MODULE SIX PROBLEM SET

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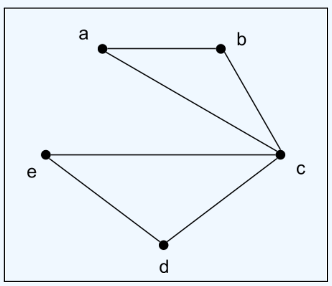
Directions: Type your solutions into this document and be sure to show all steps for arriving at your solution. Just giving a final number may not receive full credit.

Problem 1

(1)

For parts (a) and (b), indicate if each of the two graphs are equal. Justify your answer.

(a)



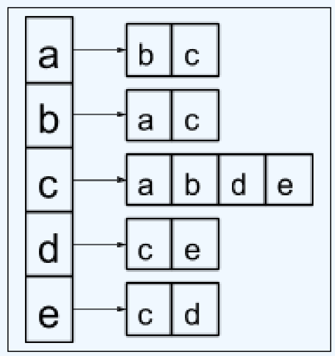


Figure 1: *Left: An undirected graph has 5 vertices. The vertices are arranged in the form of an inverted pentagon. From the top left vertex, moving clockwise, the vertices are labeled: a, b, c, d, and e. Undirected edges, line segments, are between the following vertices: a and b; a and c; b and c; c and d; e and d; and e and c.*

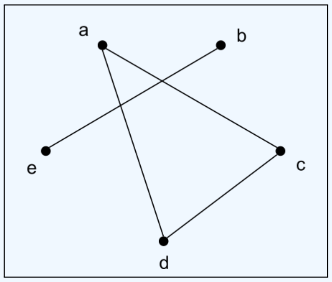
Figure 2: *Right: The adjacency list representation of a graph. The list shows all the vertices, a through e, in a column from top to bottom. The adjacent vertices for each vertex in the column are placed in a row to the right of the corresponding vertexs cell in the column. An arrow points from each cell in the column to its corresponding row on the right. Data from the list, as follows: Vertex a is adjacent to vertices b and c. Vertex b is adjacent to vertices a and c. Vertex c is adjacent to vertices a, b, d, and e. Vertex d is adjacent to vertices c and e. Vertex e is adjacent to vertices c and d.*

The figure 1 on the left is a visual representation of an undirected graph with 5 vertices labeled, "a,b,c,d,and e", vertices are arranged in a way that they make the shape of an inverted pentagon, and undirected edges, line segments connect certain pairs of vertices as followed: "a-b, a-c, b-c, c-d, e-d, and e-c".AS you can

see, the first graph is denoted by (i) and the second graph is represented by (ii). These two graphs show adjacency in a separate set organized manner.

Implications can be drawn from the connections made suggesting the adjacent graph to be true, and valid. Vertex "V" is considered adjacent to the vertex "U" if there exists a direct edge between "v" and "u" in the graph. "M" $\wedge$ "V"

(1)



$$\begin{pmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

Figure 3: *An undirected graph has 5 vertices. The vertices are arranged in the form of an inverted pentagon. Moving clockwise from the top left vertex a, the other vertices are, b, c, d, and e. Undirected edges, line segments, are between the following vertices: a and c; a and d; d and c; and e and b.*

The first graph is represented by (i) while the second graph is represented by (ii). aij $--> 1$ if there is an edge. 0 if there is no edge.
The adjacency matrix for (i) follows:

$$\begin{array}{c} 0 \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{pmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

There are no similarities between both graphs as indicated by (e.d) and as such the adjacency is False.

(1) Prove that the two graphs below are isomorphic.

