Algorithms for Decision Support

Beyond the Worst Case

Machine-Learned Advice

Outline

Deal with Uncertainty

- Online algorithms deal with optimization under uncertainty (about future input)

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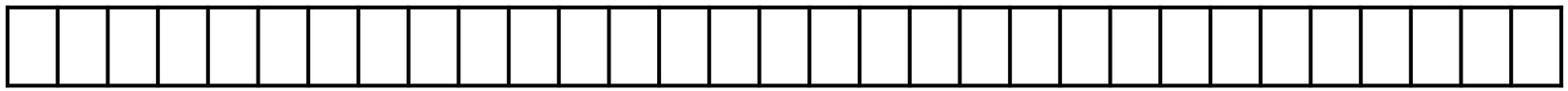
    - Example: weather forecast
    - These predictions or learned information may not be 100% correct
      - Completely trust the predictions may be a disaster
      - How can we design online algorithms with this kind of (maybe) untrusted advice?

Searching with Machine-Learned Advice

- Goal: Given an ordered sequence and a requested value, find the requested value in minimum number of steps

Searching with Machine-Learned Advice



Searching with Machine-Learned Advice

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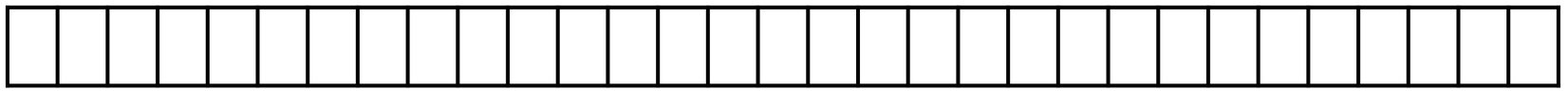
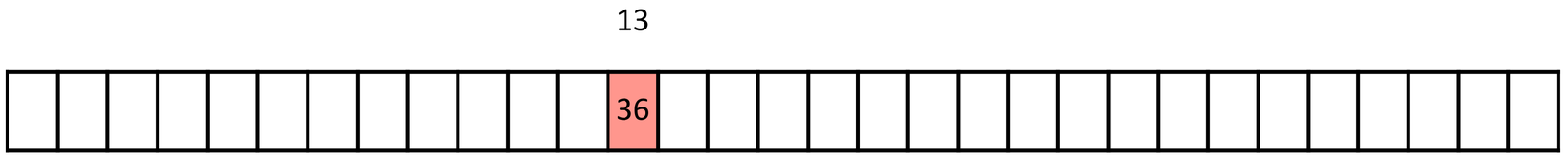


Searching with Machine-Learned Advice



Searching with Machine-Learned Advice

Cost = 1

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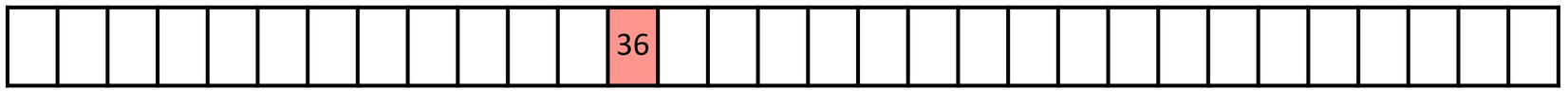

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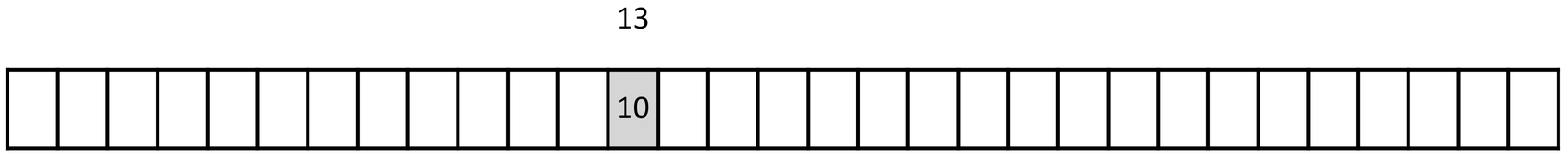

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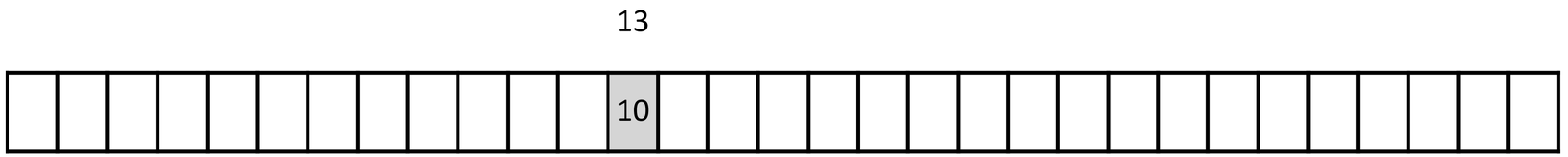

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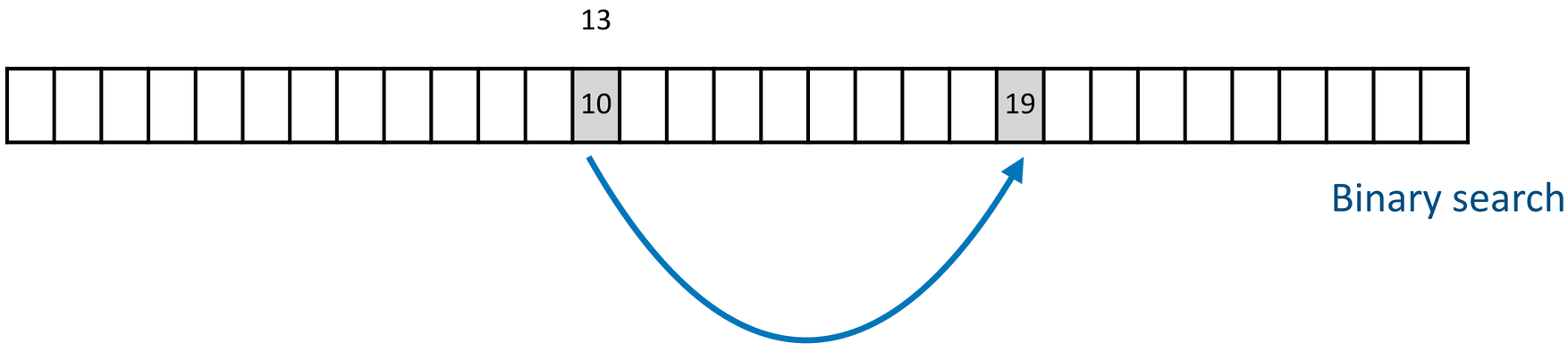

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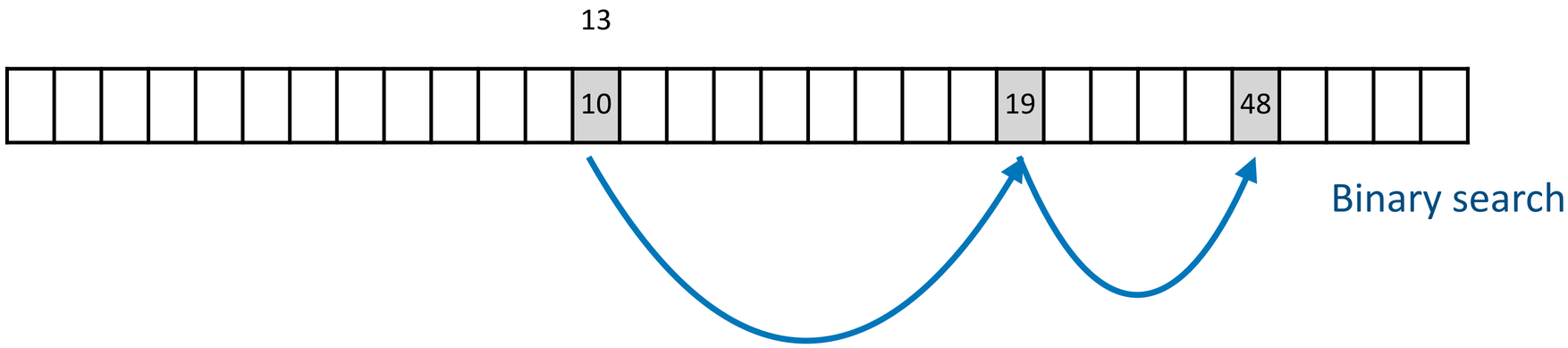

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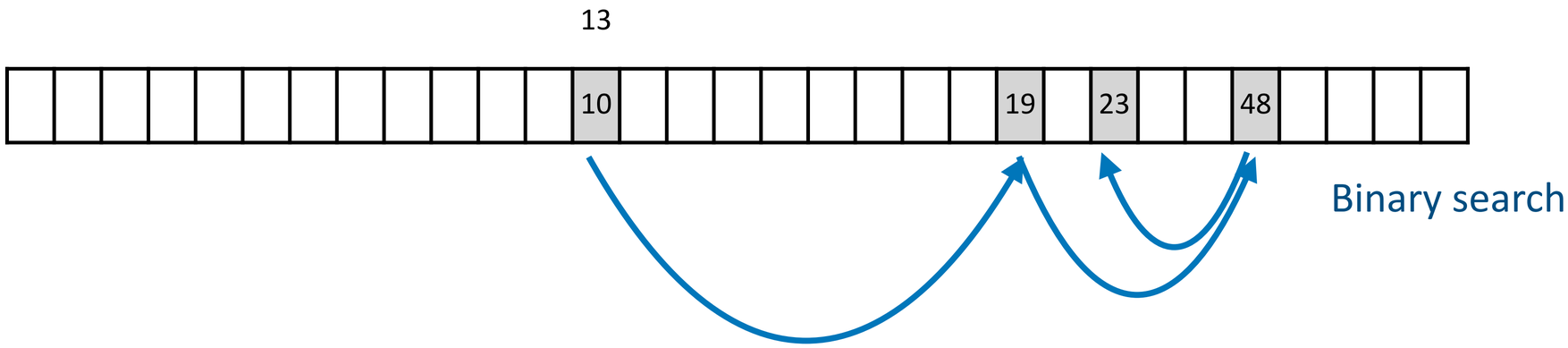

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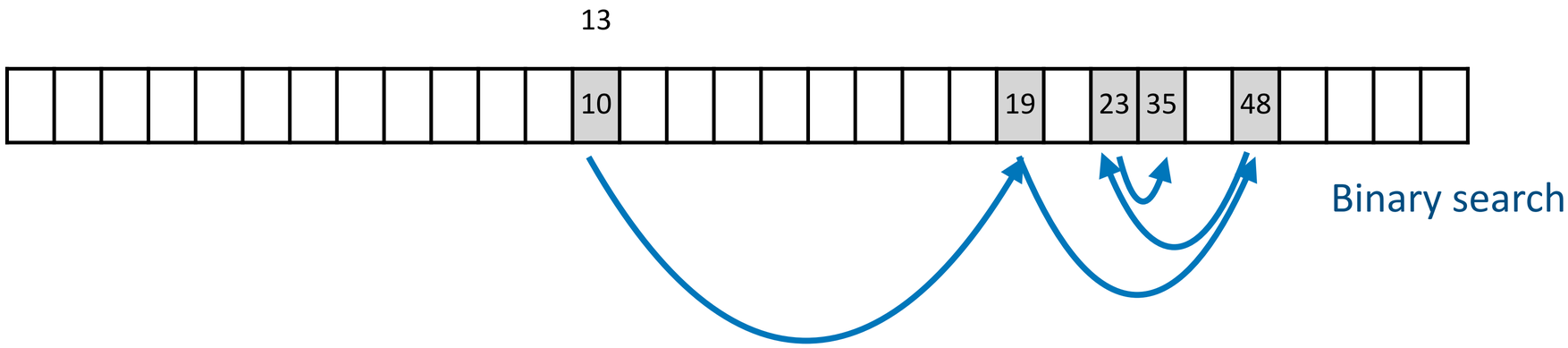

