MODULE FIVE PROBLEM SET

Marissa Lanza

Directions: Type your solutions into this document and be sure to show all steps for arriving at your solution. Just giving a final number may not receive full credit.

Problem 1

Indicate whether the two functions are equal. If the two functions are not equal, then give an element of the domain on which the two functions have different values.

(a)

$$f : \mathbb{Z} \to \mathbb{Z}, \text{ where } f(x) = x^2.$$
$$g : \mathbb{Z} \to \mathbb{Z}, \text{ where } g(x) = |x|^2.$$

Both functions result in a positive integer and as such both functions are equal.$1x12 = (1x1)2 = x^2$, therefore, for any integer x, we have $f(x) = x^2 = |x|2 = g(x)$.The two functions of f and g are in fact equal for all x in z. The square of a number, square of it's absolute value are the same.

(b)

$$f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}, \text{ where } f(x, y) = |x + y|.$$
$$g : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}, \text{ where } g(x, y) = |x| + |y|.$$

If either x or y have a opposite sign, the conclusive answer differs from one equation to the other.

Let $x = -1$
$y = 1$
$f(x,) = |1 - 1| = 0$
$g(x, y) = |1| + | - 1| = 2$
$f(x, y) = |x - y| = | - 1 - 1| = 2$
$g(x, y) = |x| + |y - x| = | - 1 + |1 - (-1)| = 2$
The input (x, y) are the two functions of f and g, and they have the same value of 2. We can conclude that the two functions, $f(x, y) = |x-y|$ and $g(x, y) = |x|+|y-x|$ are equal for this input and any input, where x, y have the same sign.

PROBLEM 2

The domain and target set of functions f and g is $\mathbb{R}$. The functions are defined as:

- $f(x) = 2x + 3$
- $g(x) = 5x + 7$

(a) $f \circ g$?

Given $f : g : R > R$
$f(x) = 2x + 3$
$g(x) = 5x + 7$
$2(5x + 7) + 3$
$10x + 14 + 3$
$10x + 17$
$f \circ g(x) = 10x + 17.$

(b) $g \circ f$?

$g(f(x))$
$5(2x + 3) + 7$
$10x + 15 + 7$
$10x + 22$
$g(f(x)) = 10x + 22.$

(c) $(f \circ g)^{-1}$?

Let $f \circ g(x) = y$
$10x + 17 = y$
$10x = y - 17$
$x = y - 17/10$
$(f \circ g(x))^{-1}(x) = x - 17/10.$

(d) $f^{-1} \circ g^{-1}$?

Let $f(x) = y$
$2x + 3 = y$
$x = y - 3/2$
Let $g(x) = y$
$5x + 7 = y$
$x = y - 7/5$
$g^{-1}(x) = (((x - 7)/5) - 3)/2$
$(x - 7 - 15/10)$

$(f^{-1} \circ g^{-1})(x) = x - 22/10.$

(e) $g^{-1} \circ f^{-1}$?

$g^{-1}(f^{-1}(x))$
$= ((x - 3/2) - 7)/5$
$= x - 3 - 14/10$
$x - 17/10$
$(g^{-1} \circ f^{-1})(x) = x - 17/10.$

Are any of the above equal?

Yes they are equal, and I will explain why. The order of operations in the outcomes of c and e are equal and the principals of functions are the same, but share contra positive natures.

Problem 3

(a) Give the matrix representation for the relation depicted in the arrow diagram. Then, express the relation as a set of ordered pairs.

The arrow diagram below represents a relation.

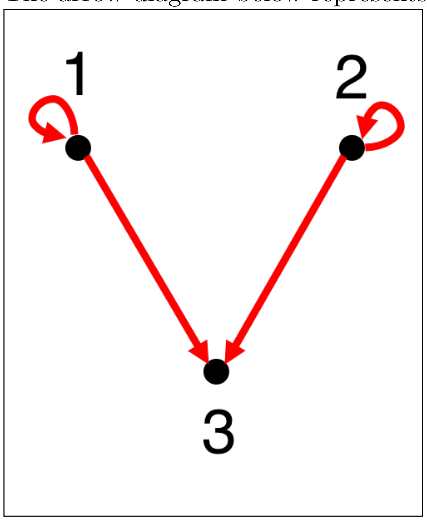

Figure 1: *An arrow diagram shows three vertices, 1, 2, and 3. An arrow from vertex 1 points to vertex 3, and another arrow from vertex 2 points to vertex 3. Two self loops are formed, one at vertex 1 and another at vertex 2.*

$R = (1,1); (1,3); (2,2); (2,3)$
$mR = [3x3]$
$[1\ 0\ 1]x = 1$
$[0\ 1\ 1]x = 1$
$[0\ 0\ 0]x = 3$
$y = 1y = 2y = 3.$

(b) Draw the arrow diagram for the relation.
The domain for the relation A is the set $\{2, 5, 7, 8, 11\}$. For x, y in the domain, xAy if $|x - y|$ is less than 2.

