



Does Rebalancing Really Pay Off??

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Investment advice is rife with nostrums that have never been backed up by adequate proof or evidence. Many of these are ritualistically repeated by advisors who assume that, because they have been repeated so often and for so long and by reputable experts, there must be a good reason for them.

No investment advice is more universally offered than the advice – originally posited by William Bernstein – to rebalance your portfolio. Yet, the evidence that this practice is beneficial is shockingly meager.

The rebalancing bonus?

The assumption that rebalancing provides a benefit in increased return is encapsulated in the phrase “the rebalancing bonus.” This catch-phrase was originated by Bernstein in an 18-year-old post on his website, efficientfrontier.com, titled “The Rebalancing Bonus.”

In this posting, Bernstein said, “An understanding of the mechanics of rebalancing is fundamental to sound portfolio management, and yet surprisingly little theoretical attention has been paid to this area.” This was true at the time and it is still true now. To this day, little work of theoretical merit has been devoted to the question of whether and when it is beneficial to engage in the practice of rebalancing. Bernstein’s posting remains the best effort to tackle the issue to date, and has had the longest-lasting effects.

Unfortunately, there are flaws in the argument. Bernstein and I have communicated about this and he is well aware of the issues. Nevertheless, many of its implications are still very much alive in the claims for rebalancing often heard voiced by advisors.

I will first point out the flaws in the 1996 posting. From there, I will go on to delineate the issues surrounding the question of rebalancing that remain after removing the misconceptions that it inadvertently perpetuated. Then I will explore these issues through a combination of empirical studies of historical returns with the results of simulations.

The trouble with Bernstein’s rebalancing bonus

The odd thing about Bernstein’s rebalancing bonus posting is that he shows what is wrong with it in

almost the same breath with which he introduces the term. He establishes a measure that he calls the “Markowitz return” by defining it as the weighted average return. But he says at the same time that “It is surprising that Markowitz considered portfolio return to be the weighted sum of the component returns.”

Yes, that is how Markowitz defined it for his one-period model and in that case it is correct. If a portfolio starts with a specified asset weighting and then holds the assets until the end of a time period without buying or selling, portfolio return over that time period is indeed the weighted average of the raw (i.e., unannualized) asset returns.

However, Bernstein went on to define the Markowitz return over a multi-year period as the average of the annualized asset returns. Over the period 1926-94 he calculated this so-called “Markowitz return” for a stock-bond portfolio with an initial 50-50 mix as 7.85%. This is in fact a meaningless number because it is not a measure of the growth of the portfolio using any discernible asset reallocation strategy, either dynamic or static. Furthermore that return, so defined, is always less than the actual annualized return on a buy-and-hold portfolio over the same time period.

Bernstein then defined the “rebalancing bonus” as the excess return of a rebalanced portfolio over the “Markowitz return.” Since the return on an annually rebalanced portfolio is 8.34% over the same time period, the “rebalancing bonus” is 0.49%. Thus, he compared the return on a rebalanced portfolio with a phantom benchmark that is always less than the return on a buy-and-hold portfolio.

In fact, in the very next sentence, Bernstein pointed out that if the portfolio had not been rebalanced, its return would have been 9.17% – 0.83% higher than the rebalanced portfolio and 1.32% higher than the Markowitz return. Nevertheless he let the definition of “rebalancing bonus” stand, as the difference between the return on a rebalanced portfolio and the Markowitz return. Here we find the accidental – and accidentally misleading – origin of the term “rebalancing bonus.” It is misleading because it identifies the benefit of rebalancing as its incremental return over a meaningless number.

The generalized mathematical problem of rebalancing

Rebalancing is generally understood to mean the practice of periodically restoring the allocation among asset classes to its original state. For example, if a portfolio began a year with 50% stocks and 50% bonds, and if at the end of the year that mix had changed to 52% stocks and 48% bonds because the return on stocks had been greater than the return on bonds, then rebalancing would entail selling 2% of the portfolio that is currently invested in stocks and using the proceeds to buy bonds, thus restoring the 50-50 mix.

The rebalancing discipline can be applied at regular time intervals, such as annually or quarterly or it could be applied when a threshold of imbalance had been reached, such as when either stocks or bonds crossed the 2% “out-of-whack” threshold (the 52% threshold in the example).

The chief problem of assessing whether rebalancing is beneficial is establishing a benchmark: compared to what is rebalancing superior? Because Bernstein chose the wrong benchmark, the comparisons that have been generally made are invalid.