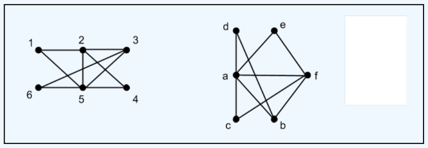


Figure 4: *Two undirected graphs. Each graph has 6 vertices. The vertices in the first graph are arranged in two rows and 3 columns. From left to right, the vertices in the top row are 1, 2, and 3. From left to right, the vertices in the bottom row are 6, 5, and 4. Undirected edges, line segments, are between the following vertices: 1 and 2; 2 and 3; 1 and 5; 2 and 5; 5 and 3; 2 and 4; 3 and 6; 6 and 5; and 5 and 4. The vertices in the second graph are a through f. Vertices d, a, and c, are vertically inline. Vertices e, f, and b, are horizontally to the right of vertices d, a, and c, respectively. Undirected edges, line segments, are between the following vertices: a and d; a and c; a and e; a and b; d and b; a and f; e and f; c and f; and b and f.*

The two graphs that you see here, (i) and (ii) are isomorphic because they have the same number of vertices and edges and if there is vertices can be relabeled in a way that preserves the adjacency and degree relationships. Graph (i) and (ii) in question have 6 vertices and 9 degrees each, they satisfy the conditions for homomorphic as their vertices have equivalent and adjacent degrees relationships that two graphs, (i) and (ii) are in fact homomorphism.

(i), (adj.degree),(ii)
(1,2),(4,5),(e)
(2,4),(2,2,3,5),(f)
(3,3),(2,4,5),(b)
(4,2),(4,5),(e)
(5,5),(2,2,2,3,4) (a)
(6,2),(3,5) (d)

(1) Show that the pair of graphs are not isomorphic by showing that there is a property that is preserved under isomorphism which one graph has and the other does not.

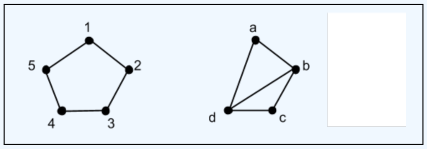


Figure 5: *Two undirected graphs. The first graph has 5 vertices, in the form of a regular pentagon. From the top vertex, moving clockwise, the vertices are labeled: 1, 2, 3, 4, and 5. Undirected edges, line segments, are between the following*

vertices: 1 and 2; 2 and 3; 3 and 4; 4 and 5; and 5 and 1. The second graph has 4 vertices, a through d. Vertices d and c are horizontally inline, where vertex d is to the left of vertex c. Vertex a is above and between vertices d and c. vertex b is to the right and below vertex a, but above the other two vertices. Undirected edges, line segments, are between the following vertices: a and b; b and c; a and d; d and c; d and b.

To show that two graphs are not isomorphic, we nee to find a property that is preserved under isomorphism, but one graph has while the second graph does not.
(i), (ii), do not have the same amount of vertices, degrees, they are not the same, thus making these graphs not isomorphic.

Problem 2

Refer to the undirected graph provided below:

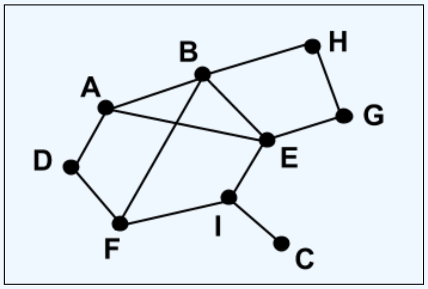


Figure 6: *An undirected graph has 9 vertices. 6 vertices form a hexagon, which is tilted upward to the right. Starting from the leftmost vertex, moving clockwise, the vertices forming the hexagon shape are: D, A, B, E, I, and F. Vertex H is above and to the right of vertex B. Vertex G is the rightmost vertex, below vertex H and above vertex E. Vertex C is the bottom most vertex, a little to the right of vertex E. Undirected edges, line segments, are between the following vertices: A and D; A and B; B and F; B and H; H and G; G and E; B and E; A and E; E and I; I and C; I and F; and F and D.*

(i) What is the maximum length of a path in the graph? Give an example of a path of that length.

The maximum path length in graph provided is 8.A path is open with no repeated edges or vertices.
As long as the start and ending vertices are different.
$D \to A \to B \to H \to G \to E \to I \to C$
This path has a length of 7(with 8 vertices).

(ii) What is the maximum length of a cycle in the graph? Give an example of a cycle of that length.

A cycle in a graph is defined as a set of edges and vertices that meet both at source and destination. Max length is 8.
$I \to E \to G \to H \to B \to A \to D \to F \to I$.

