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Coevolution of synchronization and cooperation in real networks

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As game theory thrives in networked interactions, we usually neglect the cost of information exchange between involved individuals. Individuals may decide (or refuse) to follow the state of their neighbors, which depends on the cost of the interactions. The payoff of a node's behavior is associated with the state difference between the node and its neighbors. Here, based on Kuramoto model, we investigate the collective behavior of different individuals in the game theory and the synchronization byproduct that is induced by the cooperation of connected nodes. Specially, we investigate the influence of network structure on the coevolutionary progress of cooperation and synchronization. We find that the networks with the higher average degree are more likely to reach synchronization in real networks. Strong synchronization is a sufficient, but not necessary condition to guarantee the cooperation. Besides, we show that synchronization is largely influenced by the average degree in both Erdös–Rényi (ER) and Barabási–Albert (BA) networks, which is also illustrated by theoretical analysis.

Keywords: Kuramoto synchronization; cooperation; game theory; complex network.

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1. Introduction

Synchronization describes the phenomenon of how a set of units interacts with each other and reaches the same phase.[1,2] For instance, thousands of pedestrians have caused London's Millennium Bridge to rock on its opening day.[3] The bridge rock does not attribute to the structural drawback of the bridge, but rather to the coherent behavior of the pedestrians.[4] Investigating the collective behavior of a system thoroughly is theoretically prohibitive due to the nonlinear dynamics and the complex interactions of the units. For convenience, many previous works explored the

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synchronization mechanism in the regular grid networks and showed plenty of interesting achievements.[5–7] However, since individuals interact with each other following complex connections rather than strict regular grid networks, we are promoted to investigate the synchronization in complex networks including biological systems, engineering, social sciences and so forth.[8,9]

A large number of researches merely focus on the relationship between synchronization and network structures. Barahona and Pecora[10] consider the generic synchronization of oscillator networks with arbitrary topologies, and link the linear stability of the synchronous state to an algebraic condition that is related with the Laplacian matrices of the networks. Dörfler *et al.*[11] consider the synchronization problem for the network-reduced model of a power system with nontrivial transfer conductances. Lü and Chen[12] introduce a time-varying complex dynamical model to investigate synchronization phenomena and verify that the coupling matrix determines the synchronization. However, most previous works neglect the cost for information exchange between involved individuals, such as energy loss in power grid networks, traffic jam in vehicular traffic networks, the bandwidth limitation of communication networks and so on. When the interaction cost is introduced, individuals in networks face two choices, either pay the cost and cooperate with their neighbors or ride free without the cost and take no responsibility for others.[13] Some of the network connections may fail to pass information between nodes due to the free riding behavior. In such systems, the information exchange mechanism may lead to different synchronization. Therefore, the emergence of synchronization is regarded as the byproduct of the evolutionary game.[14] Antonioni and Cardillo[15] have reported how topology is essential for synchronization and cooperation and compared the results of Erdös–Rényi (ER), Barabási–Albert (BA) and Random Geometric Graphs (RGG) networks. However, there remains a problem of how the coevolution of cooperation and synchronization performs in real networks.

Here, we address the coevolution in real networks with complex network structures. We utilize an advanced Kuramoto synchronization in which the information exchange bears a cost for an edge. Individuals may decide to cooperate (or ride free) with their neighbors based on the cost of their connected edges. Then, we analyze the synchronization strength and the fraction of individuals who are likely to cooperate with their neighbors. We find that the average degree of networks is not essential to the cooperation, but plays an important role on the synchronization. Higher average degree induces stronger synchronization. In order to illustrate the validity of the results, we conduct our experiments in both ER and BA networks. With the increase of average degree, BA and ER networks show stronger synchronization while the cooperation rarely changes. Besides, we analyze the threshold of synchronization and show the difference between simulation and theoretical results.

The organization of the paper is as follows: In Sec. 2, we describe the network models and the synchronization. In Sec. 3, we describe the datasets and compare the experimental performance in real and model networks. Besides, we theoretically analyze the threshold of synchronization. At last, the conclusion is given in Sec. 4.

2. Coevolutionary Model

In this section, ER and BA models are introduced. Then coevolutionary synchronization is presented.

