

We show the NP-hardness by reduction from VERTEXCOVER.

Given any instance of VERTEXCOVER,  $G = (V, E)$  and  $k$ , we construct an instance of STEINERTREE,  $G' = (V', E')$  and  $k'$  as follows. For any  $v_i \in V$ , there is a vertex  $v' \in V'$ . Moreover, for any edge  $(v_i, v_j)$ , there is a vertex  $v'_{i,j} \in V'$ . Finally, there is a dummy vertex  $d \in V'$ .

Now we construct the edges in  $G'$ . For any  $v'_{i,j}$ , there is an edge  $(v'_i, v'_{i,j}) \in E'$  and an edge  $(v'_j, v'_{i,j}) \in E'$ . Finally, for any  $v'_i \in V'$ , there is an edge  $(v'_i, d) \in E'$ .

In the end, we let  $k' = |E| + k$ . The construction takes polynomial time.