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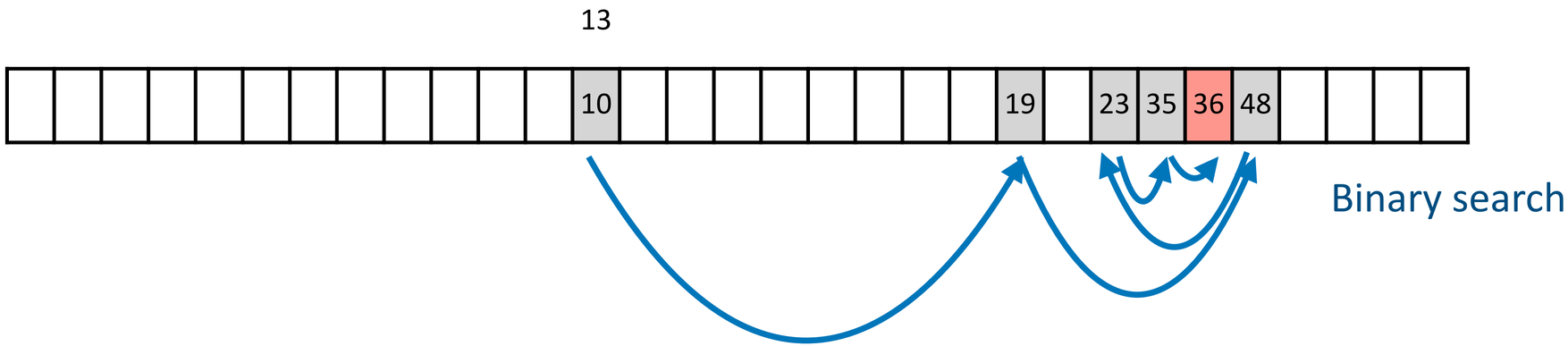
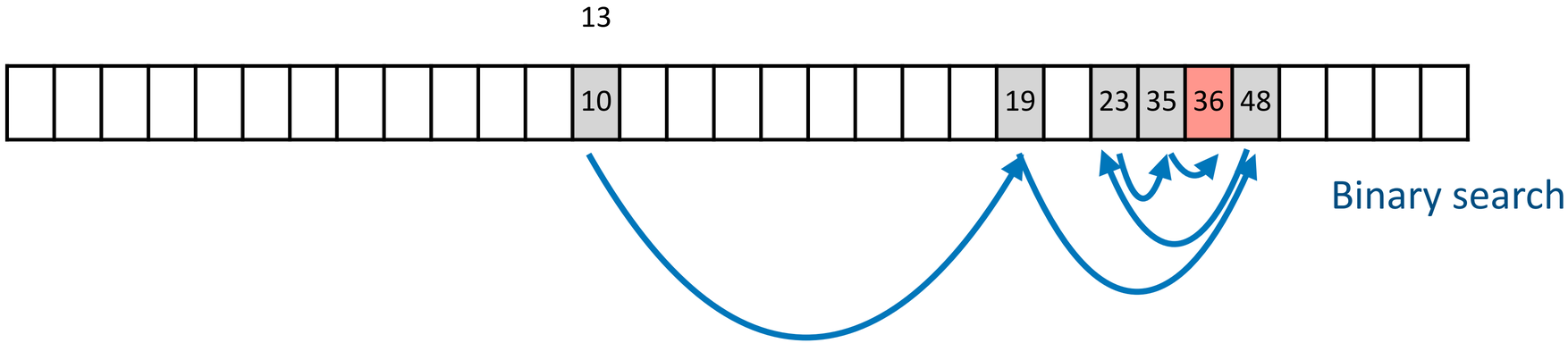


Searching with Machine-Learned Advice

Cost = $\log n$

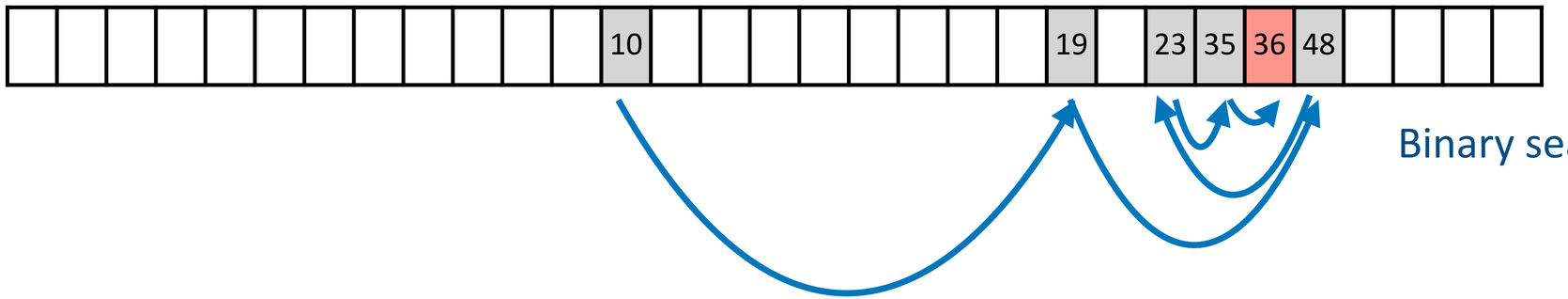


Consistency and Robustness

- We look for algorithms that
  - Are **consistent**: given more accurate predictions, the online algorithm should perform close to the optimal offline algorithm

Cost = 1

  - Are **robust**: if the prediction is wrong, the online algorithm performance should be close to the online algorithm without predictions



Cost = $\log n$

Ski Rental with Machine Learned Advice

- Imagine that you are having a ski holiday
  - The price of renting a ski is 1 per day
  - The buying price for ski is B.
  - Cost: the total money you pay
  - p: machine-learned prediction on the number of skiing days
- Suppose you want to spend money as little as possible. Should you buy the ski or rent it?

Ski Rental Algorithm 1

SKI-Rental with prediction (p)

If $p \geq B$
    Buy the ski on the first day
else ($p < B$)
    Keep **renting** for all skiing days

Ski Rental Algorithm 1

SKI-Rental with prediction (p)

```
If p ≥ B
    Buy the ski on the first day
else ( p < B )
    Keep renting for all skiing days
```

S: Actual number of skiing days

Truth: $S \geq B$ (OPT buy)	Truth: $S < B$ (OPT rent)

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If p ≥ B
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 Good prediction		
 Bad prediction		

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 Good prediction	Advice: $p \geq B$ $\frac{B}{B}$	
 Bad prediction		

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To be robust, we want an algorithm close to 2-competitive in this case

Ski Rental Algorithm 1.5

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To be robust, we want an algorithm close to 2-competitive in this case

What Happened

- Blindly trusting the prediction can cause disasters!

Trustness Parameter

- We introduce a parameter k to indicate how much the algorithm trust the advice
  - $k \in [1, B]$
    - $k = 1$: the algorithm fully trusts the advice
    - $k = B$: the algorithm does not trust the advice at all

Ski Rental Algorithm 2

SKI-Rental with prediction (p, k)

If $p \geq B$

Keep renting until the k-th day // k is our "trust parameter"

else ($p < B$)

Keep renting until the B-th day

Ski Rental Algorithm 2

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Keep renting until the B-th day

Fully trust the prediction

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	Truth: $S \geq B$ (OPT buy)	Truth: $S < B$ (OPT rent)
Good prediction	Advice: $p \geq B$ $\frac{(k-1)+B}{B}$	Advice: $p < B$
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	Truth: $S \geq B$ (OPT buy)	Truth: $S < B$ (OPT rent)
 Good prediction	Advice: $p \geq B$ $\frac{(k-1)+B}{B} = 1 + \frac{k-1}{B}$	Advice: $p < B$ $\frac{S}{S} = 1$
 Bad prediction	Advice: $p < B$ $\frac{(B-1)+B}{B} = 2 + \frac{1}{B}$	Advice: $p \geq B$ $\frac{(k-1)+B}{k-1} = 1 + \frac{B}{k-1}$

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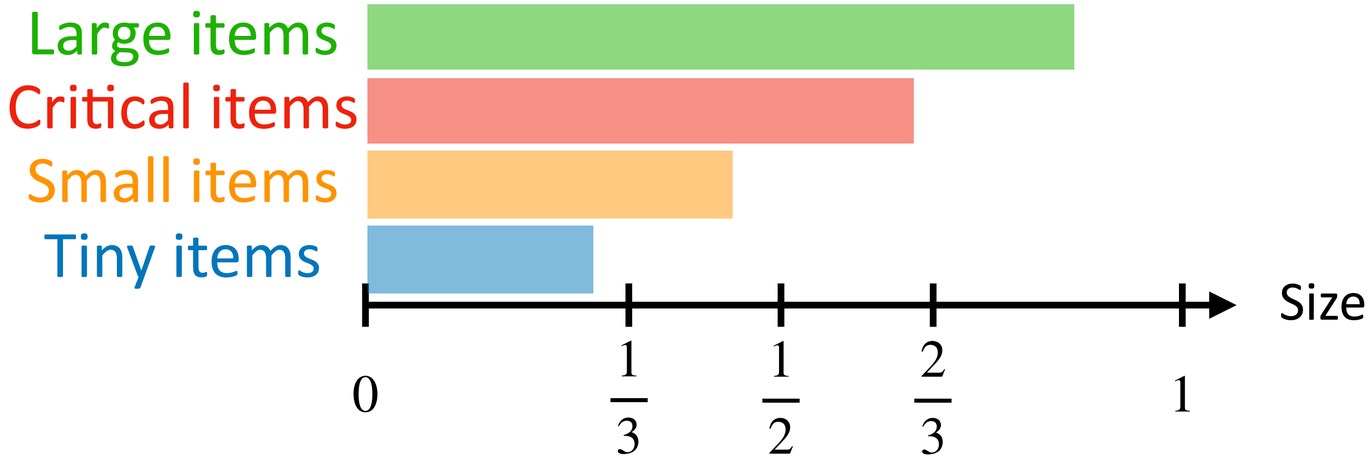
What Happened

- Use a trust parameter to *partially* trust the prediction

Bin Packing with Advice

Bin Packing with Advice



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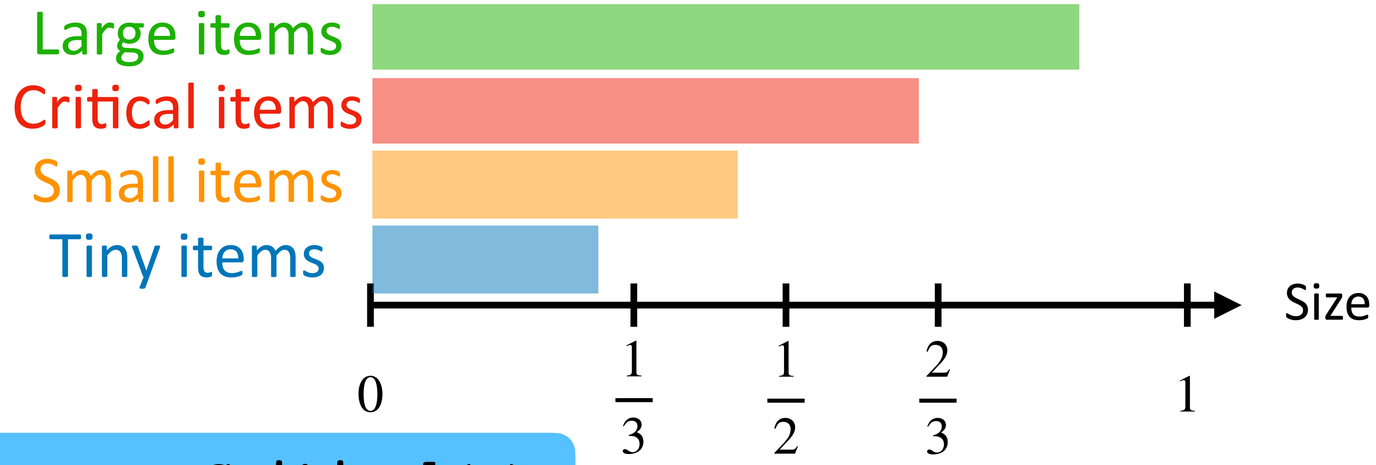
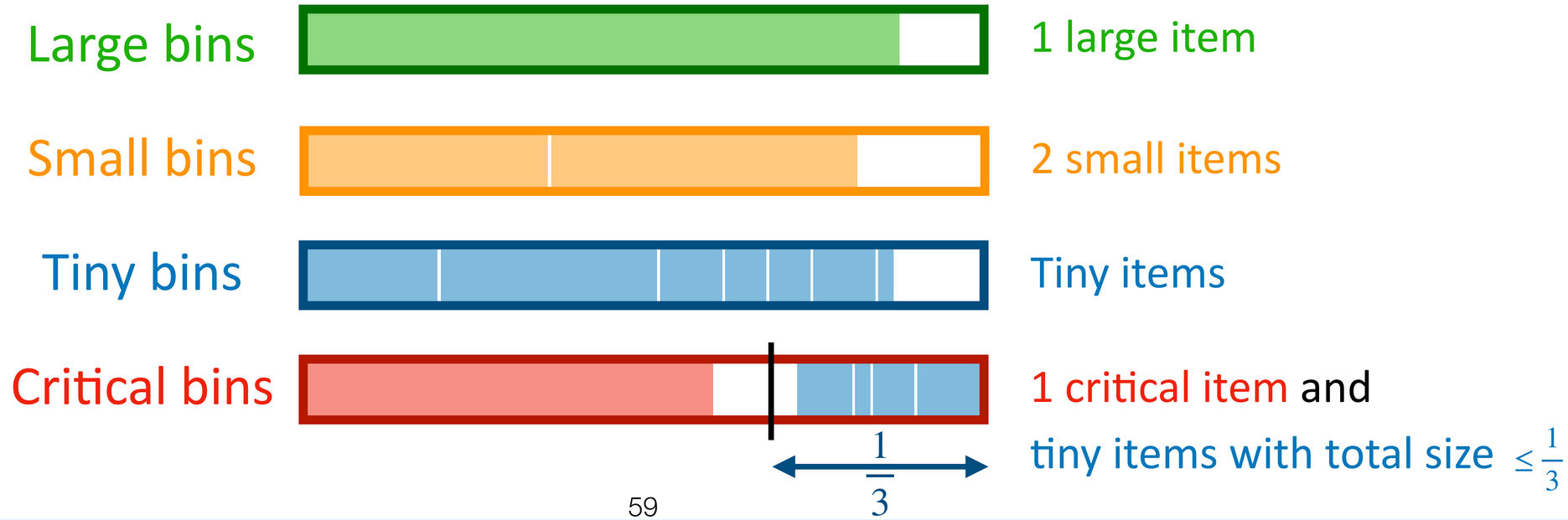


