MAT 230 EXAM ONE

Marissa Lanza

Directions: Type your solutions into this document and be sure to show all steps for arriving at your solution. Just giving a final number may not receive full credit.

PROBLEM 1

(a) The domain for all variables in the expressions below is the set of real numbers. **Determine whether each statement is true or false.**

(i) $\forall x \, \exists y \, (x + y \geq 0)$

The given statement is true.

The proof follows as:
$\forall x \in R, \exists y = -x+1 \in R.$
Similar to x+y = (-x+1)
= x - x + 1
= 1 > 0
Therefor it results in: $\forall x \in R, \exists y \, (x+y \geq 0) \equiv$ True.

(ii) $\exists x \, \forall y \, (x \cdot y > 0)$

$x \in R$ exists for least one of every single y $\in R$ such as: x * y > 0
There exist an x for every y, but if y = 0 , the conclusion proves false. Let y = 0 resulting in x * 0 > 0 Does not resolve positive and as such, it is false.

(b) **Translate each of the following English statements into logical expressions.**

(i) There are two numbers whose ratio is less than 1.

$\exists x \, \exists y \, (x/y < 1)$ As long as x or y exists and set as integer less than 1.

(ii) The reciprocal of every positive number is also positive.

$\forall x > 0.$

$1/x > 0.$

PROBLEM 2

Prove the following using the specified technique:

(a) Let x and y be two real numbers such that $x + y$ is rational. Prove by contra positive that if x is irrational, then $x - y$ is irrational.

It's proven by contra positive because x + y is rational, and x-y is rational.Consider (x + y) + (x - y) = x + x + y - y = 2x. Additional of rational numbers is rational. = 2x is rational. = x is rational thus proved.

(b) Prove by contradiction that for any positive two real numbers, x and y, if $x \cdot y \leq 50$, then either $x < 8$ or $y < 8$.

Contradiction: $x \geq 8$ *and* $y \geq 8$.
Consider, x * y ≤ 50 substituting values of x and y .
(8) * (8) ≤ 50.
64 ≤ 50.
Therefore, the assumption is false and the original statement is proved.

Problem 3

Let $n \geq 1$, x be a real number, and $x \geq -1$. **Prove the following statement using mathematical induction.**

$$(1 + x)^n \geq 1 + nx$$

Given:
n $\geq$ 1.
x $\geq$ -1.
(1 + x) ^{2} $= 1 + x, \forall x \geq -1.$

Assuming conclusion results true for n = k
(1 + x) ^{k} $\geq 1 + kx, \forall x \geq -1.$
1 + kx + x + kx $\geq$ 1 + (k + 1) x, $\forall x \geq -1.$

Conclusion, the results are true for n = k + 1.
Due to the mathematical induction, the results are n $\geq$ is true.

Problem 4

Solve the following problems:

(a) How many ways can a store manager arrange a group of 1 team leader and 3 team workers from his 25 employees?

25 Employees.
Number of methods 1TL (Team Leader) / 25 Emp. (Employees) = (1)(25)C
Number of methods 3TW (Team Worker) / 24 R. Emp. (Remaining Employees) = (1)(25)C*(3)(24)C
= 25 * 2024.
There are a total of 50,600 methods of arrangements.

(b) A states license plate has 7 characters. Each character can be a capital letter ($A - Z$), or a non-zero digit ($1 - 9$). How many license plates start with 3 capital letters and end with 4 digits with no letter or digit repeated?

Num. of methods placing capital letter first three places without letter repetition: 26*25*24.

Num. of methods of placing non-repetitive digits in the last four spots is: 9 * 8 * 7 * 6
26 * 25 * 24 * 9 * 8 * 7 * 6 w/ no digit or letter repeated.
= 47, 174, 400 plates.

(c) How many binary strings of length 5 have at least 2 adjacent bits that are the same ("00" or "11") somewhere in the string?

Total strings length is 5 = 2^5
Total strings result = 32
Conditions are violated by 0101 and 1010 as they are not pairs of "00" or "11".
Total Num. of methods = total string num. - violated conditions
32 - 2 = 30.
There are a total of 30 strings of length of 5.

PROBLEM 5

A class with n kids lines up for recess. The order in which the kids line up is random with each ordering being equally likely. There are two kids in the class named Betty and Mary. The use of the word "*or*" in the description of the events, should be interpreted as the inclusive or. That is "*A or B*" means that A is true, B is true, or both A and B are true.

What is the probability that Betty is first in line or Mary is last in line as a function of n? Simplify your final expression as much as possible and include an explanation of how you calculated this probability.

Given the total number of kids lined up would equal to: nLet, B ='Betty is first in line'
M =' Mary is last in line'

P(B or M) = P(B) + P(M) - P(B and M)
P(B) = (n-1)!/n! (fixing this position of Betty, she is first in line and arrange the rest of the n-1 kids) = 1/nP(M) = (n-1)!/n! (fixing the position of Mary is last in line, and arrange rest of the n-1 kids) = 1/n

P(B and M) = (n-2)!/n! (fixing the position of Betty is first in line and Mary is last in line, and arrange the rest of n-2 kids) = 1/n(n-1)

P(B or M) = 1/n+1/n-1/n(n-1) = ((n-1) +(n-1) + 1)/n(n-1)
= (2n-1)/n(n-1)

Problem 6

The general manager, marketing director, and 3 other employees of Company A are hosting a visit by the vice president and 2 other employees of Company B. The eight people line up in a random order to take a photo. Every way of lining up the people is equally likely.

(a) What is the probability that the general manager is next to the vice president?

The total number of people = 8
Total number of ways 8 people are lined up are: =8! = 40320
Probability that the general manager is next to the vice president = P(A)
Approach:- Combine the general manager and vice president as one of the total number of people reduced to 7
Arrange the 7 people in a line = 7! = 5040
Arrange the two-combine people(general manager and vice president) =2! = 2
P(A) = (5040 * 2)/(40320) = 1/4

(b) What is the probability that the marketing director is in the leftmost position?

Probability that the marketing director at the leftmost position = P(B)
Approach:-Fix the position of marketing director at leftmost position and arrange the rest of the 7 people by 7! ways that is 5040 ways
P(B0) = 5040/40320 = 1/8

(c) Determine whether the two events are independent. Prove your answer by showing that one of the conditions for independence is either true or false.

P(A) $\rightarrow$ refers $\rightarrow$ 1st sum

P(B) $\rightarrow$ refers $\rightarrow$ 2nd sum

They're dependent