# Notes on The Formal Semantics of Programming Languages

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October 16, 2024

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# 1 Chapter 5: some principles of induction

## 1.1 Excercise Answers

**Question 1.** *(E3.2) A string is a sequence of symbols. A string $a_1a_2\cdots a_n$ with $n$ positions occupied by symbols is said to have length $n$. A string can be empty in which case it is said to have length 0. Two strings $s$ and $t$ can be concatenated to form the string $st$. Use mathematical induction to show there is no string $u$ which satisfies $au = ub$ for two distinct symbols $a$ and $b$.*

*Proof.* by induction on the length of $u$

Let $A = \{n \mid \text{len}(u) = n \text{ and } au \neq ub\}$.

1. Base Case:
   - $0 \in A$: By the hypothesis, since $a \neq b$, it is clear that when $u$ is the empty string (i.e., length 0), $au \neq ub$. Thus, $0 \in A$.
   - $1 \in A$: If $u$ has length 1, it is easy to see that $au \neq ub$ because $a \neq b$.
   - $2 \in A$: For $u = xy$, where $\text{len}(u) = 2$, we have $axy \neq xyb$, given that $a \neq b$.
2. Inductive Step: Assume $n \in A$. We want to show that $n+1 \in A$ for $n \geq 2$.

   Proof by contradiction: Suppose $\text{len}(u) = n+1$ and $au = ub$. Since $n+1 \geq 3$, we can write $u = qu'p$, where $\text{len}(q) = \text{len}(p) = 1$. Thus, $au = aqu'p$ and $ub = qu'pb$, leading to the equation $aqu'p = qu'pb$.

   From this, we deduce that $a = q$. By eliminating the leftmost characters, we are left with $qu'p = u'pb$. Since $q = a$, the equation becomes $au'p = u'pb$, where $\text{len}(u'p) = n$. By the induction hypothesis, $au'p \neq u'pb$, leading to a contradiction.

   Therefore, $n+1 \in A$.

By mathematical induction, we conclude that there is no string $u$ such that $au = ub$ for two distinct symbols $a$ and $b$.

□

## 2 Ray's Notes Ssummary

**Ray's Note 1:**