Algorithm: **`Reserve-Critical(`**c**`)`**



Reserve-Critical Algorithm

Reserve-Critical(c)

- Open c critical bins. In each critical bin, pack one critical item and tiny items with total size $\leq 1/6$
- Pack large items into large bins, small items into small bins
- When a tiny job arrives, pack it into a critical bin if possible (without making the total size of tiny items in that bin exceeds $1/6$); otherwise, pack it into a tiny bin by First-Fit

- Theorem: If the number of critical items is known, **Reserve-Critical** is 1.5 -competitive

Reserve-Critical algorithm guarantees that every non-critical bin is at least $\frac{2}{3}$-full

(may with a constant number of exceptions

Reserve-Critical Algorithm

Reserve-Critical(c)

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- Theorem: If the number of critical items is known, **Reserve-Critical** is 1.5 -competitive

Reserve-Critical algorithm guarantees that every non-critical bin is at least $\frac{2}{3}$-full

(may with a constant number of exceptions

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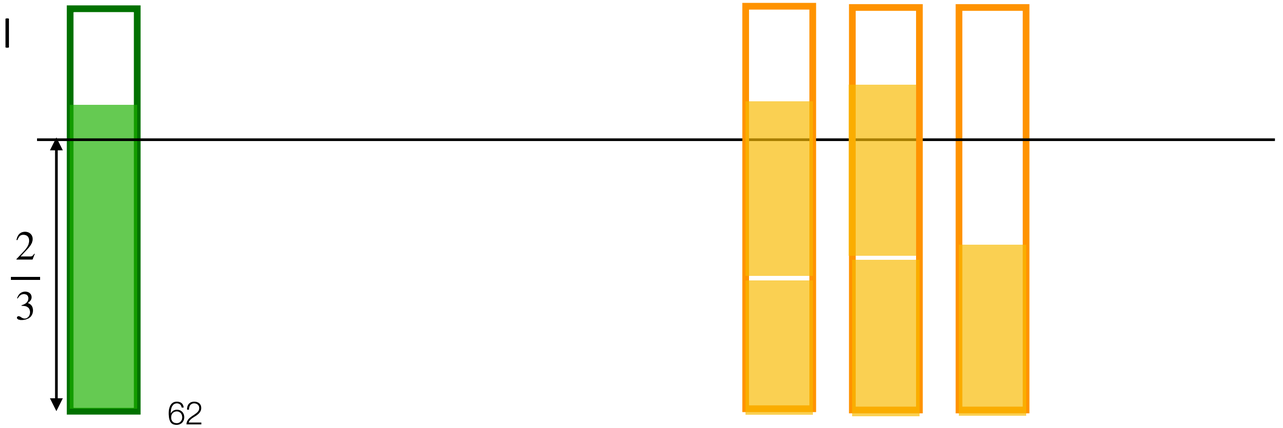

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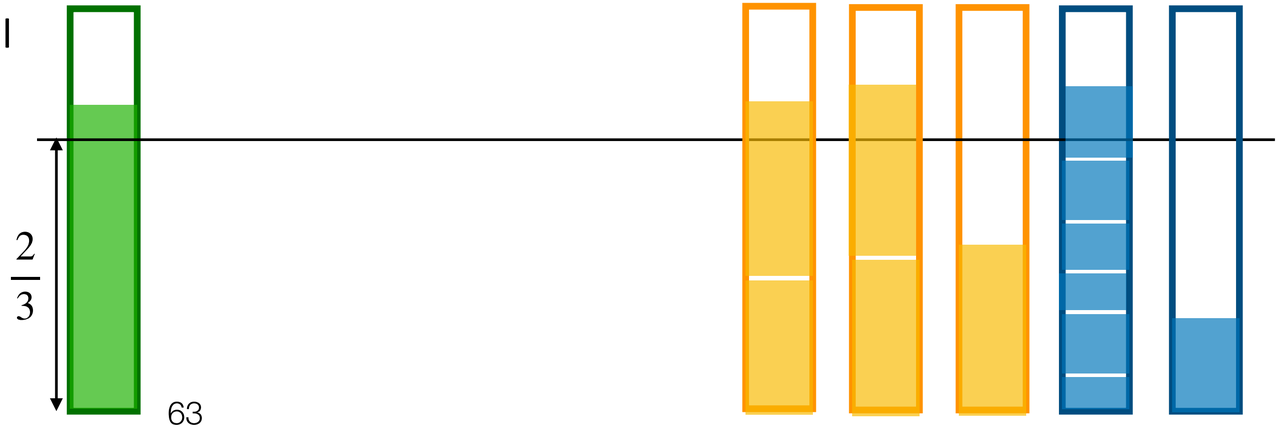

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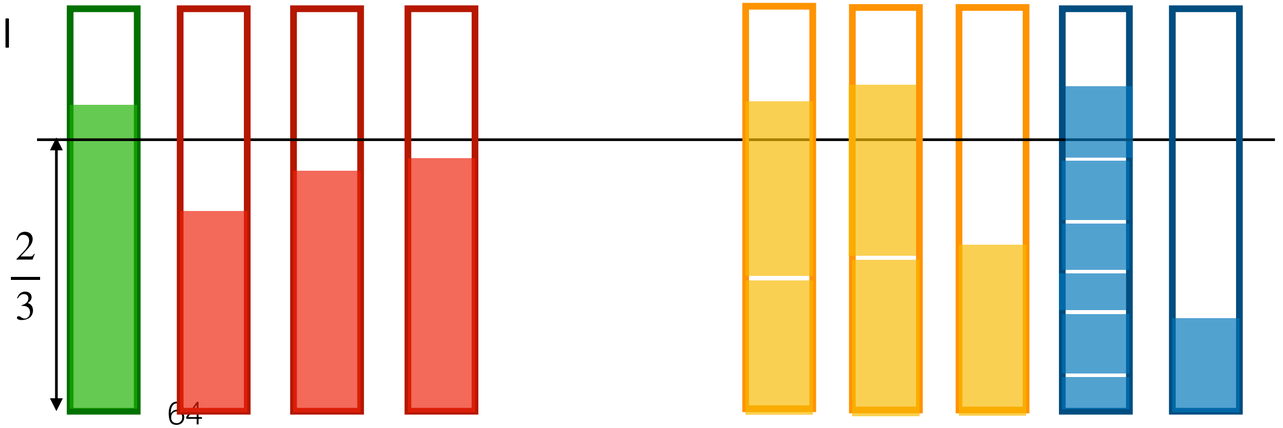

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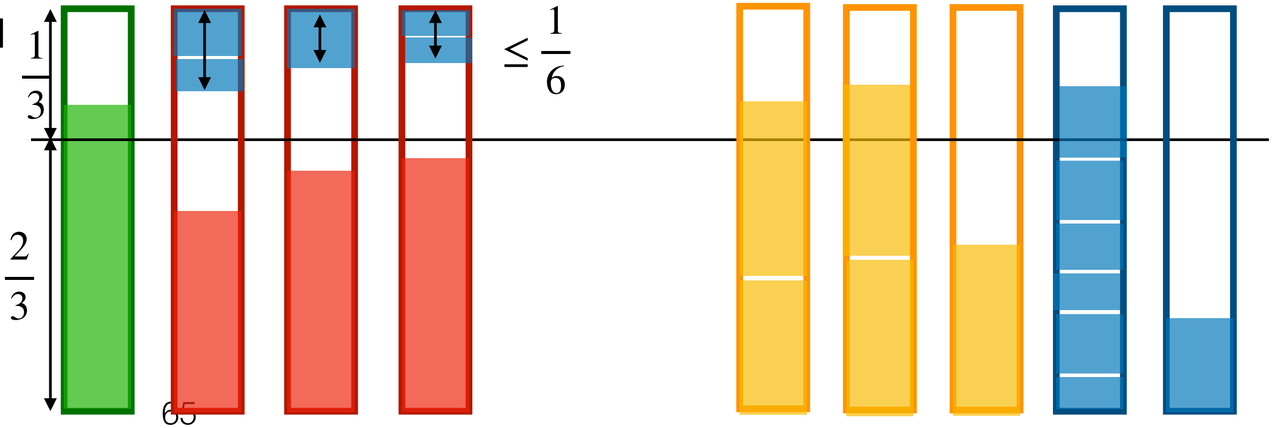

Robust-Reserve-Critical Algorithm

Robust-Reserve-Critical($r = \frac{c}{c+t}, \alpha$ **)** // c : number of critical bins, t : number of tiny bins, $\alpha \in [0,1]$

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$$\frac{c}{c+t}$$

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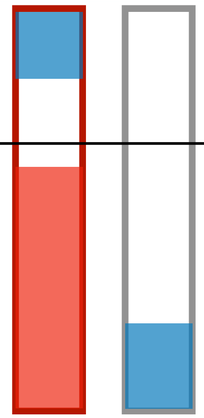

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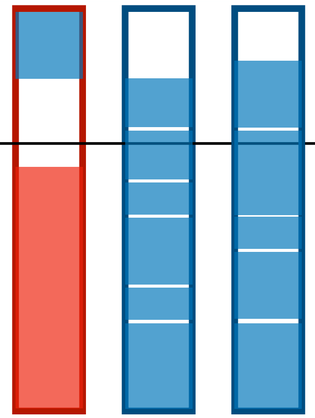

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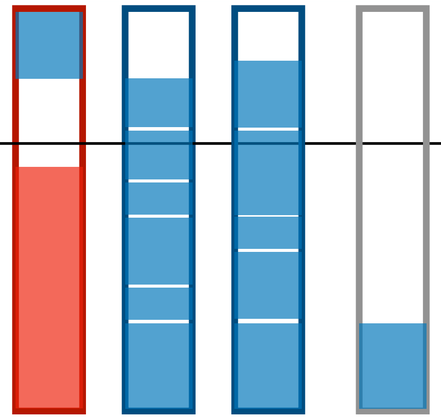

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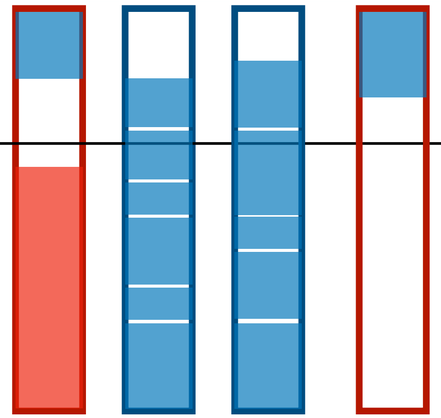

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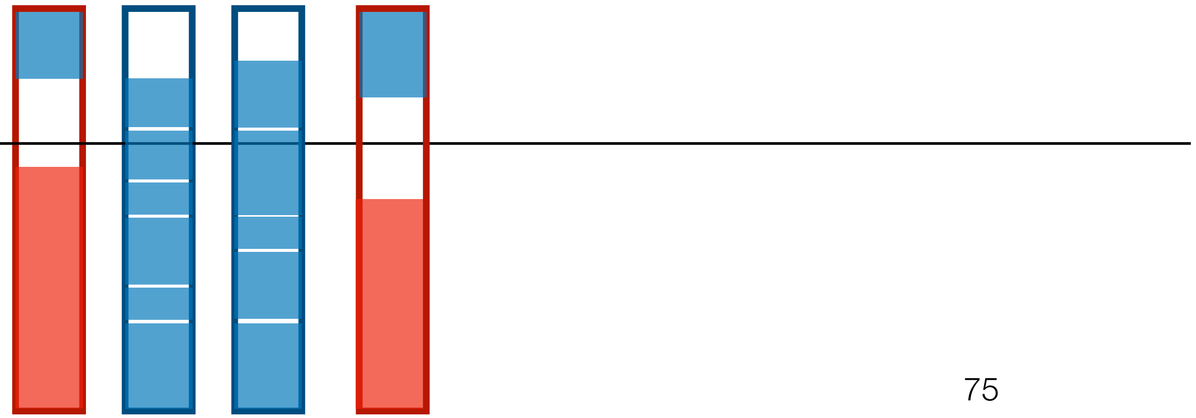

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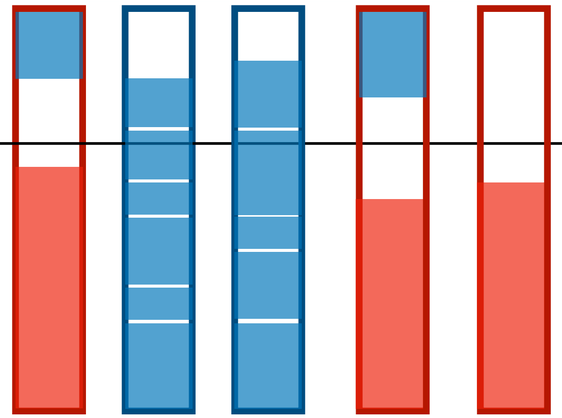

Robust-Reserve-Critical always keep the ratio $\frac{c'}{c'+t'}$ close to β

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Prediction: the number of critical bins and the number of tiny bins

$$\frac{c}{c+t}$$

Every critical bin has one critical item

We my open too many critical bins

Robust-Reserve-Critical Algorithm

- Lemma: If every critical bin has one critical item, **Robust-Reserve-Critical** is at most $1.5+\frac{1-\beta}{4-3\beta}$-competitive

By the algorithm, we can make sure that each bin is full enough

- Lemma: If there are some critical bins have no critical item, **Robust-Reserve-Critical** is at most $1.5+\frac{9\beta}{8-6\beta}$-competitive

The critical bin without a critical item must be opened because of a tiny item

$\Rightarrow$ at the time the tiny item arrives, the number of critical bins is low

Robust-Reserve-Critical Algorithm

- Theorem: If the prediction on $r = \frac{c}{c+t}$ is correct, **Robust-Reserve-Critical** is at most $1.5+\frac{1-\alpha}{4-3\alpha}$-competitive. Otherwise, **Robust-Reserve-Critical** is at most $1.5+\max\{\frac{1}{4}, \frac{9\alpha}{8-6\alpha}\}$-competitive.

