

Numerical Methods MATH-UH 3413-001 Exam

Midterm
31/10/2017

Name: _____ ID: _____

Grade: _____ /18

(office use only)

1. A very ancient binary digital computer expresses floating point numbers using three digits of mantissa and two digits of exponent. Add up $1.100 \cdot 2^{01}$ and $1.101 \cdot 2^{-01}$ as that computer would do. Has there been any loss of information in the final result?

2. The bisection method applied to the following function

$$f(x) = \frac{\sin(x)}{|x|} \log(|x|)$$

will find a zero of f starting from all but one of the following intervals.

$$[-5\pi/2, -3\pi/2], [-3\pi/2, -\pi/2], [-\pi/2, \pi/2], [\pi/2, 3\pi/2], [3\pi/2, 5\pi/2]$$

Find for which interval it will fail, and explain why.

3. Given the function

$$f(x) = \frac{x^3}{3} + 3x^2 + 8x + 1$$

which of the following values

$$-3, -2, -1, 0, 1, 2, 3$$

is not a good starting point for Newton's method, and why?

4. Find the polynomial, written in Newton's form, that passes through the following four points

$$(-1, 1), (0, 3), (1, 0), (2, -1),$$

5. Determine whether or not the following polynomials are a set of cubic splines for the three points $(0,1)$, $(1, 1)$, $(2, -1)$

$$S_1(x) = 1 + 0.5x - 0.5x^3$$

$$S_2(x) = 1 - (x - 1) - (x - 1)^2 + (x - 1)^3$$

Justify your answer.

6. Evaluate the order of error of the following approximation for the first derivative

$$f'(x) = \frac{-f(x+2h) + 4f(x+h) - 3f(x)}{2h}.$$

7. Determine the Discrete Fourier Transform of the following data set:

$$[1, 0, 1, 1]$$

8. Determine the LU decomposition of the following matrix:

$$\begin{pmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 3 & 1 & 5 \end{pmatrix}$$