$A = 2, 5, 7, 8, 11; R = A \to A; R = (x, y) \in A$
$|7 - 8| < 2, |8 - 7| < 2, |x^1 - x^1| < 2, \forall x^1$
$2 \to 2$
$5 \to 5$
$7 \to 7 \to 8$
$8 \to 8 \to 7$
$11 \to 11$

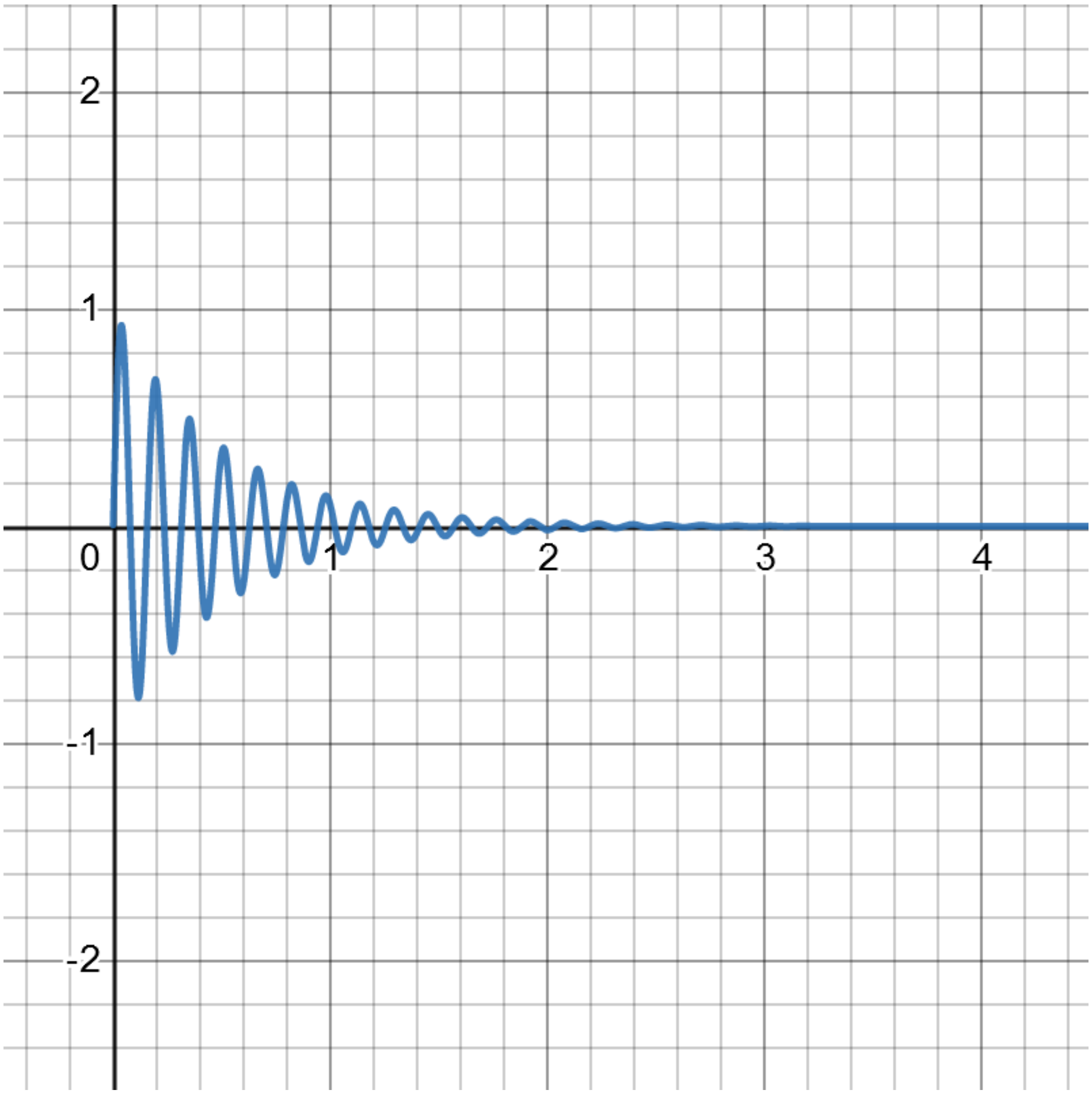

FIGURE 1. Arrow diagram.png

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PROBLEM 4

For each relation, indicate whether the relation is:

- Reflexive, anti-reflexive, or neither
- Symmetric, anti-symmetric, or neither
- Transitive or not transitive

Justify your answer.