What Happened

- Use extra information to bound the optimal cost from below

Prediction Error η

- Absolute error $\eta_1 = |p - S|$
- Squared error $\eta_2 = |p - S|^2$
- Classification error $\eta_c = 1$ if $p \neq S$

Ski Rental Algorithms Revisit: $\eta_1 = |\, p - S \,|$

- $\eta_1 = |\, p - S \,|$

Algorithm 3

If $p \geq B$ then **Buy** on the $\lceil B \cdot (1-\alpha) + \varepsilon \rceil$-th day

Else **Buy** on the $\lceil \frac{B}{1-\alpha} + \varepsilon \rceil$-th day

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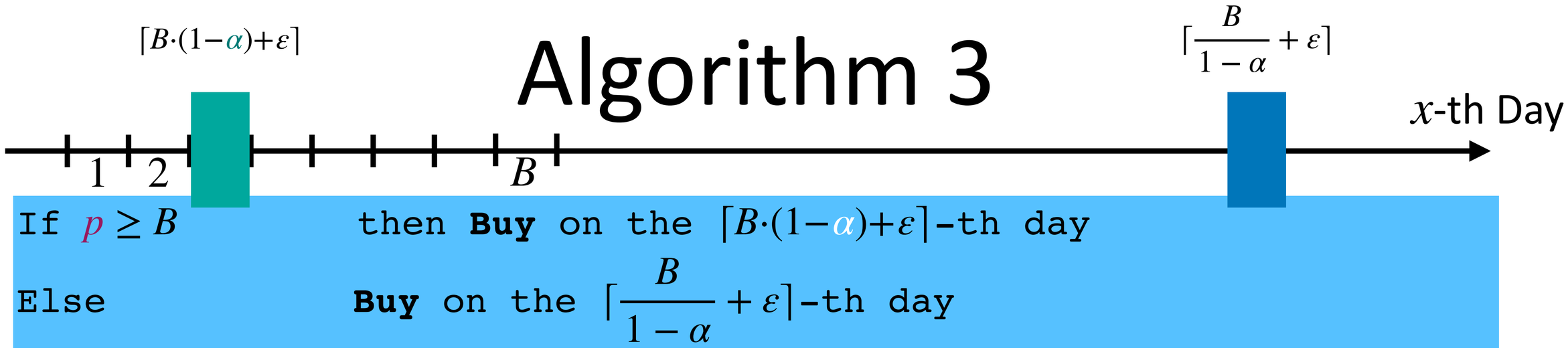
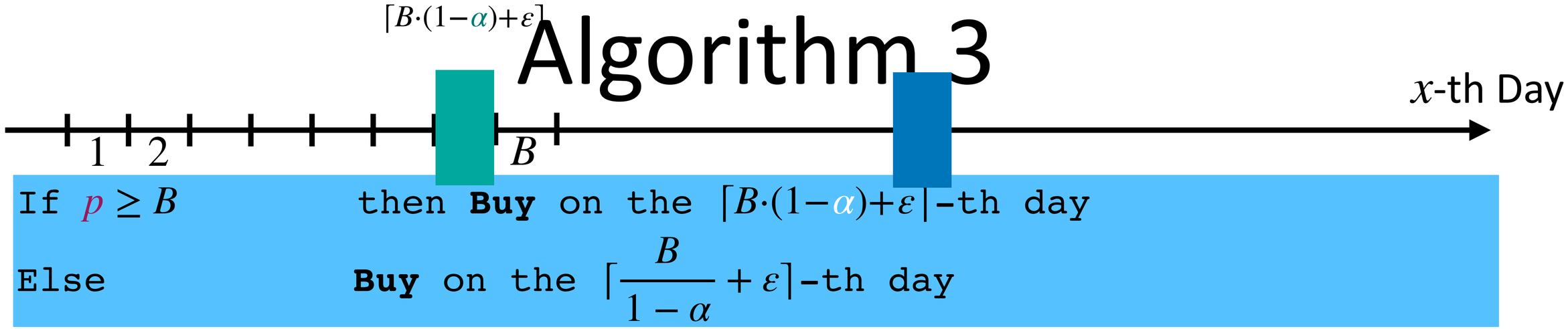


Big α

Algorithm 3



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Small α

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If $p \geq B$ then **Buy** on the $\lceil B\cdot(1-\alpha)+\varepsilon \rceil$-th day

Else **Buy** on the $\lceil \frac{B}{1-\alpha}+\varepsilon \rceil$-th day

	Truth: $S \geq B$ (OPT buy)	Truth: $S < B$ (OPT rent)
Good prediction	Advice: $p \geq B$ $\frac{B(1-\alpha)+B}{B} = 2-\alpha$	
Bad prediction		

η_1

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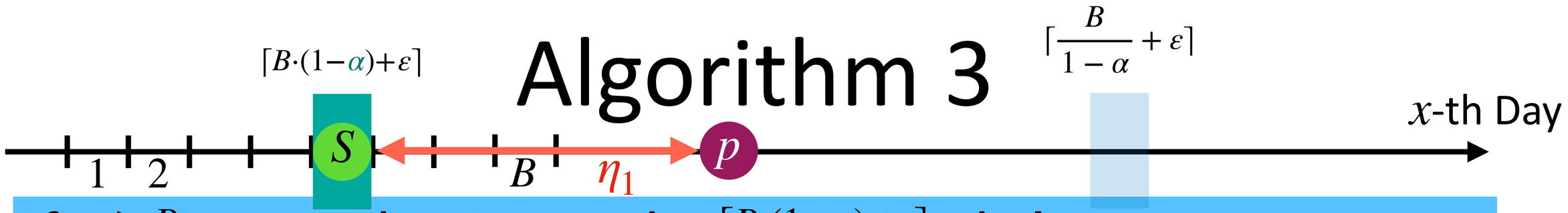

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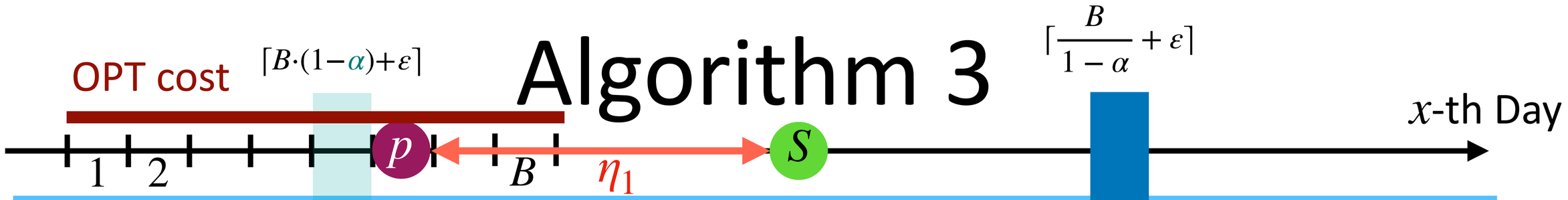
Algorithm 3



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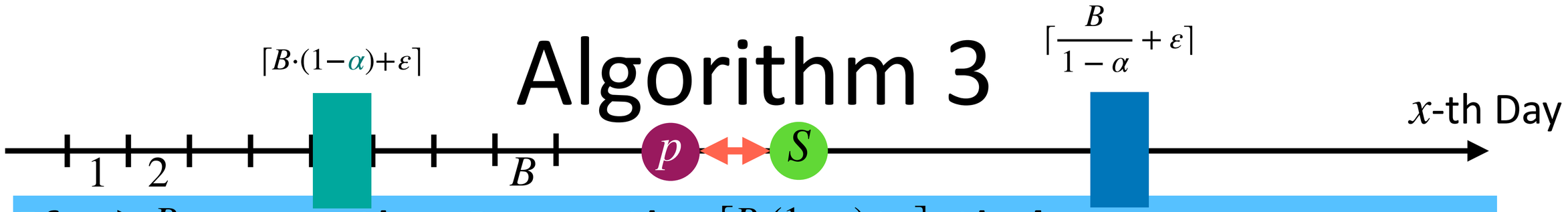

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Machine-Learned Advice

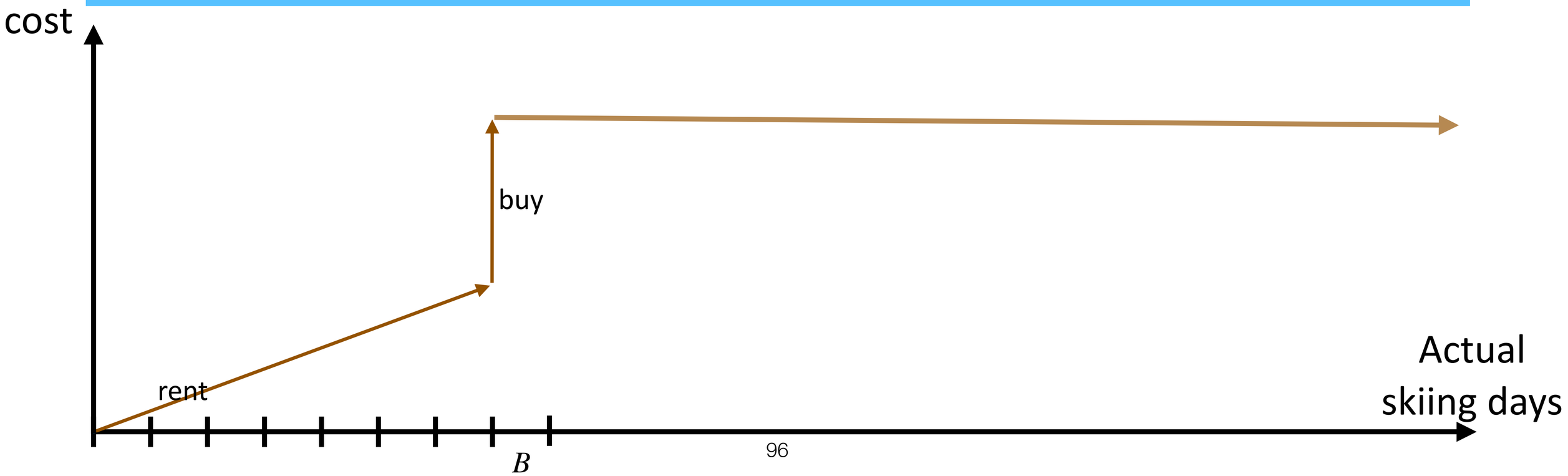
SKI-Rental with prediction (p, k)

If $p \geq B$

Keep renting until the k-th day // k is our "trust parameter"

else ($p < B$)