2.1. *ER and BA models*

ER model[16,17] is a classical random graph model: Given a graph $G(N, E)$ with N nodes and E edges, pairwise nodes have the probability p to connect together. When $n \gg p$, the ER networks follow the Poisson degree distribution, such that $p(k) = \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$, where $\langle k \rangle$ is the average degree of the whole network. Though ER networks are much different from real networks, for example, ER networks have no community and hierarchal structures, ER networks could reproduce some important properties such as small-world (SW) phenomenon and small clustering coefficient of real networks.

The degree distribution is an important property when analyzing the network structure. Since real networks (e.g. Internet, the World Wide Web and social networks) usually follow power-law (or similar) degree distribution, we are encouraged to use BA network model to reproduce the power-law degree distribution in model networks. BA model could generate networks by a preferential attachment mechanism[18] that is crucial for the power-law degree distribution. The BA model networks could also reproduce the SW phenomenon.[8] Unlike ER networks, BA networks are extremely heterogeneous. Most small degree nodes in BA networks are hinged by a few hub nodes.

2.2. *Coevolutionary method*

We consider the Kuramoto synchronization in an undirected, unweighted network $G(N, E)$ with N nodes and E edges. The network is usually represented by the adjacent matrix $A = (a_{ij})_{N \times N}$ with $a_{ij} = 1$ if $(i, j) \in E$, $a_{ij} = 0$ otherwise. In the networked synchronization, each node is a dynamical unit that interacts with its neighbors. Since every node has different frequency, the whole system may be chaotic initially. All the nodes may arrive at the coherent phase if the coupling strength satisfies some criteria.[7]

We adopt a modified Kuramoto model. In the networks, the state θ_l of a node l is determined by three ingredients: its strategy s_l, the intrinsic frequency ω_l and the phase difference between the node and its neighbors. When the strategy $s_l = 1$ ($s_l = 0$), the node l is a cooperator (defector) and prefer (not) to exchanging information with its neighbors. The modified Kuramoto model follows

$$\dot{\theta}_l = \omega_l + s_l \lambda \sum_{j=1}^{N} a_{ij} \sin(\theta_j - \theta_l), \tag{1}$$

where the intrinsic frequency ω_l and the initial θ_l are uniformly distributed over the interval $[-\pi, \pi]$, λ is the coupling strength and a_{ij} are the elements of the adjacency matrix A. The order parameter r_G is utilized to evaluate the collective synchronization, which is

$$r_G e^{i\psi} = \frac{1}{N} \sum_{N}^{j=1} e^{i\theta_j} \quad \text{with } r_G \in [0,1], \tag{2}$$

where ψ is the average phase of the system. When $r_G = 1$ ($r_G = 0$), the entire system is fully synchronized (chaotic) whereas r_G only characterizes the synchronization of the whole nodes. In order to characterize the synchronization of a node l with its neighbors, a parameter r_l is defined as

$$r_l = \frac{\sum_{m=1}^{N} a_{lm} r_{lm}}{\sum_{m=1}^{N} a_{lm}}, \tag{3}$$

where $r_{lm} e^{i(\theta_l+\theta_m)/2} = \frac{e^{i\theta_l}+e^{i\theta_m}}{2}$. In contrast with r_G, r_l represents how a node synchronizes with its neighbors. Higher r_l means stronger local synchronization. High r_G usually means high r_l. However, when r_l is high, r_G may be also small.

In traditional synchronization model, nodes communicate with their neighbors, neglecting the cost of the information exchange. Here, we suppose that a node may benefit and pay the cost for the communication. The problem of whether a node accepts the information from its neighbors depends on the balance between the benefit and the cost. If a node benefits more than the cost, it is likely to be a cooperator; otherwise a defector. In the Kuramoto model of Eq. (1), the benefit b_l and the cost c_l of node l are characterized by r_l in Eq. (3) and the absolute value of the angular acceleration $\Delta\dot{\theta}_l$, respectively. The former benefit b_l describes the convergence of an oscillator while the latter cost c_l is the power to drive the state of node l. Synthesizing the benefit b_l and the cost c_l, we introduce the payoff Π_l for node l to determine its behavior,

$$\Pi_l = r_l - \alpha \frac{c_l}{2\pi}, \tag{4}$$

where α is a tunable parameter to adjust the role of the cost. The difference of the payoff Π between two nodes is utilized to update the strategy of every unit by Fermi rules.[19,20] Fermi rule is based on Fermi distribution function[21,22] and indicates that the player l randomly chooses one of its neighbors and follows its strategy by the probability: $P(s_l \leftarrow s_m) = 1/(1 + e^{-\beta(\Pi_m - \Pi_l)})$, where β is a constant, representing noise of the game ("irrationality" of players). Note that β rarely influences the final state of the synchronization[15] and we do not discuss β much in the paper.