I will argue that – in contrast to Bernstein’s posting – the only meaningful comparison is with a buy-and-hold strategy, i.e., a strategy of not rebalancing. The reason for choosing that benchmark is that it is the simplest and most straightforward alternative to rebalancing.

Bernstein argued that while buy-and-hold will beat rebalancing when the expected return on one asset is substantially greater than the expected return on another (but at the cost of greater risk for the buy-and-hold strategy), rebalancing will always dominate when the expected returns on the assets are equal.

In fact, Bernstein supplied me with an article he coauthored with David Wilkinson, *Diversification, Rebalancing, and the Geometric Mean Frontier*, suggesting that this article showed that rebalancing will always dominate when the expected returns on the assets are the same.

This, however, is not the case, and the article does not prove it. What the article does prove (in a sense) is that if returns on all the assets over a multi-period time interval are exactly the same, then a rebalancing strategy will reap a higher return than a buy-and-hold strategy. (I say “in a sense” because Bernstein and Wilkinson’s proof is not an exact mathematical argument but uses a mathematical approximation. Nevertheless it is an ingenious way of arriving at what turns out to be a correct result – see Appendix).

What the article does not prove, however, is that the rebalancing strategy will offer a better return than buy-and-hold if the *expected* returns are the same. To use the terminology favored by economics and finance academics, Bernstein and Wilkinson show that rebalancing beats buy-and-hold if all the asset returns are the same *ex post*, but it does not show that rebalancing beats buy-and-hold if the asset returns have equal expected returns *ex ante*.

To address that question, we need to do more investigation. Bernstein told me that he thinks that mathematical arguments have limited value, and that one needs to look to the history. In fact I emphasized his respect for history in a review I wrote of his recent short book, *Deep Risk: How History Informs Portfolio Design*.

So let us turn to Bernstein’s argument based on history. That argument appears in the same posting in which he defined the term “rebalancing bonus.” On the surface, his study of rebalancing based on historical data is impressive and convincing. But here, unfortunately, there again is a problem.

Bernstein calculated the “rebalancing bonus” for quarterly returns over the period July 1988 to December 1994 for asset pairs drawn from a diversified list of asset categories as measured by indices such as the MSCI-EAFE, MSCI-EAFE Europe and MSCI-EAFE Pacific, U.S. small-company portfolio, international bonds, equity REITs and so on.

Aside from the comparison with the meaningless Markowitz return benchmark, there is another problem. Bernstein said that “A linear correction was applied to each series of quarterly returns to yield a zero annualized return for each individual asset over the whole study period; thus any return obtained from the asset pair must be due to rebalancing.”

This sounds innocent enough, but this correction would have had the effect of making all the cumulative asset returns equal *ex post*. Therefore, that correction would force the returns into precisely that pattern – and the only pattern – that guarantees that rebalancing will beat buy-and-hold.

The argument from mean-reversion and “investor irrationality”

People intuitively buy into the idea that rebalancing will succeed because they believe that investors are irrational and tend to panic and sell when the market drops and become exuberant and buy when the market soars. Therefore any strategy that goes contrary to those urges will succeed over time.

Furthermore, some studies, including particularly one for which I am grateful to Bernstein for supplying, have concluded that securities prices have historically had a tendency to mean-revert over time, with a half-life of about three and a half years (i.e., half of them mean-revert by that time). Other studies have shown that for shorter time intervals, securities prices show signs of mean-reversion’s opposite: momentum. That is, a high return over a time interval has increased the likelihood that the return will be higher than average over the next interval.

These results are in agreement with the theories of Hyman Minsky and behaviorists in finance and economics that, basically, irrational exuberance and panics do produce bubbles and crashes (i.e., momentum and mean-reversion, respectively).

However, even if these things are true, does that mean that rebalancing on any particular schedule will succeed? The rebalancing would have to occur at just the right time to catch a mean-reversion in order to succeed – and if rebalancing occurred when momentum was in charge, it might even be counterproductive.

To try to transcend such vague inferences and find out whether rebalancing would succeed in reality, I performed a few empirical studies and simulations. I will now sketch out the results.

Empirical results

I used the monthly CRSP decile returns series from January 1926 through December 2013 to create 697 rolling 30-year 360-month returns series for each CRSP decile. Forty-five two-asset portfolios, each split 50-50 were created from all possible pairings of the 10 CRSP returns series. For each two-asset portfolio and each 360-month period, returns were calculated for a buy-and-hold strategy and for six rebalancing strategies in which the portfolios were rebalanced monthly, quarterly, semi-annually, annually, every three years and every five years.