Keep renting until the B-th day



Machine-Learned Advice

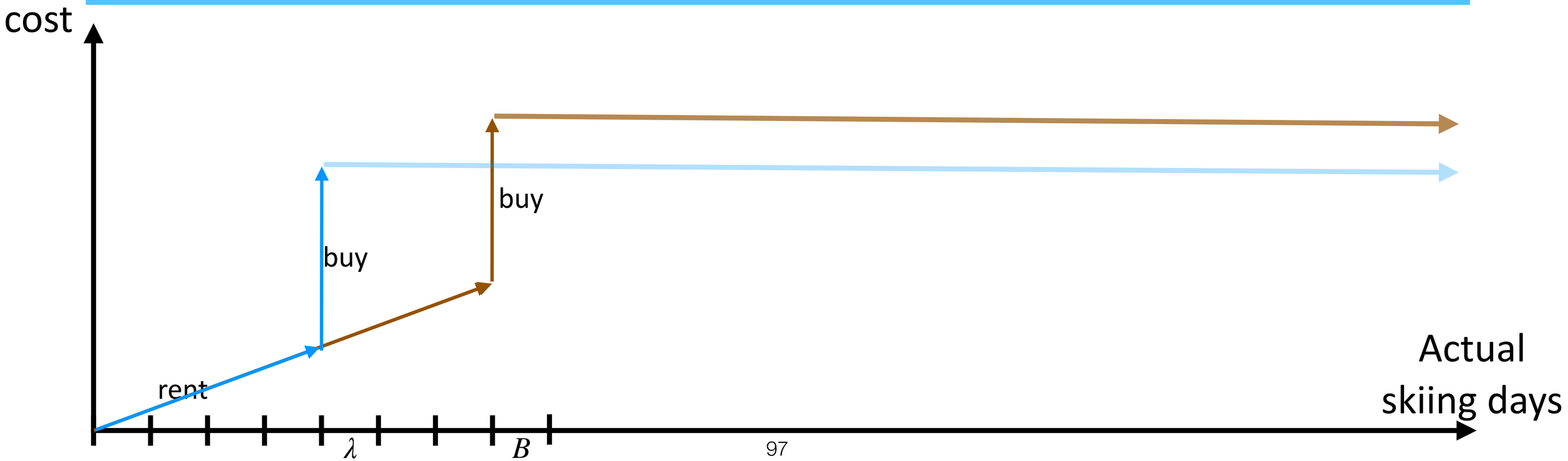
SKI-Rental with prediction (p, k)

If $p \geq B$

Keep renting until the k-th day // k is our "trust parameter"

else ($p < B$)

Keep renting until the B-th day



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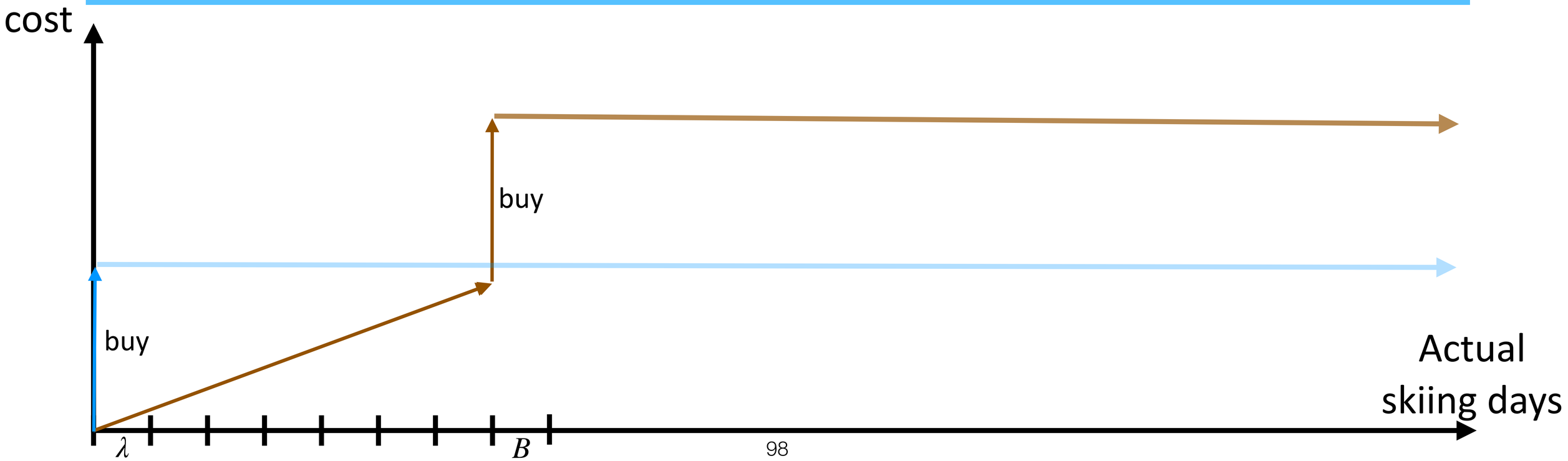

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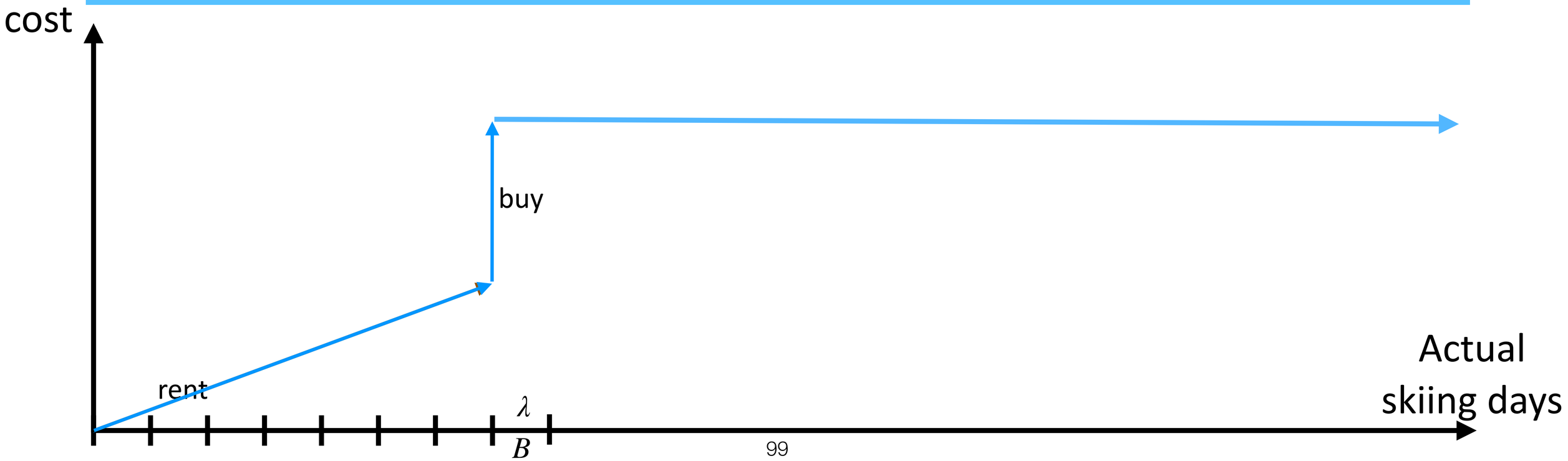

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Keep renting until the B-th day S: Actual number of skiing days

Truth: $S \geq B$ (OPT buy)	Truth: $S < B$ (OPT rent)

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 Good advice *Consistency*	Advice: $p \geq B$ $\frac{(k-1)+B}{B}$	Advice: $p < B$ $\frac{d}{d}$
 Bad advice *Robustness*	Advice: $p < B$ $\frac{(B-1)+B}{B}$	Advice: $p \geq B$ $\frac{(k-1)+B}{d}$

Randomization May Help

- If we allow the online algorithm to use randomness, smaller competitive ratios may be attainable
- A *randomized online algorithm* ALG is c-competitive against an oblivious adversary if for every oblivious adversary I_o,

$$\mathbf{E}[\text{ALG}(I_o)] \leq c \cdot \text{OPT}(I_o) + \alpha$$

  - That is, in the algorithm ALG there is some random factor; it can make random choices

RANDOMIZED-SKI-RENTAL

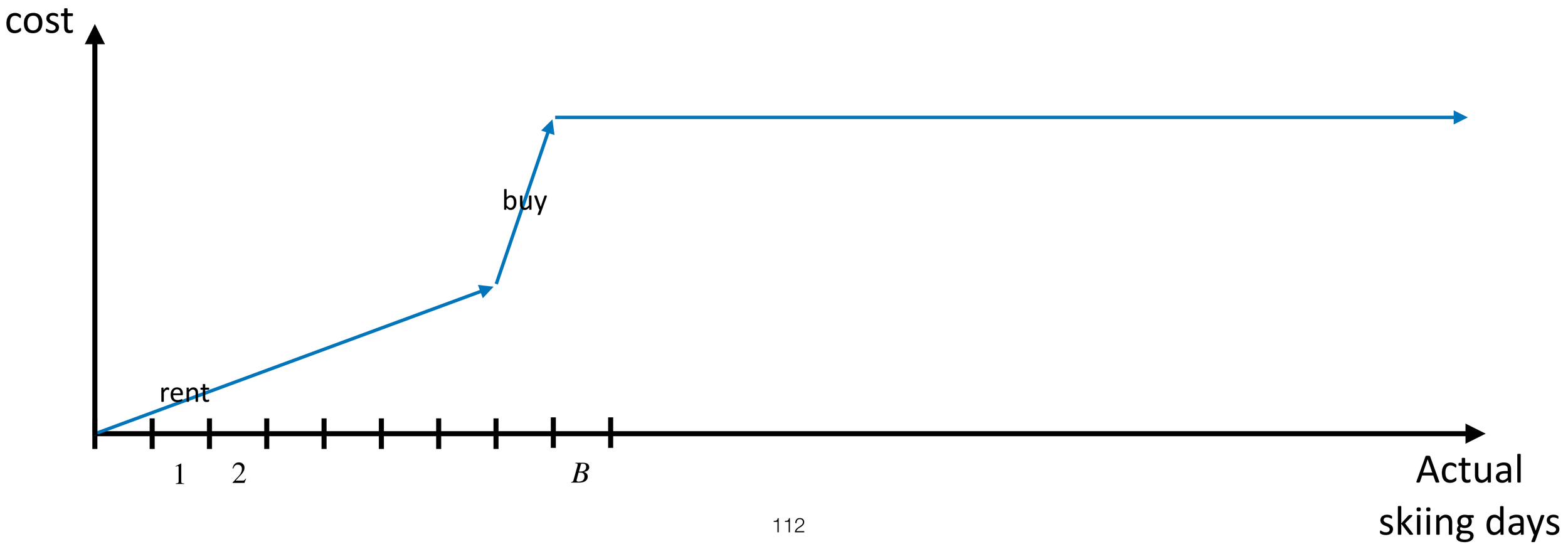
Keep renting until the $(\phi - 1) \cdot B$-th day // ϕ is the golden ratio 1.618…

On the $(\phi - 1) \cdot B$-th day, flip a coin

If it is *head*, buy the ski

Otherwise, keep renting until the B-th day

Randomized Ski-rental Algorithm



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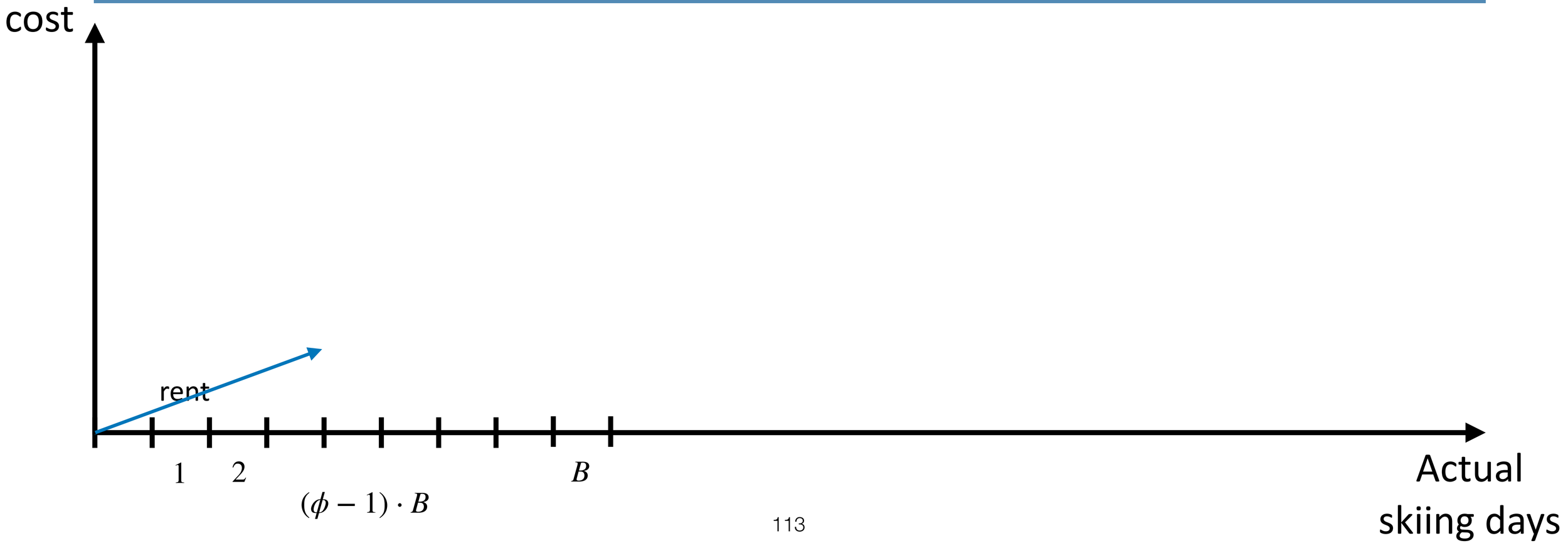

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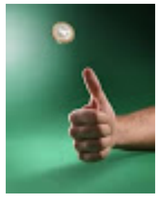

