Utrecht University
Faculty of Science
Department of Information and Computing Sciences

Practical Midterm Algorithms for Decision Support

- Switch off your smart phone, PDA and any other mobile device and put it far away.
- If you want to go to the bathroom, you have to hand in your mobile phone.
- You are allowed to use a regular calculator.
- This exam consists of **??** questions and a bonus question.
- Answers should be provided in English.
- All your answers should be clearly written down and provide a clear explanation. Unreadable or unclear answers may be judged as false.
- Please write down your name and student number on every exam paper that you hand in.

Question : Choices

1. About online algorithms, which one is NOT correct?
   (a) We don't know the future input and only make decisions based on the partial input we have seen so far.
   (b) To show that an online algorithm for a minimization problem has competitive ratio greater than or equal to 3, we only need to find an instance I such that the cost of the online algorithm on I is at least three times to the cost of the optimal algorithm on I.
   (c) To show that an online algorithm for a minimization problem is 3-competitive, we need to show that for any instance I, the cost of the online algorithm on I is at most three times to the cost of the optimal algorithm on I.
   (d) When we are analyzing the performance of an online algorithm, we shouldn't assume we know the optimal solution's behavior and develop our analysis based on this optimal solution.

2. Select the best description about the use of the Simplex algorithm.
   (a) The Simplex algorithm is an algorithm to solve Integer Linear Programming problems.
   (b) The Simplex algorithm is an algorithm to solve Linear Programming problems.
   (c) The Simplex algorithm is an algorithm to solve Mixed Integer Linear Programming problems.
   (d) The Simplex algorithm is an algorithm for the Shortest Paths problem.

Question : Hagrid's strategy for saving energy
(a: 6 pt., b: 4 pt., c: 10 pt., total: 20 points + 2 bonus.)

There is a special room of requirement with a light inside. The light in the room is automatically turned on whenever people walk in but needs to be turned off manually by the gamekeeper, Hagrid. If the light is on, it costs the energy 1 unit per timeslot. If the light is off, it costs nothing. Every time when the light is switched from off to on, it costs the energy for extra P units (where P is a natural number greater than 1). Hence it may not be the most efficient to turn the light off whenever people leave the room (see Figure 1 for examples).

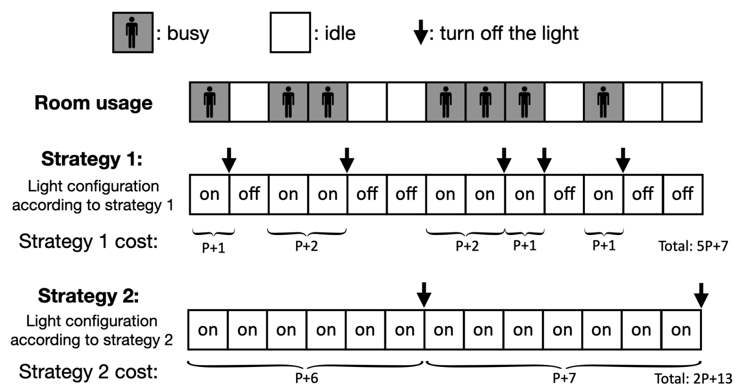

Figure 1: Two strategies

What Hagrid can do is, to check the room at the end of every timeslots and decide if he wants to turn off the light. There is no known schedule for the room, so Hagrid only knows if the room is *busy* (that is, it is used by people) or *idle* (that is, the room is not used) for the timeslots in the past. Hagrid has no idea if the room will be used in the following timeslots. He applies the following strategy:

When there are P consecutive idle timeslots, turn off the light.

Figure 2 is an illustration for a example when $P = 3$.

In this question, you can use the following parameters for your algorithm design and analysis. You can also define yours.

- n: the total number of *busy* timeslots of the room
- n_i: the total number of timeslots in the i-th period of consecutive *busy* timeslots

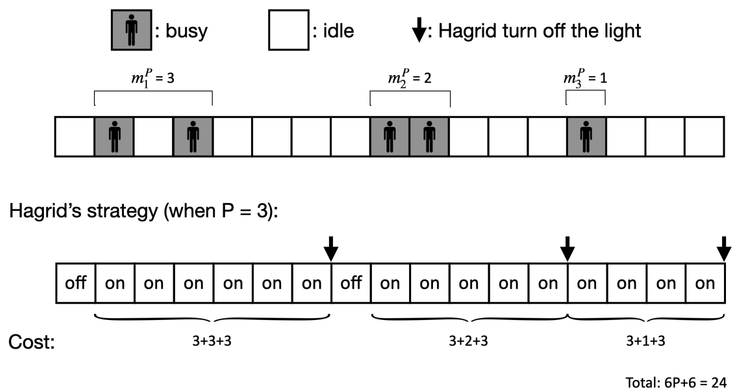

Figure 2: Hagrid's strategy when $P = 3$

- r_i: the total number of timeslots in the i-th period of consecutive *idle* timeslots
- m_i^k: if we cut the time line into fragments by the period of consecutive *idle* timeslots which has length more than k, m_i^k the total number of (*busy* or *idle*) timeslots in the i-th fragment. That is, in such a fragment, any consecutive *idle* period has length $\leq k$.

(a) **Assume the room is used for at least one timeslot, give two lower bounds of the energy cost of the optimal strategy, using different parameters mentioned above.**

(b) **Find a lower bound of the competitive ratio of Hagrid's strategy.** The higher your lower bound is, the higher grades you get.

(c) **Find an upper bound of the competitive ratio of Hagrid's strategy.** The lower your upper bound is, the higher grades you get.

(d) **If you can show that Hagrid's strategy is the best deterministic online algorithm, you get 2 bonus points.**

Question : ILP modelling: surveillance

A museum has a number of paintings that must be guarded by camera's. There are a number of possible locations for camera's; at each location, some of the paintings are guarded. In this assignment, the problem to find a smallest set of locations that guard all locations must be formulated as an integer linear program.

Given are: paintings $p_1, \ldots, p_m$, possible locations $\ell_1, \ldots, \ell_n$ for a camera, and for each possible location ℓ_i a set of paintings $P_i \subseteq \{p_1, \ldots, p_m\}$, that are guarded by a camera on location ℓ_i. The question is: find a minimum size set of locations for camera's such that each painting is guarded.

1. Model this problem as an Integer Linear Program. For each possible location ℓ_i, take a variable x_i that denotes whether there is a camera on this location or not. (10 points)
2. Write down the relaxation of your formulation. (5 points for a correct relaxation of your ILP at the previous part.)
3. The museum actually has a very corrupt director, who wants to steal one painting. Thus, instead of having an ILP that expresses that all paintings are guarded, he wants an ILP that expresses that all but one paintings are guarded. Show how to formulate this question as ILP. (I.e., we want a minimum size set of locations of camera's, such that there is exactly one painting that is not guarded. You may want to add additional variables.) Explain your solution in text.