In the experimental simulation, we use discrete synchronous updating scheme to upgrade the state of nodes. We divide the time into small time spans. At every step, benefit and cost are calculated. The payoff is utilized to upgrade its strategy s_l accordingly. We mainly investigate the final stable state of the synchronization and cooperation of the system.

3. Results

In the section, we first describe the datasets and then show the performance of different networks when using the coevolutionary method. At last, we conduct experiments in ER and BA networks and explain the difference between simulation and theoretical analysis.

3.1. *Datasets*

We consider six real networks: twitter network,[23] Facebook network,[24,25] bitcoin network,[26] power grid network,[27] road network[28,29] and CAIDA[30] (Center for Applied Internet Data Analysis) network. The statistical characteristics of the networks are shown in Table 1.

The Facebook network consists of "circles" ("friends lists") from Facebook. Facebook data are collected from survey participants using Facebook app. A Facebook user is considered as a node and the edges describe the friendship among users. Twitter network consists of "circles" (as well as "lists" in Facebook) from Twitter. Twitter data are crawled from public sources. Every account is treated as a node and edges are the follower relationship. As for bitcoin network, this is a who-trusts-whom network of people who trade using bitcoin on a platform called Bitcoin OTC. Although bitcoin users are anonymous, we do not consider users' reputation to prevent transactions with fraudulent and risky users. Instead, we discuss the accessing act among sources, which is the node in bitcoin network. Power grid network and road network are infrastructure networks. In power grid network, an edge and a node represent a power supply line and a generator in the power grid network. In the road networks, a node and an edge represent a city and a road between cities connected, respectively. CAIDA network represents AS relationships and contains 122 AS graphs from January 2004 to November 2007.

3.2. *Parameter settings*

Without the loss of generality, we randomly set half of the nodes in networks as cooperators, $s_l = 1$. Note that the initial value s_l does not affect the results, we only

Table 1. Structural properties of the different real networks, including network size (N), edge number (E), average degree $\langle k \rangle$, degree heterogeneity ($H = \langle k^2 \rangle / \langle k \rangle^2$) and the degree assortativity (r).

Networks	N	E	$\langle k \rangle$	H	r
Facebook	744	30 023	80.7070	1.6330	0.5026
Twitter	9725	171 482	35.2662	2.7790	−0.4932
Bitcoin	3775	14 120	7.4808	8.1993	−0.1687
CAIDA	26 475	53 381	4.0326	69.4951	−0.1946
Power grid	4941	6594	2.6691	1.4504	0.0035
Road	1174	1417	2.4140	0.1267	0.0042

focus on the final stable state of the system. Moreover, we set the time span $\epsilon = 0.01$ to update the payoff of all nodes.

3.3. *Experimental results*

Figures 1 and 2 describe the cooperation and synchronization as a function of α and λ in real networks. In Fig. 1, networks are synchronized when $\log(\alpha) \in (-2, 2)$ and $\log(\lambda) \in (0, 2)$, while networks in Fig. 2 are barely synchronized whatever α and λ change. Specially, Facebook network shows strongest synchronization in these networks. We have $r_G(\text{facebook}) > r_G(\text{twitter}) > r_G(\text{bitcoin}) > r_G(\text{caida}) > r_G(\text{road}) \approx r_G(\text{power grid})$. Comparing Figs. 1 and 2, social networks are more likely to reach synchronization than infrastructure networks whereas the cooperator proportion C, α and λ exert little influence on the two factors and the six networks have similar cooperation performance on the whole except Facebook network.

A problem arises that how the coevolution is influenced by the network structure? In completely random networks, the coevolution mechanism could be analyzed by the Mean Field Theory.[31] According to the theoretical analysis in Ref. 15, the coevolution is mainly determined by the degree distribution. In the empirical experiments, we also compare the influence of heterogeneity, assortativity and other factors on the coevolution and find that the average degree impacts the coevolution much stronger than other factors. Consequently, we mainly discuss the influence of average degree on the coevolution in the following experiments. In order to find out how the average degree influences the synchronization, we conduct our experiments in ER and BA networks with different average degree.

