MODULE TWO PROBLEM SET

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Directions: Type your solutions into this document and be sure to show all steps for arriving at your solution. Just giving a final number may not receive full credit.

Problem 1

Part 1. Indicate whether the argument is valid or invalid. For valid arguments, prove that the argument is valid using a truth table. For invalid arguments, give truth values for the variables showing that the argument is not valid.

(1)

$$\boxed{\begin{array}{c} (p \wedge q) \to r \\ \\ \therefore (p \vee q) \to r \end{array}}$$

I used a truth table and I found that the last two columns of the table were not equal, thus making this invalid.

p	q	r	$p \wedge q \to r$
$p \vee q \to r$ heightT	T	T	T
T T	T	F	F
F T	F	T	T
T T	F	F	T
T F	T	T	T
T F	T	F	T
T F	F	T	T
T F	F	F	T
T			

Part 2. Converse and inverse errors are typical forms of invalid arguments. Prove that each argument is invalid by giving truth values for the variables showing that the argument is invalid. You may find it easier to find the truth values by constructing a truth table.

(a) Converse error

$$\boxed{\begin{array}{c} p \to q \\ q \\ \\ \therefore p \end{array}}$$

The truth table I have laid out shows that two columns are not equal.

The argument is invalid because (p.q) $\wedge q \not\equiv p$

Inverse error

$$\begin{array}{c} p \to q \\ \neg p \\ \\ \therefore \neg q \end{array}$$

Per the truth table as you can see here, the values of p are false and thus, the argument is invalid.

Part 3. Which of the following arguments are invalid and which are valid? Prove your answer by replacing each proposition with a variable to obtain the form of the argument. Then prove that the form is valid or invalid.

(a)

The patient has high blood pressure or diabetes or both.
The patient has diabetes or high cholesterol or both.
∴ The patient has high blood pressure or high cholesterol.

The patient has high BP (p) or Diabetes (q) or both = p v q The patient has diabetes or high cholesterol (r) or both = q v r making (p v q) v (q v r) = p v q v r which means that the given statement to be not valid.

PROBLEM 2

Part 1. Which of the following arguments are valid? Explain your reasoning.

(a) I have a student in my class who is getting an A. Therefore, John, a student in my class, is getting an A.

John is not the student i.e j s could be true Therefor $\neg A(j)$ could be true and the given statement is invalid. Yes this argument is invalid.

Every Girl Scout who sells at least 30 boxes of cookies will get a prize. Suzy, a Girl Scout, got a prize. Therefore, Suzy sold at least 30 boxes of cookies.

The first hypothesis is that "For every x, S(x)-¿ P(x)" where the predicate S(x) means that x has sold at least 30 boxes of cookies and P(x) means that x got a prize.G:A Girl Scout.

C:Selling 50 boxes of cookies.

P: is getting prize.

s: Suzy

Now every girl scout who sells at least 50 boxes of cookies will get a prize: x G, C(x) ¿P(x).Consider a scenario in which there is a girl scout troop for which "For every x, S(x)-¿ P(x)" is true. Furthermore, there is a girl named Suzy in the troop such that P(Suzy) is true and S(Suzy) is false. Then all the hypothesis are true but the conclusion is false. Therefore the argument is invalid.

Part 2. Determine whether each argument is valid. If the argument is valid, give a proof using the laws of logic. If the argument is invalid, give values for the predicates P and Q over the domain a, b that demonstrate the argument is invalid.

(a)

$$\boxed{\begin{array}{c} \exists x\,(P(x)\ \wedge\ Q(x)) \\ \\ \therefore \exists x\,Q(x)\ \wedge\ \exists x\,P(x) \end{array}}$$

$(\exists x)(P(x) \wedge Q(x))$	Premise
$P(y) \wedge Q(y)$	Existential instantiation 1
$P(y)\wedge$	Simplification from 2
$Q(y)\wedge$	Simplification from 2
$(\exists x)P(x)$	Existential Generalization 3
$(\exists x)Q(x)$	Existential generalization from 4
$((\exists x)Q(x)) \wedge ((\exists x)(P(x))$	Intro. from $5, 6$

the given argument is valid.

(b)

$$\boxed{\begin{array}{c} \forall x\,(P(x)\ \vee\ Q(x)) \\ \\ \therefore \forall x\, Q(x)\ \vee\ \forall x\, P(x) \end{array}}$$

x(P(x)Q(x)) = x Q(x) x P(x) x(P(x)Q(x)) P(c) Q(c) The original argument tries to simplify the expression x(P(x)Q(x)) into x Q(x) x P(x), but this simplification is not valid due to violating the distributive property of logical operators. The simplified expressions x P(x) and x Q(x) separately do not capture the original meaning of the conjunction of the two conditions, therefore, without a valid simplification, we cannot equate the two sides of the equation in the original argument, and the argument is not valid.

Problem 3

Prove the following using a direct proof. Your proof should be expressed in complete English sentences.

If a, b, and c are integers such that b is a multiple of a^3 and c is a multiple of b^2, then c is a multiple of a^6.