(a) The domain of the relation L is the set of all real numbers. For $x,\ y \in \mathbb{R}$, xLy if $x < y$.

A1
Each element set of x does not correlate with each other and is anti-reflexive because of relation to R.
$\forall x \in R$
$(x < x)$; False,
(x, x); Does not belong to L

A2
$(x < y); (y, x)$; Does not belong to L
$(y < x)$; Is incorrect
If the first element is related to the second, then the second is also related to the first.

A3
$x < x(x, x)$; Does not belong to L
L is considered anti-symmetric as elements correlate.
Set A has relation to B, and B to A.

A4
$\forall x, y, z \in R$;
for $(x, y) \in$ L
$(x < y); (y, z) \in$ L
$(y < z); (x, z) \in$ L
Conclusion confirms $(x < z); (x < y); (y < z)$

A5
L is transitive as sets of binary relations correlate to each other in continuance.
$A \Leftrightarrow B \Leftrightarrow C \Leftrightarrow A$

(b) The domain of the relation A is the set of all real numbers. xAy if $|x - y| \leq 2$

B1
$\forall x \in$ R
$|x - x| = 0 \leq 2$; True
(x, x) belongs to A as a reflexive binary relation.
A is not anti-reflexive.

B2
$\forall x \in$ R, $(x, y) \in$ A
$|x - y| \leq 2, (y, x)$ belongs to A
$|x - y| = |y - x|$
A is symmetric

A is not anti-symmetric.

B3
$\forall(x, y, z) \in$ R, $(x, y) \in$ A
$(|x-y| \leq 2); (y, z) \in$ A
$(|y-z| \leq 2); (x, z)$; Does or Does Not belong to A.

B4
A is not transitive.

(c) The domain of the relation Z is the set of all real numbers. xZy if $y = 2x$

C1
If $x \neq 0; x = 1$
If $x \neq 2x; 1 \neq 2x * 1$
$x \neq x$
Relation is not reflexive
Relation is not anti-reflexive.

C2
Let xZy
$x = 2y$
$y \neq 2x$
$y \neq x$
The relation is not-symmetric
Anti-symmetry shows:
if aZb and bZa then $a = b$.
$xZy, yZx, x = y$

C3
Let $x = 1.y = 2.t = 4$
$(xZy); (y = 2x)(2 = 2x1)$
$(yZt); (t = 2y)(4 = 2x2)$
$(x = 1); (t = 4)(4 \neq 2x1)$
$t \neq x$
Relation is not transitive
As such the relation is also anti-symmetric.

Problem 5

The number of watermelons in a truck are all weighed on a scale. The scale rounds the weight of every watermelon to the nearest pound. The number of pounds read off the scale for each watermelon is called its measured weight. The domain for each of the following relations below is the set of watermelons on the truck. For each relation, indicate whether the relation is:

- Reflexive, anti-reflexive, or neither
- Symmetric, anti-symmetric, or neither
- Transitive or not transitive

Justify your answer.

(a) Watermelon x is related to watermelon y if the measured weight of watermelon x is at least the measured weight of watermelon y. No two watermelons have the same measured weight.

A1
x is related to x for $\forall x$ watermelon and it shows that it is a reflexive relation.
$(x = 20lbs); (y = 21lbs)$
y is related to x, but x is not related to y.
Similarly, the relation is not symmetric.

A2
As x is related to y, y is related to x.
As likely, the weight of each watermelon differs.
This relation is Anti-symmetric.

A3
x is related to y,
y is related to z,
$x \geq y \geq z$,
$x \geq z$
This relation is Transitive.

(b) Watermelon x is related to watermelon y if the measured weight of watermelon x is at least the measured weight of watermelon y. All watermelons have exactly the same measured weight.