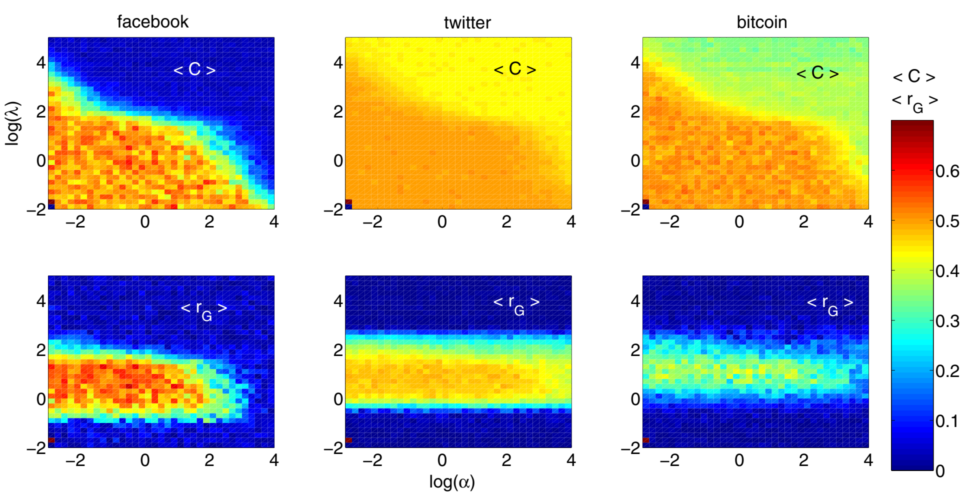


Fig. 1. (Color online) The cooperation and synchronization of networks in real social networks. The top (bottom) row describes the average level of cooperation (synchronization) $\langle C \rangle$ ($\langle r_G \rangle$) as a function of the parameter α and the coupling strength λ, where $\langle C \rangle$ is the fraction of cooperators. Each column stands for a different network, Facebook, twitter and bitcoin. The results are averages over 20 times.

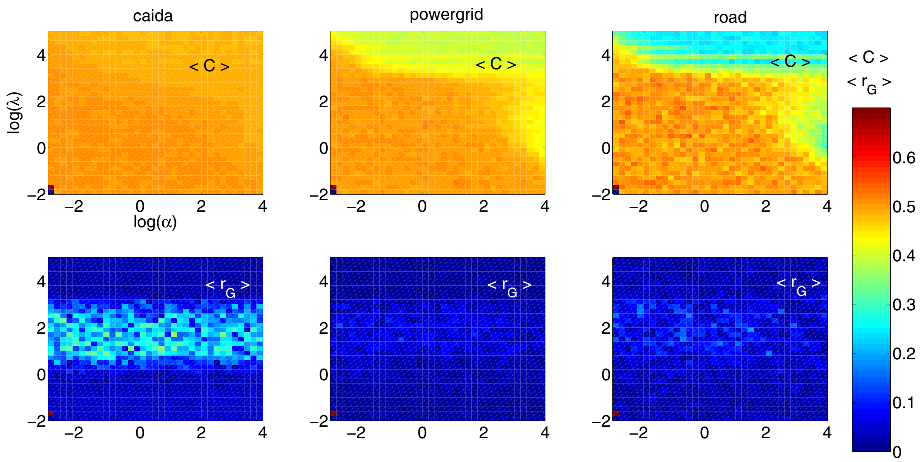


Fig. 2. (Color online) The cooperation and synchronization of networks in real infrastructure networks. The top (bottom) row describes the average level of cooperation (synchronization) $\langle C\rangle$ ($\langle r_G\rangle$) as a function of the parameter α and the coupling strength λ, where $\langle C\rangle$ is the fraction of cooperators. Each column stands for a different network, CAIDA, power grid and road. The results are averages over 20 times.