The results were surprisingly consistent across all portfolio pairs and strategies. The rebalanced portfolios did in fact beat the buy-and-hold portfolios most of the time. On average, a rebalancing strategy beat buy-and-hold about 70% of the time. However, on the occasions when buy-and-hold beat rebalancing, it was by an average margin about twice as great as the margin by which rebalancing beat buy-and-hold the rest of the time.

The end result was that for rebalancing intervals up to and including a year, the mean rebalancing /

buy-and-hold return differential was essentially zero. For rebalancing intervals of three and five years, the mean differentials were five basis points and six basis points annually, respectively. This could possibly reflect the observations that have been made of mean-reversion on approximately a three and a half year schedule.

In summary, while rebalancing beat buy-and-hold most of the time, when it did beat buy-and-hold it was by a small amount, while when buy-and-hold beat rebalancing it was by about twice as large an amount. Between these two effects, the mean result was essentially a wash.

The fact that rebalancing beats buy-and-hold most of the time but by only a small amount is consistent with the fact that rebalancing will certainly beat buy-and-hold if the whole-period returns for the constituent assets are the same. This tends to imply that when the whole-period returns for the constituent assets are *nearly* the same, rebalancing will tend to beat buy-and-hold. However, it is obvious that when the whole-period return on one asset is much greater than the whole-period return on the other asset – something that will happen randomly even when expected returns are the same or nearly the same, though less often than whole-period returns that are nearly equal – then buy-and-hold will tend to beat rebalancing by a large margin.

Nevertheless to see whether mean-reversion might have been at least partly responsible for these results, I scrambled the months over the 1926-2013 time period so that the monthly returns sequence was randomly different from its actual sequence. (This was the technique used by Eugene Fama and Kenneth French in a study they performed to test the prevalence of luck vs. skill in mutual fund alpha estimates.) Interestingly, this scrambling of the sequences did not noticeably change the overall results.

Simulation results

I also ran simulations using three different returns-generating processes: normally-distributed log-returns (i.e., the usual geometric Brownian motion model); stochastic volatility; and geometric Brownian motion but with mean-reversion. In all cases the expected returns and standard deviations, respectively, for the several simulated asset classes were all set equal.

The results varied, but in essence were all similar to the results of the empirical study. It is interesting in particular to note that introducing mean-reversion into the returns process did not cause rebalancing to beat buy-and-hold any more than it did when returns did not mean-revert.

Conclusion: no evidence as yet for a rebalancing bonus

My conclusion is that while the subject is interesting enough to warrant further research – certainly much more than has been done heretofore, given how much rebalancing is advocated by nearly all practitioners of the investment advice profession – no evidence has as yet presented itself to confirm that there is a rebalancing bonus.

Rebalancing is certainly not necessarily harmful, unless it conflicts with another risk management strategy that better suits the investor. It is better to have an investing discipline than not to have one,

and rebalancing is one acceptable default discipline – especially when the investor would fail to adhere to any discipline if his portfolio’s volatility exceeded a particular level. It should not, however, be thought of as a strategy that delivers a returns bonus as compared to other strategies.

Appendix

Comparing rebalancing to buy-and-hold can be reduced to a simply-stated mathematical question. Unfortunately, that mathematical question is very difficult to answer.

To explain what the mathematical problem is, consider the example of rates of return on four assets over four time intervals displayed below (assume each time interval is a year). Tables 1a and 1b show two ways of expressing those returns. The only difference between Table 1a and Table 1b is that Table 1b contains each rate of return plus 1, which is the growth factor. The returns for the four assets in the tables were randomly generated from four identical but cross-correlated lognormal distributions, each with an expected return of 7.2% and a standard deviation of 18.0%.

Table 1a. Rates of return on four assets in four annual periods

	Year:			
Asset:	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
A	15.68%	-3.03%	22.72%	26.84%
B	12.70%	11.31%	24.26%	31.17%
C	13.51%	-26.59%	-11.84%	9.65%
D	-5.87%	26.32%	1.16%	34.69%

Table 1b. Growth factor (one plus rate of return) on four assets in four annual periods

	Year:			
Asset:	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
A	1.1568	0.9697	1.2272	1.2684
B	1.1270	1.1131	1.2426	1.3117
C	1.1351	0.7341	0.8816	1.0965
D	0.9413	1.2632	1.0116	1.3469

Suppose that both a rebalanced portfolio and a buy-and-hold portfolio begin with an equally-weighted 25%-25%-25%-25% mix of the four assets A, B, C, and D.