If all watermelons weighed in equally, the relation results properties of:
- Reflexive Relation
- Symmetric Relation
- Transitive Relation
$w(x) \leq w(y)$,

$w(y) \leq w(x)$

Part 1. Give the adjacency matrix for the graph G as pictured below:

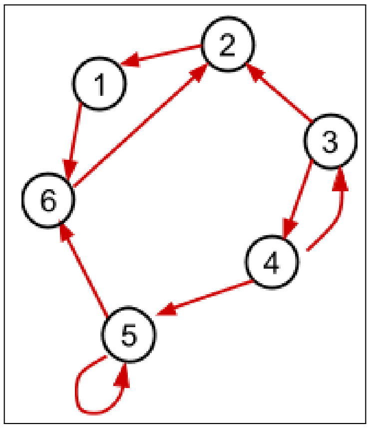

Figure 2: *A graph shows 6 vertices and 9 edges. The vertices are 1, 2, 3, 4, 5, and 6, represented by circles. The edges between the vertices are represented by arrows, as follows: 4 to 3; 3 to 2; 2 to 1; 1 to 6; 6 to 2; 3 to 4; 4 to 5; 5 to 6; and a self loop on vertex 5.*

Let G be a graph with K vertices.
$[1, 2, 3, ...k]$
K = End

The aij.matrix for G is nxn matrix
A = $[aij]$ nxn
$(Vertex)$ V Edges = aij. from vi and vj.
G contains 6 vertices, the matrix dimensions $6x6$ is applied.

$$A^2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

$[A3]ij \neq 0$
$[A3]22 = 1 \neq 0$
The only vertex that is able to be reached from vertex 2 by a walk of length 3, is vertex 2 itself.

Part 2. A directed graph G has 5 vertices, numbered 1 through 5. The 5×5 matrix A is the adjacency matrix for G. The matrices A^2 and A^3 are given below.

$$A^3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

Use the information given to answer the questions about the graph G.

(a) Which vertices can reach vertex 2 by a walk of length 3?

It basically comes down to how many vertices have the value 1 in A3 Vertices 2, 4, and 5.

(b) Is there a walk of length 4 from vertex 4 to vertex 5 in G? (Hint: $A^4 = A^2 \cdot A^2$.)

$[A4]54 = 0$

There is no walk.

$A4 = A2 {}^* A2$

Problem 7

Part 1. The drawing below shows a Hasse diagram for a partial order on the set $\{A, B, C, D, E, F, G, H, I, J\}$

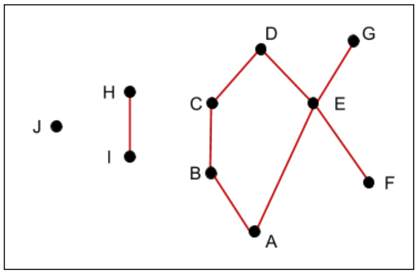

Figure 3: *A Hasse diagram shows 10 vertices and 8 edges. The vertices, represented by dots, are as follows: vertex J; vertices H and I are aligned vertically to the right of vertex J; vertices A, B, C, D, and E forms a closed loop, which is to the right of vertices H and I; vertex G is inclined upward to the right of vertex E; and vertex F is inclined downward to the right of vertex E. The edges, represented by line segments, between the vertices are as follows: Vertex J is connected to no vertex; a vertical edge connects vertices H and I; a vertical edge connects vertices B and C; and 6 inclined edges connect the following vertices, A and B, C and D, D and E, A and E, E and G, and E and F.*

(a) What are the minimal elements of the partial order?

The minimal elements are those not connected with any elements from below:
$[J, I, A, F]$

(b) What are the maximal elements of the partial order?

The Maximal elements of Hasse Diagram are those not connected with any elements from above:
$[J, H, D, G]$

(c) Which of the following pairs are comparable?

$(A,\ D)$, $(J,\ F)$, $(B,\ E)$, $(G,\ F)$, $(D,\ B)$, $(C,\ F)$, $(H,\ I)$, $(C,\ E)$

The following comparable pairs are:
$(A, D), (G, F), (D, B), (H, I)$
(B, E) are not connected
(C, E) are not connected

(C, F) are not connected.

Part 2. Each relation given below is a partial order. Draw the Hasse diagram for the partial order.

(a) The domain is {3, 5, 6, 7, 10, 14, 20, 30, 60}. $x \leq y$ if x evenly divides y.

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(b) The domain is $\{a, b, c, d, e, f\}$. The relation is the set:

$$\{(b, e), (b, d), (c, a), (c, f), (a, f), (a, a), (b, b), (c, c), (d, d), (e, e), (f, f)\}$$