In ER networks (Fig. 3), we show the influence of average degree on the cooperation and synchronization. The synchronization strength r_G increases with $\langle k\rangle$ and is positively correlated with the average degree whereas the cooperation is somewhat different from synchronization. Cooperation parameter $\langle C\rangle$ increases with

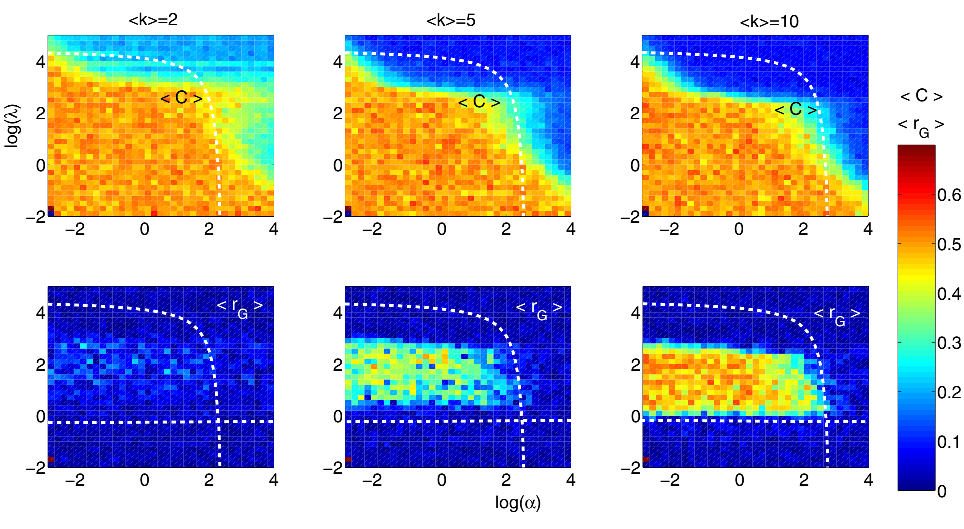


Fig. 3. (Color online) Emergence of cooperation and synchronization of ER networks. The top (bottom) row describes the average level of cooperation (synchronization) $\langle C\rangle(\langle r_G\rangle)$ as a function of the relative cost α and the coupling strength λ. Each column stands for an ER network with different average degrees, $\langle k\rangle = 2$, $\langle k\rangle = 5$ and $\langle k\rangle = 10$. The white-dotted lines in the panels are the theoretical threshold α of the cooperation that are calculated by Eq. (5). The results are averages over 20 times.

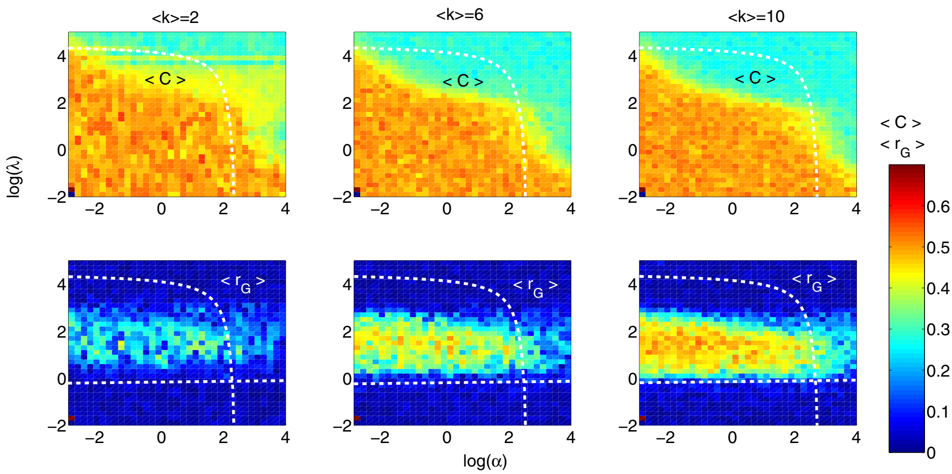
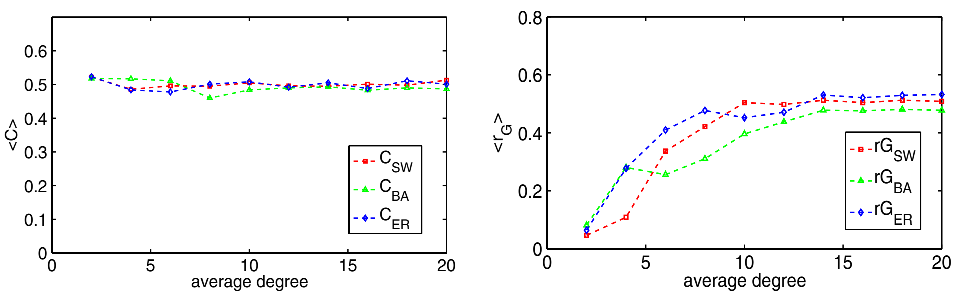