(iii) Give an example of an open walk of length five in the graph that is a trail but not a path.

A trail is defined as a walk in which edges can not be repeated but vertices can be repeated unless otherwise specified.
The sequence is: $E \to B \to A \to E \to I \to F$.
The sequence is an open trail of length of 5 in graph.

It's not a path due to the vertex E is being repeated, but it is trail because the edge is not repeated.

(iv) Give an example of a closed walk of length four in the graph that is not a circuit.

$B \to A \to D \to A \to B$ **shows that this is a close walk, that is defined by having the source and the destination meeting**
A circuit is a trail with vertices repetition but not the edges, they don't repeat.

(v) Give an example of a circuit of length zero in the graph.

Circuit refers to a closed path in a graph, where the path is a sequence of vertices connected by edges. We can consider the circuit length to be "0" as there are no edges in the circuit. We are left with a single isolated vertex. Option "C" is an isolated vertex.

Problem 3

(a) Find the connected components of each graph.

(i) $G = (V, E)$. $V = \{a, b, c, d, e\}$. $E = \emptyset$

Both points ”U” and ”V” of graph G are to be connected if ”$U \to\in$ G”.
Graph G is to be connected if every pair of points is connected where as graph U is not connected, but disconnected. Every vertex U is an individual component as there are 5 total components. (i)a, (ii)b,(iii)c, (iV)d, (v)e.

(1) $G = (V, E)$. $V = \{a, b, c, d, e, f\}$. $E = \{\{c, f\}, \{a, b\}, \{d, a\}, \{e, c\}, \{b, f\}\}$

$G = (V, E)V = [a, b, c, d, e, f], E = [[c, f], [a, b], [d, a], [e, c], [b, f]].$

The graph is connected as there exist a path between all vertices in the path d,a,b,c,f,e.

(1) Determine the edge connectivity and the vertex connectivity of each graph.

(i)

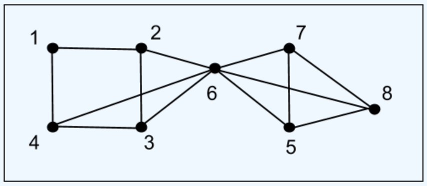


Figure 7: *An undirected graph has 8 vertices, 1 through 8. 4 vertices form a rectangular-shape on the left. Starting from the top left vertex and moving clockwise, the vertices of the rectangular shape are, 1, 2, 3, and 4. 3 vertices form a triangle on the right, with a vertical side on the left and the other vertex on the extreme right. Starting from the top vertex and moving clockwise, the vertices of the triangular shape are, 7, 8, and 5. Vertex 6 is between the rectangular shape and the triangular shape. Undirected edges, line segments, are between the following vertices: 1 and 2; 2 and 3; 3 and 4; 4 and 1; 2 and 6; 4 and 6; 3 and 6; 6 and 7; 6 and 8; 6 and 5; 7 and 5; 7 and 8; and 5 and 8.*

Edge connectivity is ”2”, while vertex connectivity is ”1”.

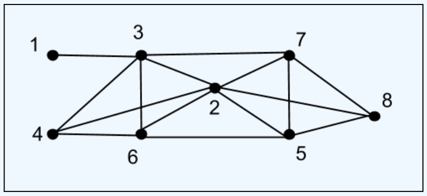
(ii) 

Figure 8: *An undirected graph has 8 vertices, 1 through 8. 4 vertices form a rectangular shape in the center. Starting from the top left vertex and moving clockwise, the vertices of the rectangular shape are, 3, 7, 5, and 6. Vertex 2 is at about the center of the rectangular shape. Vertex 8 is to the right of the rectangular shape. Vertex 1 and 4 are to the left of the rectangular shape, horizontally in-line with vertices 3 and 6, respectively. Undirected edges, line segments, are between the following vertices: 1 and 3; 3 and 7; 3 and 4; 3 and 6; 3 and 2; 4 and 2; 4 and 6; 6 and 2; 6 and 5; 2 and 5; 2 and 7; 2 and 8; 7 and 5; 7 and 8; and 5 and 8.*

Edge connectivity is "1", while the vertex connectivity is "1".