If $a,\ b,\ c$ are integers such that []b is a multiple of a^3 and c is a multiple of b^2, then c is a multiple of a^6.

a,b,c are theree integers such that b is a multiple of a3. b=ka3 where k is an integer and c is a multiple of b2 and c=k1b2 where as k is an integer or c=k1 (ka3)2 or c=k1k2a6 or c=k1a6 where k1 k2= k is an integer. C is then a multiple of a6.

Problem 4

Prove the following using a direct proof:

The sum of the squares of 4 consecutive integers is an even integer.

The sum of squares of 4 consecutive integers is an even integer.

Selecting 4 consecutive positive integers: 5, 6, 7, 8. Then:
=25

=36

=49

=64
The sum of the squares =25 + 36 + 49 + 64 = 174
Also,
Selecting 4 consecutive negative integers:-10, -11, -12, -13.Then;

=100
=121
=144
=169
The sum of the squares is =100 + 121 + 144 + 169

=534
Therefore, the sum of the squares of 4 consecutive integers is an even integer.

Problem 5

Prove the following using a proof by contrapositive:

Let x be a rational number. Prove that if xy is irrational, then y is irrational.

(a) prove that -y is irrational

(b) prove that x-y is irrational

(c) prove that y-x is irrational

(d) prove that if x not equal to 0, then xy is irrational.

There is a possibility of a different proof for part d that does not use the condition that x is not equal to 0 but still leads to the conclusion that xy is irrational.

prove that if x is not equal to zero, then x/y is irrational.

prove that if x not equal to 0, then y/x is irrational.

Problem 6

Prove the following using a proof by contradiction:

> The average of four real numbers is greater than or equal to at least one of the numbers.The average of four real numbers is greater than or equal to at least one of the numbers. Let the four numbers be: p,q,r,s. The average will be Sum of all numbers divided by count of numbers. p+q+r+s/4, but let's assume the opposite; the average is less than the all of the numbers. p+q+r+s/4 ¡p p+q+r+s/4 ¡q p+q+r+s/4 ¡r p+q+r+s/4 ¡s. Once we add these up, p+q+r+s/4 ¡p+q+r+s = 4p+q+r+s/4 ¡p+q+r+s = p+q+r+s ¡p+q+r+s. This will never be possible due to our assumption is contradicted, and our assumption is wrong. Due to our assumption being contradicted, the average of four real numbers will be greater than or equal to at least one of the four real numbers.

Problem 7

Let $q = \dfrac{a}{b}$ and $r = \dfrac{c}{d}$ be two rational numbers written in lowest terms. Let $s = q + r$ and $s = \dfrac{e}{f}$ be written in lowest terms. Assume that s is not 0.

Prove or disprove the following two statements.

a. If b and d are odd, then f is odd.

b. If b and d are even, then f is even.

q=a/b and r=c/d.

s=q+r

s=a/b + c/d = (ad+bc)/bd = e/f. e= ad+bc and f=bd.

- if b and d are odd we know that product of two odd numbers is always odd and therefore f= b*d is odd. We can prove this because b*d =b*(d-1+1) b*(d-1+1) = b*(d-1) + b since d is odd d-1 is even and therefore b*(d-1) is even. Since b is odd and sum of an odd number and even number is always odd the above product is odd.

- if b and d are even we know that product of two even numbers is even and therefore f= b*d is even.

PROBLEM 8

Define $P(n)$ to be the assertion that:

$$\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}$$

(a) Verify that $P(3)$ is true.

is true, 1+4+9=3*4*7/6 14=14.

(b) Express $P(k)$.

p/k is 11+22+32+ → +(k-1)2 +k2=(k*(k+1)(2*k+1))/6.

- Express $P(k+1)$.

11+22+32+ → +(k-1)2+k2=(k*(k+1)*(k+1+1)(2*(k+1)+1))/6.

In an inductive proof that for every positive integer n,

$$\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}$$

- what must be proven in the base case?

For this case we can prove that the formula is true, n=1 1=1.

(c) In an inductive proof that for every positive integer n,

$$\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}$$

what must be proven in the inductive step?

The induction proof must start with the base case. 1. True for n=1 base case then we can say that it is true for n=k 2. Turn for n=k we can prove n=k+1 and it's known as inductive step 3. Show true for n=k+1 it's an inductive step.

(d) What would be the inductive hypothesis in the inductive step from your previous answer?

Our assumptions n=k known as induction hypothesis by that we say it's ture as well for n=k+1.

(e) Prove by induction that for any positive integer n,

$$\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}$$

We know that n=k is trtue, k=k is true, n=k+1 is true, sk+ak+1 =k(k+1)(2k+1)/6+ak+1 we need a common denominator so we need to multiply by 6 now simplify factor it out sk+1=(k+1[k(2k+1)+6(k+1)]/6 now factor in 2k2+7k+6 sk+1=(k+1)(k+2)(2k+3)/6.The principal of mathmatical induction 1+4+9+......+n2=n2(n+1)(2n+1)/6 for all positive integers n.