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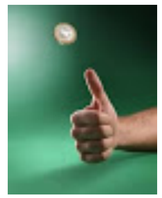

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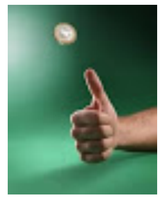

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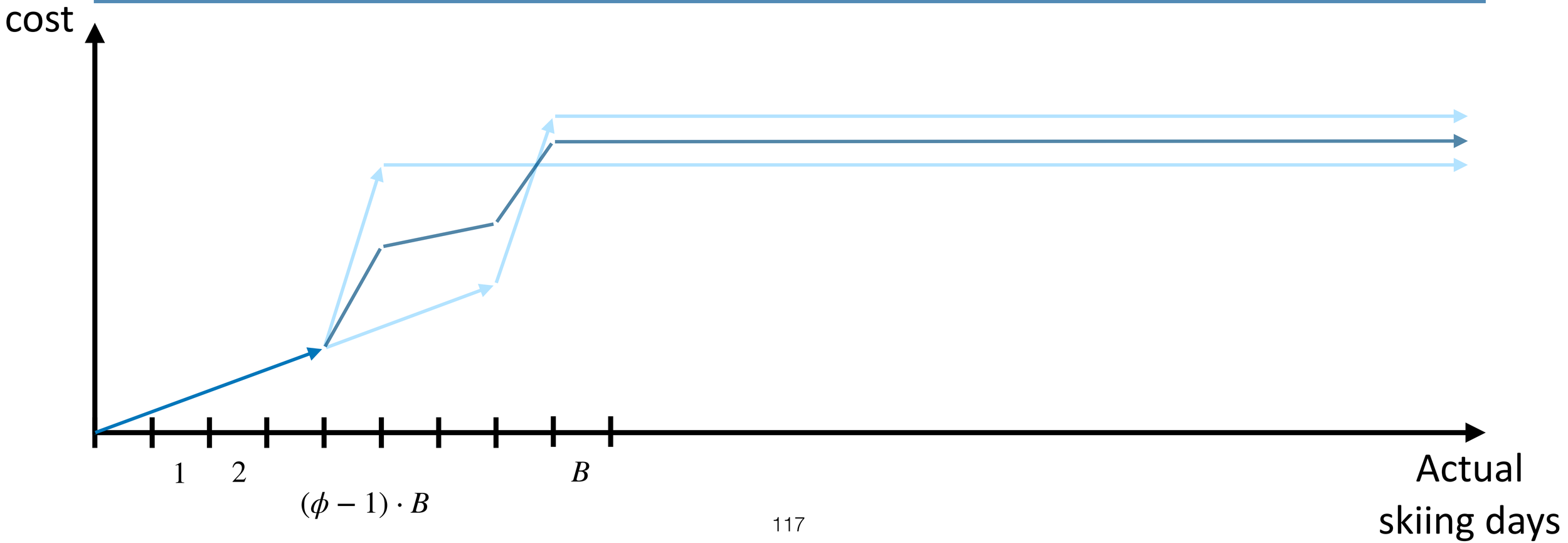

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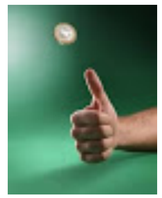

cost

1 2 $(\phi - 1) \cdot B$ B

Actual skiing days

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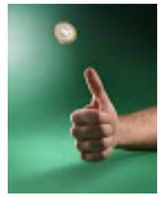

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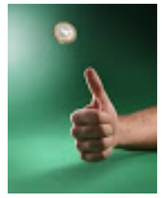

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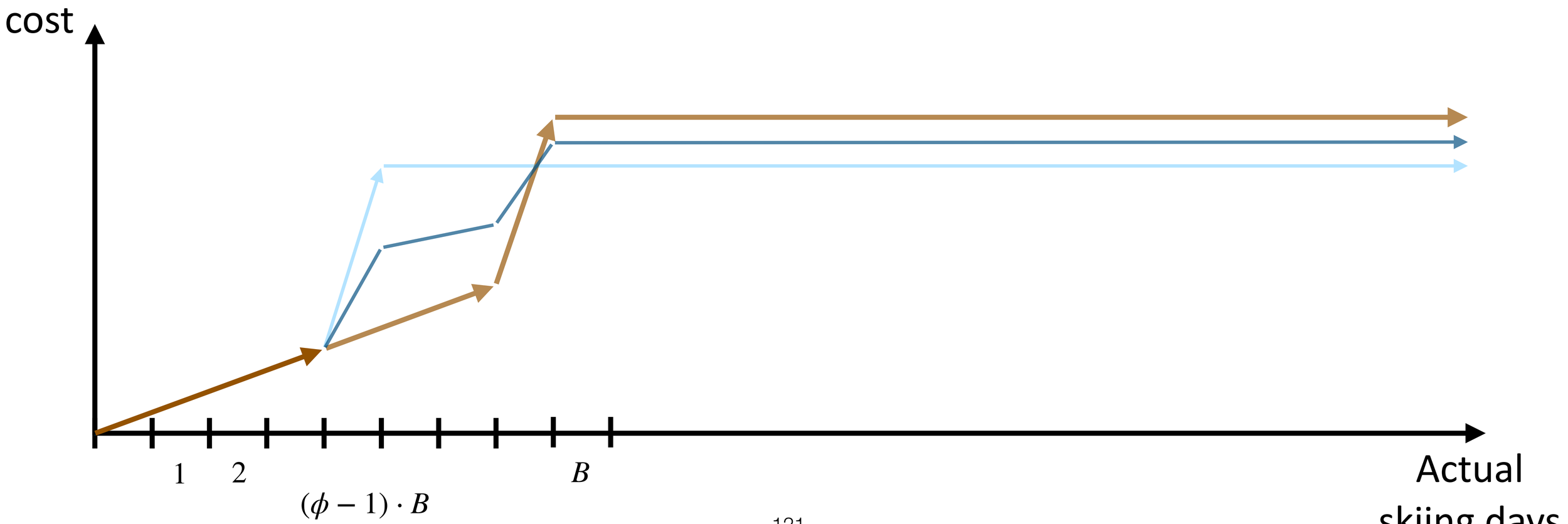
Deterministic Ski-Rental ALG competitive ratio:



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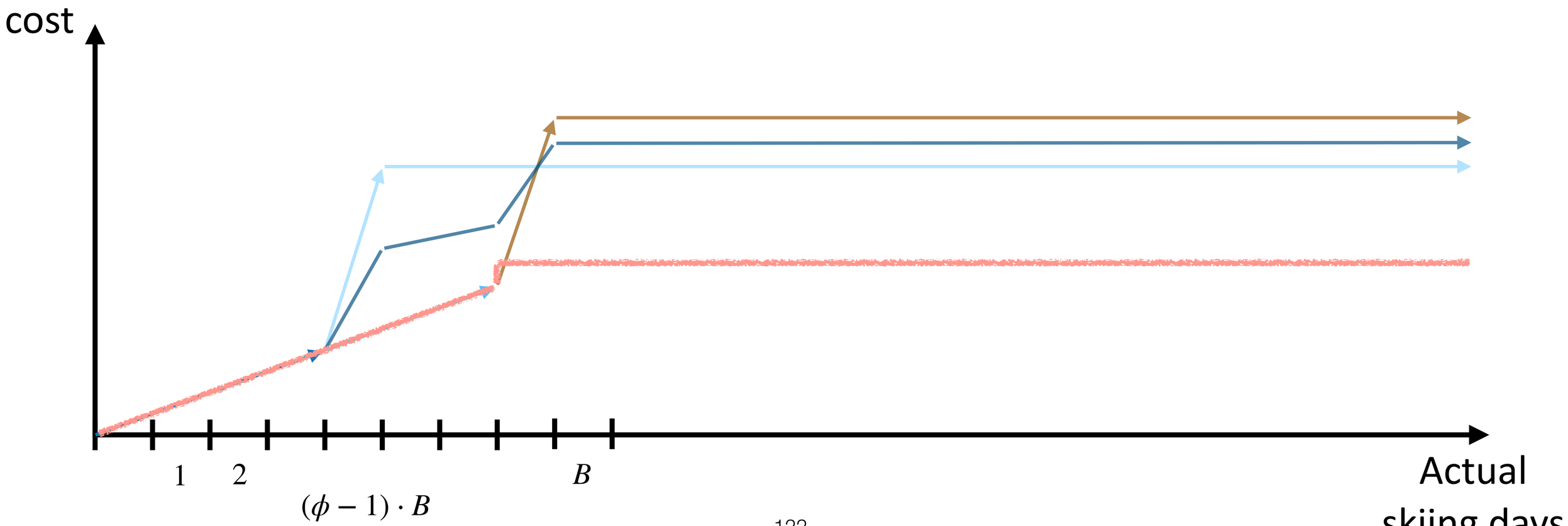

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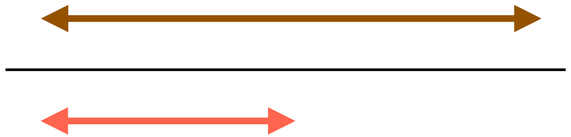

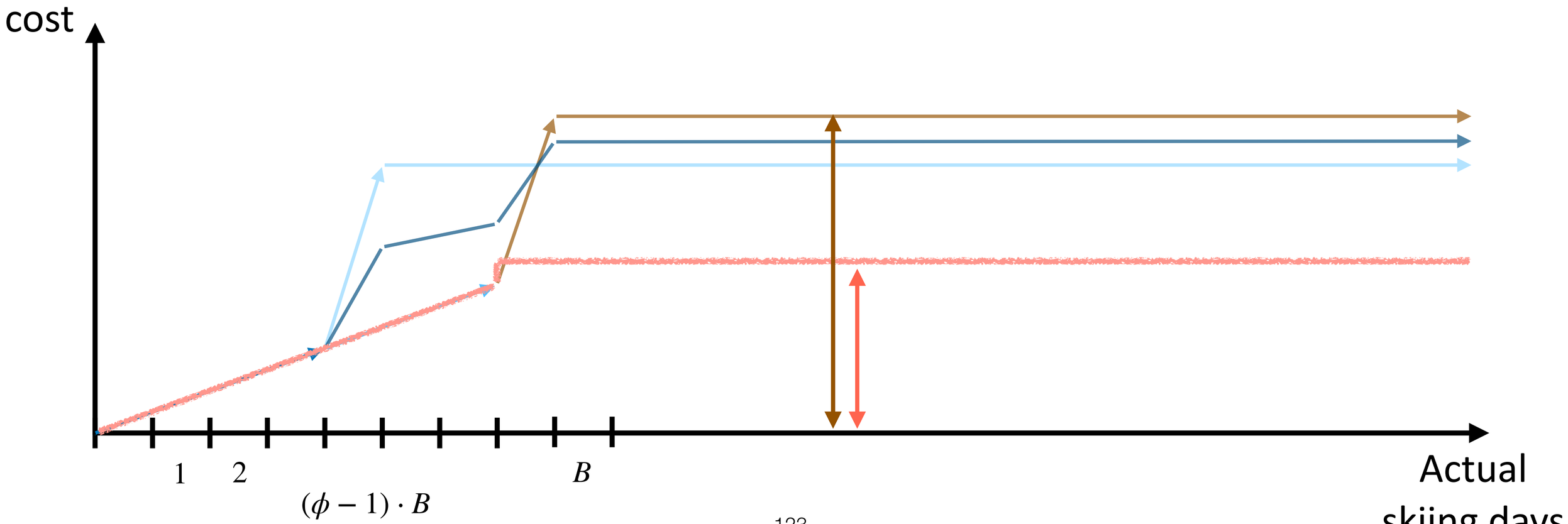

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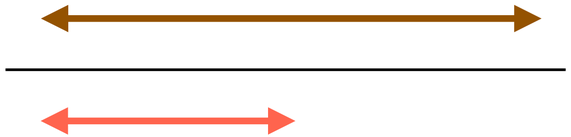