PROBLEM 4

For parts (a) and (b) below, find an Euler circuit in the graph or explain why the graph does not have an Euler circuit.

(a)

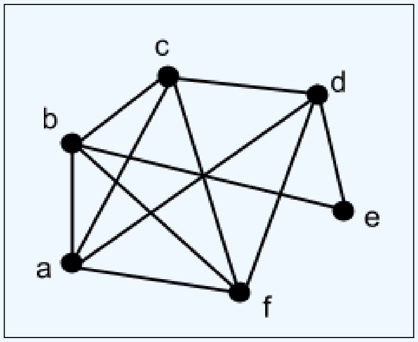


Figure 9: *An undirected graph has 6 vertices, a through f. 5 vertices are in the form of a regular pentagon, rotated 90 degrees clockwise. Hence, the top vertex becomes the rightmost vertex. From the bottom left vertex, moving clockwise, the vertices in the pentagon shape are labeled: a, b, c, e, and f. Vertex d is above vertex e, below and to the right of vertex c. Undirected edges, line segments, are between the following vertices: a and b; a and c; a and d; a and f; b and f; b and c; b and e; c and d; d and e; and d and f. Edges c f, a d, and b e intersect at the same point.*

Euler circuit is true and exists, due to the vertices being all even.

$E \to B \to A \to C \to F \to B \to C \to D \to F \to A \to D \to E$.
Yes this graph satisfy the Euler Circuit path and I will explain why.Every vertex in the above graph, Figure 9:, has an even degree. This graph has a degree of 6, an even number, therefore, the Euler circuit exist in this graph which each vertex is even and is a path that visits every edges of the graph exactly once and starts and ends at the same vertex.

(1)

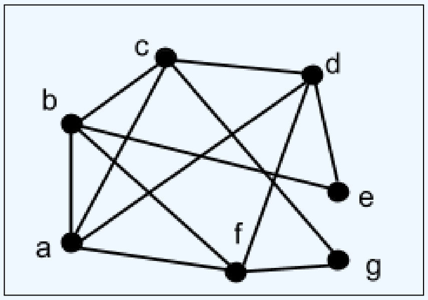


Figure 10: *An undirected graph has 7 vertices, a through g. 5 vertices are in the form of a regular pentagon, rotated 90 degrees clockwise. Hence, the top vertex becomes the rightmost vertex. From the bottom left vertex, moving clockwise, the vertices in the pentagon shape are labeled: a, b, c, e, and f. Vertex d is above vertex e, below and to the right of vertex c. Vertex g is below vertex e, above and to the right of vertex f. Undirected edges, line segments, are between the following vertices: a and b; a and c; a and d; a and f; b and f; b and c; b and e; c and d; c and g; d and e; d and f; and f and g.*

The graph, Figure 10:, is an Euler Circuit because all of the vertices have even degrees and it has 7 vertices, a-g. Each vertices, a,b,c,d,e,f, are connected to the three other vertices, while vertices d, and g, are each connected to the two other vertices.
$E \to B \to A \to C \to B \to F \to G \to C \to D \to F \to A \to D \to E.$

(1)

For each graph below, find an Euler trail in the graph or explain why the graph does not have an Euler trail.

(Hint: One way to find an Euler trail is to add an edge between two vertices with odd degree, find an Euler circuit in the resulting graph, and then delete the added edge from the circuit.)