The “product” of numbers means the result of multiplying them together. The product of the four growth factors in row A of Table 1b, for example, is equal to the four-year growth factor. The four-year growth factor is also equal to the rate of return on asset A compounded over the four years, plus one.

With a buy-and-hold strategy (see Table 2a) the portfolio’s four-year growth factor will be equal to the average of the four-year growth factors for the four assets, that is, the average of the products of the four 1-year growth factors.

Table 2a. Growth factor (one plus rate of return) on four assets in four annual periods

	Year:								
Asset:	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	Product	(4-year growth factor)			
A	1.1568	0.9697	1.2272	1.2684	1.7461				
B	1.1270	1.1131	1.2426	1.3117	2.0449				
C	1.1351	0.7341	0.8816	1.0965	0.8056				
D	0.9413	1.2632	1.0116	1.3469	1.6201				
					1.5542	=Average of products (buy&hold portfolio growth)			
Average:	1.0901	1.0200	1.0908	1.2559	1.5232	=Product of averages (rebalanced portfolio growth)			

On the other hand, if the four assets are rebalanced to a 25%-25%-25%-25% mix every year, then each year the portfolio's growth factor will be the equally-weighted average of the growth factors in that year's column (see last row in Table 2a). The rebalanced portfolio's growth factor over the four-year period will then be equal to the product of those annual average growth factors.

Table 2b shows the same information as in Table 2a but in terms of rates of return instead of growth factors.

Table 2b. Rates of return on four assets in four annual periods

	Year:				Compound return				
Asset:	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	(unann)	(annlized)			
A	15.68%	-3.03%	22.72%	26.84%	74.61%	14.95%			
B	12.70%	11.31%	24.26%	31.17%	104.49%	19.58%			
C	13.51%	-26.59%	-11.84%	9.65%	-19.44%	-5.26%			
D	-5.87%	26.32%	1.16%	34.69%	62.01%	12.82%			
				Average:	55.42%				
					11.65%	=Annualized average			
						(Buy-and-hold portfolio return)			
Average:	9.01%	2.00%	9.08%	25.59%	11.09%	=Compounded avgs, annualized			
						(Rebalanced portfolio return)			

The problem of comparing rebalancing to buy-and-hold then reduces to the following mathematical question. When is the product of averages greater than the average of products?

This problem is stated mathematically as follows.

Let G_{reb} , the growth factor for the rebalanced portfolio, equal

$$\prod_{j=1}^n \sum_{i=1}^m \frac{X(i,j)}{m}$$

and let G_{bh} , the growth factor for the buy-and-hold portfolio equal

$$\frac{\sum_{i=1}^m \prod_{j=1}^n X(i, j)}{m}$$

where $X(i, j)$ is the growth factor for the i 'th asset in the j 'th time interval. The \prod symbol means "product" and the \sum symbol means "sum".

Then the question is, when is G_{reb} greater than G_{bh} ($G_{reb} > G_{bh}$)?¹

One answer, at least, can be given. If all of the four-year growth factors (that is, $\prod_{j=1}^n X(i, j)$ for each i) are equal, then G_{reb} will be greater than or equal to G_{bh} . This is not easy to prove mathematically, though it was approximately proven by Bernstein and Wilkinson and can be confirmed by simulations. It is fairly easy, however, to show that it is true by means of an economic argument. Suppose that the growth of one of the assets, call it X, at the end of one of the years one through three is less than the growth of another asset, call it Y. Since we know that all the assets will end the four-year period with the same growth factor, the remaining growth of X must surely be greater than the remaining growth of Y. Therefore it will be advantageous to increase the investment in X relative to Y. (In fact it would be even more advantageous to move all of the holdings in investment Y to investment X than merely to rebalance the two.)

Unfortunately, it is very difficult to draw further general inferences about the relationship between G_{reb} and G_{bh} . I have been unable as yet to locate any rigorous mathematical analyses of this problem.

1. This is a completely general statement of the rebalancing vs. buy-and-hold problem even though in the mathematical statement the portfolios are equal-weighted. This is because to replicate an unequal-weighted portfolio one need only include multiple perfectly-correlated copies of each asset in the formula.