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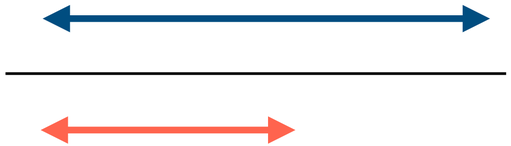
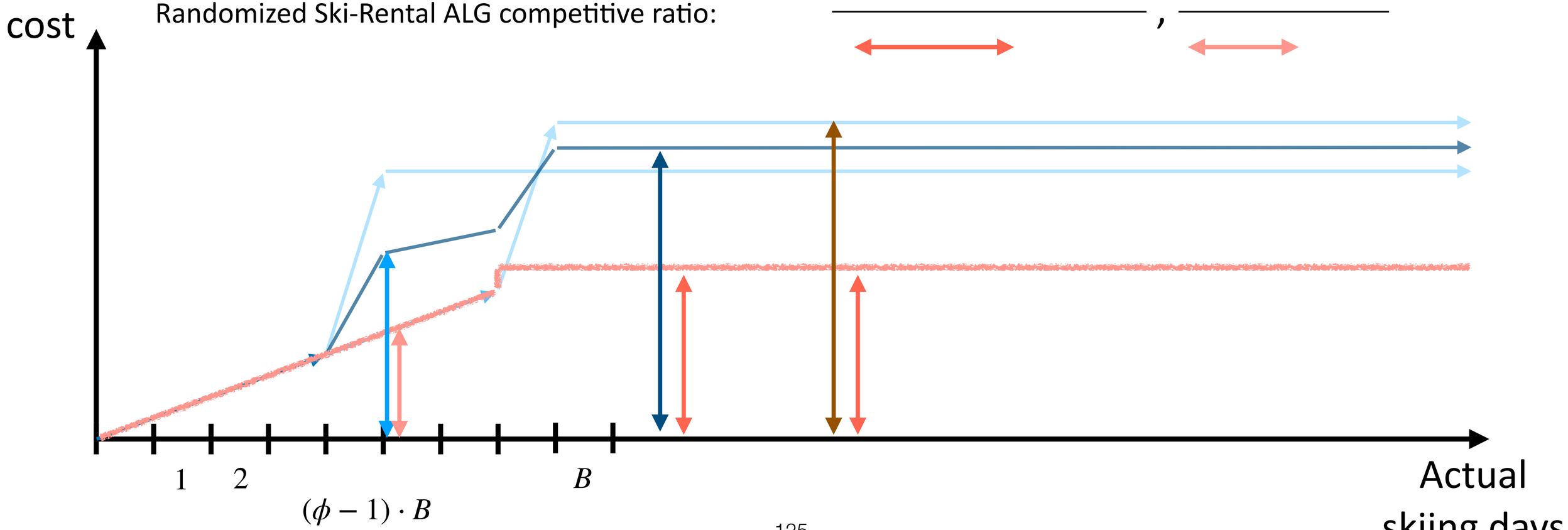


Randomized Ski-rental Algorithm

Deterministic Ski-Rental ALG competitive ratio: $\frac{2}{1}$

Randomized Ski-Rental ALG competitive ratio: $\frac{2}{1}, \frac{2}{1}$



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Deterministic Ski-Rental ALG competitive ratio: $\frac{\longleftrightarrow}{\longleftrightarrow}$

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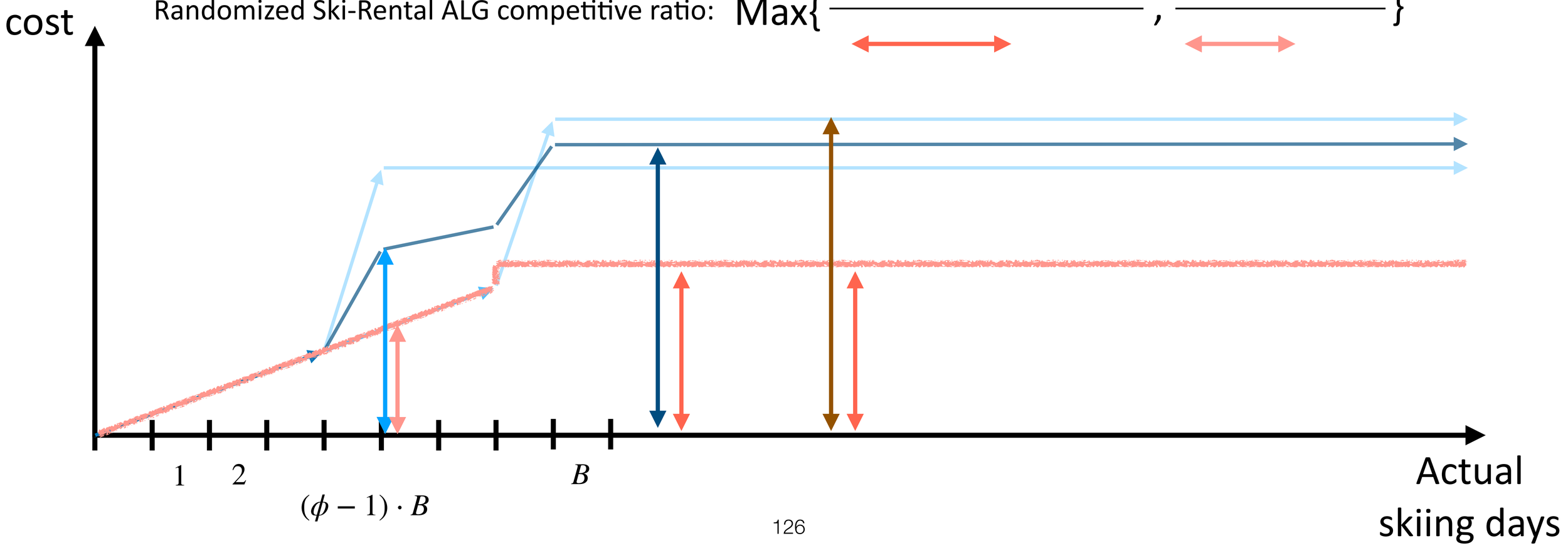

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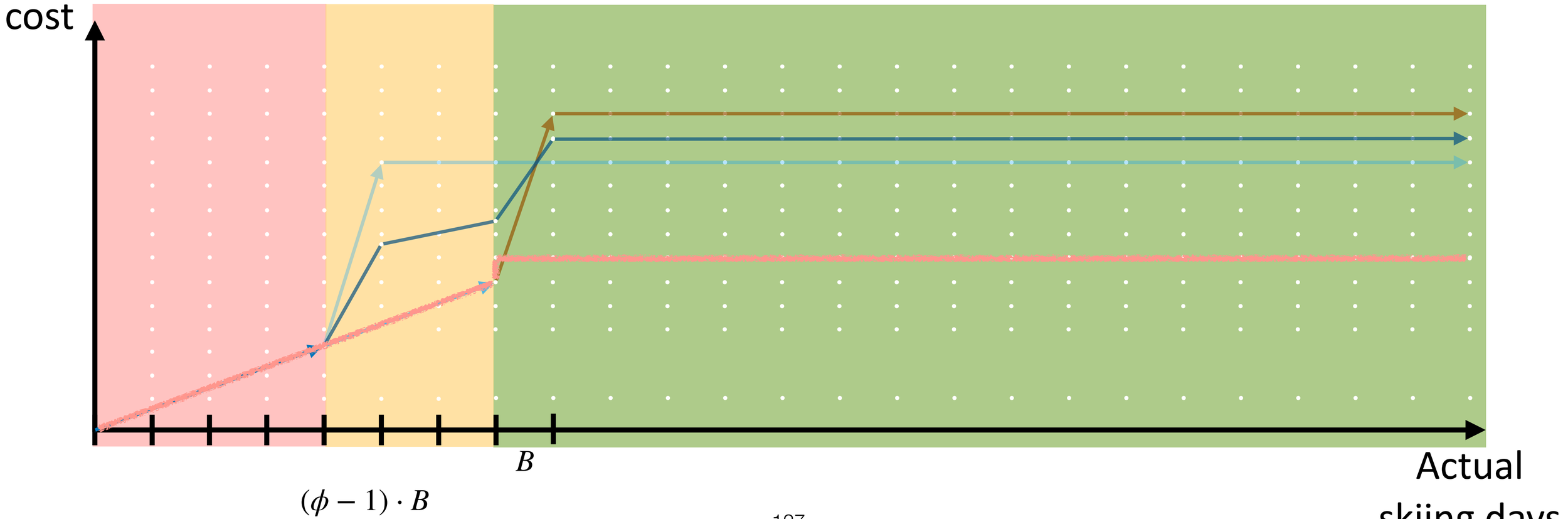

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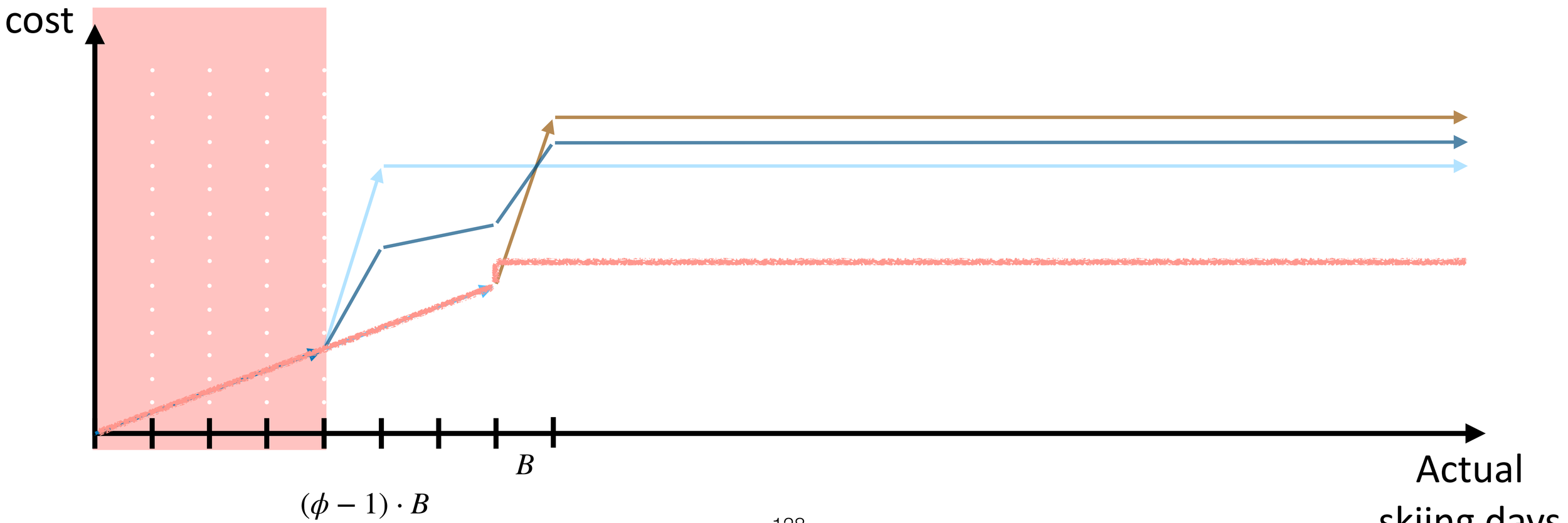
If #actual skiing days $< (\phi - 1) \cdot B$: ratio = 1



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If #actual skiing days $\in [(\phi - 1) \cdot B, B)$:

ratio = $\frac{1}{2} \cdot \frac{(\phi - 1) \cdot B - 1 + B}{D} + \frac{1}{2} \cdot \frac{D}{D} \leq 1.809$

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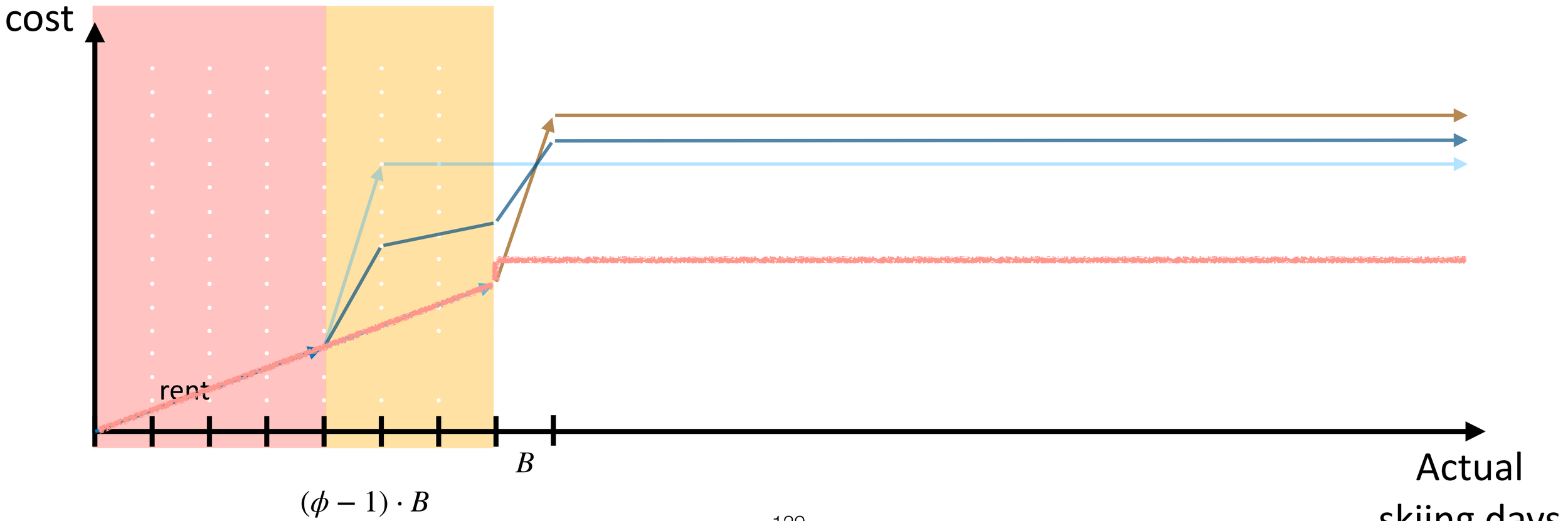

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If #actual skiing days $\geq B$:

