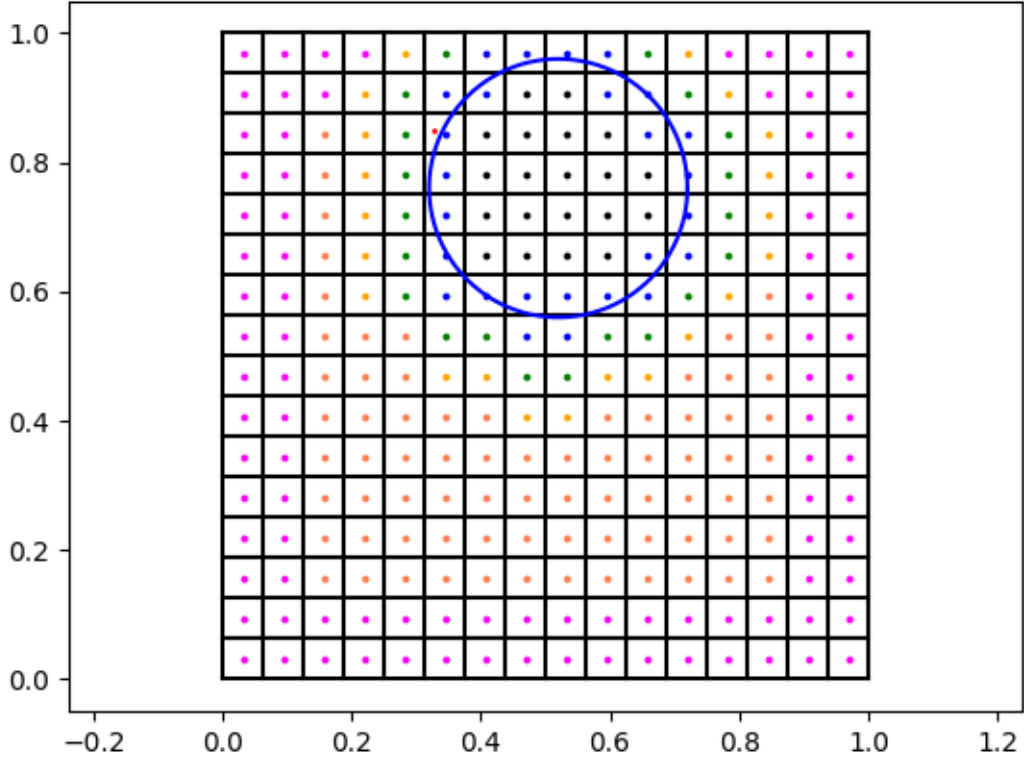


Figure 2: Cell Division

References

- [1] Marsha Berger and Andrew Giuliani. A state redistribution algorithm for finite volume schemes on cut cell meshes. *Journal of Computational Physics*, 428:109820, 2021.
- [2] Daniel Hartmann, Matthias Meinke, and Wolfgang Schröder. A strictly conservative cartesian cut-cell method for compressible viscous flows on adaptive grids. *Computer Methods in Applied Mechanics and Engineering*, 200(9):1038–1052, 2011.
- [3] Nathaniel Overton-Katz, Xinfeng Gao, Stephen Guzik, Oscar Antepara, Daniel T. Graves, and Hans Johansen. A fourth-order embedded boundary finite volume method for the unsteady stokes equations with complex geometries. *SIAM Journal on Scientific Computing*, 45(5):A2409–A2430, 2023.
- [4] Ruben Sevilla, Sonia Mendez, and Antonio Huerta. Nurbs-enhanced finite element method (nefem). *International Journal for Numerical Methods in Engineering*, 76:56–83, 10 2008.

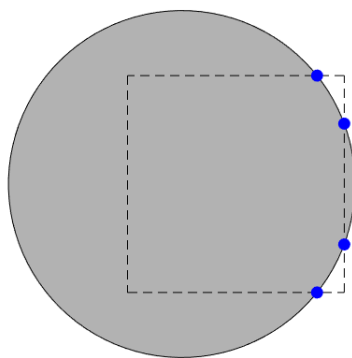


Figure 3: Example: A cell is cut by the curve twice.