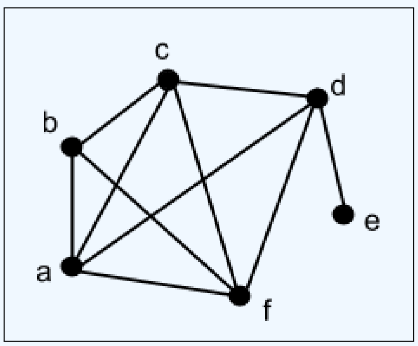
(i) 

Figure 11: *An undirected graph has 6 vertices, a through f. 5 vertices are in the form of a regular pentagon, rotated 90 degrees clockwise. Hence, the top vertex becomes the rightmost vertex. From the bottom left vertex, moving clockwise, the vertices in the pentagon shape are labeled: a, b, c, e, and f. Vertex d is above vertex e, below and to the right of vertex c. Undirected edges, line segments, are between the following vertices: a and b; a and c; a and d; a and f; b and f; b and c; c and d; c and f; d and e; and d and f.*

In (i):figure 11:, this graph has 6 vertices a-f and does not contain 2 odd degree vertices.The image you see here is arranged in a regular rectangle pentagon shape with a vertex d above e and below and the right of c. It has 10 undirected edges.We calculated the degree of each vertex has and found that the vertices b and e, have odd degrees, thus making this graph <u>Not an Euler circuit.</u>
An Euler Trail does not exist for the same reason given above.

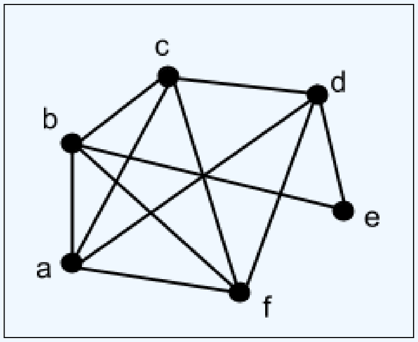
(ii) 

Figure 12: *An undirected graph has 6 vertices, a through f. 5 vertices are in the form of a regular pentagon, rotated 90 degrees clockwise. Hence, the top vertex becomes the rightmost vertex. From the bottom left vertex, moving clockwise, the vertices in the pentagon shape are labeled: a, b, c, e, and f. Vertex d is above vertex e, below and to the right of vertex c. Undirected edges, line segments, are between the following vertices: a and b; a and c; a and d; a and f; b and f; b and c; b and e; c and d; d and e; and d and f. Edges c f, a d, and b e intersect at the same point.*

This graph shows a pentagon shape with 6 vertices, a-f. However, due to having odd degrees instead of even degrees, the vertices d, f, have even degrees which are connected to an even numbers of edges. Vertex d is connected to three edges, a-d, d-e, d-f, while vertex f is connected to four edges namely, a-f, b-f, d-f, and c-f. Both vertices b and f have a degree of 4.It has two vertices with odd degrees, therefore, this graph, Figure 12:, can not be an Euler circuit.

The graph is connected and every vertex in the graph as even degree. Therefore, the graph must have an Euler trail. The Euler circuit is also an Euler trail.

Problem 5

Consider the following tree for a prefix code:

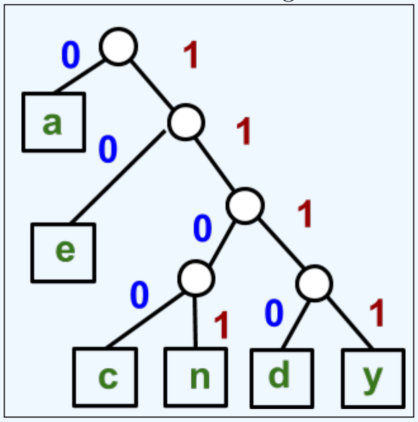


Figure 13: *A tree with 5 vertices. The top vertex branches into character, a, on the left, and a vertex on the right. The vertex in the second level branches into character, e, on the left, and a vertex on the right. The vertex in the third level branches into two vertices. The left vertex in the fourth level branches into character, c, on the left, and character, n, on the right. The right vertex in the fourth level branches into character, d, on the left, and character, y, on the right. The weight of each edge branching left from a vertex is 0. The weight of each edge branching right from a vertex is 1.*

(a) Use the tree to encode "day".

The (Decode) and (Encode) are illustrated below: a

(a).(0)
(e).(10)
(c).(1100)
(n).(1101)
(d).(1110)
(y).(1111)
The encoding for "Day" results as follows:
"111001111"

(b) Use the tree to encode "candy".