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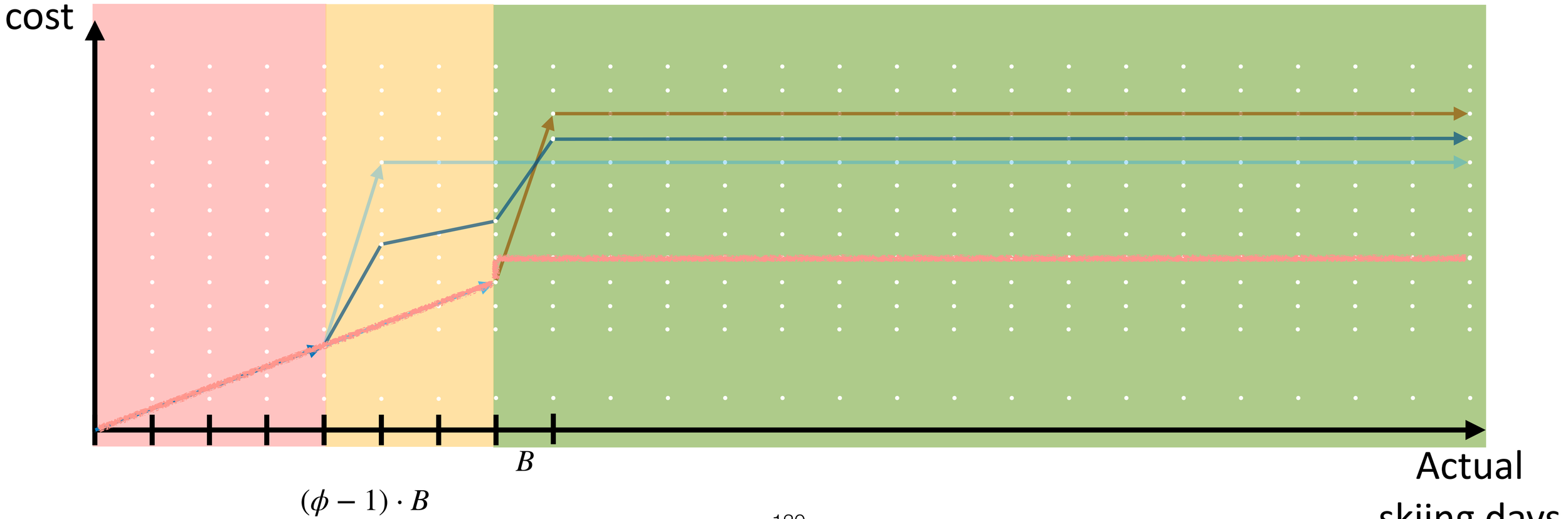

Outline

- Research cycle of online algorithms — Using **BinPacking** as an example
  - BinPacking and FirstFit algorithm
  - FirstFit is 2-competitive
  - There is no 1.333-competitive deterministic online algorithm for BinPacking
  - FirstFit is at least 1.667-competitive
  - FirstFit $\leq 1.7 \cdot \mathrm{OPT} + 2$
- Different variants of an online problem — Using **MachineMinimization** as an example
  - There is no $O(1)$-competitive algorithm for the general case
  - For special cases, MachineMinimization admits better competitive algorithms
- Randomization may helps
- Machine-learned advices

Machine-Learned Advice

- Online algorithms deal with optimization under uncertainty (about future input)

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    - Example: weather forecast
    - These predictions or learned information may not be 100% correct
      - Completely trust the predictions may be a disaster
      - How can we design online algorithms with this kind of (maybe) untrusted advice?

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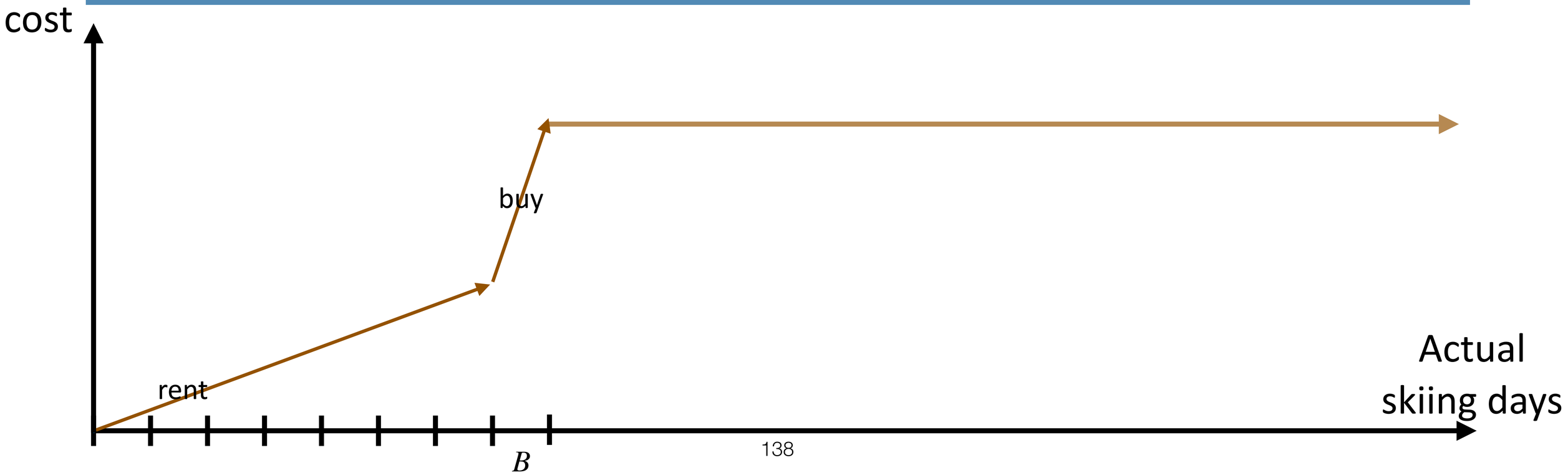

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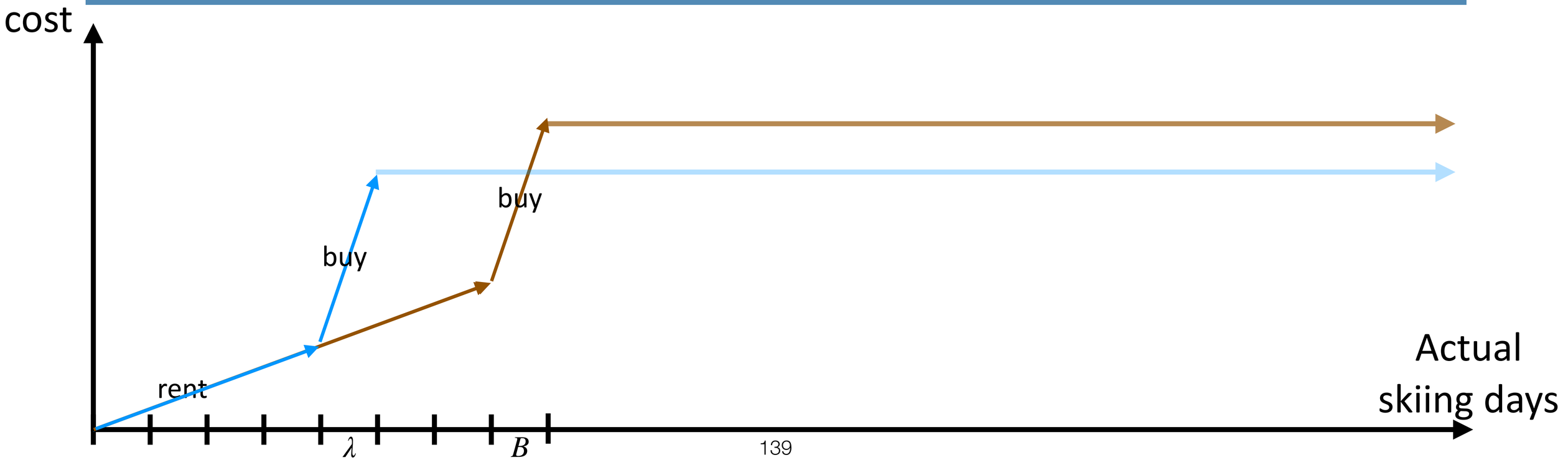

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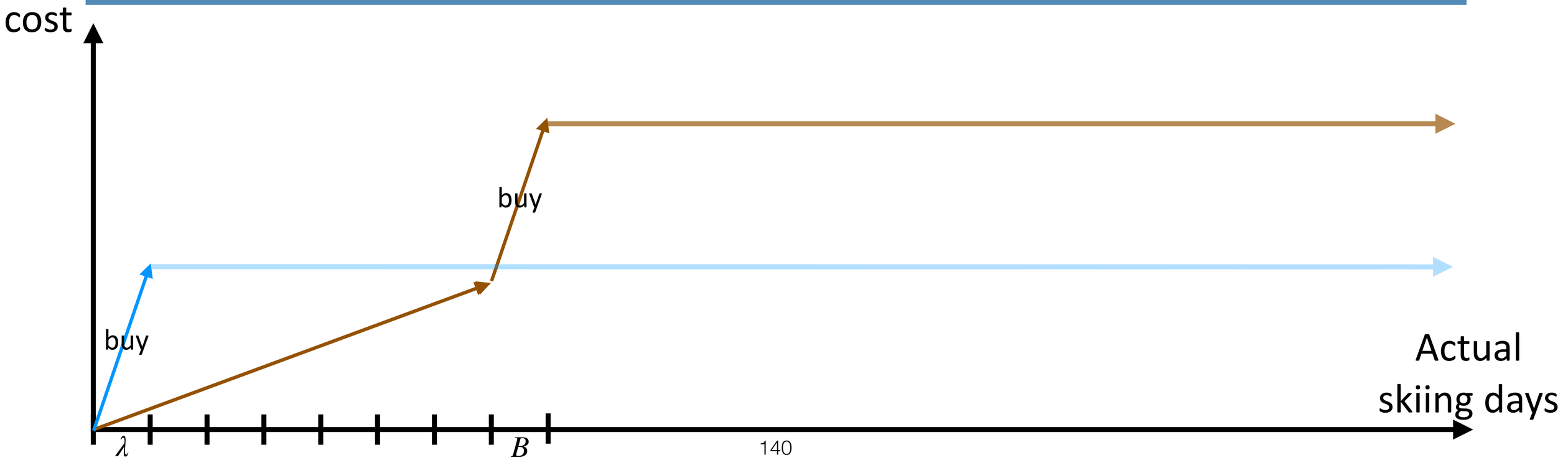
Keep renting until the λ-th day // λ is our "trust parameter"

Otherwise ($a < B$)

Keep renting until the B-th day



Machine-Learned Advice

SKI-Rental with advice (a)

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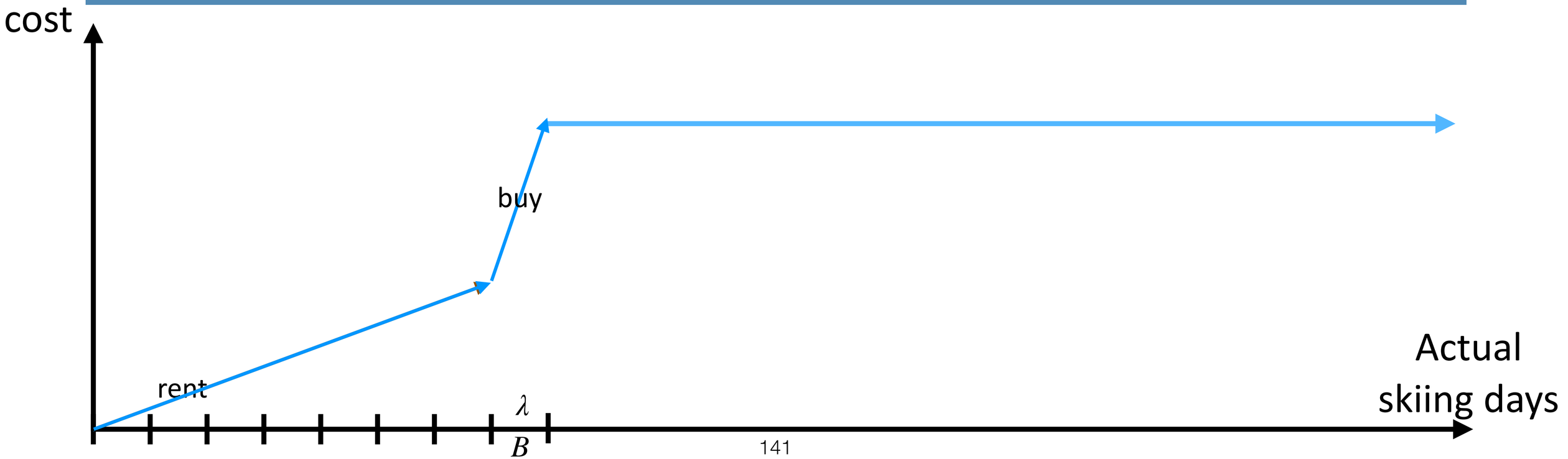

November 1 (Tue)

November 3 (Thu)

Both okay