Exercise Solution: P and NP

1. Consider the following problems, give a certificate that helps proving the NP membership.

   (a) PARTITION $= \{\langle S\rangle | S = \{x_1, x_2, \cdots, x_k\}$ and there is a partition of $S, T = \{x_{i_1}, \cdots, x_{i_t}\} \subseteq S$ such that the sum in T is equal to the sum in $S \setminus T\}$

   We can use a subset of S, which is one of the two parts, as a certificate.

   (b) HAMPATH $= \{\langle G, s, t\rangle | G$ is a directed graph with a Hamiltonian path from s to $t\}$

   We can use a permutation of vertices in G, which in order forms a Hamiltonian path from s to t, as a certificate.

2. Show that PARTITION is in NP.

   First, for any yes-instance (that is, a set of numbers with a equal-sum bipartition) we define a certificate C as a subset of elements in one of the two parties.

   Next, we design a polynomial-time verifier for PARTITION.

   $V =$ "On input $\langle\langle S\rangle, C\rangle$:

   1. Check if C is a subset of S. If it is not true, *reject*.

   2. Check if the sum of elements in C is equal to the sum of elements in $S \setminus C$.

   If it is not true, *reject*

   3. If both Steps 1 and 2 pass, *accept*."

   Finally, we show that V can verify PARTITION in polynomial time. For Step 1, V scans the input string for at most $\min\{|C|, |S|\}$ times. For Step 2, V scans the input string twice. Hence, the total running time of V is polynomial in the size of input string.

3. Show that 3SAT is in NP.