MATH-UH 2010 Ordinary Differential Equations	Spring 2024
Recitation 2 Worksheet	February 02 2024

Question 1. Determine (without solving the problem) an interval in which the solution of the given initial value problem is certain to exist:

$$(\ln t)y' + 2y = \cot t,\ y(2) = 3$$

Question 2. State where in the ty-plane the hypotheses of theorem 2.4.1 are satisfied:

$$y' = \frac{4+t^2}{2y-y^2}$$

Question 3. Determine if the given differential equation is exact. If it is exact, find the solution. If it is not exact, use an appropriate integrating factor to solve it:

a. $(\tan x - \sin x \sin y)dx + \cos x \cos y dy = 0$

b. $(y^2 \cos x - 3x^2 y - 2x)dx + (2y \sin x - x^3 + \ln y)dy = 0,\ y(0) = e$

c. $6xydx + (4y + 9x^2)dy = 0$

d. $\cos xdx + (1 + \frac{2}{y}) \sin xdy = 0$

e. $xdx + (x^2y + 4y)dy = 0, y(4) = 0$

f. $\frac{dy}{dx} + \frac{2y^2+6xy-4}{3x^2+4xy+3y^2} = 0$

Question 4. Suppose that a certain population has a growth rate that varies with time and that this population satisfies the differential equation:

$$\frac{dy}{dt} = (0.3 + \sin t)\frac{y}{3}$$

a. If $y(0) = 1$, find (or estimate) the time τ at which the population has doubled. Choose other initial conditions and determine whether the doubling time τ depends on the initial population.

b. Suppose that the growth rate is replaced by its average value 1/10. Determine the doubling time τ in this case.

c. Suppose that the term $\sin t$ in the differential equation is replaced by $\sin 2\pi t$; that is, the variation in the growth rate has a substantially higher frequency. What effect does this have on the doubling time τ?

d. Plot the solutions obtained in parts a, b, and c on a single set of axes.

Question 5. Consider the differential equation:

$$ty' + 2y = \frac{1}{2}t^2,\ y(2) = 1$$

a. Draw a direction field for the given differential equation.

b. Use theorem 2.4.1 to find an interval in which the initial value problem has a unique solution.

c. Solve the differential equation

d. Then do the same when the initial condition is changed to $y(-2) = 1$