The encoding for "Candy" as follows: "11000110111101111"

(c) Use the tree to decode “1110101101″.

The decoding for ”1110101101”, is resulted in the word or string, ”den”.

(d) Use the tree to decode “111001101110010″.

The decoding of ‘111001101110010’, results in the word or string, ”dance”.

Problem 6

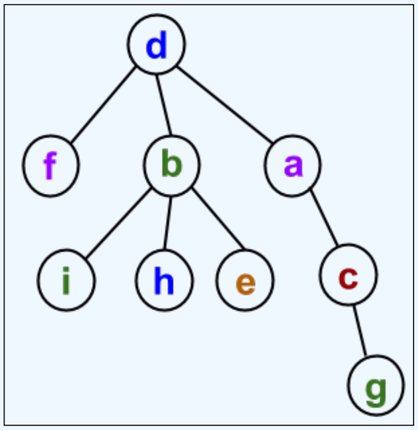


Figure 14: *A tree diagram has 9 vertices. The top vertex is d. Vertex d has three branches to vertices, f, b, and a. Vertex b branches to three vertices, i, h, and e. Vertex a branches to vertex c. Vertex c branches to vertex g.*

(a) Give the order in which the vertices of the tree are visited in a post-order traversal.

The process of post-order-traversal indicates we will count the children nodes first from left to right and then include the parent nodes.As follows:

$F \to I \to H \to E \to B \to G \to C \to A \to D$

(b) Give the order in which the vertices of the tree are visited in a pre-order traversal.

Pre-order traversal indicates that we begin with the parent nodes down to the children nodes from left to right.

$D \to F \to B \to I \to H \to E \to A \to C \to G$

Problem 7

Consider the following tree. Assume that the neighbors of a vertex are considered in alphabetical order.

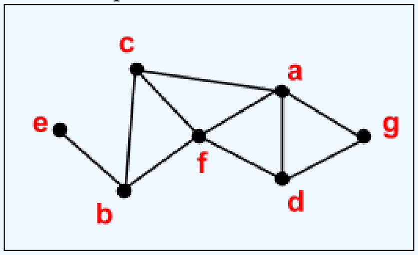


Figure 15: *A graph has 7 vertices, a through g, and 10 edges. Vertex e on the left end is horizontally inline with vertex g on the right end. Vertex b is below and to the right of vertex e. Vertex c is above vertex e and to the right of vertex b. Vertex f is between and to the right of vertices c and b. Vertex f is horizontally inline with vertices e and g. Vertex a is above and to the right of vertex f. Vertex d is below and to the right of vertex f. Vertex a is vertically inline with vertex d. Vertex g is between and to the right of vertices a and d. The edges between the vertices are as follows: e and b; b and c; c and f; c and a; a and d; b and f; f and a; f and d; a and g; and d and g.*

(a) Give the tree resulting from a traversal of the graph below starting at vertex a using BFS.

I have uploaded the tree image for part (a) in brightspace, please check it out.

The current applied queue for revisions is: $a, c, d, f, g, b, e \rightarrow [-a, -c, -d, -f, -g, -b, e]$

Staring from a, we add all neighbors alphabetically: c,then d, then f, then g. Vertex b is the only one a distance of 2 away from a, thereby adding edge (c,b). Finally, vertex b has only one un-visited neighbor, vertex e.

The BFS 'algorithm indicates the following:(a,c), (a,d), (a,f), (a,g),(c,b), (b,e)

1.Find the root node.

2. Display the adjacent vertices found.

3.Print the appropriate node.

4.Move onto the next column.

5. Repeat steps.

(b) Give the tree resulting from a traversal of the graph below starting at vertex a using DFS.

I have uploaded the second tree image for part (b) onto brightspace, please check it out.

The traversal graph DFS results in the following:
Starting from a, the first neighbor to visit alphabetically is c. Vertex c has neighbors a, b, e, and f. Since a has been visited, the next vertex is b. Vertex b has neighbors c, e, and f. Since c has been visited, the next vertex is e. Vertex e only has b as a neighbor, so the next vertex after backtracking to b is f. Vertex f has only one neighbor that has not been visited, vertex d. Finally, vertex d has only one un-visited neighbor, vertex g. Therefore, DFS: (a,c), (c,b), (b,e), (e,b), (b,f), (f,d), (d,g)

The algorithm applied instructions.