Fig. 4. (Color online) Emergence of cooperation and synchronization of BA networks. The top (bottom) row describes the average level of cooperation (synchronization) $\langle C\rangle$ ($\langle r_G\rangle$) as a function of the relative cost α and the coupling strength λ. Each column stands for an ER network with different average degrees, $\langle k\rangle = 2$, $\langle k\rangle = 5$ and $\langle k\rangle = 10$. The white-dotted lines in the panels are the theoretical threshold α of the cooperation that are calculated by Eq. (5). The results are averages over 20 times.

$\langle k\rangle$ for small $\langle k\rangle$. However, when $\langle k\rangle$ is large (for example $\langle k\rangle > 5$), $\langle C\rangle$ fluctuates little with the change of $\langle k\rangle$ (see the scenarios of $\langle k\rangle = 5$ and $\langle k\rangle = 10$ in Figs. 3 and 4). Then in Fig. 5, we discuss the cooperation and synchronization of model SW, ER and BA networks as a function of average degree under the case of $\alpha = 1$ and $\lambda = 10$. Note that SW networks show similar performance with other two kinds of networks. We can see that the cooperator proportion $\langle C\rangle$ in SW, BA and ER networks is close to 0.5, and fluctuate little with the average degree $\langle k\rangle$. However, synchronization $\langle r_G\rangle$ of SW, BA and ER networks is sensitive to the average degree $\langle k\rangle$.



(a) (b)

Fig. 5. (Color online) (a) Cooperator fraction $\langle C\rangle$ in SW, ER and BA networks as a function of the average degree when $\alpha = 1$ and $\lambda = 10$. (b) Collective synchronization $\langle r_G\rangle$ in SW, ER and BA networks as a function of the average degree when $\alpha = 1$ and $\lambda = 10$. The results are averages over 20 times.

Furthermore, we explain how the average degree influences the cooperation. Antonioni and Cardillo[15] derive the cooperation threshold of α,

$$\frac{\sqrt{2+2\sin(\epsilon\lambda)}-\sqrt{2}}{\epsilon\lambda\langle k\rangle}\pi > \alpha, \tag{5}$$

where ϵ is the time span, λ is the coupling strength and α is the tunable parameter in the payoff. Equation (5) is obtained from random homogeneous networks[14] and holds on SW and ER networks. However, we find that Eq. (5) is largely suitable for heterogeneous networks in the simulation.

Cooperation emerges when α satisfies Eq. (5). With the increase of the average degree $\langle k\rangle$, the upper limit of α (i.e. the left-hand side of Eq. (5)) is reduced, meaning that the benefit b_l contributes more to the payoff Π_l than the cost. Based on the Fermi rule, large Π_l promotes the cooperation between nodes. Furthermore, more cooperator nodes mean that more edges participate in the information exchange, and hence, the collective synchronization is promoted. However, Eq. (5) is not a strict threshold to characterize the cooperation. Based on the empirical analysis, the large average degree means the more edges and information exchange, which benefits collective synchronization. Besides, we plot the theoretical threshold in Figs. 3 and 4 (the white-dotted line), which illustrates the validity of our analysis.

4. Conclusions

In conclusion, we investigate the coevolution of cooperation and synchronization in both artificial and real networks based on the advanced Kuramoto model. Different from previous synchronization researches that mostly investigate the influence of network structures on synchronization, we consider that information exchange bears the cost for every edge. The cost mechanism brings about the cooperation and free-riding behavior of individuals. In our synchronization model, we particularly observe the cooperation fraction and synchronization strength and the relationship between the two. We find that networks with the higher average degree are more likely to reach synchronization. However, cooperation behavior is irrelevant to the average degree. Strong synchronization is a sufficient, but not necessary condition to guarantee the high cooperation. We theoretically explain the phenomenon in both ER and BA model networks.

In this paper, we mainly consider the influence of average degree on the synchronization. However, other network structures (e.g. community structure, assortativity) may also influence the cooperation and synchronization. Our experiments show that other factors influence the coevolution much less than average degree factor in most cases, which we do not show in this paper. Therefore, our work provides a deep understanding of coevolution problem and may help better analyze the nonlinear dynamics in real networks.

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