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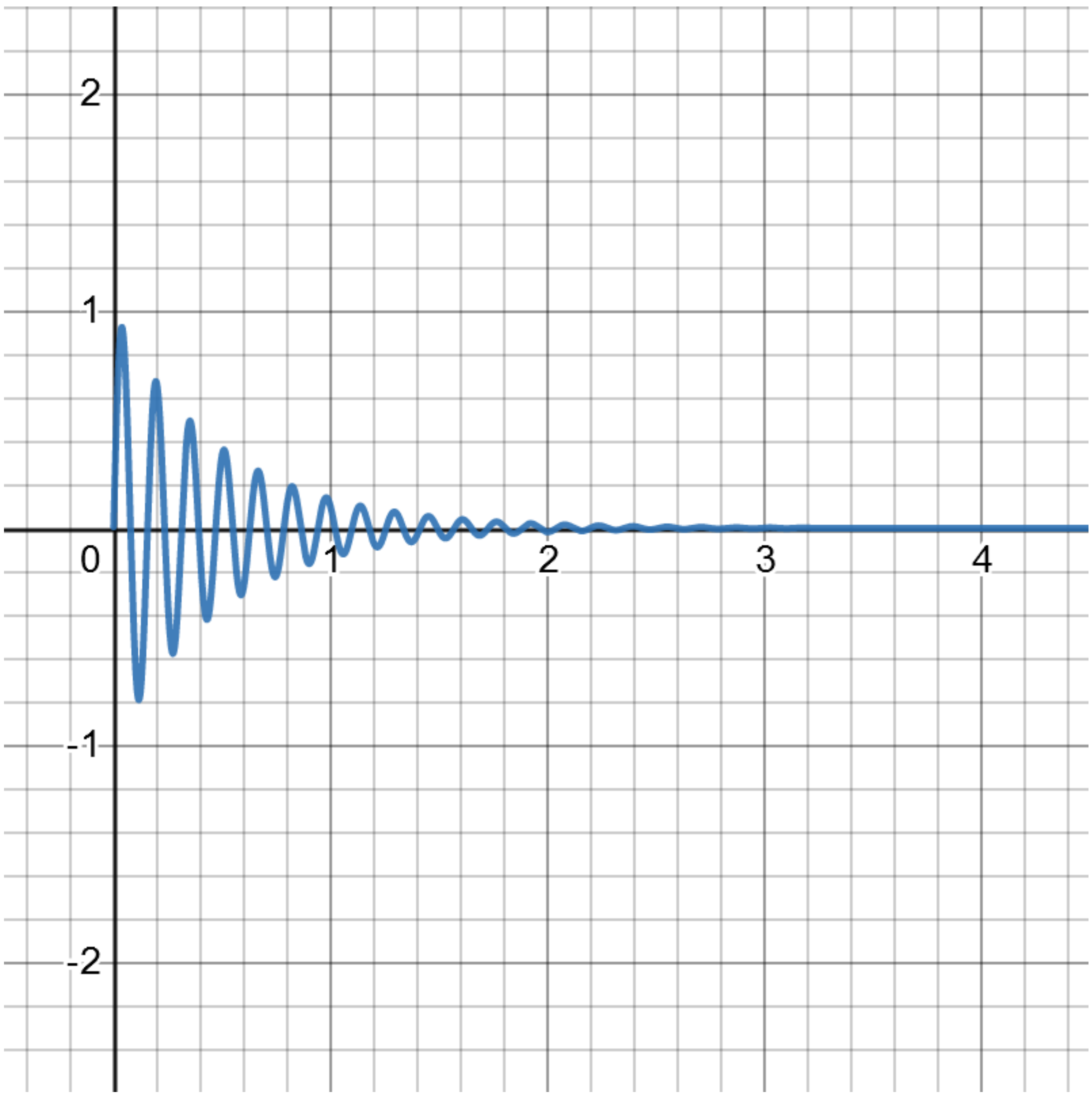

Figure 2. Arrow diagram.png

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Problem 8

Determine whether each relation is an equivalence relation. Justify your answer. If the relation is an equivalence relation, then describe the partition defined by the equivalence classes.

(a) The domain is a group of people. Person x is related to person y under relation M if x and y have the same favorite color. You can assume that there is at least one pair in the group, x and y, such that xMy.

M is reflective
If xMy means x and y have the same favorite color, y and x have the same favorite color
$(yMx);(xMz)$
M is Symmetric
M is Transitive

(b) The domain is the set of all integers. xEy if $x+y$ is even. An integer z is even if $z=2k$ for some integer k.

E is reflexive, $x+x=2x(even), xEx$
E is symmetric, $x+y(even), xEy; y+x(even, yEx)$
E is transitive, $xEyandyEx$
$x+y=2k,$
$y+z=2k2$
$x+2y+z=2k1+2k[2]$
$x+z=2(k1+k2-y)=2k[3]$
$k[3]=k[1]+k[2]-y$
$x+z(even)$
xEz

TotalNum.of partition $=2;[0],[1]$
$x=y|x+y(even)$
$0-2k|k\in\forall=[(elipsis)...,-4,-2,0,2,4,...(elipsis)]$
$1=[2k-1|K\in\forall=[(elipsis)...,-3,-1,1,3,...(elipsis)]$