1.Find the root node.

2.Stock one adjacent vertex.

3.Print the appropriate node.

4.Move on to the next column.

5. Repeat steps 6. If any errors or complications are evident, then go back and retrace the steps please and search in the outer adjacent vertex for the previous column and this should fix your error.

Problem 8

An undirected weighted graph G is given below:

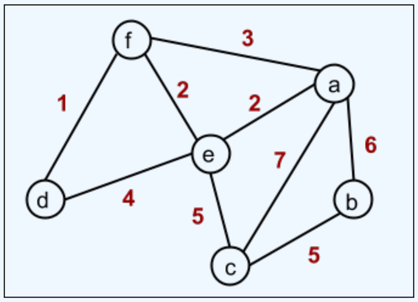


Figure 16: *An undirected weighted graph has 6 vertices, a through f, and 9 edges. Vertex d is on the left. Vertex f is above and to the right of vertex d. Vertex e is below and to the right of vertex f, but above vertex d. Vertex c is below and to the right of vertex e. Vertex a is above vertex e and to the right of vertex c. Vertex b is below and to the right of vertex a, but above vertex c. The edges between the vertices and their weight are as follows: d and f, 1; d and e, 4; f and e, 2; e and a, 2; f and a, 3; e and c, 5; c and a, 7; c and b, 5; and a and b, 6.*

(a) Use Prim's algorithm to compute the minimum spanning tree for the weighted graph. Start the algorithm at vertex a. Show the order in which the edges are added to the tree.

Starting on vertex a, we start at the root node "a", then take all available paths and take key with minimum key value such as in this case "e".

$(f, a), (3).$

$(a, e), (2).$

$(a, c), (7).$

$(a, b), (6).$

Then take all paths previous and new to choose minimum key value: (e,f),(2), which seems to be the minimum key value.

(e,f),(2) E primary Node.

(e,d)(4),

(e,c)(5)

Check all key from "f" greater than the minimum key value $i > 1$ which results in (f,d). Yes, and after (f,d), edges (e,c) and (c,b) are selected.

Based on the given logic statements, it seems that the path begins with E and goes to F, then from F to D, and finally from D to A. Let's clarify the steps in the logical reasoning.

E implies F, and F implies A: ($E \to F \to A$. D implies F, F implies E, and E implies A: ($D \to F(1), F \to E(2), E \to A(2)$. E implies C: ($E \to C(5)$. C implies B: ($C \to B(5)$.

Add weight of 2 to edge (e,f), add weight of 1 to edge (f,d), add weight of 5 to edge (e,c), add weight of 1 to edge (c,b) to avoid cycle formation.

(b) What is the minimum weight spanning tree for the weighted graph in the previous question subject to the condition that edge $\{d, e\}$ is in the spanning tree?

The graph should not form any loops.
$E \to F \to E \to A$.

The order in which the edges are added are: (d,e), (d,f), (a,e),(c,e), (b,c)

The existing edge from $E \to F$ is removed and the edge from $E \to D$ is implemented.With the following edges and weights: $D \to F$ with a weight of 1, $F \to E$ with a weight of 2, and $E \to A$ with a weight of 2, additionally, there is another edge: $E \to C$ with a weight of 5.The path starts at E, then it goes to F, then back to E, then finally to A, with the corresponding weights on each edge as mentioned above.

$F \to D(1), D \to E(1), E \to A(2), E \to C(5), C \to B(5)$. **This tree has the correct minimum weight of 17.**

(c) How would you generalize this idea? Suppose you are given a graph G and a particular edge $\{u, v\}$ in the graph. How would you alter Prim's algorithm to find the minimum spanning tree subject to the condition that $\{u, v\}$ is in the tree?

We would do this by implementing the $[U, V]$ edge and utilize the prism algorithm reaching every available vertex without

any repeated loops.